Kinetics of muon catalysis processes in a mixture of deuterium and tritium

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Simple analytic expressions are obtained for the kinetics of the processes of muon catalysis in a molecular mixture of deuterium and tritium. Conditions are obtained under which the highest efficiency of the muon catalysis is attained. Various types of experiments necessary for the measurements of the basic characteristics of the processes are discussed.

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1. INTRODUCTION

It follows from theoretical calculations[1] that in the mesic molecule $dt\mu$ there exists an excited rotational-vibrational level ($J=1, v=1$) with very low binding energy $\varepsilon_{11}\approx 0.7$ eV,[2] as a result of which resonant production of mesic molecules $dt\mu$ is possible in the reactions

$$t\mu+D_2\to[(dt\mu)d2e]_{1,1}^{*}, \quad \text{(Ia)}$$

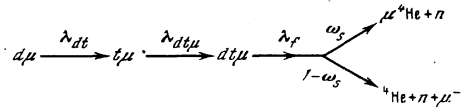
$$t\mu+DT\to[(dt\mu)t2e]_{1,1}^{*}. \quad \text{(Ib)}$$

The existence of an analogous resonant mechanism in pure deuterium

$$d\mu+D_2\to[(dd\mu)d2e]_{1,1}^{*}$$

was suggested long ago[3] and recently demonstrated experimentally[4] and theoretically.[1]

The rate of resonant production of $dt\mu$ molecules should depend on the temperature of the D_2+T_2 mixture, and the maximum rate is $\lambda_{dt\mu}\sim 10^8\ \text{sec}^{-1}$. Under these conditions, the efficiency of the muon catalysis of the nuclear fusion reaction in the chain of processes



(Ic)

is quite large, so that the question of the feasibility of practical use of the μ-catalysis phenomenon can be raised.[5]

By virtue of the large expected rate of production of the $dt\mu$ molecules (comparable with the $d\mu\to t\mu$), transfer rate, the kinetics of the reactions in the D_2+T_2 mixture has specific features that distinguish it from the kinetics of the reactions in the H_2+D_2 mixture in pure deuterium.[6] Bearing in mind experiments planned for the study of the μ-catalysis phenomenon in a D_2+T_2 mixture, we obtain in this paper simple analytic formulas that describe the kinetics of the processes that consitute the catalysis, and discuss the conditions under which their principal characteristics can be measured.[4)]

2. SYMBOLS AND SYSTEM OF EQUATIONS

The general scheme of the mesic-molecular processes in the D_2+T_2 mixture, which holds true for an arbitrary density and concentrate of the components, is shown in Fig. 1. We have introduced here the following symbols: $\lambda_0=0.455\times10^6\ \text{sec}^{-1}$ is the rate of decay of the μ^- meson; $\varphi=N/N_0$ is the ratio of the density of the nuclei of the mixture (N) to the density of the liquid hydrogen ($N_0=4.25\times10^{22}\ \text{cm}^{-3}$); λ_a, ($\lambda_a{}^0$) are the rates of capture of the μ^- meson by the K orbits of the mesic atoms $d\mu$ and tm, respectively, at a mixture density N and at a liquid-hydrogen density N_0; $\lambda_{dt}(\lambda_{dt}{}^0)$ are the rates of the isotropic exchange $d\mu+t\to t\mu+d$; $\lambda_{dt\mu}, \lambda_{dd\mu}$ and $\lambda_{tt\mu}(\lambda^0_{dt\mu}, \lambda^0_{dd\mu}$ and $\lambda^0_{tt\mu})$ are the rates of formation of the mesic molecules $dt\mu$, $dd\mu$, and $tt\mu$; C_d and C_t are the concentrations of the deuterium and tritium nuclei ($C_d+C_t=1$); λ_f, λ_{fd}, and λ_{ft} are the rates of the nuclear fusion reactions respectively in the mesic molecules $dt\mu$, $dd\mu$, and $tt\mu$ (averaged over the spin states of the mesic molecules); $\omega_d, \omega_d', \omega_t, \omega_s$ are the sticking coefficients of the μ^- meson to the charged fusion products in the reactions represented in Fig. 1; $N_\mu=N_\mu(t)$ is the number of free μ^- mesons at the instant of time t; $N_{d\mu}$ and $N_{t\mu}$ are the numbers of the mesic atoms $d\mu$ and $t\mu$; $N_{dt\mu}$, $N_{dd\mu}$, and $N_{tt\mu}$ are the numbers of the corresponding mesic molecules; N_n, N_{nd}, and N_{nt} are the numbers of neutrons emitted in the fusion reactions in the mesic molecules $dt\mu$, $dd\mu$, and $tt\mu$, respectively.

The system of equations describing the kinetics of the processes shown in Fig. 1, takes the form

$$-dN_\mu/dt=(\lambda_0+\lambda_a)N_\mu-\lambda_f(1-\omega_s)N_{dt\mu}$$
$$-\lambda_{fd}[1-{}^1/_2(\omega_d+\omega_d')]N_{dd\mu}-\lambda_{ft}(1-\omega_t)N_{tt\mu}, \quad (1.0)$$

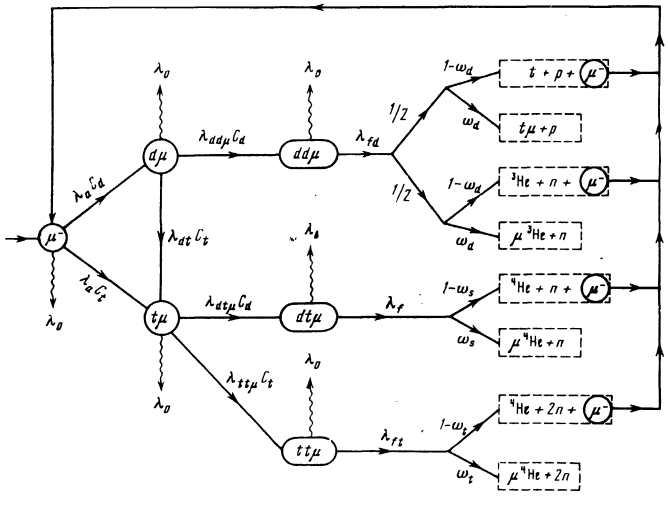


FIG. 1. Scheme of mesic-molecular processes that take place in a mixture of deuterium and tritium.

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$$-dN_{d\mu}/dt=(\lambda_0+\lambda_{dt}C_t+\lambda_{dd\mu}C_d)N_{d\mu}-\lambda_aC_dN_\mu, \quad (1.1)$$

$$-dN_{t\mu}/dt=(\lambda_0+\lambda_{dt\mu}C_d+\lambda_{tt\mu}C_t)N_{t\mu}-\lambda_{dt}C_tN_{d\mu}-\lambda_aC_tN_\mu-{}^1/_2\lambda_{fd}\omega_d'N_{dd\mu}, \quad (1.2)$$

$$-dN_{dt\mu}/dt=(\lambda_0+\lambda_f)N_{dt\mu}-\lambda_{dt\mu}C_dN_{t\mu}, \quad (1.3)$$

$$-dN_{dd\mu}/dt=(\lambda_0+\lambda_{fd})N_{dd\mu}-\lambda_{dd\mu}C_dN_{d\mu}, \quad (1.4)$$

$$-dN_{tt\mu}/dt=(\lambda_0+\lambda_{ft})N_{tt\mu}-\lambda_{tt\mu}C_tN_{t\mu}, \quad (1.5)$$

$$-dN_n/dt=-\lambda_fN_{dt\mu}, \quad (1.6)$$

$$-dN_{nd}/dt=-\frac{1}{2}\lambda_{fd}N_{dd\mu}, \quad (1.7)$$

$$-dN_{nt}/dt=-2\lambda_{ft}N_{tt\mu}. \quad (1.8)$$

At the initial instant of time, i.e., at the instant when the muon falls into the D_2+T_2 mixture, we have $N_\mu(0)=1$, and the concentration of all the remaining components is $N_i(0)=0$. In view of the resonant character of the production of the mesic molecules $dt\mu$, the rates $\lambda_{dt\mu}^{(a)}$ and $\lambda_{dt\mu}^{(b)}$ of the process (Ia) and (Ib) can differ greatly. The quantity $\lambda_{dt\mu}C_d$ in Eqs. (1) actually means

$$\lambda_{dt\mu}C_d=\lambda_{dt\mu}^{(a)}2C_{D_2}+\lambda_{dt\mu}^{(b)}C_{DT}, \quad (2)$$

where C_{D_2} and C_{DT} are the concentrations of the molecules D_2 and DT. Inasmuch as isotopic exchange takes place in the mixture D_2+T_2 even at room temperatures, and an equilibrium concentration is established:

$$C_{D_2}:C_{DT}:C_{T_2}=C_d:2C_dC_t:C_t^2, \quad (2.1)$$

the effective rate $\lambda_{dt\mu}$ in the kinetic equations (1), under the condition (2.1), is

$$\lambda_{dt\mu}=\lambda_{dt\mu}^{(a)}C_d+\lambda_{dt\mu}^{(b)}C_t. \quad (2.2)$$

Similarly

$$\lambda_{dd\mu}=\lambda_{dd\mu}^{(a)}C_d+\lambda_{dd\mu}^{(b)}C_t,$$

where $\lambda_{dd\mu}^{(a)}$ and $\lambda_{dd\mu}^{(b)}$ are the rates of the resonant exchange of the mesic molecules $dd\mu$ in collisions of the $d\mu$ atoms with the deuterium nuclei in the molecules D_2 and DT.

It should be noted that by virtue of the resonant mechanism of the production of the $dt\mu$ molecules, the quantities $\lambda_{dt\mu}^{(a)}$ and $\lambda_{dt\mu}^{(b)}$ can have a complicated dependence on the temperature, on the density, and the cocentration of the mixture components. Primarily, their temperature dependence will be more complicated than the simplest case considered in Ref. 1, inasmuch as actually even at moderate temperatures the reactions (Ia) and (Ib) proceed not only from the ground state but also from the excited rotational states of the molecules D_2 and DT into the excited rotational states of the complexes $[(dt\mu)d2e]_v^*$ and $[(dt\mu)t\,2e]_v^*$.

It must also be taken into account that the formation of the mesic molecules $dt\mu$ takes place in various states of their hyperfine structure. According to calculations,[9] the level $(J=1, v=1)$ is split by the interaction of the particle spins and the orbital momentum J of the mesic molecules into 10 levels, the maximum spacing between which is ~0.2 eV. The rates of production of the $dt\mu$ molecules in each of the states hfs has its own resonant dependence on the temperature. In addition, these probabilities depend substantially on the initial population of the levels of the hyperfine structure of the atom $t\mu$, which in turn depends on the rate of spin flip in the reaction $t\mu(\uparrow\uparrow)+t\to t\mu(\uparrow\downarrow)+t$, i.e., on the density φ of the mixture and on the concentration C_t of the tritium nuclei.

In the entire discussion above it was implicitly assumed that the $t\mu$ atoms in the reactions (Ia) and (Ib) are thermalized. At sufficiently low tritium concentrations ($C_t \ll 1$), however, the $t\mu$ atoms are produced in the isotopic-exchange reaction $d\mu+t\to t\mu+d$, with energy ≈ 20 eV, and do not manage to slow down to thermal velocities. This can lead to a change in the character of the temperature dependences of the μ-catalysis process.[5] In this case the system of equations (1) must be solved simultaneously with the equations that describe the kinetics of the slowing down of the mesic atoms $t\mu$.

It is important to take into account all the foregoing effects in the theoretical calculation of the rates $\lambda_{dt\mu}^{(a)}$ and $\lambda_{dt\mu}^{(b)}$ in the detailed calculation of the kinetics pf the μ-catalysis process. In the phenomenological approach used in the present paper, the rates $\lambda_{dt\mu}^{(a)}$ and $\lambda_{dt\mu}^{(b)}$ can be regarded as certain averaged characteristics of the process. They are assumed to be unknown functions of φ and C_t, which must be determined from experiment.

As to the dependence of the rate of the nuclear reactions on the spin state of the mesic molecules, in the case when all these rates are high ($\lambda_f \gtrsim 10^{11}$ sec^{-1}), the kinetics of the intermolecular processes is independent of them at a time $t \gg \lambda_f^{-1}$ (see Sec. 3). It suffices therefore to use in the system (1) the values of λ_f averaged over the states of the hyperfine structure of the mesic molecules.

For the numerical estimates that follow, we assume the following tentative values of the rate constants of the μ-mesic-molecular processes, referred to the density N_0 of the liquid hydrogen.[8] The rates of the reactions

$$\lambda_{dt\mu}^0\sim 10^8 \text{ sec}^{-1}, \quad \lambda_{dd\mu}^0\sim(0.08-0.8)\cdot 10^6 \text{ sec}^{-1} \quad (3.1)$$

depend on the temperature of the medium;

$$\lambda_{tt\mu}=3\times10^6 \text{ sec}^{-1},$$
$$\lambda_{dt}^0=2\times10^8 \text{ sec}^{-1},$$
$$\lambda_f\sim10^{12} \text{ sec}^{-1}, \quad \lambda_{fd}\sim\lambda_{ft}\sim10^{11} \text{ sec}^{-1}, \quad (3.2)$$
$$\lambda_a^0=10^{10} \text{ sec}^{-1};$$

the sticking coefficients are

$$\omega_a\approx10^{-2}, \quad \omega_d=0.13, \quad \omega_d'=0.0027, \quad \omega_t\sim0.1; \quad (3.3)$$

$\varphi=1.28\times10^{-3}P$ (atm), where P is the pressure in the mixture at the normal temperature $T=273$ K.[6]

3. SOLUTION OF THE SYSTEM OF EQUATIONS

Under the experimental conditions it is usually difficult to observe events at instants of time $t<t_0=10^{-8}$ sec after the stopping of the μ^- meson. We confine ourselves therefore to a description of the kinetics of μ-mesic-molecular processes at $t>t_0\gg\lambda_f^{-1}$, and consider, for example, Eq. (1.3). The solution of this equation can be written in the form

$$N_{dt\mu}(t)=\lambda_{dt\mu}C_de^{-\lambda_ft}\int_0^t e^{\lambda_ft'}N_{t\mu}(t')\,dt'.$$

At $t\gg\lambda_f^{-1}$ the quantity $N_{t\mu}(t)$ varies relatively slowly,

like

$$N_{t\mu}(t)=B_1e^{-\lambda_1 t}+B_2e^{-\lambda_2 t},$$

with $\lambda_1 \ll \lambda_f$ and $\lambda_2 \ll \lambda_f$. We therefore have, accurate to terms $\sim\lambda_i/\lambda_f$,

$$N_{dt\mu}(t)\approx\frac{\lambda_{dt\mu}C_d}{\lambda_f}N_{t\mu}(t). \tag{4}$$

The physical meaning of this relation is clear: the concentration of the molecules $dt\mu$ is determined by the slowest process of production of the atoms $t\mu$, inasmuch as after a time $t\sim\lambda_f^{-1}$ a quasi-equilibrium is established between the concentrations $N_{dt\mu}$ and $N_{t\mu}$, with a proportionality factor $\lambda_{dt\mu}C_d/\lambda_f$ equal to the ratio of the rates of the production and destruction of the $dt\mu$ molecules. (This result can be obtained directly by taking into account the quasi-equilibrium condition $dN_{dt\mu}/dt\approx 0$, which means that the rate of change of $N_{dt\mu}$ is much less than the rate of λ_f).

Analogously, from (1.4) and (1.5) we obtain the relations

$$N_{dd\mu}\approx\frac{\lambda_{dd\mu}C_d}{\lambda_{fd}}N_{d\mu}, \tag{5}$$

$$N_{tt\mu}\approx\frac{\lambda_{tt\mu}C_t}{\lambda_{ft}}N_{t\mu}, \tag{6}$$

and from (1.0) at $t\gg\lambda_a^{-1}$ and not too small φ, when $\lambda_a\gg\lambda_0$, we have[7)]

$$N_\mu\approx\lambda_a^{-1}\left\{\lambda_{dd\mu}C_d\left(1-\frac{\omega_d+\omega_d'}{2}\right)N_{d\mu}+[\lambda_{dt\mu}C_d(1-\omega_s)+\lambda_{tt\mu}C_t(1-\omega_t)]N_{t\mu}\right\}. \tag{7}$$

Taking relations (4)–(7) into account, the temporal distribution of the neutrons is expressed by the formulas

$$dN_n/dt=\lambda_{dt\mu}C_dN_{t\mu}, \tag{8.1}$$

$$dN_{nd}/dt={}^1/_2\lambda_{dd\mu}C_dN_{d\mu}, \tag{8.2}$$

$$dN_{nt}/dt=2\lambda_{tt\mu}C_tN_{t\mu}, \tag{8.3}$$

and the functions $N_{d\mu}(t)$ and $N_{t\mu}(t)$ are determined from the system of equations

$$\begin{aligned}-dN_{d\mu}/dt&=a_{11}N_{d\mu}-a_{12}N_{t\mu},\\ -dN_{t\mu}/dt&=-a_{21}N_{d\mu}+a_{22}N_{t\mu}\end{aligned} \tag{9}$$

with initial conditions (at $t_0\gg\lambda_a^{-1}$)

$$N_{d\mu}(t_0)=C_d,\quad N_{t\mu}(t_0)=C_t,\quad C_d+C_t=1. \tag{10}$$

The coefficients a_{ij} of Eqs. (9) are equal to

$$\begin{aligned}a_{11}&=\lambda_0+\lambda_{dt}C_t+\lambda_{dd\mu}C_d-\lambda_{dd\mu}C_d^2[1-{}^1/_2(\omega_d+\omega_d')],\\ a_{12}&=\lambda_{dt\mu}C_d^2(1-\omega_s)+\lambda_{tt\mu}C_dC_t(1-\omega_t),\\ a_{21}&=\lambda_{dt}C_t+\lambda_{dd\mu}C_dC_t[1-{}^1/_2(\omega_d+\omega_d')]+{}^1/_2\lambda_{dd\mu}C_d\omega_d',\\ a_{22}&=\lambda_0+\lambda_{dt\mu}C_d+\lambda_{tt\mu}C_t-\lambda_{dt\mu}C_dC_t(1-\omega_s)-\lambda_{tt\mu}C_t^2(1-\omega_t),\end{aligned} \tag{11}$$

and the general solution of the system (9) takes the form

$$\begin{aligned}N_{d\mu}&=\frac{1}{\lambda_2-\lambda_1}(B_{11}e^{-\lambda_1 t}+B_{12}e^{-\lambda_2 t}),\\ N_{t\mu}&=\frac{1}{\lambda_2-\lambda_1}(B_{21}e^{-\lambda_1 t}+B_{22}e^{-\lambda_2 t}).\end{aligned} \tag{12}$$

The explicit forms of expressions λ_i and B_{ij} in terms of the coefficients a_{ij} are given in the Appendix.

The temporal distribution of the neutrons, with allowance for the relations (1.6) and (4), is expressed in terms of $N_{t\mu}$:

$$dN_n/dt=\lambda_{dt\mu}C_dN_{t\mu}. \tag{13}$$

The integrated yield of the neutrons with energy 14.1 MeV from the fusion reaction in the $dt\mu$ molecule, $Y_{dt\mu}=N_n(\infty)$ (per stopped μ^- meson) can be represented, using the formulas of the Appendix, in the form

$$\begin{aligned}Y_{dt\mu}=W_{t\mu}W_{dt\mu}\Big[1-(1-\omega_s)W_{t\mu}W_{dt\mu}\\ -(1-\omega_t)W_{t\mu}W_{tt\mu}-\left(1-\frac{\omega_d+\omega_d'}{2}\right)W_{d\mu}W_{dd\mu}\Big]^{-1},\end{aligned} \tag{14}$$

where the quantities $W_{t\mu}$, $W_{dt\mu}$, $W_{tt\mu}$, $W_{dd\mu}$ have a simple physical meaning:

$$W_{t\mu}=C_t+\frac{\lambda_{dt}C_tC_d+{}^1/_2\omega_d'\lambda_{dd\mu}C_d^2}{\lambda_0+\lambda_{dt}C_t+\lambda_{dd\mu}C_d},\qquad W_{d\mu}=C_d, \tag{15.1}$$

$$W_{dt\mu}=\frac{\lambda_{dt\mu}C_d}{\lambda_0+\lambda_{dt\mu}C_d+\lambda_{tt\mu}C_t}, \tag{15.2}$$

$$W_{tt\mu}=\frac{\lambda_{tt\mu}C_t}{\lambda_0+\lambda_{dt\mu}C_d+\lambda_{tt\mu}C_t}, \tag{15.3}$$

$$W_{dd\mu}=\frac{\lambda_{dd\mu}C_d}{\lambda_0+\lambda_{dt}C_t+\lambda_{dd\mu}C_d}. \tag{15.4}$$

$W_{t\mu}$ is the probability of production of the mesic atom $t\mu$ (the first term corresponds to direct landing of the μ^- meson on the tritium, the second to the charge exchange $d\mu\to t\mu$, and the third to regeneration of $t\mu$ in the reaction $dd\mu\to t\mu+p$); $W_{dt\mu}$, $W_{tt\mu}$ and $W_{dd\mu}$ are real respectively to the probabilities of production of the mesic molecules $dt\mu$ and $dd\mu$ from the $t\mu$ atoms, and of the mesic molecules $dd\mu$ from the $d\mu$ atoms. Equation (14) takes into account the return of the muons to the catalysis cycle after the nuclear fusion reaction in the mesic molecules $dt\mu$, $tt\mu$, and $dd\mu$, as can be seen from the structure of its denominator. Similar expressions for $Y_{dd\mu}$ and $Y_{tt\mu}$ for a number of $dd\to{}^3\mathrm{He}+n$ and $tt\to{}^4\mathrm{He}+2n$ reactions in the mesic molecules $dd\mu$ and $tt\mu$ are obtained from (14) by replacing the products $W_{t\mu}W_{dt\mu}$ in the numerator by $\frac{1}{2}W_{d\mu}W_{dd\mu}$ and $W_{t\mu}W_{tt\mu}$, respectively.

Formula (14) can also be represented in the form[8)]

$$Y_{dt\mu}=\frac{Y_{dt\mu}^{(0)}}{1+\omega_sY_{dt\mu}^{(0)}+\omega_tY_{tt\mu}^{(0)}+(\omega_d+\omega_d')Y_{dd\mu}^{(0)}}, \tag{16}$$

where $Y_{dt\mu}^{(0)}$, $Y_{dd\mu}^{(0)}$, and $Y_{tt\mu}^{(0)}$ are the numbers of the fusion reactions with neutron emission in the mesic molecules $dt\mu$, $dd\mu$, and $tt\mu$, respectively, in the absence of sticking to the produced helium nucleus ($\omega_s=\omega_d=\omega_t=0$)

$$Y_{dt\mu}^{(0)}=\frac{\lambda_{dt\mu}C_d}{\lambda_0}\frac{(\lambda_0+\lambda_{dt}+\lambda_{dd\mu}C_d)C_t+{}^1/_2\omega_d'\lambda_{dd\mu}C_d^2}{\lambda_0+(\lambda_{dt}+\lambda_{dd\mu}C_d)C_t+(\lambda_{dt\mu}C_d+\lambda_{tt\mu}C_t)C_d} \tag{17}$$

and analogously for $Y_{dd\mu}^{(0)}$ and $Y_{tt\mu}^{(0)}$.

4. KINETICS OF THE PROCESSES AT MODERATE DENSITIES OF THE D_2+T_2 MIXTURE

At pressures $P=5$–100 atm in the mixture, concentrations $C_t\leq 0.5$, and at the assumed values of the rates of the mesic-molecular processes (3), the following inequalities hold:

$$\begin{aligned}&\lambda_{dt\mu}C_d\gg\lambda_{tt\mu}C_t,\quad \lambda_{dt}\gg\lambda_{dd\mu}C_d,\\ &\{\omega_s\lambda_{dt\mu}C_t,\ \omega_d\lambda_{dd\mu}C_d,\ \omega_t\lambda_{tt\mu}C_t\}\ll\lambda_0.\end{aligned} \tag{18}$$

The last condition means that the number of fusion reactions produced by a single μ^- meson is limited by its lifetime, and the influence of the muon sticking to the helium nuclei can be neglected.

Taking the inequalities (18) into account, it follows from (17) that

$$Y_{dt\mu} \approx Y_{dt\mu}^{(0)} \approx \frac{(\lambda_0+\lambda_{dt})\lambda_{dt\mu}C_dC_t}{\lambda_0(\lambda_0+\lambda_{dt}C_t+\lambda_{dt\mu}C_d^2)}. \quad (19)$$

The expression (13) for dN_n/dt also becomes in this case much simpler:

$$\frac{dN_n}{dt} \approx \frac{\lambda_{dt\mu}C_dC_t}{\lambda_{dt}C_t+\lambda_{dt\mu}C_d^2}[\lambda_{dt}e^{-\lambda_1 t}+C_d(\lambda_{dt\mu}C_d-\lambda_{dt})e^{-\lambda_2 t}], \quad (20)$$

where

$$\lambda_1 \approx \lambda_0, \quad \lambda_2 \approx \lambda_0+\lambda_{dt}C_t+\lambda_{dt\mu}C_d^2. \quad (21)$$

Expressions (19)–(21) can be used for an experimental determination of the rates λ_{dt} and $\lambda_{dt\mu}$ at the different mixture temperatures. The measured value of the argument λ_2 of the exponential yields one combination of the rates λ_{dt} and $\lambda_{dt\mu}$, and the ratio of the preexponential factors yields another combination. Taking (2.1) into account, measurements of these quantities at the different concentrations C_t yields also the rates $\lambda_{dt\mu}^{(a)}$ and $\lambda_{dt\mu}^{(b)}$ at a given temperature and density of the mixture.

The observed form of the temporal distribution of dN_n/dt depends significantly on the ratio of the rates λ_{dt} and $\lambda_{dt\mu}$. The optimal conditions for the experimental determination of the exponent λ_2 are reached if the relation $\lambda_{dt}C_t+\lambda_{dt\mu}C_d^2 \sim \lambda_0$ is satisfied. Assuming that $\lambda_{dt}=\lambda_{dt}^0\varphi$ and $\lambda_{dt\mu}=\lambda_{dt\mu}^0\varphi$,[9] and using the rates (3), we find that the indicated relation is satisfied at $\varphi \sim 10^{-2}$, i.e., at pressures $P \sim 5$ atm and at a normal temperature of the mixture.

Figure 2 shows the temporal distribution of dN_n/dt, calculated from Eq. (20), at a mixture pressure 10 atm, a tritium concentration $C_t=0.1$, at $\lambda_{dt}^0=2\cdot10^8$ sec^{-1}, and at values $\lambda_{dt\mu}^0=(0.5-4.0)\cdot10^8$ sec^{-1}. It is seen from this figure that at $\lambda_{dt\mu}^0C_d=\lambda_{dt}^0$ the character of the temporal distribution of the neutrons changes strongly.

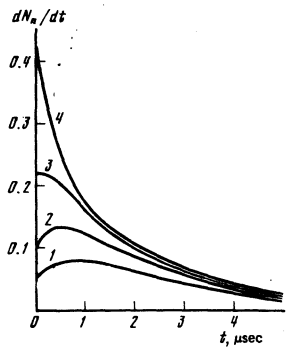

FIG. 2. Temporal distributions (20) of neutrons with energy 14.1 MeV emitted in the fusion reaction $d+t \to {}^4He+n$ in the mesic molecule $dt\mu$ at a pressure $P=10$ atm, tritium concentration $C_t=0.1$, value $\lambda_{dt}^0=2\times10^8$ sec^{-1}, and at the following values of $\lambda_{dt\mu}^0$: curve 1—0.5×10^8 sec^{-1}, 2—1.0×10^8 sec^{-1}, 3—2.0×10^8 sec^{-1}, 4—4×10^8 sec^{-1}.

Measurements of the integrated yield $Y_{dt\mu}$ as a function of the concentration C_2 also make it possible to determine the rates of the processes, particularly at $\varphi \gg 10^{-2}$.

At mixture densities $\varphi < 10^{-3}$, i.e., at pressures $P<1$ atm, the condition $\lambda_a \gg \lambda_0$ is no longer satisfied and the analytic expressions (19)–(21) in the general case are no longer valid, especially at the instants of time $t \lesssim \lambda_a^{-1}$. The numerical solution of the system of equations (1) shows that in this case at $t<\lambda_a^{-1}$ the temporal distribution of dN_n/dt differs strongly from that shown in Fig. 2, and from the character of this difference it is possible in principle to determine the value of λ_a. Measurements of the rate of the atomic capture of the μ^- mesons have so far not been performed, partially because all the known rates of the mesic-atom processes (with the exceptions of $\lambda_{dt\mu}^0$) were larger than λ_a by several orders of magnitude.

5. KINETICS OF THE PROCESSES AT HIGH MIXTURE DENSITIES

The effectiveness of muon catalysis depends substantially on the sticking coefficient ω_s (with allowance for the "jarring" of the μ^- meson, see footnote 6). To determine this quantity from measurements of dN_n/dt and $Y_{dt\mu}$, the experiments must be performed at large mixture densities. In this case we can neglect in (12) the "fast" exponent (λ_2), while the "slow one" (λ_1) can be represented in the form

$$\lambda_1 \approx \lambda_0+\frac{\lambda_{dt\mu}\lambda_{dt}C_dC_t}{\lambda_{dt}C_t+\lambda_{dt\mu}C_d^2}\left(\omega_s+\omega_t\frac{\lambda_{tt\mu}C_t}{\lambda_{dt\mu}C_d}\right). \quad (22)$$

Equation (22) makes it possible to determine ω_s if the remaining quantities are known.

At $\varphi > 0.1$ and $C_t \sim 0.5$, the following relations hold

$$\lambda_{dt\mu}C_d \gg \lambda_0, \quad \lambda_{dt}C_t \gg \lambda_0, \quad (23)$$

and formula (14) for the integrated yield of the neutrons (from which it is also possible to determine ω_s) can be represented in the form

$$\frac{1}{Y_{dt\mu}} \approx \frac{\lambda_0}{\lambda_{dt\mu}C_d}+\frac{\lambda_0C_d}{\lambda_{dt}C_t}+\omega_s+\frac{\lambda_{tt\mu}C_t}{\lambda_{dt\mu}C_d}\omega_t. \quad (24)$$

The maximum neutron yield $Y_{dt\mu}^{max}$ is reached at the concentration

$$C_t \approx (1+\gamma)^{-1}, \quad \gamma=(\lambda_{dt}/\lambda_{dt\mu})^{1/2} \quad (25)$$

and is equal to

$$\frac{1}{Y_{dt\mu}^{max}} \approx \omega_s+\frac{\lambda_0}{\lambda_{dt\mu}\gamma}\left\{2+\gamma+\omega_t\frac{\lambda_{tt\mu}}{\lambda_0}\right\}. \quad (26)$$

Substituting in this formula the values of the constants (3) and assuming that $\lambda_{dt\mu}=\lambda_{dt\mu\varphi}^0$ etc., we get

$$\frac{1}{Y_{dt\mu}^{max}} \approx \omega_s+\frac{\lambda_0}{\lambda_{dt\mu}^0\varphi\gamma}(2+\gamma+0.7\varphi). \quad (27)$$

Figure 3 shows plots of $Y_{dt\mu}^{max}(\omega_s, \varphi)$ constructed for $\omega_s=10^{-2}$ and 0.7×10^{-2} and for $\lambda_{dt\mu}^0=10^8$ and 10^9 sec^{-1} at $\lambda_{qt}^0=2\cdot10^8$ sec^{-1}. It is seen from the figure that already at $\varphi \sim 0.1$ the sticking of the μ^- meson to the helium influences noticeably the neutron yield $Y_{dt\mu}^{max}$ and

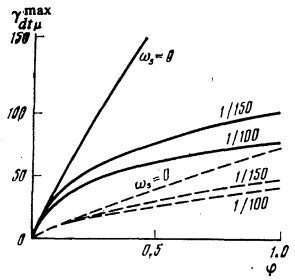

FIG. 3. Effectiveness of muon catalysis in a mixture of deuterium and tritium, i.e., the maximum yield of the neutrons $\gamma_{dt\mu}^{max}$ per μ^- meson, as a function of the mixture density φ at optimal tritium concentration $C_t=(1+\gamma)^{-1}$, $\gamma=(\lambda_{dt}^0/\lambda_{dt\mu}^0)^{1/2}$ at $\lambda_{dt}^0=2\times10^8$ sec^{-1}, $dt\mu$-molecule formation rates $\lambda_{dt\mu}^0=10^9$ sec^{-1} (solid curves, $\lambda_{dt\mu}^0=10^8$ sec^{-1} (dashed curves), and different values of the sticking coefficient ω_s (marked on the curves).

decreases it by 20–30%. At $\mu\sim1$, i.e., at the density of liquid hydrogen, the value of $Y_{dt\mu}^{max}$ is determined mainly by the value of ω_s. Therefore the experimental determination of ω_s, and also of $\lambda_{dt\mu}^0$, $\lambda_{tt\mu}^0$, and ω_t, is of paramount significance for the determination of the efficiency of muon catalysis. At the constant values (3) and at $\lambda_{dt}^0\sim10^9$ sec^{-1} the neutron yield can reach a value $Y_{dt\mu}^{max}\sim\omega_s^{-1}$, i.e., one μ^- meson in a mixture of deuterium and tritium can effect ~100 cycles of catalysis of the fusion reaction of the nuclei $d+t\to{}^4\mathrm{He}+n+17.6$ MeV.[11]

We note that in the case when the hypothesis that there is no thermalization of the $t\mu$ atoms turns out to be correct, a study of the μ-catalysis at densities $\varphi\sim1$ can be carried also in a liquid D_2+T_2 mixture; this is apparently much simpler than experiments with a gas mixture at high pressures $P\sim500$–1000 atm.

6. EXPERIMENTS IN PURE DEUTERIUM AND TRITIUM

It is known that the reaction of the fusion of deuterium nuclei in the mesic molecule $dd\mu$ proceeds with equal probability via two channels (see Fig. 1), in one of which an ^{3}He nucleus is produced, and a μ^- meson stick to it with a probability $\omega_d\approx0.13$. A measurement of ω_d is very important for checking the correctness of the theoretical calculations of the sticking coefficients, and, in particular, the coefficient ω_s. Up to now, only an upper-bound experimental estimate has been obtained for ω_d.[12]

At $C_t=0$, formulas (12) and (14) become simpler:

$$\frac{dN_{nd}}{dt}\approx\frac{1}{2}\lambda_{dd\mu}\exp\left\{-\left(\lambda_0+\frac{1}{2}\omega_d\lambda_{dd\mu}\right)t\right\},\qquad Y_{dd\mu}\approx\frac{1}{2}\frac{\lambda_{dd\mu}}{\lambda_0+{}^1/_2\omega_d\lambda_{dd\mu}}. \tag{28}$$

From (28) we can determine $\lambda_{dd\mu}$ and ω_d.

At $C_d=0$ we obtain the expressions

$$dN_{nt}/dt=2\lambda_{tt\mu}\exp\{-(\lambda_0+\omega_t\lambda_{tt\mu})t\},\qquad Y_{tt\mu}\approx2\lambda_{tt\mu}/(\lambda_0+\omega_t\lambda_{tt\mu}), \tag{29}$$

from which we can extract the values of $\lambda_{tt\mu}$ and ω_t.

For the mixtures D_2+ He and T_2+ He, the temporal distribution and the integrated yield of the neutrons from the reactions $dd\to{}^3\mathrm{He}+n$ and $tt\to{}^4\mathrm{He}+2n$ are calculated as before from formulas (28) and (29), with the substitutions

$${}^1/_2\omega_d\lambda_{dd\mu}\to{}^1/_2\omega_d\lambda_{dd\mu}C_d+\lambda_{d\mu\mathrm{He}}C_{\mathrm{He}},\qquad \omega_t\lambda_{tt\mu}\to\omega_t\lambda_{tt\mu}C_t+\lambda_{t\mu\mathrm{He}}C_{\mathrm{He}},$$

where $\lambda_{d\mu\mathrm{He}}$ and $\lambda_{t\mu\mathrm{He}}$ are the rates of formation of the molecules $d\mu$He and $t\mu$He in the reactions $d\mu+\mathrm{He}\to d\mu\mathrm{He}$ and $t\mu+\mathrm{He}\to t\mu\mathrm{He}$, while C_{He} is the concentration of the helium nuclei in the mixture ($C_d+C_{\mathrm{He}}=1$ and $C_t+C_{\mathrm{He}}=1$). By varying C_{He}, we can obtain from the measured values of $dN_{d\mu}/dt$, $Y_{dd\mu}$ and dN_{nt}/dt, $Y_{tt\mu}$, with the aid of (28) and (29), the values of ω_d, $\lambda_{dd\mu}$, $\lambda_{d\mu\mathrm{He}}$ and ω_t, $\lambda_{tt\mu}$, $\lambda_{t\mu\mathrm{He}}$.

7. CONCLUSION

The kinetics of the μ-catalysis processes in a mixture of deuterium and tritium has up to now hardly been investigated: there are at present only several calculations of the mesic-molecular processes[6,8] and only a single experiment.[7]

In the present paper we paid principal attention to a discussion of the possibility of extracting information on the μ-catalysis process by measuring the integrated yield and the temporal distribution of the neutrons from the synthesis reaction $d+t\to{}^4\mathrm{He}+n$. In this way it is possible to separate at least four experiments that differ in their tasks and in the extracted information.

a) Experiments at moderate mixture densities ($\varphi\sim3\cdot10^{-3}-10^{-2}$) and at low tritium concentrations ($C_t\sim0.2-0.1$) make it possible to measure the rates $\lambda_{dt\mu}^{(a)}$ and λ_{dt}.

b) Experiments at $\varphi\sim3\cdot10^{-3}-10^{-2}$ and $C_t\sim0.5-0.9$ make it possible to measure the rate $\lambda_{dt\mu}^{(b)}$ and, in addition, to determine the role of thermalization of the $t\mu$ atoms in the course of resonant production of the mesic molecules $dt\mu$.

(c) Experiments at high mixture densities ($\varphi\sim0.1-1$) and tritium concentrations $C_t\sim0.5$ make it possible to measure the sticking coefficient ω_s as well as determine directly the maximum number of catalysis cycles $Y_{dt\mu}^{max}$ that can be effected by a single μ^- meson.

d) Experiments in pure deuterium and tritium make it possible to measure the rates $\lambda_{dd\mu}$ and $\lambda_{tt\mu}$ as well as the sticking coefficients ω_d and ω_t (so far, only the rate $\lambda_{dd\mu}$ has been measured[4]).

This paper does not deal with certain processes that can change the picture of the μ-catalysis kinetics, for example, the ortho–para transition in a $t\mu$ atom colliding with tritium nuclei, the process of formation of the mesic molecules $d\mu$He and $t\mu$He,[13] the "jarring" of the μ^- mesons in the reaction $\mu{}^4\mathrm{He}+t\to t\mu+{}^4\mathrm{He}$, etc. All call for additional investigations.

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APPENDIX

The solution of the system (9) with initial conditions (10) takes the form (12), where

$$\lambda_{1,2}=\frac{a_{11}+a_{22}}{2}\mp\left\{\left(\frac{a_{11}-a_{22}}{2}\right)^2+a_{12}a_{21}\right\}^{1/2}, \tag{A.1}$$

$$B_{11}=-(a_{11}-\lambda_2)C_d+a_{12}C_t,\quad B_{12}=(a_{11}-\lambda_1)C_d-a_{12}C_t,$$
$$B_{21}=a_{21}C_d-(a_{22}-\lambda_2)C_t,\quad B_{22}=-a_{21}C_d+(a_{22}-\lambda_1)C_t. \tag{A.2}$$

The integrated yields of the neutrons are equal to

$$Y_{dt\mu}=\lambda_{dt\mu}C_d\frac{a_{21}C_d+a_{11}C_t}{a_{11}a_{22}-a_{12}a_{21}}$$
$$Y_{dd\mu}=\frac{1}{2}\lambda_{dd\mu}\frac{a_{22}C_d+a_{12}C_t}{a_{11}a_{22}-a_{12}a_{21}}, \tag{A.3}$$
$$Y_{tt\mu}=2\frac{\lambda_{tt\mu}}{\lambda_{dt\mu}}\frac{C_t}{C_d}Y_{dt\mu}.$$

Taking into account the smallness of the sticking coefficients ω_i, we can obtain simple analytic expressions for the quantities (A.1) and (A.2). Introducing the notation

$$a_{11}=\lambda_0+a_1+(\delta_1+\delta_1')C_d,\quad a_{12}=a_2-\delta_2C_d,$$
$$a_{21}=a_1-\delta_1C_t+\delta_1'C_d,\quad a_{22}=\lambda_0+a_2+\delta_2C_t,$$

where

$$a_1=(\lambda_{dt}+\lambda_{dd\mu}C_d)C_t,\quad a_2=(\lambda_{dt\mu}C_d+\lambda_{tt\mu}C_t)C_d, \tag{A.4}$$
$$\delta_1={}^1/_2\omega_d\lambda_{dd\mu}C_d,\quad \delta_1'={}^1/_2\omega_d'\lambda_{dd\mu}C_d, \tag{A.5}$$
$$\delta_2=\omega_s\lambda_{dt\mu}C_d+\omega_t\lambda_{tt\mu}C_t,$$

we obtain, accurate to terms of first order in the coefficients ω_i:

$$\lambda_1\approx\lambda_0+\delta,\quad \lambda_2\approx\lambda_0+a_1+a_2-\delta+\delta', \tag{A.6}$$
$$B_{11}\approx a_2-\delta C_d,\quad B_{12}\approx a_1C_d-a_2C_t+(\delta'-\delta)C_d, \tag{A.7}$$
$$B_{21}\approx a_1+\delta_1'C_d-\delta C_t,\quad B_{22}\approx a_2C_t-a_1C_d-\delta_1'C_d+(\delta'-\delta)C_t,$$

where

$$\delta=(a_1\delta_2+a_2\delta_1)/(a_1+a_2),\quad \delta'=(\delta_1+\delta_1')C_d+\delta_2C_t. \tag{A.8}$$

[1)] Institute of High Energy Physics, Protvino.
[2)] Leningrad Institute of Nuclear Physics, Gatchina.
[3)] I. V. Kurchatov Atomic Energy Institute, Moscow.
[4)] At the Nuclear Physics Problems Laboratory of JINR they have just completed an experiment[7] in which the nuclear-fusion reaction was registered for the first time ever in the $dt\mu$ mesic molecules and the lower bound of the rate of production of these molecules was obtained ($\lambda_{dt\mu}^0>10^8$ sec^{-1}); this agrees with the theoretical predictions.[1] The measured rate $\lambda_{dt}{}^0=(2.7\pm0.9)\cdot10^8$ sec^{-1} of the $d\mu\to t\mu$ process also agrees with the theoretical calculations.[8]
[5)] This circumstance was pointed out by P. F. Ermolov[7] (see also Ref. 10). It is easy to show that 6–8 $t\mu+d\to t\mu+d$ collisions are necessary to slow down a $t\mu$ atom from an energy ≈20 eV to ~0.04 eV. Taking into account the cross section of this process, $\sigma=1.5\cdot10^{-19}$ cm^2 (Ref. 8), we find that the $t\mu$-atom thermalization time is $\sim(10^8\varphi)^{-1}$ sec. Thus, not all the $t\mu$ atoms manage to become thermalized if the rates of production of the $dt\mu$ molecules are high ($\lambda_{dt\mu}\gtrsim\tau^{-1}$, i.e., $\lambda_{dt\mu}^0\gtrsim10^8$ sec^{-1}).
[6)] The presented values of the sticking coefficients ω_s, ω_d, etc. do not take into account the "jarring" of the μ^- mesons in collisions of the produced helium mesic atoms, for example, in the reactions

$$\mu^4He+t\to t\mu+{}^4He,\quad \mu^4He+d\to{}^4He+d+\mu^-,$$

which decrease effectively the values of ω_s, ω_d, ω_d', and ω_t.
[7)] At low density φ, the value of λ_a is determined by the time of the mesic-atom transitions to higher orbits, which for mesic hydrogen atoms take place mainly on account of the Auger effect on the "foreign" atoms. Therefore $\lambda_a\approx\gamma\varphi$, where $\gamma\sim10^{10}$ sec^{-1}, i.e., the assumed approximation is not valid at too small $\varphi(\varphi\lesssim10^{-3})$.
[8)] Allowance for the terms $\sim\omega_d'$ is important only for experiments at exceedingly small tritium concentrations ($C_t\lesssim10^{-5}$) or in pure deuterium (see below).
[9)] Possible violations of these relations were discussed in Sec. 2.

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