fields that are applied along the constant field H and along the z axis and lead to the same result when their frequency corresponds to the transition $\psi_1 \rightleftharpoons \psi_2$. The constant field H is close to the direction of the [110] axis. The alternating low-frequency equalizes the populations of the spin levels of the As^{75} nuclei, for which the z_i' axes coincide with the axes $[1\bar{1}1]$ and $[\bar{1}11]$ of the crystal. The alternating field H_{1h} gives rise to transitions $\psi_1 \rightleftharpoons \psi_2$ corresponding to $\Delta m = \pm 1$, on account of the admixture of the states $|\mp\frac{1}{2}\rangle$ to the states $\chi_{\pm 3/2}$, as a result of non-axiality of the EFG. Those components of the field H_{1z} which are parallel to the z_i' axes give rise to $\Delta m = 0$ transitions if the states $\chi_{+3/2}$ and $\chi_{-3/2}$ are mixed.

As seen from Fig. 4, in a narrow region of angles φ the field H_{1z} is much more effective than the field H_{1h}, this being proof of the mixing of the states $\chi_{+3/2}$ and $\chi_{-3/2}$. The shift of the maximum on the curve of Fig. 4 relative to the point $\varphi = 0$ is determined by the field of the electrons.

The mixing of the states $\chi_{+3/2}$ and $\chi_{-3/2}$ at an orientation H along the [110] axis turns off part of the field of the As^{75} nuclei with one substituted neighboring atom. The direction of the summary nuclear field due to the quadrupole interaction differs in this case from the direction of the vector $\langle \mathbf{S} \rangle$. A consistent allowance for the effect of mixing of the spin wave function is made in the theory of D'yakonov, Merkulov, and Perel', who explained the sharp magnetic anisotropy and the onset of the ambiguity of the polarization following optical orientation as being due to the influence of the level anti-crossing.

The described experiments confirm the validity of the premises and of the conclusions of this theory.

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The hydrodynamics of two- and one-dimensional liquids

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We evaluate the (first and second) viscosity coefficients and the thermal conductivity coefficient of an arbitrary two-dimensional (nonsuperfluid) liquid. The kinetic coefficients are mainly produced by the contribution of the long-wavelength thermal fluctuations and can be expressed completely in terms of thermodynamic functions. We find a thermodynamic inequality, the violation of which must lead to a singularity in the kinetic coefficients. We evaluate the sound absorption coefficient in a one-dimensional liquid.

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The existence of long-wavelength weakly damped thermal fluctuations (of sound waves, and also of entropy and viscous waves) in liquids causes the occurrence of a number of non-local kinetic effects.[1-9] In particular, power-law "tails" appear in the expressions for the damping in space or time of such excitations which are usually characterized by exponential damping laws.

In the literature, a large of amount of special attention has been paid to a study of the power-law decrease in time of the auto-correlation function of the particle velocity in a liquid, which was detected by Alder and Wainwright[6] as the result of a numerical calculation, and which, as was shown in Refs. 2 to 5, is caused by long-wavelength fluctuations. This law is so slow that there appear (see Refs. 2, 4, 5, 7) formal divergences of the coefficients in the Barnett correction terms to the Navier-Stokes equation (in the three-dimensional case) and of the viscosity coefficients (in the two- and one-dimensional cases).

The present author[1,9] has shown that the system of equations which describes the dynamic properties of a

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liquid, taking into account the long-wavelength fluctuations consists of hydrodynamic kind of equations and Boltzmann kinetic equations for the fluctuation distribution functions, which are related to each other. One must solve this system of equations taking into account the boundary conditions, for instance, the condition for diffusive reflection, which are satisfied by the distribution function at the boundaries of the liquid since concrete examples show[1,8] that the main role is usually played by fluctuations with a mean free path of the order of the characteristic length of the problem. It is important that then the wavelength of the fluctuations is small.

The complete set of equations can be reduced[9] to equations of a purely hydrodynamic type in the particular case when we are dealing with linearized equations in an unbounded liquid. In that case the fluctuation distribution functions can be evaluated in the general form and eliminated from the equations. As a result one obtains the equations of hydrodynamics taking the fluctuation correction terms into account,[9] which in the three-dimensional case can be expressed in terms of thermodynamic functions and the kinetic coefficients of the liquid and which are non-local and appreciably larger than the Barnett correction terms.

In the present paper a similar program is carried out for a two-dimensional liquid. In that case the contribution from the fluctuations is appreciably more important—all kinetic coefficients of the liquid are basically of a fluctuation character and can thus completely be expressed in terms of thermodynamic functions. The two-dimensional hydrodynamic equations, taking dissipation into account, turn out to be non-local which is connected with the logarithmic dependence of the kinetic coefficients on the frequency and the wavevector. A qualitatively similar conclusion follows, of course, (see Refs. 4, 5, 7, 10) from the presence of the above-mentioned formal (logarithmic) divergence of the kinetic coefficients. It is interesting to note that the condition that the expressions for the kinetic coefficients found below are positive imposes some restriction on the thermodynamic functions of a two-dimensional liquid.

The situation in a one-dimensional liquid turns out to be appreciably more complicated. In that case the essential role is played by fluctuations with a wavelength comparable to the characteristic length of the problem. A hydrodynamic formulation is, essentially, impossible as it is impossible to introduce quantities which are averaged over volumes with linear dimensions which are larger than the wavelengths of the fluctuations, but smaller than the characteristic length of the problem. Below we shall evaluate sound absorption in a one-dimensional liquid which turns out to be proportional to $\omega^{3/2}$ (ω is the sound frequency) i.e., which differs greatly from the hydrodynamic quadratic law.

In a previous paper[9] we omitted incorrectly from the kinetic equations for the shear and entropy waves several terms, which, if taken into account, change the coefficients in the final results. We give therefore in the Appendix of the present paper the corrected formulae for the correction terms to the hydrodynamic equations of a three-dimensional liquid and for the sound dispersion.

1. BASIC EQUATIONS

We shall start from the equations, obtained in a previous paper,[9] which express the time derivatives of the hydrodynamic quantities in terms of the deviations $\delta n(\mathbf{q})$, $\delta f_{\alpha\beta}(\mathbf{q})$, $\delta g(\mathbf{q})$ of the distribution functions of, respectively, the sound, shear, and entropy fluctuations from their equilibrium values:

$$\begin{gathered}\dot\rho+\rho\operatorname{div}\mathbf{v}=0,\\ \rho\dot v_i+\frac{\partial P}{\partial x_i}+\frac{\partial\pi_{ik}}{\partial x_k}=0,\\ \rho T\dot\sigma+\operatorname{div}\mathbf{Q}=0.\end{gathered}\tag{1}$$

Here

$$\begin{gathered}\pi_{ik}=\delta_{ik}c(\varphi-\psi)\int d\tau q\delta n-\delta_{ik}\frac{\rho\psi}{2}\int d\tau\,\delta f_{\alpha\alpha}\\ -\delta_{ik}\frac{T\rho}{2c_p}\theta\int d\tau\,\delta g+c\int d\tau\frac{q_iq_k}{q}\delta n+\rho\int d\tau l_{\alpha i}l_{\beta k}\delta f_{\alpha\beta},\\ \mathbf{Q}=c^2\int d\tau\mathbf{q}\delta n,\end{gathered}\tag{2}$$

ρ, P, and v are the density, pressure, and velocity of the liquid, T is the temperature, and σ the entropy per unit mass,

$$c^2=\left(\frac{\partial P}{\partial\rho}\right)_\sigma,\quad\varphi=\frac{\rho}{c}\left(\frac{\partial c}{\partial\rho}\right)_\sigma,\quad\psi=\frac{\rho}{T}\left(\frac{\partial T}{\partial\rho}\right)_\sigma,\quad\theta=\frac{\rho}{c_p}\left(\frac{\partial c_p}{\partial\rho}\right)_\sigma,$$

c_p is the heat capacity per unit mass at constant pressure, $d\tau=d^dq/(2\pi)^d$, and d is the dimensionality of the space.

The indices α, β number the directions of the polarization of the shear waves. In the three-dimensional case there are two directions of polarization determined by the mutually orthogonal unit vectors $\mathbf{l}_\alpha(\alpha=1,2)$ which lie in the plane perpendicular to the direction of the wavevector q and which satisfy the condition $l_{\alpha i}l_{\alpha k}=\delta_{ik}-q_iq_k/q^2$. In the two-dimensional case we can drop the indices α and β, since there is only one polarization direction determined by the vector $l_i=e_{ik}q_k/q$, where e_{ik} is the antisymmetric unit tensor.

The deviation $\delta n(\mathbf{q})$ of the sound fluctuation distribution function from its equilibrium value is determined by the formula[9]

$$\begin{gathered}\delta n(\mathbf{q})=-\frac{\rho T}{cq}\frac{1}{\gamma_sq^2-i\rho(\omega-c\mathbf{nk})}\left\{\frac{c}{T}\mathbf{n}\nabla T\right.\\ \left.+n_in_k\frac{\partial v_k}{\partial x_i}+(\varphi-\psi)\operatorname{div}\mathbf{v}\right\},\end{gathered}\tag{3}$$

where $\mathrm{n}=\mathbf{q}/q$, and ω, k are the frequency and wavevector of the hydrodynamic motion. The quantity γ_s is in the three-dimensional case equal to

$$\gamma_s={}^4/_3\eta+\zeta+\varkappa(1/c_v-1/c_p),$$

where η, ζ are the first and second viscosity coefficients, $\varkappa$ is the heat conductivity coefficient, and c_v is the heat capacity per unit mass at constant volume. In the two-dimensional case we have

$$\gamma_s\equiv\gamma_s(q)=\eta_s(q)+\zeta_s(q)+\varkappa_s(q)\,(1/c_v-1/c_p),$$

where $\eta_s(q)$, $\zeta_s(q)$, and $\varkappa_s(q)$ are the values of the coefficients of first ($\eta(\omega, q)$) and second ($\zeta(\omega, q)$) viscosity and the heat conductivity ($\varkappa(\omega, q)$) for $\omega = cq$.

The distribution functions $f_{\alpha\beta}(\mathbf{q})$ and $g(\mathbf{q})$ of the shear and entropy fluctuations satisfy the equations

$$\begin{gathered}
\dot{f}_{\alpha\beta}-\frac{T}{\rho}\delta_{\alpha\beta}\operatorname{div}\mathbf{v}+\frac{T}{\rho}l_{\alpha i}l_{\beta k}\left(\frac{\partial v_i}{\partial x_k}+\frac{\partial v_k}{\partial x_i}\right) \\
+\frac{2\eta_0 q^2}{\rho}\left(f_{\alpha\beta}-\delta_{\alpha\beta}\frac{T}{\rho}\right)=0, \\
\dot{g}-\frac{c_p}{\rho}\operatorname{div}\mathbf{v}+2\chi_0 q^2\left(g-\frac{c_p}{\rho}\right)=0,
\end{gathered} \tag{4}$$

where $\eta_0 = \eta(0, q)$, $\chi_0 = \varkappa(0, q)/\rho c_p$. In a previous paper[9] we erroneously omitted in Eqs. (4) the terms with an explicit dependence on the velocity gradients. These terms [the second and third in the first and the second in the second of Eqs. (4)] are obtained in the usual way (see Appendix to the paper by Meĭerovich and the present author[8]) from the non-linear terms $(\mathbf{v}\nabla)\mathbf{v}$ and $\mathbf{v}\nabla\sigma$ in the hydrodynamic equations for $\dot{\mathbf{v}}$ and $\dot{\sigma}$.

From Eqs. (4) we easily find the deviations

$$\begin{gathered}
\delta f_{\alpha\beta}=\frac{T}{2\eta_0 q^2-i\omega\rho}\left\{\delta_{\alpha\beta}\psi\operatorname{div}\mathbf{v}-l_{\alpha i}l_{\beta k}\left(\frac{\partial v_i}{\partial x_k}+\frac{\partial v_k}{\partial x_i}\right)\right\}, \\
\delta g=\frac{1}{2\chi_0 q^2-i\omega}\left(\frac{\partial c_p}{\partial\rho}\right)_\sigma\operatorname{div}\mathbf{v}
\end{gathered} \tag{5}$$

of the distribution functions of the shear and entropy waves from their equilibrium values which are, respectively, equal to $\delta_{\alpha\beta}T/\rho$ and c_p/ρ.

2. KINETIC COEFFICIENTS OF TWO-DIMENSIONAL LIQUIDS

Substitution of Eqs. (3) and (5) into Eqs. (2) for the dissipative momentum and energy fluxes gives in the two-dimensional case

$$\begin{gathered}
\pi_{ik}=-\eta(\omega,k)\left(\frac{\partial v_i}{\partial x_k}+\frac{\partial v_k}{\partial x_i}-\delta_{ik}\frac{\partial v_l}{\partial x_l}\right)-\zeta(\omega,k)\delta_{ik}\frac{\partial v_l}{\partial x_l}, \\
\mathbf{Q}=-\varkappa(\omega,k)\nabla T,
\end{gathered} \tag{6}$$

where the kinetic coefficients are defined by the formulae

$$\begin{gathered}
\eta(\omega,k)={}^1/_8\rho T(I_1+2I_2), \\
\zeta(\omega,k)=\rho T(\varphi-\psi+{}^1/_2)^2 I_1+{}^1/_2 T\theta^2 I_3+{}^1/_2\rho T(\psi-1)^2 I_2, \\
\varkappa(\omega,k)={}^1/_2\rho c^2 I_1,
\end{gathered} \tag{7}$$

in which

$$\begin{gathered}
I_1=\frac{1}{(2\pi)^2}\int\frac{d^2q}{\gamma_s(q)q^2-i\rho(\omega-c\mathbf{nk})}, \\
I_2=\frac{1}{(2\pi)^2}\int\frac{d^2q}{2\eta_0(q)q^2-i\omega\rho},\quad I_3=\frac{1}{(2\pi)^2}\int\frac{d^2q}{2\chi_0(q)q^2-i\omega}.
\end{gathered} \tag{8}$$

All integrals in (8) are logarithmic as γ_s, η_0, χ_0 depend only logarithmically on q, as we shall see below. To evaluate them with logarithmic accuracy we can neglect in the denominators of the integrands the terms with ω and $\mathbf{k}$. We must clearly in all integrals take as upper limit the quantity $q \sim 1/a$, where a is the interatomic distance. The lower limit of the integral I_1 must be that value of q for which the quantity $\gamma_s q^2$ equals $\rho(\omega - c\mathbf{n}\cdot\mathbf{k})$, i.e.,

$$qa\sim\max((ka)^{1/2},\ (\omega a/c)^{1/2}).$$

The analogous role in I_2 and I_3 is played by the value $qa \sim (\omega a/c)^{1/2}$, but it is necessary to remember that Eqs. (8) were obtained under the assumption that $q \gg k$. Therefore, the lower limit in I_2 and I_3 is, in fact, the value

$$qa\sim\max(ka,\ (\omega a/c)^{1/2}).$$

Introducing the logarithmic variables

$$x=-\ln ka,\quad y=-\ln\frac{\omega a}{c},\quad z=-\ln qa,$$

we get from (7) and (8) the following equations:

$$\begin{gathered}
\eta(x,y)=\frac{\rho T}{16\pi}\left\{\int_0^L\frac{dz}{\gamma_s(z)}+\int_0^\Lambda\frac{dz}{\eta_0(z)}\right\}, \\
\zeta(x,y)=\frac{\rho T}{2\pi}\left(\varphi-\psi+\frac{1}{2}\right)^2\int_0^L\frac{dz}{\gamma_s(z)}+\frac{\rho T}{8\pi}(\psi-1)^2\int_0^\Lambda\frac{dz}{\eta_0(z)} \\
+\frac{\rho T c_p}{8\pi}\theta^2\int_0^\Lambda\frac{dz}{\varkappa_0(z)}, \\
\varkappa(x,y)=\frac{\rho c^2}{4\pi}\int_0^L\frac{dz}{\gamma_s(z)},
\end{gathered} \tag{9}$$

where $L = \frac{1}{2}\min(x, y)$, $\Lambda = \min(x, y/2)$. Since

$$\begin{gathered}
\eta_0(x)=\eta(x,\infty),\quad \varkappa_0(x)=\varkappa(x,\infty), \\
\gamma_s(x)=\eta(x,x)+\zeta(x,x)+(T\psi^2/c^2)\varkappa(x,x),
\end{gathered}$$

Eqs. (9) form a closed set. Through simple transformations we get from them the following set of equations for the functions $\varkappa_0(x)$, $\eta_0(x)$, and $\gamma_s(x)$:

$$\begin{gathered}
\varkappa_0(x)=\frac{\rho c^2}{4\pi}\int_0^{x/2}\frac{dz}{\gamma_s(z)},\quad \eta_0(x)=\frac{\rho T}{16\pi}\left\{\int_0^{x/2}\frac{dz}{\gamma_s(z)}+\int_0^{x}\frac{dz}{\eta_0(z)}\right\} \\
\gamma_s(x)=\frac{\rho T}{16\pi}\left\{\left[1+8\left(\varphi-\psi+\frac{1}{2}\right)^2+4\psi^2\right]\int_0^{x/2}\frac{dz}{\gamma_s(z)}\right. \\
\left.+[1+2(\psi-1)^2]\int_0^{x/2}\frac{dz}{\eta_0(z)}+2c_p\theta^2\int_0^{x/2}\frac{dz}{\varkappa_0(z)}\right\}.
\end{gathered}$$

The solution has the following form

$$\begin{gathered}
\varkappa_0(x)=c^2u\left[\frac{\rho x}{\pi T(u+2)}\right]^{1/2},\quad \eta_0(x)=\left[\frac{\rho T}{16\pi}(u+2)x\right]^{1/2}, \\
\gamma_s(x)=\left[\frac{\rho T}{8\pi}\frac{u+2}{u^2}x\right]^{1/2},
\end{gathered}$$

where

$$\begin{gathered}
u=b^{-1}\{[a^2+4b(1-\bar\theta^2)]^{1/2}-a\}, \\
a=2(\psi-1)^2+\bar\theta^2,\quad b=\sqrt{2}\{1+8(\varphi-\psi+{}^1/_2)^2+4\psi^2\}, \\
\bar\theta=\left(\frac{T}{2c_p}\right)^{1/2}\frac{\rho}{c}\left(\frac{\partial c_p}{\partial\rho}\right)_\sigma.
\end{gathered}$$

After substitution into (9) we get the final expressions for the kinetic coefficients:

$$\begin{gathered}
\eta(x,y)=\left\{\frac{\rho TL}{8\pi(u+2)}\right\}^{1/2}\left\{u+\left(\frac{2\Lambda}{L}\right)^{1/2}\right\}, \\
\zeta(x,y)=\frac{1}{u}\left[\frac{\rho TL}{2\pi(u+2)}\right]^{1/2}\left\{4u^2\left(\varphi-\psi+\frac{1}{2}\right)^2+(au+2\bar\theta^2)\left(\frac{\Lambda}{2L}\right)^{1/2}\right\}, \\
\varkappa(x,y)=uc^2\left[\frac{2\rho L}{\pi T(u+2)}\right]^{1/2}.
\end{gathered} \tag{10}$$

All kinetic coefficients increase as $\omega, k \to 0$ in proportion to $\ln^{1/2}\omega$ or $\ln^{1/2}k$. This justifies the original assumption that the main contribution to them comes from the long-wavelength fluctuations.

There follows from Eq. (10) an interesting conclusion that in order that the kinetic coefficients are positive it is necessary that the thermodynamic inequality $\bar{\theta}^2 < 1$ or

$$\left(\frac{\partial c_p}{\partial \rho}\right)_0^2 < \frac{2c^2}{T\rho^2} c_p.$$

This condition is always satisfied in a rarefied system (in a gas), where c_p is independent of the density. However, in the general case there may exist in the phase diagram of a two-dimensional liquid a peculiar singular line on which $\bar{\theta}^2 = 1$ and such that when one approaches it, $\zeta \to \infty$, $\varkappa \to 0$, while $\eta = \text{const.}$, as can be seen from (10).

3. SOUND ABSORPTION IN A ONE-DIMENSIONAL LIQUID

In the one-dimensional case there exist altogether two kinds of long-wavelength fluctuations—sound waves and entropy waves. There are no shear waves. However, a more important characteristic is the well known resonance character of the interaction between sound waves which propagate in one of two possible directions in that case the conditions for conservation of energy and of momentum are identically the same. Below we shall evaluate the sound absorption coefficient and we show that due to the resonance condition the main contribution to the absorption comes from thermal sound fluctuations with a wavelength of the order of the sound wavelength. The contribution from the entropy waves can be neglected as there is no resonance.

Sound absorption is determined properly by the phonon energy function

$$\Sigma(\omega, k) = G_R^{-1}(\omega, k) + \omega_k - \omega.$$

Here $\omega_k = c|k|$, $G_R(\omega, k)$ is the retarded Green function defined in the usual way in terms of the complex operator of the sound field $\Sigma_k a_k e^{ikr}$, where a_k are the phonon annihilation operators. The use of the function G_R instead of the usual D function is here more convenient as the well known hydrodynamic operator (see Ref. 11) of the phonon-phonon interaction cannot be expressed directly in terms of the real phonon field.

As the interaction between long-wavelength phonons is weak, the main contribution to Σ comes in the Matsubara technique from the diagrams shown in the figure in which the bare hydrodynamic vertex occurs (see Ref. 11)

$$V(k_1, k_2, k_3) = \left(\frac{\omega_{k_1}\omega_{k_2}\omega_{k_3}}{4\rho c^2}\right)^{1/2} \left\{\varphi + \frac{1}{2}\left(\frac{k_1k_2}{|k_1k_2|} + \frac{k_1k_3}{|k_1k_3|} + \frac{k_2k_3}{|k_2k_3|} - 1\right)\right\}$$

and the exact Green functions. After analytical continuation with imaginary frequencies we get

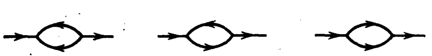

FIG. 1.

$$\Sigma(\omega, k) = \int_{-\infty}^{+\infty} \frac{dq}{2\pi} \int_{-\infty}^{+\infty} dx \{V^2(k, q, -k-q)[(1+n(x))\rho(-k-q, x)$$
$$\times G_A(-\omega-x, q) + n(x)\rho(q, x) G_A(-\omega-x, -k-q)]$$
$$+2V^2(k, q, k+q) n(x)[\rho(q, x) G_R(\omega+x, k+q) + \rho(k+q, x) G_A(x-\omega, q)]$$
$$+V^2(k, q, k-q) n(x)[\rho(k-q, x) G_R(\omega-x, q) + \rho(q, x) G_R(\omega-x, k-q)]\},$$

where $\rho(k, \omega) = (2\pi i)^{-1}\{G_R(\omega, k) - G_A(\omega, k)\}$ is the spectral function (see Ref. 12), G_A is the advanced Green function, and $n(x) = (e^{x/T} - 1)^{-1}$.

It will be clear from the result that the quantity $\Sigma_k = \Sigma(ck, k)$ is appreciably smaller than ω_k. In that case the main contribution to the integral over x comes from the poles of the Green functions in the integrand which lie close to the real axis. As a result we get

$$\Sigma_k = -T \int_{-\infty}^{+\infty} \frac{dq}{2\pi} \left\{ V^2(k, q, -k-q) \left[\frac{1}{\omega_{k+q}} G_A(-\omega_k - \omega_{k+q} + \Sigma_{k+q}^{\bullet}, q) \right.\right.$$
$$\left. + \frac{1}{\omega_q} G_A(-\omega_k - \omega_q + \Sigma_q^{\bullet}, -k-q) \right] + 2V^2(k, q, k+q)$$
$$\times \left[\frac{1}{\omega_q} G_R(\omega_k + \omega_q - \Sigma_q^{\bullet}, k+q) + \frac{1}{\omega_{k+q}} G_A(-\omega_k + \omega_{k+q} - \Sigma_{k+q}, q) \right]$$
$$\left. + V^2(k, q, k-q) \left[\frac{1}{\omega_{k-q}} G_R(\omega_k - \omega_{k-q} + \Sigma_{k-q}, q) + \frac{1}{\omega_q} G_R(\omega_k - \omega_q + \Sigma_q, k-q) \right] \right\},$$

where we have used the fact that in the case considered of a classical liquid we must assume that the condition $n(x) = T/x \gg 1$ is satisfied.

In the integral over q the main contribution also comes from the pole terms, especially those of them which have a resonance nature, i.e., which tend to infinity at $\Sigma = 0$. If, to fix the ideas, we assume that $k > 0$, such pole terms give terms with $V(k, q, k+q)$ when $q > 0$ and terms with $V(k, q, k-q)$ when $0 < q < k$. After simple transformations we get

$$\Sigma_k = -\frac{T}{2\rho c^2}(\varphi+1)^2\omega_k^2 \left\{ \int_0^{\infty} \frac{dq}{2\pi} \frac{1}{\Sigma_{k+q} - \Sigma_q^{\bullet}} + \frac{1}{2} \int_0^{k} \frac{dq}{2\pi} \frac{1}{\Sigma_q + \Sigma_{k-q}} \right\}. \quad (11)$$

Separating the real and imaginary parts in (11) we easily check that Σ_k is pure imaginary so that the damping is much larger than the dispersion.

We can write the solution of Eq. (11) in the form $\Sigma_k = i\Gamma(k)$ where the sound absorption coefficient $\Gamma(k)$ is equal to

$$\Gamma(k) = k^{3/2}|\varphi+1| \left\{ \frac{T}{4\pi\rho} \left(\int_0^{\infty} \frac{dx}{x^{3/2} + (1+x)^{3/2}} \right.\right.$$
$$\left.\left. + \frac{1}{2} \int_0^{1} \frac{dx}{x^{3/2} + (1-x)^{3/2}} \right) \right\}^{1/2} = 0{,}394|\varphi+1| \left(\frac{T}{\rho}\right)^{1/2} k^{3/2}. \quad (12)$$

One easily checks from Eqs. (11) and (12) that, indeed, the main contribution to the sound absorption comes from the interaction with thermal sound fluctuations with wavelengths of the order of the sound wavelength.

I express my gratitude to I. M. Lifshitz for a useful discussion.

APPENDIX

If we use instead of Eqs. (10) and (11) of Ref. 9 Eqs. (5) the expression for the corrections $\delta\eta_{iklm}$ to the viscosity tensor is changed. In the notation of Ref. 9 we have

$$\delta\eta_{iklm}(\omega,\mathbf{k})=\frac{T}{6\pi}\left(\frac{\rho}{2\gamma}\right)^{3/2}(i-1)\,\omega^{1/2}\Big\{\frac{k_ik_kk_lk_m}{k^4}(3F_1-15F_2+35F_3)$$
$$+\frac{1}{k^2}(\delta_{ik}k_lk_m+\delta_{lm}k_ik_k)\left[-F_1+3F_2-5F_3+(\varphi-\psi)(-4F_1+6F_2)\right]$$
$$+\frac{1}{k^2}(\delta_{il}k_kk_m+\delta_{im}k_kk_l+\delta_{kl}k_ik_m+\delta_{km}k_ik_l)(-F_1+3F_2-5F_3) \tag{p}$$
$$+(\delta_{il}\delta_{km}+\delta_{im}\delta_{kl})\left[F_1-F_2+F_3+\frac{7}{40}\left(\frac{2\gamma}{\eta}\right)^{3/2}\right]$$
$$+\delta_{ik}\delta_{lm}\Big[F_1-F_2+F_3+4(\varphi-\psi)(2F_1-F_2)+8(\varphi-\psi)^2F_1$$
$$+\frac{3}{2}\theta^2\left(\frac{\gamma c_p}{2\varkappa}\right)^{3/2}+3\left(\frac{\gamma}{2\eta}\right)^{3/2}\left[\left(\psi-\frac{2}{3}\right)^2-\frac{14}{45}\right)\Big]\Big\}.$$

The phase velocity of the sound $c(\omega)$ and its absorption $\Gamma(\omega)$ are given by the formulae

$$c(\omega)=c\left\{1+\frac{T}{12\pi\rho c^2}\left(\frac{\rho\omega}{\gamma}\right)^{3/2}\Phi\right\},$$
$$\Gamma(\omega)=\frac{\gamma\omega^2}{2\rho c^2}\left\{1-\frac{\rho T}{6\pi\gamma^2}\left(\frac{\rho\omega}{\gamma}\right)^{1/2}\Phi\right\}, \tag{q}$$

where

$$\Phi=\frac{1024}{14553}+\frac{48}{175}\left(\psi-\frac{2}{9}\right)^2+\left(\varphi-\frac{6}{5}\psi+\frac{11}{35}\right)^2$$
$$+\frac{3}{2}\theta^2\left(\frac{\gamma c_p}{4\varkappa}\right)^{3/2}+3\left(\frac{\gamma}{4\eta}\right)^{3/2}\left[\left(\psi-\frac{2}{3}\right)^2+\frac{28}{45}\right]. \tag{r}$$

As Φ is positive it follows from this that in any liquid one should find at sufficiently low frequencies a positive sound dispersion.

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Interaction of moving dislocation with a soft phonon mode in a displacement-type phase transition

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We consider the anomalies of dynamic slowing down of dislocations as a result of their interaction with the soft phonon mode in a displacement-type phase transition. It is shown that when the phase transition temperature T_c is approached and the corresponding critical frequency ω_c decreases, the dislocation dragging coefficient B_c increases in proportion to $\ln(\omega_s/\omega_c)$, where ω_s is of the order of the Debye frequency. The possibilities of observing in experiment the predicted anomalies in the temperature dependence of the viscous component of dislocation friction is discussed.

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It is known[1,2] that in phase transitions of the displacement type there appear in many crystals the so-called phonon modes, which are characterized by a dip in the dispersion law $\omega_k=\omega(\mathbf{k})$ in the vicinity of a certain wave vector[1)] $\mathbf{k}=\mathbf{k}_0$. It is important that as the phase-transition temperature T_c is approached the depth of this dip increases, and the corresponding frequency $\omega_c(T)=\omega(\mathbf{k}_0)$ tends to zero as $T\to T_c$. The width of the energy level of the soft mode (T)—the reciprocal phonon relaxation time—increases and near T_c it can even exceed the frequency ω_c.[2] Recognizing that the lattice-anharmonicity-induced scattering of a phonon by a dislocation from a state with wave vector $\mathbf{k}$ and frequency ω_k into a state $\{\mathbf{k}',\omega_{k'}\}$ is characterized by a matrix element $\Gamma_{kk'}\propto(\omega_k\omega_{k'})^{-1/2}$, and also the fact that near the transition the density of the critical phonons increases in proportion to ω_c^{-1}, it is natural to expect the intensity of the phonon-dislocation interaction to increase in the vicinity of the point $\mathbf{k}=\mathbf{k}_0$ of reciprocal space near the transition temperature T_c. This can manifest itself in integral fashion in the appearance of singularities of the phonon slowing down of the dislocations at $T\approx T_c$. We attempt below to investigate the character of these singularities within the framework of a simple model.

Ultrasound damping anomalies of similar origin were investigated earlier in a number of studies (see, e.g., the review by Garland[3]). Unfortunately, these results

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