Coexistence of magnetism and superconductivity in the two-band model

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It is shown that superconductivity and ferromagnetism can coexist in systems that are unstable to restructuring of the single-particle electron spectrum with formation of a Bose condensate of triplet electron-hole pairs (spin wave density) and containing localized moments. The ordering of the localized magnetic moments is due to both intraband and interband RKKY interactions. The latter can be treated as exchange of triplet electron-hole pairs. In this system the coexistence is possible even at arbitrarily weak exchange scattering. The Curie temperature in this system is higher than the superconducting-transition temperature.

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INTRODUCTION

The coexistence of superconductivity and ferromagnetism has been the subject of many experimental and theoretical studies. Within a phenomenological framework, this question was first discussed by Ginzburg[1] and Zharkov.[2] They have shown that in typical ferromagnets, owing to the large internal fields (compared with the critical fields of superconductors) no superconductivity can appear in a ferromagnetic phase.

This question was considered in a microscopic model by Gor'kov and Rusinov.[3] They have shown that in superconductors with sufficiently low magnetic-impurity concentration there can appear a narrow region of mixed phase. The impurity becomes magnetized because of exchange interaction with conduction electrons. The region of existence of the mixed phase is narrow because the Cooper pairs are produced in the superconductor in the singlet state with a finite binding energy. Therefore at $T=0$ (disregarding scattering) the exchange interaction of the electrons becomes impossible. On the basis of this fact, Baltensperger[4] has concluded that realization of a mixed phase is impossible. However, according to Gor'kov and Rusinov,[3] allowance for exchange scattering and spin-orbit scattering by a nonmagnetic impurity leads to a nonzero paramagnetic susceptibility of the superconductor even at $T=0$. Consequently exchange interaction between the localized moments via the conduction electrons is possible.

Upon appearance of ferromagnetic ordering, the exchange interaction moves apart the Fermi surfaces of electrons with different spin projections, and this effect appears in first order in the exchange-interaction constant.

At the same time, scattering by a paramagnetic impurity (which leads to a nonzero paramagnetic susceptibility of the electrons in the superconductor) is an effect of second order in J. The spin-orbit scattering intermingles the Fermi surfaces of electrons having different spin orientation, and decreases the moving-apart effect of the exchange interaction.[3] We note that the narrowness of the coexistence region is actually due to the fact that the effect of the magnetic ordering of the paramagnetic impurities in a superconductor is the consequence of scattering effects that are quadratic in J. The suppression of the superconductivity by the exchange magnetization of the electrons is linear in J.

Andreĭchenko and Burmistrov[5] treated the question of the coexistence of superconductivity and ferromagnetism within the framework of a phenomenological scheme as applied to ternary superconducting compounds that exhibit magnetic ordering.[6] They call attention to the fact that in transition metals the effective scattering time in the presence of several (s and d) bands increases if there is no exchange interaction with electrons of the d band. This increases the critical concentration of the magnetic impurity and facilitates the magnetic ordering in the superconductor. The appearance of an inhomogeneous magnetic structure decreases the influence of moving-apart effect. The question of the coexistence of superconductivity and ferromagnetism with helicoidal structure was discussed by Blount and Varma, as well as by Bulaevskiĭ, Kulich, and Rusinov.[7]

In this paper we demonstrate the possibility of coexistence of ferromagnetism and superconductivity in systems with an electron spectrum of special form. The coexistence turns out to be possible within the framework of the assumed model even at arbitrarily weak exchange scattering. Therefore in contrast to other models, the appearance of superconductivity in the ferromagnetic phase is possible.

Volkov and Mnatsakanov[8] have shown that in systems that are unstable to formation of a Bose condensate of electron-hole pairs in the triplet state (spin density waves, SDW), it is possible to have, besides the usual Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction[9] between the localized magnetic moments, also exchange via triplet electron-hole pairs. This leads to magnetization of the localized moments, i.e., to ferromagnetism. In their opinion this mechanism is apparently responsible for a number of magnetic pulse transitions in R_3Al_2 compounds (R is a rare-earth element).[10]

We shall study below the behavior of magnetic substitutional impurities in systems that are unstable to formation of SDW and to Cooper pairing. The coexis-

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tence of a singlet superconducting and a dielectric pairing within the framework of a model of a semimetal with almost coinciding electron and hole Fermi surfaces at $T=0$ was investigated in Ref. 11. It can be shown that the equations describing the coexistence of SDW, i.e., of antiferromagnetism and singlet Cooper pairing (and the absence of localized moments) reduces to the same system of equations as in Ref. 11. When magnetic impurities are introduced in such a system they become magnetized via the SDW, i.e., ferromagnetism appears.

The suppression of superconductivity by the moving-apart effect increases with increasing intraband exchange-interaction constant, which we shall assume to be arbitrarily small. In the scheme proposed in Ref. 3, coexistence is possible only at $T_{C0} \lesssim T_{c0}$ (T_{C0} and T_{c0} are the Curie and superconducting-transition temperatures), with $\tau_{cx}\Delta_0 \gtrsim 1$ (τ_{ex} is the exchange-scattering time). But it is precisely in this region that a strong suppression of the superconductivity takes place because of the exchange scattering, and when magnetic order appears there is also magnetization of the electrons, and this effect is linear in J.

The conditions for the coexistence of ferromagnetism and superconductivity in the single-band scheme are therefore very stringent.

HAMILTONIAN AND FUNDAMENTAL EQUATIONS

We consider a model of a doped semimetal that is unstable to electron-hole and Cooper pairing. We write the Hamiltonian of the system in the form

$$H=H_0+H_{int},$$
$$H_0=\sum_{i,\mathbf{p},\alpha}\varepsilon_i(\mathbf{p})a_{i\alpha}^+(\mathbf{p})a_{i\alpha}(\mathbf{p}), \tag{1}$$

where H_0 is the Hamiltonian of the free electrons in the first and second zones; the electron spectrum is assumed for simplicity to be isotropic.

$$\varepsilon_{1,2}(\mathbf{p})=-\mu\pm(p^2/2m-\varepsilon_F),$$

where μ is the shift of the Fermi surface by the doping;

$$H_{int}=\sum_{\substack{\mathbf{p},\mathbf{p}',\mathbf{q}\\ \alpha,\beta}}\{V_1(\mathbf{q})a_{1\alpha}^+(\mathbf{p}+\mathbf{q})a_{2\beta}^+(\mathbf{p}'-\mathbf{q})a_{2\beta}(\mathbf{p}')a_{1\alpha}(\mathbf{p})$$
$$+V_2(\mathbf{q})[a_{1\alpha}^+(\mathbf{p}+\mathbf{q})a_{2\beta}^+(\mathbf{p}'-\mathbf{q})a_{1\beta}(\mathbf{p}')a_{2\alpha}(\mathbf{p})$$
$$+a_{1\alpha}^+(\mathbf{p}+\mathbf{q})a_{1\beta}^+(\mathbf{p}'-\mathbf{q})a_{2\beta}(\mathbf{p}')a_{2\alpha}(\mathbf{p})+\text{э. c.}]\}$$
$$+\sum_{i,\mathbf{p},\mathbf{p}'}V_c(\mathbf{p},\mathbf{p}')a_{i\alpha}^+(\mathbf{p})\overset{+}{a}_{i-\alpha}(-\mathbf{p})a_{i\alpha}(\mathbf{p}')a_{i-\alpha}(-\mathbf{p}') \tag{2}$$

is the Hamiltonian of the electron-electron interaction, in which we have retained only the terms responsible for the exciton and Cooper instability within the limits of each band.

As usual, we shall assume the high-density approximation $na_B^3 \gg 1$ (n is the carrier density in the semimetal and a_B is the Bohr radius of the electron). In this limit the Fourier components of the potentials $V_{1,2}(\mathbf{q})$ can be replaced respectively by the constants $g_{1,2}$. This replacement is possible because $V_1(\mathbf{q})$ and $V_2(\mathbf{q})$ are short-range, the first because of screening and the second because of the orthogonality of the Bloch wave functions from different bands. As usual, we replace also $V_c(\mathbf{p},\mathbf{p}')$ by the constant g_c. Without loss of generality, the constants $g_{1,2,c}$ can be regarded as real.[11]

The interaction of the electrons with the localized moments of the impurity will be written in the form

$$H_1=\sum_{\substack{i,j,\mathbf{p},\mathbf{q}\\ \alpha,\beta}}U_{ij}^{\alpha\beta}(\mathbf{q})a_{i\alpha}^+(\mathbf{p})a_{j\beta}(\mathbf{p}+\mathbf{q}),$$
$$U_{ij}^{\alpha\beta}(\mathbf{q})=-\int\sum_a J_{ij}(\mathbf{r}-\mathbf{r}_a)\sigma_{\alpha\beta}\mathbf{S}_a\varphi_{i\mathbf{p}}^*(\mathbf{r})\varphi_{j\mathbf{p}+\mathbf{q}}(\mathbf{r})\,d\mathbf{r}, \tag{3}$$

where $\varphi_{i\mathbf{p}}(\mathbf{r})$ are the Bloch wave functions of the i-band, and $\mathbf{S}_a$ is the moment of the a-th impurity atom.

Magnetic ordering gives rise in the system of localized moments to a nonzero mean value $\mathbf{S}=\langle\mathbf{S}_a\rangle$. After averaging over the positions of the impurity atoms[12] we have

$$H_1=-\sum_{\substack{i,j,\mathbf{p}\\ \alpha,\beta}}J_{ij}(\sigma_{\alpha\beta}\mathbf{S})a_{i\alpha}^+(\mathbf{p})a_{j\beta}(\mathbf{p}),$$
$$J_{ij}=N_{im}\int J_{ij}(\mathbf{r})\varphi_{i\mathbf{p}}^*(\mathbf{r})\varphi_{j\mathbf{p}}(\mathbf{r})\,d\mathbf{r}, \tag{4}$$

where N_{im} is the total number of atoms with localized moments. We assume next that the relation between the coupling constants g_1 and g_2 for electron-hole pairing is such that an excitonic dielectric phase with triplet pairing is realized, and the temperature of the dielectric transition is higher than the temperature T of the superconducting pairing (T_{c0} is the temperature of the transition to the superconducting state in the absence of other interactions).

As shown by Volkov and Mnatsakanov,[8] the transition to the excitonic-dielectric phase with triplet order parameter (at a nonzero interband exchange-interaction constant and at electron and hole band extrema that coincide in momentum space) is the necessary and sufficient condition for ferromagnetic ordering of the localized moments.

If the electron and hole Fermi surfaces are not fully congruent, the excess carriers remain above the gap after the dielectric transitions. At certain conditions a superconducting pairing is also possible. We shall investigate below the stability of a doped ferromagnetic excitonic dielectric with a triplet order parameter with respect to Cooper pairing. We assume that the characteristic cutoff energy for the superconducting pairing satisfies the condition $\varepsilon_F \gg \omega_D \gg \Delta$ (ε_F, ω_D, Δ are respectively the Fermi energy of the semimetal, the Debye frequency, and the dielectric gap). In this limiting case the carriers of the completely filled valence band also take part in the Cooper pairing.

In view of the great computational difficulties, we are forced to confine ourselves to an investigation of the stability of the ferromagnetic excitonic phase to Cooper pairing. The complete phase diagram cannot be constructed even for $T=0$.

It is convenient to solve the problem by the method of temperature Green's functions.[12] We consider first the transition from a semimetal in a state with SDW. We assume that the extrema of the electron and hole bands coincide in momentum space. The period of the resultant SDW is then equal to the period of the initial lattice.

The equations of motion for the Green's functions take, after Fourier transformation with respect to imaginary time, the form

$$(i\omega-\varepsilon_1(\mathbf{p})\pm I_a)G_{11}^{\pm}(\omega,\mathbf{p})\pm(\Delta+I_b)G_{21}^{\pm}(\omega,\mathbf{p})=1,$$
$$(i\omega-\varepsilon_2(\mathbf{p})\pm I_a)G_{21}^{\pm}(\omega,\mathbf{p})\pm(\Delta+I_b)G_{11}^{\pm}(\omega,\mathbf{p})=0, \quad (5)$$

where ± corresponds to the up and down spin projections on the z axis,

$$I_a=J_aS,\quad I_b=J_bS;$$
$$J_{11}=J_{22}=J_a,\quad J_{12}=J_{21}=J_b.$$

The quantities in (5) are determined in a self-consistent manner. The self-consistency equation for the triplet order parameter is of the usual form

$$\Delta=\frac{1}{2}g_tT\sum_\omega\int \mathrm{Sp}\,\hat{\sigma}G_{21}(\omega,\mathbf{p})\frac{d\mathbf{p}}{(2\pi)^3}, \quad (6)$$

where $g_t=g_1+g_2$ is the coupling constant responsible for the triplet electron-hole pairing. The expression for the average magnetization of the localized moments takes according to Ref. 3 the form

$$\mathbf{S}=\mathrm{Sp}\,[\hat{\mathbf{S}}\exp(-E_{int}(\hat{\mathbf{S}})/T)]/\mathrm{Sp}\,[\exp(-E_{int}(\hat{\mathbf{S}})/T)]. \quad (7)$$

The interaction energy of the localized moments of the atoms with the magnetized electrons is obtained by averaging the Hamiltonian H_1 in (4) over the state with SDW. We have

$$E_{int}=\langle H_1\rangle=-\mathrm{Sp}\left\{T\sum_{\omega,i,j}\sigma_{\alpha\beta}\mathbf{I}_{ij}\int G_{ij}^{\alpha\beta}(\omega,\mathbf{p})\frac{d\mathbf{p}}{(2\pi)^3}\right\}. \quad (8)$$

After the calculation we obtain for (6) the following result:

$$\Delta\ln\left(\frac{T_{D0}}{T}\right)+\frac{1}{2}\int_0^\infty\frac{\Delta+I_b}{E}\{n(-E-\mu-I_a)-n(E-\mu-I_a)$$
$$+n(-E-\mu+I_a)-n(E-\mu+I_a)\}d\varepsilon-\Delta\int_0^\infty\frac{n(-\varepsilon)-n(\varepsilon)}{\varepsilon}d\varepsilon=0, \quad (9)$$

where

$$E(\mathbf{p})=(\varepsilon^2(\mathbf{p})+(\Delta+I_b)^2)^{1/2},\quad n(x)=(e^{x/T}+1)^{-1},$$
$$S=LB_L\left(\frac{LJ_as_a+LJ_bs_b}{T}\right),\quad B_L(x)=\frac{2L+1}{2L}\,\mathrm{cth}\left(\frac{2L+1}{2L}x\right)-\frac{1}{2L}\,\mathrm{cth}\left(\frac{x}{2L}\right). \quad (10)$$

Here L is the localized moment, T_{D0} is the temperature of the transition into the state of an undoped excitonic dielectric in the absence of magnetic impurities. The quantities s_a and s_b in (10) have the following meaning: s_a determines the polarization of the spins of the quasiparticles located above the gap on account of doping and thermal excitation, and s_b characterizes the polarization of the spins of the electron-hole pairs. It is seen from (10) that the interaction with the triplet electron-hole pairs also contributes to the polarization of the local moments. The quantities s_a and s_b are given by

$$s_a=2N(0)\int_0^\infty d\varepsilon\{n(E-\mu-I_a)-n(E-\mu+I_a)+n(-E-\mu-I_a)-n(-E-\mu+I_a)\}, \quad (11)$$

$$s_b=N(0)\int_0^{\varepsilon_F}\frac{\Delta+I_b}{E}\{n(-E-\mu-I_a)-n(E-\mu-I_a)$$
$$+n(-E-\mu+I_a)-n(E-\mu+I_a)\}d\varepsilon, \quad (12)$$

where $N(0)$ is the state density on the Fermi level.

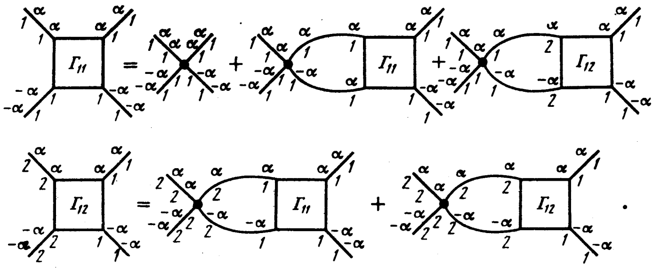
We assume in addition that the difference between the electron and hole densities in the system is given. Therefore Eqs. (9)–(12) must be supplemented by the electroneutrality condition

$$2n=\int_0^\infty d\varepsilon\{n(E-\mu-I_a)+n(-E-\mu-I_a)+n(E-\mu+I_a)+n(-E-\mu+I_a)-2\}, \quad (13)$$

where $N=4N(0)n$ is the difference between the electron and hole densities.

We proceed now to consider the possibility of superconducting pairing in the ferromagnetic excitonic dielectric phase. An investigation of the stability of such a system to Cooper pairing of the electrons within the limits of each reduces to a system of equations for the vertex functions $\Gamma_{11}(q)$ and $\Gamma_{12}(q)$ at a small total momentum.

The system of equations for the vertex functions takes at $q=0$ the form



The solution of this system at $q=0$ is

$$\Gamma_{11}=\frac{g_c(1-g_c\Pi_{22})}{\det},\quad \Gamma_{12}=\frac{g_c^2\Pi_{21}}{\det}$$
$$\det=(1-g_c\Pi_{11})(1-g_c\Pi_{22})-g_c^2\Pi_{12}\Pi_{21}. \quad (14)$$

The polarization operators Π_{11} and Π_{12} are defined as follows:

$$\Pi_{11}(q)=T\sum_\omega\int G_{11}^{\alpha\alpha}(q-p)G_{11}^{-\alpha-\alpha}(p)\frac{d\mathbf{p}}{(2\pi)^3},$$
$$\Pi_{12}(q)=T\sum_\omega\int G_{12}^{\alpha\alpha}(q-p)G_{12}^{-\alpha-\alpha}(p)\frac{d\mathbf{p}}{(2\pi)^3}, \quad (15)$$
$$q=(\mathbf{q},\omega_0),\quad p=(\mathbf{p},\omega).$$

The Green's functions contained in the polarization operators satisfy the system (5).

The superconducting transition temperature is determined from the condition that the vertex function become infinite, i.e., from the condition det = 0. We note that det is a quadratic function of the coupling constant g_c for the Cooper pairing. Therefore the equation det = 0 has, generally speaking, two solutions for T_c. To clarify this situation, we write down the equations for the superconducting order parameters in bands 1 and 2. At $T=T_c$ we have

$$\Delta_{11}=g_c\Pi_{11}\Delta_{11}+g_c\Pi_{12}\Delta_{22},$$
$$\Delta_{22}=g_c\Pi_{22}\Delta_{22}+g_c\Pi_{21}\Delta_{11}. \quad (16)$$

In analogy with Ref. 13, it can be shown that the system (16) has two solutions: $\Delta_{11}=\pm\Delta_{22}$. We shall call the solution $\Delta_{11}=\Delta_{22}$ symmetrical and $\Delta_{11}=-\Delta_{22}$ asymmetrical. We have respectively for these cases the following equations for the superconducting transition temperature:

1) $\ln(T/T_{c0})-(J_1+(\mu-(\Delta+I_b)^2/\mu)J_2)=0,$ (17)

2) $\ln(T/T_{c0})-(J_1+\mu J_2)=0,$ (18)

where the quantities $J_{1,2}$ are defined as follows:

$$J_1=\frac{1}{2}\int_0^\infty d\varepsilon\left\{\frac{1}{E}[n(-E-\mu-I_a)-n(E+\mu-I_a)+n(-E+\mu-I_a)-n(E-\mu-I_a)]-\frac{2n(-\varepsilon)-2n(\varepsilon)}{\varepsilon}\right\}, \quad (19)$$

$$J_2=\frac{1}{2}\int_0^\infty\frac{d\varepsilon}{E}\left\{\frac{n(-E+\mu-I_a)-n(E-\mu-I_a)}{E-\mu}-\frac{n(-E-\mu-I_a)-n(E+\mu-I_a)}{E+\mu}\right\}. \quad (20)$$

We note that at $J_a=J_b=0$ Eqs. (17) and (18) go over into the relations obtained in a study[11] of the coexistence of singlet Cooper and electron-hole pairings. The symmetrical ($\Delta_{11}=\Delta_{22}$) and the asymmetrical ($\Delta_{11}=-\Delta_{22}$) cases then change places. The reason is that in the case of triplet exciton pairing the dielectric order parameter satisfies the relation $\Delta_{\uparrow\downarrow}=-\Delta_{\downarrow\downarrow}$ (in the z representation in which we work), whereas in the singlet pairing it satisfies the relation $\Delta_{\uparrow\uparrow}=\Delta_{\downarrow\downarrow}$.

We shall investigate below jointly the self-consistency equations (9), (11–13), and (17), (18) by numerical methods.

RESULTS OF NUMERICAL CALCULATIONS

The large number of parameters ($T_{c0}, T_{D0}, J_a, J_b, n$) makes the construction of the complete phase diagram impossible. We confine ourselves therefore to a determination of the dependences of the superconducting transition temperature in a ferromagnetic excitonic dielectric on the other parameters of the system. To determine T_c we solved first numerically Eqs. (9) and (11)–(13). These yielded the temperature dependences of $\Delta(T)$, $\mu(T)$, $I_a(T)$, $I_b(T)$, which were then used to solve Eqs. (17) and (18) for T_c.

Figure 1 shows plots of T_c against the bare temperature T_{c0} for different doping levels. Figure 1a pertains to the antisymmetrical case, and the values of the parameters n/T_{D0} for curves 1–7 are: 0.1, 0.2, 0.3, 0.4, 0.5, 0.7, and 0.9. Curve 8 shows the temperature dependence of the dielectric transition for the doping level $n/T_{D0}=1.1$. Figure 1b corresponds to the symmetrical case, and for curves 1–9 the degree of doping n/T_{D0} is respectively 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, and 1.5. Line 10 yields the temperature of the dielectric transition at a doping $n/T_{D0}=9$. The values of J_a, J_b, L are the same for the curves of Figs. 1a and 1b and are equal to $N(0)J_a^2/T_{D0}=0.25$; $N(0)J_b^2/T_{D0}=0.04$; $L=\frac{1}{2}$.

Inasmuch as in this scheme the contribution to the magnetization of the localized moments is made (in addition to the particles above the gap) also by the triplet electron-hole pairs, the restrictions on the lower bound of J_a are not so stringent (J_a can literally be arbitrarily small). Therefore, at sufficiently low J_a the appearance of superconductivity in the ferromagnetic phase is possible.

However, in the presence of dielectric pairing there is another mechanism that impedes the superconductivity. In the dielectric transition, a dielectric gap is produced on the Fermi level and leads to a decrease of the energy interval for the superconducting pairing. This circumstance should lead to a decrease of T_c in the exciton phase, but the state density on the Fermi level increases in the dielectric phase, and this should lead to an increase of T_c. As shown in Ref. 11, there exist optimal conditions at which T_c exceeds the value in the absence of dielectric pairing. For the system parameters chosen by us, T_c in the ferromagnetic phase does not exceed T_{c0}, as seen from Fig. 1.

With increasing doping, the condition for the nucleation of the superconductivity become more favorable, and for the symmetrical case ($\Delta_{11}=\Delta_{22}$) the suppression of the superconductivity with decreasing doping is stronger than for the antisymmetrical case ($\Delta_{11}=-\Delta_{22}$). This is easily seen from a comparison of Figs. 1a and 1b. The formal reason is that the second term with Π_{12} has opposite signs in Eq. (16) for T_c in the symmetrical and antisymmetrical cases, and it is this which decreases the effective coupling constant in the symmetrical case.

Figure 2 shows plots of $T_c(T_{c0})$ for the antisymmetrical and symmetrical cases at various doping levels. The values of J_a, J_b, and L, which are the same for all the curves of Fig. 2, are $N(0)J_a^2/T_{D0}=0.25$, $N(0)J_b^2/T_{D0}=0.16$, and $L=\frac{1}{2}$. The doping n/T_{D0} for curves 1–6 of Fig. 2a are 0.1, 0.3, 0.5, 0.7, 0.9, and 1.3. Line 7 shows the temperature of the dielectric state at a doping $n/T_{D0}=1.3$. The degrees of doping n/T_{D0} for curves

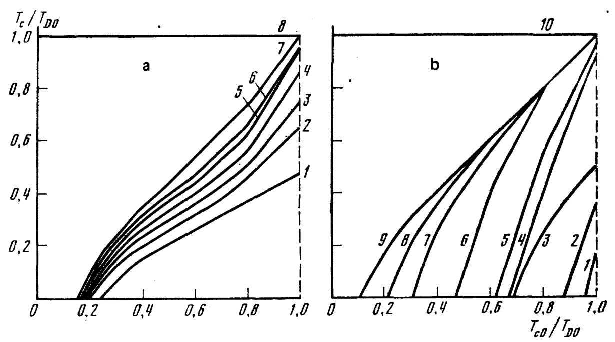


FIG. 1. Plots of the temperature of the superconducting transition in the ferromagnetic phase: a—antisymmetrical case, b—symmetrical case.

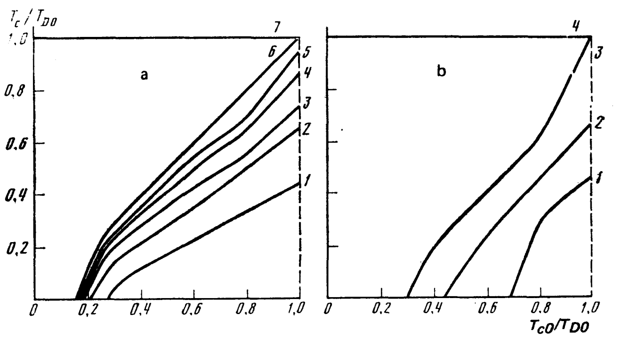


FIG. 2. Plots of the superconducting transition in the ferromagnetic phase: a—antisymmetrical case, b—symmetrical case.

1–3 of Fig. 2b are 0.9, 1.1, and 1.3. Line 4 gives the temperature of the transition into the state of the ferromagnetic excitonic dielectric for a doping $n/T_{D0}=1.3$.

From a comparison of Figs. 1a, 2a with Figs. 1b, 2b it follows that T_c decreases with increasing interband exchange-interaction constant, and in the symmetrical case the decrease of T_c manifests itself more strongly than in the asymmetrical case. We note that T_c is more sensitive to a change of doping to variation of J_b. Because of the complexity of the equations, we did not investigate the behavior of the system at temperatures lower than T. One cannot exclude here the following possibility. With decreasing temperature, $T < T_c$, the dielectric gap increases and with it the magnetization $S \sim B_L(2J_b\Delta L/g_tT)$. Therefore at sufficiently low temperatures the superconductivity can be suppressed on account of the paramagnetic effect, since the quantity I_a responsible for this effect at $T \ll T_c$, is proportional to $J_aB_L(2J_b\Delta L/g_tT)$.

We have attempted here to demonstrate (within the framework of the given model) the possibility of the onset of superconductivity in a ferromagnetic phase. It appears, however, that the inverse situation is also possible. The system first becomes superconducting, and the ferromagnetic ordering appears at lower temperatures via the mechanism proposed in Ref. 8.

CONCLUSION

We have thus shown that in systems that are unstable to a restructuring of the spectrum with formation of a Bose condensate of triplet electron-hole pairs (SDW), and containing localized magnetic moments, superconductivity and ferromagnetism can coexist. The coexistence is possible even at arbitrarily weak exchange and spin-orbit scattering, which lead to nonzero paramagnetic susceptibility of the superconductor. The reason is that a substantial contribution to the ferromagnetic ordering of the localized moments is made by triplet electron-hole pairs. In addition, within the framework of the model, the weak exchange scattering makes possible to onset of superconductivity in the ferromagnetic phase, i.e., T_c is lower than the Curie temperature.

We have considered so far the model of a semimetal with energy-band extrema that coincide in momentum space. In this case the period of the produced SDW is equal to the lattice period. Therefore the magnetic moments of the atoms (or the moments of the substitutional impurities) are located at points having the same value of the SDW amplitude. This is precisely why the moments of the atoms align themselves in the same direction as the SDW, i.e., ferromagnetism sets in. On the other hand if the band extrema are separated in momentum space by a distance equal to half the reciprocal-lattice vector, then the SDW period doubles. Therefore the magnetic moments at neighboring sites are in antiphase. This produces two magnetic lattices, i.e., antiferromagnetism. In the antiferromagnetic phase the appearance of superconductivity is also possible, and furthermore more easily, since there is no effect of the spin separation ($I_a=0$). This situation is also described by the presented model, except that we must put $I_a=0$ in (17) and (18).

We have considered here a homogeneous situation; allowance for diamagnetic effects brings about a more complicated spatial structure.[7]

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