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Spatial development of the instability of a dense beam of negative ions in a rarefied gas

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The mechanism of the appearance of the decompensation effect of a dense (~ 50 mA/cm^2) beam of negative ions in a rarefied gas is elucidated. The effect is due to singularities in the development in the resultant ion-ion plasma, of the instability of oscillations that are almost perpendicular to the beam velocity. It is shown that the spatial characteristics of the instability depend in turn on the depth of the stationary negative potential well that determines the mean transverse energy of the positive ions. It is established experimentally for the first time that the longitudinal phase velocity of the excited oscillations is much smaller than the beam velocity.

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Upon propagation of a beam of negatively charged particles in a gas and its ionization as a result of the accumulation of positive ions in the potential well of the beam and of the rapid departure from it of electrons at a sufficiently low gas pressure, a two-beam plasma is formed.

Interest in the study of the stability of such beams relative to transverse perturbations is stimulated first by applications, such, for example, as the acceleration of positive ions by means of electron rings,[1-3] electron-beam welding,[4,5] and, in recent times, the contemplated use of beams of negative ions in fast-arom injectors for thermonuclear devices.[6] At the beam current densities (~ 50 mA/cm^2) and low gas pressure required for injectors, as was discovered in Ref. 7, effect of strong decompensation of the beam arises as a result of the instability of the transverse oscillations of the ion-ion plasma; this hinders the effective beam transport.

In the present paper we study those characteristic features of the spatial development of the instability in such a plasma which make clear the mechanism that produces the effect.

Before proceeding to the esposition of the experimental results, we consider the buildup of oscillations in an unbounded plasma, consisting of immobile positive ions with mass M_+ and a beam of negative ions (or electrons) with mass m_-, propagating with velocity v_b in the z direction. It is known that the natural oscillations of the charge density of each of the components, in the frame of reference in which it is at rest, take place with the frequencies $\omega_p=(4\pi e^2 n_+/M_+)^{1/2}$ and $\omega_b=(4\pi e^2 n_b/m_-)$, where $n_+=n_b=n$ are the densities of the components. In the case of resonance of these oscillations, i. e., $|\omega_p-k_z v_b|\approx\omega_b$, their buildup occurs; here $k_z=2\pi/\lambda_z$ is the longitudinal wave number of the oscillations.

An important consequence then follows, namely, that in the case $\omega_p/\omega_b=(m_-/M_+)^{1/2}\ll 1$,

$$k_z\approx\omega_b/v_b, \tag{1}$$

$$\omega/k_z\approx(m_-/M_+)^{1/2}v_b\ll v_b. \tag{2}$$

The growth rates of the buildup can be obtained from the dispersion equation. Usually the temporal problem is solved in the approximation of a cold plasma,[1-4] the analysis of which was first given by Buneman.[8] However, this approximation, as is well known, is inapplicable in finding the spatial growth rate. Account of thermal motion of the positive ions[9] leads to a more complicated equation:

$$1+\frac{1}{k^2d_+^2}\left[1+i\pi^{1/2}\frac{\omega}{|k|v_T}W\left(\frac{\omega}{|k|v_T}\right)\right]-\frac{\omega_b^2}{(\omega-k_z'v_b)^2}=0, \tag{3}$$

where $v_T=(2T_+/M_+)^{1/2}$, $d_+=(T_+/4\pi e^2 n)^{1/2}$, $W(\omega/|k|v_T)$ is a Kramp function, $k^2=|k_z'|^2+k_\perp^2$, and $k_z'=k_z+i\varkappa$ must be determined.

The problem simplifies slightly at $\omega/|k|v_T\gg 1$; then

$$1-\frac{\omega_p^2}{\omega^2}\left(1+\frac{3}{2}\frac{k^2v_T^2}{\omega^2}\right)+\frac{i\pi^{1/2}}{k^2d_+^2}\frac{\omega}{|k|v_T}\exp\left(-\frac{\omega^2}{k^2v_T^2}\right)-\frac{\omega_b^2}{(\omega-k_z'v_b)^2}=0. \tag{4}$$

Upon satisfaction of the condition $k_z\ll k_\perp$, the maximum value of the spatial growth rate $\varkappa$ and its corresponding longitudinal wave number k_z and resonance frequency ω_{res} are connected by the following relations, as follows from the solution of (4):

$$\omega_{res}=\omega_p\left(1+\frac{3}{2}\frac{k^2v_T^2}{\omega_{res}^2}\right)^{1/2},\quad -\varkappa+k_z\approx\frac{\omega_{res}}{v_b},$$

$$\varkappa = \frac{\omega_b}{v_b} \frac{1}{(2\Gamma)^{1/2}},$$

or

$$\Gamma = \frac{\pi^{1/2}}{k^2 d_+^2} \frac{\omega_{res}}{|k| v_T} \exp\left(-\frac{\omega_{res}^2}{k^2 v_T^2}\right). \tag{5}$$

It is important to note that with decrease in the ion temperature, $\varkappa$ and k_z increase until the condition $k_z \ll k_\perp$ begins to be violated and the increase of k manifests itself and stops the growth.

RESULTS OF EXPERIMENTS

The investigations were carried out with the apparatus described in Ref. 7. A beam of H^- ions with a current up to 100 mA and energy ≈14 keV was obtained from a pulsed source, shaped into a weakly divergent beam with initial density ≈50 mA/cm^2 with the aid of a sector magnet, and then propagated freely in a gas whose pressure was regulated from 3×10^{-6} Torr upwards. The pulse length $\tau_p = 0.6$ μsec was much greater than the time required for establishing the quasistatic state, $\tau_k = (\sigma_i n_a v_b)^{-1}$; here σ_i is the ionization cross section, and n_a is the density of atoms of the gas.

The beam current was measured with the help of two Rogowski bands of inside diameter 5 cm; one of them was located at a distance $z = 8$ cm from the sector magnet and the second could be displaced along z. The investigations were carried out using a source in which no pulsations of beam current at $z = 8$ cm were present during most of the pulse.

The beam current density was measured by means of a diaphragmed Faraday cylinder with aperature diameter 3 mm, which could be displaced along z. A cylindrical four-grid analyzer of slow particles, along the axis of which the beam propagated, served for measurement of the radial current and the energy spectrum of the positive ions; its length was 5 cm, inner diameter 7 cm, and outer diameter 5 cm. It was placed in one of two positions along the beam coordinate: $z = 18$ cm or $z = 38$ cm. Because of the large time constant in the collector circuit ($RC \approx 5$μsec), the time-averaged current of positive ions was measured. (R is the resistance of the load, C the capacitance of the cable.)

With the help of two movable (along z) capacitive probes of special construction, described in Ref. 7, the potential was measured simultaneously at two points of the beam cross section, separated from one another by a distance of 1 cm. This made it possible to monitor the phase distribution and consequently also the transverse wave length of the excited oscillations.

In spite of the absence of pulsations of the beam current at small z, it turned out to be modulated in density. Along with the spontaneous modulation, stimulated modulation of the transverse velocity can also be achieved, by applying an alternating potential on a metallic probe located in the middle of the beam cross section at the coordinate $z = 0$; modulation of the transverse velocity leads to some component of the perturbation of the potential, whose initial amplitude is much smaller than the amplitude of the potential fed to the probe.

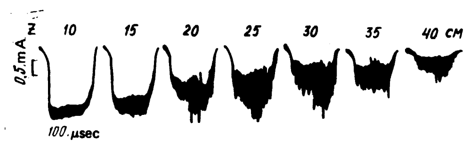

FIG. 1. Oscillograms of the beam current density, $p = 6.7\times10^{-6}$ Torr.

Figure 1 shows typical oscillograms of the current density at the center of the beam as a function of z for the case of spontaneous modulation. With increase in z, the amplitude of the pulsations of the current density increases appreciably, reaches a maximum near $z = 27$ cm and thereafter falls off. At large amplitudes, the pulsations have an irregular character and separate blips appear. The characteristic frequency of the amplified pulsations, determined by the fast sweep of the oscilloscope, amounts to ≈1 MHz even when the 10 MHz frequency predominates in the initial pulsations of the spectrum. Measurement of the beam current density allows us to determine an important instability parameter ω_b/v_b, which is equal to 0.25 cm^{-1} for the conditions of Fig. 1 and at $z = 10$ cm. The spatial growth rate, determined over a part of the exponential growth of the amplitude, is equal to $\varkappa = 0.16$ cm$^{-1} = 0.7\omega_b/v_b$.

Measurement of the beam current with the help of the movable Rogowski band allows us to establish an important fact: the development of the instability leads to significant pulsations of the total current, and their amplitude increases even after the current density amplitude had reached its maximum. The corresponding pulsation coefficients reach the values $\eta_{tot} = I_{max} - I_{min}/I_{av} \approx 15\%$, and $\eta_{pl} = (j_{max} - j_{min})/j_{av} \approx 85\%$. This fact testifies to the significant increase in the electric field component E_z in the beam, which also leads to longitudinal grouping of the ions of the beam.

The dependences of the indicated coefficients on the gas pressure (argon) (Fig. 2) differ considerably from one another: η_{pl} does not change very much, while η_{tot}, along with the amplitude of the pulsations of the potential $\tilde{\varphi}$, falls off materially upon increase in the pressure above a certain value p_0. It follows from the measurements also that at $p < p_0$ the oscillations of the potential within the beam are in phase, and at $p \geq p_0$ they are no longer in phase. This indicates a decrease in the transverse wavelength.

The longitudinal wavelength was measured from the change, along z, of the phase of the potential oscilla-

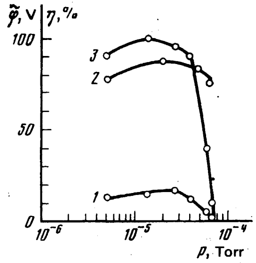

FIG. 2. Dependences of the amplitude of potential oscillations $\tilde{\varphi}$ (3) and the pulsation coefficient η of the total current (1) and current density (2) on the gas pressure, $z = 45$ cm.

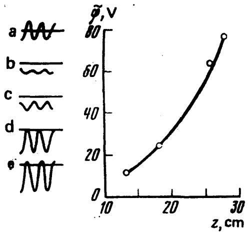

FIG. 3. Oscillograms of the modulating potential (*a*), the potential in the beam (*b–e*) and the dependence of its amplitude on *z*: $p=3.5\times10^{-5}$ Torr, $U_{mod}=125$ V, $f_{mod}=0.7$ MHz; *z* is equal to: *b*—13.5, *c*—18.5, *d*—25.5, *e*—28 cm.

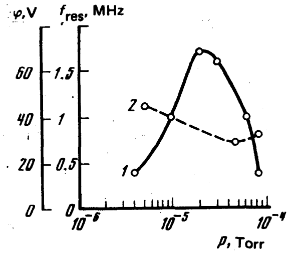

FIG. 5. Dependence of the resonance frequency (2) and amplitude of the potential pulsations (1) on the gas pressure: $1-z=28$ cm, $2-z=20$ cm, $U_{mod}\approx100$ V.

tions excited at the resonance frequency by the stimulated modulation (Fig. 3). The value determined in this way is $\lambda_z=25$ cm ($k_z=0.25$ cm^{-1}), and $v_{ph\,z}=1.8\times10^7$ cm/sec at a velocity of the beam $v_b=1.7\times10^8$ cm/sec. We note that this fact is the first experimental proof of excitation, by a two-beam mechanism, of waves with longitudinal phase velocity much less than the velocity of the beam.

The value of k_z measured in such fashion agrees with the value obtained from the ratio η_{pl}/η_{tot} by equating it to $k_\perp/k_z$ and using the measured value $k_\perp=1.6$ cm^{-1}; at $p=4\times10^{-5}$ Torr (Fig. 2), we obtain $k_z\approx0.27$ cm^{-1}.

The dependences of $\varkappa$, k_z and also f_{res} on the pressure are given in Figs. 4 and 5. They reveal the following features. First, $\varkappa\sim k_z$ and decreases with decrease in the pressure; second, with decrease in the pressure, f_{res} increases in the entire range of pressures below 5×10^{-5} Torr. Such a character of the dependence of f_{res} contradicts those observed in Ref. 10 in a stationary beam of negative ions with current density ~1 mA/cm^2 and in Ref. 5 in a beam of electrons. Along with the noted dependences of $\varkappa$ and k_z, it can also be connected with the change in the depth of the negative potential well in the initial portion of the beam, shown in Fig. 4 (the reason for its stationary behavior will be given below). It must be taken into account that in the case of continuous formation and departure of positive ions from the beam, the depth of the well determines the scatter of the ions with respect to the transverse energies, which can be characterized by a certain effective velocity $v_{eff}\sim(2e\varphi_0/M_+)^{1/2}$. Its value diminishes with increase in the pressure. In correspondence with the relations (5), this should lead to a decrease in f_{res} and an increase in the growth rate $\varkappa$ and k_z, as is also observed experimentally throughout almost the entire range of pressures where a negative potential exists in the plasma. At pressures $p\sim p_0$, as has already been noted, a decrease in the transverse wavelength also begins ($k_\perp$ increases), because of the growing influence of the electrons, which becomes decisive at $p>p_0$ (the characteristics of the instability in this very region of pressures in the electron beam were apparently investigated in Ref. 5). The increase in $k_\perp$ noted above leads to a decrease in the initial amplitude of the perturbation of the potential; therefore the dependence of the amplitude of the pulsations of the potential $\tilde{\varphi}(z)$ on the pressure in the portion of the beam where $\tilde{\varphi}(z)=\tilde{\varphi}(0)\exp\varkappa x$ has a maximum (Fig. 5, curve 1) in spite of the fact that the growth rate continues to increase.

The radial boundedness of the beam and the associated inhomogeneity of the density of positive ions, and also the lack of knowledge of its distribution function over the beam, makes a quantitative comparison with the results of the theoretical model difficult. What has been said applies especially to the value of the growth rate of the oscillations, which is determined by the Landau damping, and which, as was shown experimentally in Ref. 11, is brought about by a very small number of particles moving in the direction of the wave and having the phase velocity of the wave.

It is known that the depth of the stationary negative potential well is determined by the balance between the rate of formation of the positive ions on an individual portion of the beam and the rate of their departure. If the ions depart only in the radial direction, then the balance equation has the form

$$I_+=I_-\sigma_i n_a L,$$

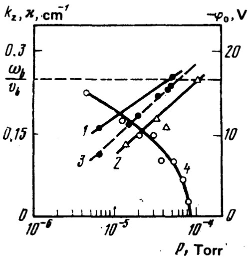

FIG. 4. Dependence of the growth rate $\varkappa$ (1, 2) longitudinal wave number k_z (3) and depth of the potential well φ_0 (4) at $z=18$ cm on the gas pressure: 1—spontaneous modulation, 2, 3, 4—stimulated modulation.

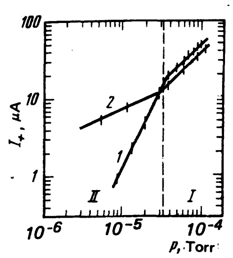

FIG. 6. Dependence of the radial current of positive ions on the pressure in two coordinates along the beam length at $z=18$ cm (curve 1) and $z=38$ cm (2) (spontaneous modulation).

where I_-, I_+ are the beam current and the radial current of the positive ions, respectively, L is the length of the portion of the beam, n_a is the atomic concentration, proportional to the pressure, and the experimental dependence of I_+ on the pressure reveals the following characteristic feature (Fig. 6). In the region of low pressures (II) the current at small z is much smaller and at large z greater than the quantity $I_-\sigma_i n_a L$. This fact indicates the existence of a longitudinal intense flux of positive ions in the direction of propagation of the beam, which is in fact the reason for the existence of a rather deep potential well in its initial part.

As is seen from the drawing, the formation of the flux is in turn connected with an increased departure of the ions in the radial direction in that part of the beam in which the amplitude of the oscillations is sufficiently great. As a result, the mean value of the potential increases along the length.

The effect of the amplitude of the increasing oscillations on the value of the radial current of positive ions has been observed experimentally with stimulated modulation. The dependence of the radial current on the frequency of modulation, together with the dependence at the same point in the beam of the amplitude of the potential oscillations, is shown in Fig. 7. Both dependences have a resonant character. A change in the amplitude of the potential supplied to the modulating probe, up to 200 V, led to a proportional increase in the amplitude of the pulsations of the potential and of the radial current of positive ions, the value of which reached at the maximum $\approx I_-\sigma_i n_a L$.

The basic results of the research were the following:

1. It was established that the transverse oscillations excited in the beam of negatively charged particles in a rarefied gas become, at a sufficiently high amplitude, the decisive mechanism of radial departure of the positive ions; here the rate of departure is almost proportional to the amplitude of the oscillations. As a result, the mean value of the negative potential in the beam increases along its length.

2. The generation of an intense flux of positive ions in the direction of the beam is observed and is associated with this increase in the potential.

3. A singularity of the spatial development of the instability of the beam of positive ions is revealed: the instability leads to the formation of a negative potential well in the beam, on whose depth depend in turn the spatial characteristics of the instability ($\varkappa$, k_z, ω_{res}). The character of all the dependences agrees with the

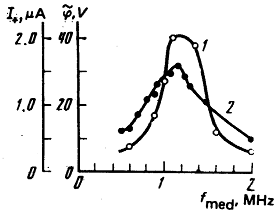

FIG. 7. Dependence of the radial current of positive ions (1) of the amplitude of the potential pulsations (2) on the frequency of modulation: $p = 4\times10^{-6}$ Torr, $z = 18$ cm, $U_{mod} = 140$ V.

theory of oscillations of a radially unbounded beam if we take it into account that the depth of the well determines the mean transverse energy of the positive ions.

4. It was established that, in such a self-regulating plasma, the values of $\varkappa$ and k_z change in a relatively short interval near the value $k_z \approx \omega_b/v_b$ corresponding to the simple condition of resonance of the oscillations of the density of the positive ions and the beam ions. This fact is the first experimental proof of excitation, by the two-beam mechanism, of waves with a phase velocity much less than the velocity of the beam.

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