$$R(t) \approx s_0 \exp(-i\Omega - 2\Gamma^3/D_0^2)\,t. \qquad (29)$$

We see therefore that at $\Gamma \ll D_0$ the damping coefficient is determined by the expression $2\Gamma^3/D_0^2 \ll \Gamma$. Thus, the effect of suppression of the microscopic inhomogeneity of the HFF[11, 12] is realized even for a quadratic Lorentz function.

If the system is acted upon by a rectangular HF-field pulse, then the transform of the function $R(t)$ is given by

$$\hat{R} = \frac{-i\omega_N[1-\exp(-p\tau)](p+i\Delta+\Gamma_n+2\Gamma)}{p[(p+i\Delta+\Gamma_n+\Gamma)^2 - iD_0(p+i\Delta+\Gamma_n+2\Gamma)]}, \qquad (30)$$

where τ is the pulse duration. The pole at the point $p=0$ corresponds to a stationary solution as $\tau \to \infty$. The two other poles describe the frequency and the damping coefficients in the transient processes. These poles are obtained from an algebraic equation for (24) only in the substitution $\langle\omega_n\rangle \to \Delta$. It is therefore clear that at $t > \tau$ the frequencies and the damping coefficients of the DFP are determined by expressions (26) and (27) and do not depend on the duration and carrier frequency of the exciting pulse.

The authors are deeply grateful to V. A. Ignatchenko and M. I. Kurkin for constant interest in the work and for helpful advice.

[1] D. Hone, V. Jaccarino, T. Ngwe, and P. Pinculs, Phys. Rev. **186**, 291 (1969).
[2] J. Barak, I. Siegelstein, A. Gabai and N. Kaplan, Phys. Rev. B **8**, 5282 (1973).
[3] W. Zinn, Atom. Energy Rev. **12**, 709 (1974).
[4] M. I. Kurkin and V. V. Serikov, Fiz. Tverd. Tela (Leningrad) **12**, 3524 (1970) [Sov. Phys. Solid State **12**, 2862 (1971)].
[5] V. A. Ignatchenko, V. K. Mal'tsev, and V. I. Tsifrinovich, Zh. Eksp. Teor. Fiz. **75**, 217 (1978) [Sov. Phys. JETP **48**, 108 (1978)].
[6] E. A. Glozman, V. A. Ignatchenko, and V. I. Tsifrinovich, Fiz. Tverd. Tela (Leningrad **20**, 3112 (1978) [Sov. Phys. Solid State **20**, 1794 (1978)].
[7] V. I. Tsifrinovich, Fiz. Tverd. Tela (Leningrad) **20**, 1657 (1978) [Sov. Phys. Solid State **20**, 959 (1978)].
[8] E. A. Turov, M. I. Kurkin, and V. V. Nikolaev, Zh. Eksp. Teor. Fiz. **64**, 283 (1973) [Sov. Phys. JETP **37**, 147 (1973)].
[9] M. P. Petrov, G. A. Smolenskii, A. A. Petrov, and S. I. Stepanov, Fiz. Tverd. Tela (Leningrad) **15**, 184 (1973) [Sov. Phys. Solid State **15**, 126 (1973)].
[10] V. P. Chekmarev and M. P. Petrov, Zh. Eksp. Teor. Fiz. **71**, 377 (1976) [Sov. Phys. JETP **44**, 197 (1976)].
[11] E. A. Turov and M. I. Kurkin, in: Problemy magnitnogo rezonansa (Problems of Magnetic Resonance), Nauka, 1978, p. 271.
[12] M. I. Kurkin, Pis'ma Zh. Eksp. Teor. Fiz. **28**, 675 (1978) [JETP Lett. **28**, 628 (1978)].

Translated by J. G. Adashko

Absorption of sound in dilute magnetic alloys

Yu. M. Ivanchenko and G. I. Trush

Donets Physicotechnical Institute, Ukrainian Academy of Sciences

(Submitted 20 May 1979)
Zh. Eksp. Teor. Fiz. **78**, 1767–1780 (May 1980)

The electronic absorption of sound in metals with magnetic impurities is considered. An expression is obtained for the absorption coefficient of transverse sound α_t with account of effects connected with the motion of impurities in the field of the sound wave. This expression is connected with the electron self-energy part $\Sigma(\omega)$. Two types of dilute magnetic alloys are analyzed. In alloys with disordered magnetic impurities, the characteristic temperature dependence of the absorption coefficient is obtained. In spin glasses there is an anomalous frequency dependence of α_t. At frequencies at which $\alpha_t \propto \omega^{-3}$ in metals without magnetic impurities the relation of spin glasses is $\alpha_t \propto \omega^{-1}$. The latter result is obtained without recourse to any specific model of the spin glass and is the result of the impurity spin's being "frozen in". The proposed method can also be employed in calculating the absorption of longitudinal sound.

PACS numbers: 75.80. + q, 75.30.Hx

1. INTRODUCTION

A number of interesting effects, which are due to the interaction of the conduction electrons with the magnetic impurities, have been observed in dilute magnetic alloys (see, for example, Refs. 1 and 2). The specific character of this interaction leads to a characteristic dependence of the lifetime of the electronic excitations on the energy and temperature, and also to rearrangement of the electron spectrum in the case in which any of the types of magnetic ordering of the impurities (ferromagnetic, antiferromagnetic, spin glass, see, for example, Refs. 3 and 4) is established. Inasmuch as the basic contribution to the sound absorption at low temperatures is made by the interaction of the conduction electrons with the incident sound wave, it is natural to expect that the characteristic features of the dilute magnetic alloys are essentially manifested in the sound absorption.

The problem of the sound absorption has been posed and solved in many researches. There exist several approaches to its solution. One of them—the phenomenological—is based on the use of the kinetic equation of Boltzmann with a single relaxation time τ.[5,6] This approach is distinguished by its simplicity and clarity, but its essential inadequacy is that it is difficult to express τ in terms of the microscopic characteristics of the system. This inadequacy is especially evident if there are several relaxation mechanisms for the electrons or, just as in the present case, if the times of the electronic excitations depend significantly on the energy of the electron.

Another approach consists in the calculation of the phonon polarization operator P with account of all the interactions, the imaginary part of which determines the sound absorption.[7,8] However, since there are randomly distributed impurities, over the coordinates of which averaging must be carried out, then it is not clear which quantity determines the sound absorption: $\operatorname{Im}\bar{P}$ (the bar denotes averaging over the locations of the impurities) or $\operatorname{Im}\tilde{P}$, where $\tilde{P}=D_0^{-1}-(\bar{D})^{-1}$ (D_0 is the zero phonon Green's function, $\bar{D}$ is the exact phonon Green's function averaged over the impurities), since $\bar{P}\neq\tilde{P}$. Moreover, as a rule, the interaction of the electrons with the lattice vibrations is taken into account in these calculations only by the Fröhlich Hamiltonian with the electron-phonon interaction constant $g_{q\lambda}$ ($\mathbf{q}$ is the momentum of the phonon, λ is the index enumerating the branches of the vibrations). But, as is well known, at $q \ll p_0$ (p_0 is the Fermi momentum, $g_{q\lambda}\approx 0$ for the transverse branches, i.e., it is impossible to calculate the absorption of transverse sound directly by such a method.

There is an approach, based on the microscopic consideration of the interaction of the electrons with the sound wave, with account of the deformation of the crystal and of the vortical electric fields. The kinetic equation for the distribution function of the electrons is obtained in this case from the equation for the matrix density, and the τ approximation is not used (see Refs. 9 and 10). In principle, such an approach would allow us to consider the specifics of dilute magnetic alloys and to calculate the sound absorption correctly. However, it is very difficult to solve the problem of the description of the behavior of the electrons interacting with the magnetic impurities in the nonequilibrium case. Therefore, for the calculation of the sound absorption, we use an approach which recalls the method employing the theory of linear response (see Refs. 11 and 12). Just as in Refs. 11 and 12, we shall find the linear response to the perturbation produced by the sound wave; however, we shall determine the sound absorption not from the amount of energy transferred to the electron subsystem, but from the equation which describes its distribution in the crystal. We write down the quantum equation of motion for the displacement of the atoms in the presence of the sound wave. This equation will contain the nonequilibrium electron Green's function [for the definition, see (2.11), cf. Ref. 13] and the mean value of the intensity of the vortical electric field. Expanding the Green's function in the perturbations produced by the sound wave, up to linear terms [see (2.12)], and eliminating the electromagnetic field, we arrive at a homogeneous equation of the form

$$\sum_{\lambda_1}\hat{\mathscr{L}}_{\lambda\lambda_1}(\mathbf{q},\omega)\,\varphi_{\mathbf{q}\lambda_1}(\omega)=0, \tag{1.1}$$

where $\varphi_{\mathbf{q}\lambda_1}(\omega)$ is connected in linear fashion with the mean displacements of the atoms [see (2.5)], and the summation in (1.1) extends over all branches of the vibrations. From the condition of the existence of nontrivial solutions of (1.1), we find the damping of the sound. Such an approach is convenient in the given problem because, first, it allows us to express the sound absorption coefficient in terms of the equilibrium Green's functions of the dilute magnetic alloy, averaged over the locations of the impurities. We assume these functions to be known.[1,2,4] Second, we can calculate the absorption of both transverse and longitudinal sound by such a method. Finally, the employed method permits us to take into account the effects connected with the motion of the impurities in the field of the sound wave, without use of a comoving set of coordinates. These effects are important in principle, since without their consideration even the order of magnitude of the sound absorption is incorrect.

2. BASIC RELATIONS

Let us consider a system of electrons and ions in the field of a sound wave. The longitudinal electromagnetic interaction of the charges is described, as usual, by the Coulomb potential, and the transverse interaction, with the help of the vortical fields which arise upon the displacement of the ions from their equilibrium positions. The Hamiltonian of such a system has the form

$$H=H_e+H_p+H_{ep}+H_{im}+H_j. \tag{2.1}$$

The Hamiltonian of the electrons is

$$H_e=\int d\mathbf{r}\,\psi^+(\mathbf{r})\left\{\frac{1}{2m}\left[\hat{\mathbf{p}}+\frac{e}{c}\hat{\mathbf{A}}(\mathbf{r})\right]^2+\sum_m V(\mathbf{r}-\mathbf{R}_m{}^0)\right.$$
$$\left.+\frac{1}{2}\int d\mathbf{r}'\psi^+(\mathbf{r}')V_{ee}(\mathbf{r}-\mathbf{r}')\psi(\mathbf{r}')\right\}\psi(\mathbf{r}). \tag{2.2}$$

Here $\psi^+(\mathbf{r})$, $\psi(\mathbf{r})$ are the field operators of creation and annihilation of the electron $\hat{\mathbf{p}}$ is the momentum operator of the electron, $\hat{\mathbf{A}}(\mathbf{r})$ is the vector potential operator of the transverse electromagnetic field, $e>0$ is the charge of the electron, $V(\mathbf{r}-\mathbf{R}_m^0)$ is the interaction potential of the electron with an ion found at the equilibrium position $\mathbf{R}_m^0$, $V_{ee}(\mathbf{r}-\mathbf{r}')$ is the Coulomb potential of the electron-electron interaction.

The Hamiltonian of the ions is

$$H_p=\sum_m\left\{\frac{1}{2M}\left(\hat{\mathbf{P}}_m-\frac{e}{c}\hat{\mathbf{A}}(\mathbf{R}_m)\right)^2+\frac{1}{2}\sum_{m'\neq m}W(\mathbf{R}_m,\mathbf{R}_{m'})\right\}, \tag{2.3}$$

where $\hat{\mathbf{P}}_m$ is the momentum operator of the m-th ion, $\mathbf{R}_m$ is the position of this ion (for simplicity, a metal with a single atom in the elementary cell is considered), $W(\mathbf{R}_m,\mathbf{R}_{m'})$ is the potential of ion-ion interaction, M is the mass of the ion.

The Hamiltonian of the electron-phonon interaction is

$$H_{ep}=\int d\mathbf{r}_1 d\mathbf{r}_2\sum_\lambda g_\lambda(\mathbf{r}_2-\mathbf{r}_1)\hat{\varphi}_\lambda(\mathbf{r}_1)\psi^+(\mathbf{r}_2)\psi(\mathbf{r}_2). \tag{2.4}$$

Here $q_\lambda(\mathbf{r}_2-\mathbf{r}_1)$ is the constant of electron-phonon interaction,

$$\hat{\varphi}_\lambda(\mathbf{r}_1)=\sum_{\mathbf{q}}e^{i\mathbf{q}\mathbf{r}_1}\varphi_{\mathbf{q}\lambda},\quad \varphi_{\mathbf{q}\lambda}=a_{\mathbf{q}\lambda}+a^+_{-\mathbf{q}\lambda},$$

where $a^{+}_{\mathbf{q}\lambda}$ and $a_{\mathbf{q}\lambda}$ are the creation and annihilation operators of a phonon with momentum **q** in the λ-th branch of vibrations. The operator $\hat{\varphi}_{\mathbf{q}\lambda}$ is connected with the ion-displacement operators by the canonical transformation[14]

$$\hat{\mathbf{U}}_m=\mathbf{R}_m-\mathbf{R}_m{}^0=\sum_{\mathbf{q},\lambda}(2Mn\Omega_{\mathbf{q}\lambda})^{-1/2}\varepsilon_{\mathbf{q}\lambda}\hat{\varphi}_{\mathbf{q}\lambda}\exp(i\mathbf{q}\mathbf{R}_m{}^0), \tag{2.5}$$

where n is the number of electrons in a unit volume, $\Omega_{\mathbf{q}\lambda}$ are the "bare" natural frequencies of the phonons, $\varepsilon_{\mathbf{q}\lambda}$ are the polarization vectors. We note that the Hamiltonian of the electron-phonon interaction, chosen in the form (2.4), actually does not include the interaction of the electrons with longwave transverse phonons, since $g_{\mathbf{q}t}\approx 0$ at $q\ll p_0$ (see, for example, Ref. 14). For just this reason, the transverse electromagnetic interaction is included in the complete Hamiltonian.

The Hamiltonian of electron-impurity interaction has the form

$$H_{im}=\int d\mathbf{r}\psi^{+}(\mathbf{r})\left\{\sum_i V_{im}(\mathbf{r}-\mathbf{R}_i)-\frac{J}{2n}\sum_k \sigma\mathbf{S}_k\delta(\mathbf{r}-\mathbf{R}_k)\right\}\psi(\mathbf{r}), \tag{2.6}$$

where $V_{im}(\mathbf{r}-\mathbf{R}_i)$ is the interaction potential of the electron with a nonmagnetic impurity located at the point $\mathbf{R}_i$. The interaction of electrons with magnetic impurities is described as usual by the s-d exchange Hamiltonian.

The Hamiltonian of the electromagnetic field is

$$H_t=\int\frac{d\mathbf{r}}{8\pi}(\hat{\mathbf{E}}^2+\hat{\mathbf{H}}^2)=\int\frac{d\mathbf{r}}{8\pi}\left\{\frac{1}{c^2}\left(\frac{\partial\hat{\mathbf{A}}}{\partial t}\right)^2+(\operatorname{rot}\hat{\mathbf{A}})^2\right\}. \tag{2.7}$$

We again emphasize that a purely transverse electromagnetic field is considered (the entire longitudinal part of the electromagnetic interaction is included in the other components of the complete Hamiltonian). This field is regarded as a quantum field in order to avoid an explicit dependence of the complete Hamiltonian on the time. Since the vectors $\hat{\mathbf{A}}$ and $\partial\hat{\mathbf{A}}/\partial t$ are transverse, it is natural that their commutator should be expressed in terms of a transverse tensor, i.e.,

$$[\hat{A}^{\alpha}(\mathbf{r}),\partial\hat{A}^{\beta}(\mathbf{r}')/\partial t]_{-}=4\pi c^2 iP_{\alpha\beta}(\mathbf{r},\mathbf{r}');\quad \alpha,\beta=(x,y,z). \tag{2.8}$$

The tensor $P_{\alpha\beta}$ should, on the one hand, possess "unitary" properties (i.e., commutation relations among the generalized coordinates and momentum) and, on the other hand, should guarantee fulfillment of the condition $\operatorname{div}\hat{A}=0$ and its consequences. In other words, $P_{\alpha\beta}$ should be a projection operator on the transverse states. The indicated properties are possessed by the operator

$$P_{\alpha\beta}(\mathbf{r},\mathbf{r}')=-\frac{1}{4\pi}\left(\delta_{\alpha\beta}\nabla^2-\frac{\partial^2}{\partial r^{\alpha}\,\partial r^{\beta}}\right)\frac{1}{|\mathbf{r}-\mathbf{r}'|}. \tag{2.9}$$

We introduce the mean value of the operator $\hat{\varphi}_{\lambda}(1)$, which is defined on the imaginary time axis $0\leqslant it\leqslant\beta$, where $\beta^{-1}=T$, T is the temperature of the system;

$$\varphi_{\lambda}(1)=\frac{\langle\hat{T}\Phi\hat{\varphi}_{\lambda}(1)\rangle}{\langle\hat{T}\Phi\rangle},\qquad 1=(\mathbf{r}_1,t_1).$$

Here $\hat{T}$ is the time-ordering operator on the imaginary axis,[13] which depends on the time operators in the Heisenberg representation with the Hamiltonian (2.1),

$$\Phi=\exp\left\{-i\int_0^{-i\beta}d\bar{1}\left[\bar{U}(\bar{1})\rho(\bar{1})+\sum_{\lambda}V_{\lambda}(\bar{1})\hat{\varphi}_{\lambda}(\bar{1})\right.\right.$$
$$\left.\left.+\mathbf{C}(\bar{1})\hat{\mathbf{A}}(\bar{1})+\sum_k\mathbf{\Gamma}_k(t)\mathbf{S}_k\delta(\mathbf{r}-\mathbf{R}_k)\right]\right\},$$

$\bar{1}=(\bar{r},\bar{t})$, $\rho(\bar{1})=\psi^{+}(\bar{1})\psi(\bar{1})$, $\bar{U}(\bar{1})$, $V_{\lambda}(\bar{1})$, $\mathbf{C}(\bar{1})$, $\mathbf{\Gamma}_k(t)$ are arbitrary functions which, after obtaining the corresponding equations, can be set equal to zero. The angular brackets denote averaging over the statistical ensemble with the complete Hamiltonian in the presence of a sound wave, and also averaging over the random distribution of the impurities. Averaging the Heisenberg equation of motion for the operation $\varphi_{\lambda}(1)$ in the same way, we obtain

$$\left[\frac{\partial^2}{\partial t^2}+\Omega_{\lambda}{}^2(-i\nabla)\right]\varphi_{\lambda}(1)-4\Omega_{\lambda}(-i\nabla)g_{\lambda}(2-1)iG(2,2^{+})+\mathscr{E}_{\lambda}(1)=0; \tag{2.10}$$

here

$$\Omega_{\lambda}(-i\nabla)g_{\lambda}(2-1)iG(2,2^{+})=i\sum_q\Omega_{q\lambda}\int d\mathbf{r}_1{}'\int d2g_{\lambda}(\mathbf{r}_2-\mathbf{r}_1{}')$$
$$\times G(2,2^{+})e^{i\mathbf{q}(\mathbf{r}_1-\mathbf{r}_1{}')}\delta(t_2-t_1),$$

$$\mathscr{E}_{\lambda}(1)=\sum_{q,\alpha}\frac{e}{M}(2Mn\Omega_{q\lambda})^{1/2}\varepsilon^{\alpha}_{-q\lambda}\int d\mathbf{r}_1{}'e^{-i\mathbf{q}(\mathbf{r}_1-\mathbf{r}_1{}')}\left\{-\frac{1}{c}\frac{\partial A^{\alpha}(r_1{}',t)}{\partial t}\right\},$$

$$A^{\alpha}(r',t)=\frac{\langle\hat{T}\Phi\hat{A}^{\alpha}(r')\rangle}{\langle\hat{T}\Phi\rangle},\qquad G(1,1')=\frac{\langle\hat{T}\Phi\psi(1)\psi^{+}(1')\rangle}{i\langle\hat{T}\Phi\rangle};$$
$$2^{+}=(r_2,t_2+0). \tag{2.11}$$

We note that the equation for $\varphi_{\lambda}(1)$ in the form (2.10) was obtained under the assumption that the $\mathbf{R}^0_n$ are the real equilibrium positions of the ions with account of the presence of impurities and their vibrations in the crystal.

The sound wave in the crystal leads to the appearance of a transverse electromagnetic field, a perturbation of the electron density, and a nonzero mean displacement of the ions. Assuming these perturbations to be small, we can write the following expansion for the Green's function of the crystal with the sound wave:

$$G(1,1')=\tilde{G}(1-1')+\int\left[\frac{\delta G(1,1')}{\delta U_{ef}(3)}\bigg|_0 U_{ef}(3)\right.$$
$$\left.+\sum_{\lambda}\frac{\delta G(1,1')}{\delta\varphi_{\lambda}(3)}\bigg|_0\varphi_{\lambda}(3)+\sum_{\alpha}\frac{\delta G(1,1')}{\delta A^{\alpha}(3)}\bigg|_0 A^{\alpha}(3)\right]d3. \tag{2.12}$$

$\tilde{G}(1-1')$ is the Green's function of the dilute magnetic alloy without the sound wave, the subscript 0 of the variational derivatives means that after the derivative is taken, all the perturbations are assumed to be equal to zero, the value of U_{ef} is determined by the equation

$$U_{ef}(3)=\bar{U}(3)-2i\int d\bar{2}V_{ee}(r_3-r_{\bar{2}})\delta(t_3-t_{\bar{2}})[G(\bar{2},\bar{2}^{+})-\tilde{G}(\bar{2}-2^{+})]. \tag{2.13}$$

From the commutation relations (2.8) we obtain the equation for **A**:

$$\operatorname{rot}\operatorname{rot}\mathbf{A}(1)-\frac{1}{c^2}\frac{\partial^2\mathbf{A}(1)}{\partial t_1{}^2}=\frac{4\pi}{c}\mathbf{j}_t(1), \tag{2.14}$$

where

$$j_t{}^{\alpha}(1)=\int P_{\alpha\beta}(\mathbf{r}_1,\mathbf{r}_1{}')j^{\beta}(\mathbf{r}_1{}',t_1)d\mathbf{r}_1{}',$$

$\mathbf{j}(1)$ is the total current in the system. For $\mathbf{j}_t(1)$ we can write an expansion similar to (2.12). Here it is necessary to take it into account that, in the absence of a sound wave, the total current in the system is equal to zero:

$$j_t^\alpha(1)=\int\left[\frac{\delta j_t^\alpha(1)}{\delta U_{ef}(3)}\bigg|_0 U_{ef}(3)+\sum_\lambda\frac{\delta j_t^\alpha(1)}{\delta\varphi_\lambda(3)}\bigg|_0\varphi_\lambda(3)+\sum_\beta\frac{\delta j_t^\alpha(1)}{\delta A^\beta(3)}\bigg|_0 A^\beta(3)\right]d3. \tag{2.15}$$

We note that, since U_{ef} characterizes the change in the electron density and the screening of the charges, i.e., purely longitudinal effects, it follows that in the isotropic case, which will be considered further, $\delta j_t^\alpha(1)/\delta U_{ef}(3)|_0=0$.

We introduce the notation

$$Q_{\alpha\beta}(1-3)=\frac{\delta j_t^\alpha(1)}{\delta A^\beta(3)}\bigg|_0,\quad C_\lambda^\alpha(1-3)=\frac{\delta j_t^\alpha(1)}{\delta\varphi_\lambda(3)}\bigg|_0. \tag{2.16}$$

Carrying out the Fourier transformation

$$K(\mathbf{p},z)=\int d\mathbf{r}\int_0^{-i\beta}dt\exp[-i\mathbf{pr}+izt]K(\mathbf{r},t),\qquad z=2\pi mTi$$

(m is an integer) we obtain in Eq. (2.14), with account of (2.15) and (2.16):

$$\left[\left(\frac{z^2}{c^2}-p^2\right)\delta_{\alpha\beta}+\frac{4\pi}{c}Q_{\alpha\beta}(\mathbf{p},z)\right]A^\beta(\mathbf{p},z)=-\frac{4\pi}{c}\sum_{\lambda_1}C_{\lambda_1}^\alpha(\mathbf{p},z)\varphi_{\mathbf{p}\lambda_1}(z),$$

whence it follows that

$$A^\alpha(\mathbf{p},z)=-\frac{4\pi}{c}\sum_{\lambda_1}C_{\lambda_1}^\alpha(\mathbf{p},z)\varphi_{\mathbf{p}\lambda_1}(z)\left[\frac{z^2}{c^2}-p^2+\frac{4\pi}{c}Q(\mathbf{p},z)\right]^{-1}, \tag{2.17}$$

where

$$Q(\mathbf{p},z)=\frac{1}{2}\sum_{\substack{\alpha,\beta,\gamma\\ \beta_1,\gamma_1}}e_{\alpha\beta\gamma}e_{\alpha\beta_1\gamma_1}n_{\gamma_1}n_\gamma Q_{\beta\beta_1}(\mathbf{p},z)\qquad n_\gamma=\frac{p^\gamma}{|\mathbf{p}|}; \tag{2.17a}$$

$e_{\alpha\beta\gamma}$ is a unitary antisymmetric tensor.

We also introduce the permittivity:

$$\varepsilon(1-2)=\delta(1-2)+2i\int d3V_{ee}(\mathbf{r}_3-\mathbf{r}_2)\delta(t_3-t_2)\frac{\delta G(1,1^+)}{\delta U_{ef}(3)}\bigg|_0 \tag{2.18}$$

and denote

$$K_\lambda(1-3)=\frac{\delta G(1,1^+)}{\delta\varphi_\lambda(3)}\bigg|_0,\qquad K^\alpha(1-3)=\frac{\delta G(1,1^+)}{\delta A^\alpha(3)}\bigg|_0. \tag{2.19}$$

Taking the Fourier transform in Eq. (2.10), we obtain

$$\sum_{\lambda_1}\Bigg\{(z^2-\Omega_{q\lambda}^2)\delta_{\lambda\lambda_1}-4i\Omega_{\mathbf{q}\lambda}g_{-\mathbf{q}\lambda}\Bigg[K_{\lambda_1}(\mathbf{q},z)\varepsilon^{-1}(\mathbf{q},z)-\frac{4\pi}{c}K^\alpha(\mathbf{q},z)\frac{\varepsilon^{-1}(\mathbf{q},z)C_{\lambda_1}^\alpha(\mathbf{q},z)}{z^2/c^2-q^2+4\pi c^{-1}Q(\mathbf{q},z)}\Bigg]$$
$$-\left(\frac{m}{M}\right)^{1/2}\frac{2\sqrt{2}\pi z}{c^2}(\omega_{pe}^2\Omega_{\mathbf{q}\lambda})^{1/2}\varepsilon_{q\lambda}^\alpha\frac{C_{\lambda_1}^\alpha(\mathbf{q},z)}{z^2/c^2-q^2+4\pi Q(\mathbf{q},z)/c}\Bigg\}\varphi_{\mathbf{q}\lambda_1}(z)=0, \tag{2.20}$$

ω_{pe} is the electron plasma frequency.

After finding the quantities K_λ, ε, $\mathbf{C}_\lambda$, $\mathbf{K}$, and Q, we must carry out the analytic continuation $z\to\omega+i\delta$, $\delta=+0$. This is connected with the fact that Eq. (2.2) is obtained for the Fourier transform of the function φ_λ, which is defined on the imaginary time axis, while the quantity $\varphi_\lambda(t)$, which is connected with the mean displacements, is defined for real times, with $\varphi_\lambda(t)\to 0$ as $t\to-\infty$. After analytic continuation, the sound attenuation coefficient is found from the condition of existence of nontrivial solutions of (2.20).

3. CALCULATION OF THE VARIATIONAL DERIVATIVES

We first calculate $C_\lambda^\alpha(q,z)$. Inasmuch as the total current consists of the electron and ion currents, then

$$C_\lambda^\alpha(1-3)=\frac{\delta j_t^\alpha(1)}{\delta\varphi_\lambda(3)}\bigg|_0=\frac{\delta j_{ti}^\alpha(1)}{\delta\varphi_\lambda(3)}\bigg|_0+\frac{\delta j_{te}}{\delta\varphi_\lambda(3)}\bigg|_0=C_{\lambda i}^\alpha(1-3)+C_{\lambda e}^\alpha(1-3),$$
$$\mathbf{j}_i(1)=e\sum_m\delta(\mathbf{r}-\mathbf{R}_m^0)\dot{\mathbf{U}}_m(t), \tag{3.1}$$

whence

$$\varepsilon_{q\lambda}^\alpha\sum_{\lambda_1}C_{\lambda_1 i}^\alpha(\mathbf{q},z)\varphi_{\mathbf{q}\lambda_1}(z)=-izen(2Mn\Omega_{\mathbf{q}\lambda})^{-1/2}\varphi_{\mathbf{q}\lambda}(z)(1-\delta_{\lambda l}) \tag{3.2}$$

($\lambda=l$ for the longitudinal branch of the vibrations). Since the expression for the electron current has the form

$$\mathbf{j}_e(1)=-\frac{ne^2}{mc}\mathbf{A}(1)-\frac{ie}{m}[(\hat{\mathbf{p}}'-\hat{\mathbf{p}})G(1,1')]_{1'\to1^+},\quad\hat{\mathbf{p}}=-i\frac{\partial}{\partial\mathbf{r}}, \tag{3.2a}$$

it follows that

$$C_{\lambda e}^\alpha(1-3)=-\frac{ie}{m}\int P_{\alpha\beta}(1-2)\left[(\hat{p}_2^{\beta'}-\hat{p}_2^\beta)\frac{\delta G(2,2')}{\delta\varphi_\lambda(3)}\bigg|_0\right]_{2'\to2^+}d2. \tag{3.3}$$

We can obtain the following expression for the Green's function $G(1,1')$:

$$\int d\bar{1}\{G_0^{-1}(1,\bar{1})-\Sigma_{ee}(1,\bar{1})-\Sigma_{ep}(1,\bar{1})-\Sigma_{im}(1,\bar{1})\}G(\bar{1},1')=\delta(1-1'), \tag{3.4}$$

where

$$G_0^{-1}(1,\bar{1})=\Bigg\{i\frac{\partial}{\partial t_1}-\frac{\hat{p}^2}{2m}-\frac{e}{mc}\mathbf{A}(1)\hat{\mathbf{p}}-U_{ef}(1)-\frac{J}{2n}\sum_{\mathbf{k}}\boldsymbol{\sigma}\mathbf{M}_k\delta(r-R_k)-\sum_\lambda\int d2g_\lambda(1-2)\varphi_\lambda(2)\Bigg\}\delta(1-\bar{1}), \tag{3.4a}$$
$$\mathbf{M}_k=\langle\hat{T}\Phi\mathbf{S}_k\rangle/\langle\hat{T}\Phi\rangle,$$

Σ_{ee}, Σ_{ep}, Σ_{im} are the self-energy parts connected with electron-electron, electron-phonon, and electron-impurity interactions, respectively. The function Σ_{ee} no longer contains the long-range part of the electron-electron interaction and is purely collisional. For simplicity, we consider the approximation of almost free electrons.

Since we are interested in the sound absorption connected with electron scattering from impurities, the variational derivative in (3.3) can be calculated without account of Σ_{ee} and Σ_{ph}, assuming them to be small. However, it must be taken into account that under the action of the sound wave, the impurities also are displaced. Account of this fact, as will be seen below, yields a contribution to the absorption of transverse sound, corresponding to the contribution made in the phenomenological theory by terms appearing when a transition to a comoving set of coordinates is made.

For the impurity self-energy part, we can obtain the expression[4,15]

$$\Sigma_{im}(1,1')=c_i\sum_l T_l(1,1'),$$

where c_i is the relative impurity concentration, the summation is over all the lattice sites, and the function $T_l(1,1')$ is defined by the equation

$$T_l(1,1')=V(1,1';l)+\int V(1,\bar{1};l)\tilde{G}(\bar{1},2)T_l(2,1')\,d\bar{1}\,d2.$$

Here $V(1,1';l)$ is the potential of the impurity located at the l-th site. With accuracy to terms that are linear in the displacements,

$$\begin{aligned}V(1,1';l)&=V(\mathbf{r}-\mathbf{R}_l)\delta(1-1')=V(\mathbf{r}-\mathbf{R}_l^0)\delta(1-1')\\&-\mathbf{U}_l(t)\nabla_rV(\mathbf{r}-\mathbf{R}_l^0)\delta(1-1')=V_0(1,1';l)+V_1(1,1';l),\end{aligned}$$

$T_l(1,1')=T_l^0(1,1')+T_l^1(1,1')$, while it is not difficult to show that T_l^1 is determined in the following fashion (for brevity, we write down this and several succeeding expressions in the matrix form):

$$T_l^1=\tilde{\Pi}_lV_1\Pi_l,\quad \tilde{\Pi}_l=1+\tilde{G}T_l^0,\quad \Pi_l=1+T_l^0\tilde{G}.$$

After simple transformations, we get

$$\begin{aligned}C_{\lambda e}^{\alpha}(\mathbf{q},z)&=\frac{ec_in}{m}(2Mn\Omega_{\mathbf{q}\lambda})^{-1/2}P_{\alpha\beta}(\mathbf{q})\int\frac{d\mathbf{p}_1}{(2\pi)^3}\\&\times\frac{1}{(-i\beta)}\sum_{z_1}(2p_1^{\beta}-q^{\beta})\varepsilon_{\mathbf{q}\lambda}{}^{\gamma}\tilde{G}(\mathbf{p}_1,z_1)\\&\times\tilde{G}(\mathbf{p}_1-\mathbf{q},z_1-z)\int\frac{d\mathbf{p}_2\,d\mathbf{p}_3}{(2\pi)^6}\tilde{\Pi}(\mathbf{p}_1,\mathbf{p}_2,z_1)\\&\times(p_2{}^{\gamma}-p_3{}^{\gamma})V_0(\mathbf{p}_2-\mathbf{p}_3)\Pi(\mathbf{p}_2,\mathbf{p}_1-\mathbf{q},z_1-z).\end{aligned}$$

We note that $P_{\alpha\beta}=\delta_{\alpha\beta}-n_\alpha n_\beta$ and $\Pi_l(1,1')$ is connected with $\Pi(\mathbf{p}_1,\mathbf{p}_2,z)$ in the following fashion (similarly for $\tilde{\Pi}$):

$$\begin{aligned}\Pi_l(1,1')&\equiv\Pi_l(\mathbf{r}_1,\mathbf{r}_1',t_1-t_1')=\int\frac{d\mathbf{p}_1\,d\mathbf{p}_2}{(2\pi)^6}\frac{1}{(-i\beta)}\\&\times\sum_{z_1}\exp[i\mathbf{p}_1(\mathbf{r}_1-\mathbf{R}_l^0)-i\mathbf{p}_2(\mathbf{r}_1'-\mathbf{R}_l^0)-iz_1(t_1-t_1')]\Pi(\mathbf{p}_1,\mathbf{p}_2,z_1).\end{aligned}$$

Using the properties of the functions Π and $\tilde{\Pi}$, we can show that

$$\begin{aligned}\Pi(\mathbf{k},\mathbf{q},z)&=(2\pi)^3\delta(\mathbf{k}-\mathbf{q})+\tilde{G}(\mathbf{k},z)T^0(\mathbf{k},\mathbf{q},z),\\\tilde{\Pi}(\mathbf{k},\mathbf{q},z)&=(2\pi)^3\delta(\mathbf{k}-\mathbf{q})+\tilde{G}(\mathbf{q},z)T^0(\mathbf{k},\mathbf{q},z).\end{aligned}$$

The Fourier components $T^0(\mathbf{k},\mathbf{q},z)$ are determined in the the same way as $\Pi(\mathbf{k},\mathbf{q},z)$. We then get immediately

$$\begin{aligned}&\int\frac{d\mathbf{p}_2\,d\mathbf{p}_3}{(2\pi)^6}\tilde{\Pi}(\mathbf{p}_1,\mathbf{p}_2,z_1)(p_2{}^{\gamma}-p_3{}^{\gamma})V_0(\mathbf{p}_2-\mathbf{p}_3)\Pi(\mathbf{p}_2,\mathbf{p}_1-\mathbf{q},z_1-z)\\&=p_1{}^{\gamma}[T^0(\mathbf{p}_1,\mathbf{p}_1-\mathbf{q},z_1-z)-T^0(\mathbf{p}_1,\mathbf{p}_1-\mathbf{q},z_1)]-qT^0(p_1,p_1-q,z_1)\\&+\int\frac{d\mathbf{p}_2}{(2\pi)^3}p_2{}^{\gamma}[\tilde{G}(\mathbf{p}_2,z_1)-\tilde{G}(\mathbf{p}_2,z_1-z)]T^0(\mathbf{p}_1,\mathbf{p}_2,z_1)T^0(\mathbf{p}_2,\mathbf{p}_1-\mathbf{q},z_1-z).\end{aligned}$$

We shall consider a δ-function potential of the impurities. Then $T^0(p,q,z)=T^0(z)$, $nc_iT^0=\Sigma_{im}$. Then

$$\begin{aligned}C_{\lambda e}^{\alpha}(q,z)&=\frac{e}{m}(2Mn\Omega_{\mathbf{q}\lambda})^{-1/2}\frac{1}{(-i\beta)}\sum_{z_1}\int\frac{d\mathbf{p}_1}{(2\pi)^3}P_{\alpha\beta}(\mathbf{q})\\&\times(2p_1^{\beta}-q^{\beta})(\varepsilon_{\mathbf{q}\lambda}\mathbf{p}_1)\tilde{G}(\mathbf{p}_1,z_1)\tilde{G}(\mathbf{p}_1-\mathbf{q},z_1-z)[\Sigma_{im}(z_1-z)-\Sigma_{im}(z_1)].\end{aligned}\tag{3.4b}$$

Starting out from the expressions (3.2) and (3.4b), it is easy to establish the fact that the component containing $C_{\lambda 1}^{\alpha}$ in Eq. (2.20) does not entangle the branches of the vibrations in the isotropic case. If we consider only the absorption of transverse sound, then the remaining variational derivatives cannot be calculated because of the fact that for the transverse branches $g_{-qt}\approx 0$ at $q\ll p_0$. Taking it into account that the transverse branches for the isotropic metal differ only in the direction of the polarization vectors, we have

$$\left[z^2-\Omega_{\mathbf{q}t}{}^2-\frac{m}{M}\frac{\omega_{pe}{}^2z^2(1-Q_1(\mathbf{q},z))}{z^2-c^2q^2+4\pi cQ(\mathbf{q},z)}\right]\varphi_{\mathbf{q}t}(z)=0,\tag{3.5}$$

$$\begin{aligned}Q_1(\mathbf{q},z)&=-\frac{i}{zmn}\frac{1}{(-i\beta)}\sum_{z_1}\int\frac{d\mathbf{p}_1}{(2\pi)^3}p_1{}^2(1-x^2)\tilde{G}(\mathbf{p}_1,z_1)\\&\times\tilde{G}(\mathbf{p}_1-\mathbf{q},z_1-z)[\Sigma_{im}(z_1-z)-\Sigma_{im}(z_1)],\end{aligned}\tag{3.6a}$$

$$\begin{aligned}Q(\mathbf{q},z)&=-\frac{e^2n}{mc}\\&+\frac{i}{c}\left(\frac{e}{m}\right)^2\frac{1}{(-i\beta)}\sum_{z_1}\int\frac{d\mathbf{p}}{(2\pi)^3}p_1{}^2(1-x^2)\tilde{G}(\mathbf{p}_1,z_1)\tilde{G}(\mathbf{p}_1-\mathbf{q},z_1-z);\end{aligned}\tag{3.6b}$$

here $x=\cos(\widehat{\mathbf{q},\mathbf{p}_1})$. The expression (3.6b) is obtained from the expressions (2.16), (2.17a), (3.2a) and (3.4a).

After carrying out analytic continuation we shall assume in the study of the existence of a nontrivial solution that the damping is small in comparison with the frequency, that is, $\omega\gg\gamma$ for the complex frequency $\bar{\omega}=\omega-i\gamma$. Also taking into account the fact that $m/M\ll 1$, we obtain the following expression for the damping coefficient γ:

$$\gamma=\frac{m}{2M}\frac{\omega_{pe}{}^2}{\omega}\operatorname{Im}\frac{1-Q_1(\mathbf{q},\omega+i\delta)}{1-c^2q^2/\omega^2+4\pi cQ(\mathbf{q},\omega+i\delta)/\omega^2};\tag{3.7}$$

$\omega=\Omega_{qt}=qs_t$, where s_t is the speed of transverse sound. The absorption coefficient of transverse sound α_t is connected with γ by the simple relation

$$\alpha_t=2\gamma/s_t.$$

The relations (3.6) and (3.7) allow us to compute the absorption coefficient of transverse sound in dilute magnetic alloys.

4. ABSORPTION COEFFICIENT

The Green's function of a metal containing magnetic impurities, in the absence of a sound wave, has the form

$$\tilde{G}(\mathbf{p}_1,z_1)=[z_1-\xi_{\mathbf{p}_1}-\Sigma(z_1)]^{-1};\tag{4.1}$$

here $z_1=(m+1)\pi iT$, m is an integer, $\xi_{\mathbf{p}1}$ is the energy of the electron with momentum p_1, calculated from the Fermi energy ε_0. Since we consider an isotropic metal in the free-electron approximation, we have $\xi_{\mathbf{p}1}=p_1^2/2m-\varepsilon_0$.

It is not difficult to see that the functions $Q(\mathbf{q},z)$ and $Q_1(\mathbf{q},z)$ which determine the sound absorption are expressed in terms of integrals of the form

$$F(\mathbf{q},z)=\frac{1}{-i\beta}\sum_{z_1}\int_{-1}^{1}dx\varphi(x)\int_{-\infty}^{\infty}d\xi_{\mathbf{p}_1}\frac{N(\xi_{\mathbf{p}_1})}{2}p_1{}^k\tilde{G}(\mathbf{p}_1-\mathbf{q},z_1-z)\tilde{G}(\mathbf{p}_1,z_1),\tag{4.2}$$

where k is an integer, $\varphi(x)$ is some function of x, $N(\xi_{\mathbf{p}1})$ is the density of electron states. For improving the convergence, we rewrite (4.2) in the form

$$\begin{aligned}F(\mathbf{q},z)&=\frac{1}{-i\beta}\sum_{z_1}\int_{-1}^{1}dx\varphi(x)\int_{-\infty}^{\infty}d\xi_{\mathbf{p}_1}\frac{N(\xi_{\mathbf{p}_1})}{2}p_1{}^k\\&\times[\tilde{G}(\mathbf{p}_1-\mathbf{q},z_1-z)\tilde{G}(\mathbf{p}_1,z_1)-\tilde{G}_0(\mathbf{p}_1-\mathbf{q},z_1-z)\tilde{G}_0(\mathbf{p}_1,z_1)]\\&+\int_{-1}^{1}dx\varphi(x)\int_{-\infty}^{\infty}d\xi_{\mathbf{p}_1}\frac{N(\xi_{\mathbf{p}_1})}{2}p_1{}^k\frac{1}{(-i\beta)}\sum_{z_1}\tilde{G}_0(\mathbf{p}_1-\mathbf{q},z_1-z)\tilde{G}_0(\mathbf{p}_1,z_1),\end{aligned}\tag{4.3}$$

where $\bar{G}_0(p,z)=[z-\xi_{\mathbf{p}}]^{-1}$. The second component in (4.3) is calculated in elementary fashion and yields

$$\frac{N(0)ip_0{}^k}{2}\int_{-1}^{1} dx\,\frac{v_0qx\varphi(x)}{z-v_0qx}$$

($v_0=p_0/m$ is the Fermi velocity). The expression in square brackets in the first term in (4.3) differs from zero in a narrow region $\sim\Sigma\ll\varepsilon_0$ near the Fermi energy; therefore,

$$F(\mathbf{q},z)=\int_{-1}^{1}dx\varphi(x)\frac{N(0)p_0{}^k}{2}\frac{1}{-i\beta}\sum_{z_1}\int d\xi_{p_1}$$
$$\times\tilde{G}(\mathbf{p}_1-\mathbf{q},z_1-z)\tilde{G}(\mathbf{p}_1,z_1)-\frac{N(0)ip_0{}^k}{2}\int_{-1}^{1}dx\varphi(x). \tag{4.4}$$

Calculating the second term in (4.4) and using the expressions (3.6a) and (3.6b), and also the fact that the total sound vector q is connected with the frequency ω by the relation $\omega=qs_t$, we have

$$Q(\mathbf{q},\omega+i\delta)=-\frac{e^2n}{mc}\tilde{Q}(\omega)=\frac{3}{4}\frac{e^2n}{mc}\int_0^1 dyJ(\omega,y), \tag{4.5}$$

$$Q_1(\mathbf{q},\omega+i\delta)=Q_1(\omega)=-\frac{3}{4\omega}\int_0^1 dy\Phi_0(\omega,y)J(\omega,y), \tag{4.6}$$

$$\Phi_0(\omega,y)=\Sigma_{im}(\omega y+i\delta)-\Sigma_{im}(\omega y-\omega-i\delta), \tag{4.7}$$

$$J(\omega,y)=\frac{\omega\tau(\omega,y)}{a^3}\left\{\frac{1}{2}(a^2-b^2+1)\ln\frac{1+(a+b)^2}{1+(a-b)^2}\right.$$
$$+2b[a-\operatorname{arctg}(a+b)-\operatorname{arctg}(a-b)]-$$
$$\left.-i\left[(a^2-b^2+1)(\operatorname{arctg}(a+b)+\operatorname{arctg}(a-b))+b\ln\frac{1+(a+b)^2}{1+(a-b)^2}-2a\right]\right\}, \tag{4.8}$$

$$a\equiv a(\omega,y)=v_0\omega\tau(\omega,y)/s_t,$$
$$b\equiv b(\omega,y)=[\omega-\operatorname{Re}\Phi_0(\omega,y)]\tau(\omega,y), \tag{4.9}$$

$$\tau(\omega,y)=-\{\operatorname{Im}\Phi_0(\omega,y)\}^{-1}. \tag{4.10}$$

The relations (3.7) and (4.5)–(4.10) allow us to calculate the transverse sound absorption by the electron subsystem, by starting from the electron self-energy part $\Sigma_{im}(\omega)$.

We now consider dilute magnetic alloys with disordered magnetic impurities. It can be shown that for such a system $\operatorname{Re}\Sigma_{im}$ is negligibly small, while $\operatorname{Im}\Sigma_{im}(\omega+i\delta)=-i\Gamma(\omega)/2$, where $\Gamma(\omega)>0$ is the reciprocal of the lifetime of the elementary excitations. In the expressions (4.5)–(4.10) we have $\omega y\lesssim\omega$ and $\omega-\omega y\lesssim\omega$. If $\omega\lesssim T$, then it can be assumed that the quantity Γ does not depend on the frequency and is equal to

$$\Gamma(0)=\frac{c_sn}{\pi N(0)}\left[1+\ln\left(\frac{T_k}{T}\right)\left\{\ln^2\left(\frac{T_k}{T}\right)+\pi^2S(S+1)\right\}^{-1/2}\right]+\Gamma_i, \tag{4.11}$$

$\Gamma_i=\text{const}$ is the reciprocal of the lifetime of the electron excitations connected with scattering of the electrons by nonmagnetic impurities, $T_k\approx\varepsilon_0\exp[-n/JN(0)]$ is the Kondo temperature. Here the quantities a, b, Φ_0, and τ become constants, which facilitates the problem of calculation of the integrals in (4.5) and (4.6). In the limit $\omega\ll\Gamma(0)$ we obtain

$$\alpha_t=\frac{nm\Gamma(0)}{\rho s_t}\left[\frac{1}{\eta(a)}-1\right]; \tag{4.12}$$

ρ is the density of the metal, $a=v_0\omega/s_t\Gamma(0)$:

$$\eta(a)=3[(1+a^2)\operatorname{arctg}a-a]/2a^3. \tag{4.13}$$

The presented expression for α_t is identical with the expression obtained in the phenomenological theory at $\tau^{-1}=\Gamma(0)$. Such an identity will always be the case when the self-energy part of the electrons can be represented in the form

$$\Sigma(\omega)=-i(2\tau)^{-1}\operatorname{sign}\omega,$$

τ is the relaxation time, that is, in those cases in which the τ approximation works well. We note that the function $\Gamma(\omega)$ is constant only in the region $\omega\lesssim T$. The fact that the expression for the absorption coefficient α_t contains $\Gamma(0)$ follows from actual calculations. But, generally speaking, there is no basis in the phenomenological approach for setting $\tau=\Gamma^{-1}(0)$. Moreover, it is evidently always possible to choose the parameters ω and T so that, on the one hand, ω would be greater than T, and on the other, $\omega\ll\Gamma$. Then (4.12) becomes incorrect and α_t must be found by numerical integration of (4.5) and (4.6).

We also note that in the region $\omega\ll T$, where the relations (4.12) and (4.13) are valid, at sufficiently low sound frequencies $[\omega\ll s_t\Gamma(0)/v_0]$ α_t falls off logarithmically with decrease in temperature, and at $\omega\gg(s_t/v_0)\Gamma(0)$ it does not depend on the temperature. There is an essential difference here between dilute magnetic alloys with disordered magnetic impurities and metals with nonmagnetic impurities. In the latter, the sound absorption, which is connected with scattering of the electrons from the impurities, does not depend on the temperature throughout the entire frequency range.

We further consider dilute magnetic alloys with ordering of the spin-glass type. As is known, spin glasses are characterized by a finite value of the magnetization at each impurity site in the absence of a total magnetization of the sample. Thus there exists a certain random internal field which "freezes in" the impurity spins. It is interesting that the dependence of the character of the behavior of the electron sound absorption coefficient on the frequency can be elucidated without taking recourse to a particular model of the glass. For this purpose, we consider the dependence $\Gamma(\omega)=-2\operatorname{Im}\Sigma_{im}(\omega+i\delta)$. In low frequency regions, the quantity Γ is determined by elastic processes of electron scattering without spin flip and, consequently, Γ is equal to some constant Γ_0 independent of the frequency. In the high frequency region, $\omega\gg E$, where E is the mean value of the energy necessary to change the orientation of the spin of the impurity,[1)] the value of Γ is also constant, while $\Gamma=\Gamma_\infty>\Gamma_0$, since the quantity Γ_∞ is determined also by inelastic processes of electron scattering with spin flip. In the intermediate frequency range $\omega\sim E$, a smooth increase of Γ takes place from the purely elastic value Γ_0 to the value Γ_∞. The region of frequencies where the quantity Γ depends significantly on the frequency is obviously determined by the dispersion of the internal field ΔE. The values of the parameters Γ_0, Γ_∞, E, ΔE depend on the specific model of the spin glass. Knowing the dependence of $\Gamma(\omega)$ from the dispersion relations, it is not difficult to find $\operatorname{Re}\Sigma_{im}(\omega)$, which has a peak at $\omega\sim E$. In connection with the fact that the quantities $Q(\omega)$ and $Q_1(\omega)$ are connected with $\Gamma(\omega)$ by the integral relations (4.5) and (4.6), in the limit in which $\Delta E \ll E$, the character of the approximation $\Gamma(\omega)$ in the transitional region is unimportant. Naturally, the integrals in (4.5) and (4.6) cannot be calculated analytically. However, we can analyze the behavior of the absorption coefficient in the different frequency regions.

1) At $\omega \ll E$, $\Gamma(\omega)=\Gamma_0$, $\mathrm{Re}\Sigma_{im}=0$, and the expression (4.12) with $\tau=\Gamma_0^{-1}$ is obtained for the sound absorption coefficient. We note that Γ_0 for spin glass is not identical with $\Gamma(0)$ for a metal with disordered magnetic impurities.

2) The region $\omega \gtrsim E$. Since the characteristic values of the quantity E have orders from tenths of a degree to tens of degrees, it is not difficult to estimate that at $\omega \gtrsim E$ the inequality $c^2q^2/\omega^2 \gg 4\pi c\,|\tilde{Q}(\omega)|/\omega^2$ holds. Then, under the assumption that $\omega\tau(\omega, y) \gtrsim 1$, we obtain the following expression for α_t with accuracy to within terms of order s_t^2/v_0^2:

$$\alpha_t=\frac{nm}{\rho s_t}\frac{1}{\varkappa^4\omega^3}\{(1-\mathrm{Re}\,Q_1(\omega))\,\mathrm{Im}\,\tilde{Q}(\omega)+\mathrm{Im}\,Q_1(\omega)\,\varkappa^2\omega^2\}, \qquad (4.14)$$

where the function $Q_1(\omega)$ and $\tilde{Q}(\omega)$ are given by the expressions (4.5)–(4.10) with Σ_{im} for the spin glass and $a(\omega, y) \gg b(\omega, y) \geqslant 1$, $\varkappa=c/s_t\omega_{pe}$. In the considered frequency range we have in metals without magnetic impurities $\alpha_t \propto \omega^{-3}$.[9] This result can be obtained from (4.14) if we use the corresponding electron self-energy part for the calculation of the functions Q_f and $\tilde{Q}$. In the case of nonmagnetic impurities, for example, $\mathrm{Re}\Sigma_{im}$ is negligibly small, in this case $\mathrm{Im}Q_1 \approx 0$, $\mathrm{Im}\tilde{Q} \sim s_t/v_0$ and depends weakly on ω, and $\mathrm{Re}Q_1(\omega) \sim s_t/v_0$, i.e., the principal term in α_t is proportional to ω^{-3}. The situation is different in spin glasses. The electron spectrum in these systems is significantly rearranged and as a consequence of this, $\mathrm{Im}Q_1(\omega) \sim \mathrm{Im}\tilde{Q}(\omega) \sim s_t/v_0$ in (4.14). This means that $\alpha_t \propto \omega^{-1}$ at $\varkappa\omega \sim 1$.

5. CONCLUSION

We estimate under what limitations on the introduced parameters do the specific features of the diluted magnetic alloys appear in the sound absorption. In the calculation of α_t we have neglected the quantities Σ_{ee} and Σ_{ph} in comparison with Σ_{im}. The quantity Σ_{ee} (we recall that this is only the collisional part of the electron-electron self-energy) in good metals is always much less than Σ_{im}. So far as the quantity Σ_{ph} is concerned, its smallness in comparison with Σ_{im} can always be insured by choice of a sufficiently low temperature.

In disordered dilute magnetic alloys, the principal feature is the temperature dependence of the absorption coefficient. It appears at low sound frequencies $[\omega \ll \Gamma(0)s_t/v_0,\ \omega \ll T]$. For temperatures $T \sim 1$ K and concentrations of the magnetic impurities $c_s \sim 10^{-3}$, the indicated temperature dependence should be observed for sound with frequency $\omega \lesssim 10^7-10^8$ sec^{-1}.

In spin glasses, as analysis of the relation (4.14) shows, the absorption coefficient at $\omega \sim E$ can vary as ω^{-1}. The relation (4.14) is valid at $\varkappa^2\omega^2 \gg s_t/v_0$, i.e., for frequencies $\omega \gtrsim 10^9-10^{10}$ sec^{-1}, while $\alpha_t \propto \omega^{-1}$ at $\varkappa\omega \gtrsim 1$, i.e., $\omega \gtrsim 10^{10}-10^{11}$ sec^{-1}. For $E \sim 0.1-1$ K (10^9-10^{10} sec^{-1}) the range of frequencies $\omega \gtrsim E$ again corresponds to the region of validity of the relation (4.14). This result is obtained without the use of a specific model of spin glass, i.e., it is a manifestation of the basic, determinative properties of such systems. However, it must be noted that upon further increase in the frequency (4.14) ceases to be valid, since the sound wave vector q becomes comparable with p_0 and, consequently, $g_{qt} \neq 0$. Then the relations (4.5)–(4.10) are no longer operative. A purely deformation mechanism of sound absorption begins to dominate, leading to a dependence of the absorption coefficient on frequency, $\alpha_t \sim \omega$.

[1)] In the corresponding units, E is equal to the mean value of the modulus of the internal field acting on the impurity.

---

[1] K. Fisher, Phys. St. Sol. 46, 11 (1971).
[2] G. Grüner and A. Zawadowski, Rep. Prog. Phys. 37, 1497 (1974).
[3] J. Kondo, Progr. Theor. Phys. 33, 575 (1965).
[4] Yu. M. Ivanchenko and A. A. Lizyanskiĭ, Zh. Eksp. Teor. Fiz. 71, 2131 (1976) [Sov. Phys. JETP 44, 1121 (1976)].
[5] A. B. Pippard, Phil. Mag. 46, 1104 (1955).
[6] A. I. Akhiezer, M. I. Kaganov and G. Ya. Lyubarskiĭ, Zh. Eksp. Teor. Fiz. 32, 837 (1957) [Sov. Phys. JETP 5, 685 (1957)].
[7] S. Liu and A. M. Toxen, Phys. Rev. 138A, 487 (1965).
[8] A. Ramakanth and S. K. Ghatak, J. Phys. F.: Met. Phys. 3, 1796 (1973).
[9] V. L. Gurevich, I. G. Lang and S. T. Pavlov, Zh. Eksp. Teor. Fiz. 59, 1679 (1970) [Sov. Phys. JETP 32, 914 (1971)].
[10] Yu. M. Gal'perin, Zh. Eksp. Teor. Fiz. 67, 2195 (1974) [Sov. Phys. JETP 40, 1088 (1975)].
[11] L. Kadanoff and I. Falko, Phys. Rev. 136A, 1170 (1964).
[12] T. Tsuneto, Phys. Rev. 121, 402 (1961).
[13] L. Kadanov and G. Baym, Quantum statistical Mechanics, New York, N. Y. 1962. Russian translation, Mir, 1964.
[14] J. Schrieffer, Theory of Superconductivity, New York, N. Y., 1964. Russian translation, Nauka, 1970.
[15] Yu. M. Ivanchenko and G. I. Trush, Zh. Eksp. Teor. Fiz. 67, 351 (1974) [Sov. Phys. JETP 40, 175 (1975)].

Translated by R. T. Beyer