determined by the anharmonicity. The anharmonicity constant for the LO turned out to be close to the value of the constant for the high-frequency oscillations of the crystal lattice.

[1)]The authors of Ref. 5, who measured the spectrum of neutrons scattered by two-phase Cu-Be samples, were unable to observe this singularity because of the experimental conditions.

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Nonlinear waves and the dynamics of domain walls in weak ferromagnets

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Nonlinear waves are investigated in a two-sublattice antiferromagnet with noncollinear sublattices. Simple magnetization waves, describing the motion of 180-degree domain walls of two types, are considered far from the spin-flip region. The specific nature of nonlinear magnetization waves within the spin-flip region is investigated. Solutions are obtained that describe two-parameter magnetic solitons, to which corresponds a periodic oscillation of the magnetization in a reference system moving with the wave. The properties of N-soliton solutions, describing the interaction of nonlinear waves, are discussed. Formulas are obtained that describe the velocity of steady-state motion of a domain wall under the action of an external magnetic field.

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In a description of the nonlinear dynamics of magnetically ordered crystals, there almost always arises the problem of the properties of isolated magnetization waves. The interest in this problem is due to the broad possibility of describing the nonlinear dynamics of a magnetic material in terms of nonlinear waves. In particular, nonlinear isolated waves describe an effect that is important practically: the motion of domain walls (DW) and of isolated magnetic domains during the magnetization reversal of magnets.[1]

The theory of nonlinear waves has been developed in greatest detail for the case of a magnet with a single sublattice. It is known, however, that in magnets with two equivalent sublattices the dynamics of nonlinear waves is different in a number of important features. Among the specific features of this system must be included the exchange character of the limiting velocity of motion of DW,[2,3] which attains tens of kilometers per second,[4-6] and the presence of certain types of nonlinear waves that correspond to single boundary conditions.[2,7]

A very interesting class of such magnets is the weak ferromagnets (WFM) (in particular, the rare-earth orthoferrites (REO)), in which the Dzyaloshinskiĭ interaction leads to noncollinearity of the sublattices; that is, there is a weak ferromagnetic moment. The presence of a magnetic moment makes it possible to control the motion of SW in REO by means of an external magnetic field, just as is done in ordinary ferromagnets.[4-6] We note that control of the motion of DW in compensated antiferromagnets is considerably more complicated (see Ref. 8).

The present paper carries out a detailed study of the dynamics of nonlinear magnetization waves in WFM. Analytic solutions of the type of a simple magnetization wave are obtained, describing the motion of DW, and also more complicated isolated waves (two-parameter magnetic solitons); the possibility is indicated of constructing N-soliton solutions, describing interaction of several isolated waves.

The specific nature of nonlinear magnetization waves in the spin-reorientation range of REO is investigated. Formulas are obtained that describe the velocity of stationary motion of DW under the action of an external magnetic field.

1. EFFECTIVE EQUATIONS OF MAGNETIZATION DYNAMICS IN A WEAK FERROMAGNET

In a study of the nonlinear dynamics of WFM, it is unusual to start from the equations for the normalized vectors of ferromagnetism $\mathbf{m}$ and antiferromagnetism $\mathbf{l}$, which are connected with the sublattice magnetization vectors $\mathbf{M}_1$ and $\mathbf{M}_2$ by the relations

$$\mathbf{m}=\frac{1}{2M_0}(\mathbf{M}_1+\mathbf{M}_2),\quad \mathbf{l}=\frac{1}{2M_0}(\mathbf{M}_1-\mathbf{M}_2), \tag{1}$$

where $|\mathbf{M}_1|=|\mathbf{M}_2|=M_0$. By virtue of (1), the vectors $\mathbf{m}$ and $\mathbf{l}$ satisfy the relations

$$\mathbf{ml}=0,\quad \mathbf{m}^2+\mathbf{l}^2=1. \tag{2}$$

We shall write the energy of a WFM of rhombic symmetry (for example, REO) in the usual form[9,10]:

$$W=M_0^2\int d\mathbf{r}\left\{\frac{\delta}{2}\mathbf{m}^2+\frac{\alpha}{2}(\nabla\mathbf{l})^2+\frac{\alpha'}{2}(\nabla\mathbf{m})^2+\frac{\beta_1 l_x^2}{2}+\frac{\beta_3 l_z^2}{2}\right.$$
$$\left.+\frac{\beta_1'}{4}l_x^4+\frac{\beta_2'}{4}l_x^2l_z^2+\frac{\beta_3'}{4}l_z^4+d_1m_xl_z-d_3m_zl_x\right\}. \tag{3}$$

Here α and α' are the constants of nonuniform and δ of uniform exchange; β_1 and β_3 are the second-order, β_1', β_2', and β_3' the fourth-order anisotropy constants; d_1 and d_3 are the Dzyaloshinskiĭ constants.

The equations of motion for the vectors $\mathbf{m}$ and $\mathbf{l}$ can be easily obtained from the Landau-Lifshitz equations for the sublattice magnetization vectors $\mathbf{M}_1$ and $\mathbf{M}_2$ (see, for example, Refs. 11 and 12):*

$$\frac{2}{gM_0}\frac{\partial\mathbf{m}}{\partial t}=[\mathbf{m}\times\mathbf{H}_m]+[\mathbf{l}\times\mathbf{H}_l],\quad \frac{2}{gM_0}\frac{\partial\mathbf{l}}{\partial t}=[\mathbf{m}\times\mathbf{H}_l]+[\mathbf{l}\times\mathbf{H}_m], \tag{4}$$

where g is the gyromagnetic ratio, and where

$$\mathbf{H}_m=-\frac{1}{M_0^2}\frac{\delta W\{\mathbf{m},\mathbf{l}\}}{\delta\mathbf{m}},\quad \mathbf{H}_l=-\frac{1}{M_0^2}\frac{\delta W\{\mathbf{m},\mathbf{l}\}}{\delta\mathbf{l}}. \tag{5}$$

Using the specific form (3) of the energy, we can write the equations of motion of the magnetization of a WFM in the following form:

$$\frac{2}{gM_0}\frac{\partial\mathbf{m}}{\partial t}=\alpha[\mathbf{l}\times\Delta\times\mathbf{l}]+\alpha'[\mathbf{m}\times\Delta\mathbf{m}]-d_1(l_z[\mathbf{m}\times\mathbf{e}_x]+m_x[\mathbf{l}\times\mathbf{e}_z])+d_3(l_x[\mathbf{m}\times\mathbf{e}_z]$$
$$+m_z[\mathbf{l}\times\mathbf{e}_x])-(\beta_1l_x+\beta_1'l_x^3+{}^1/_2\beta_2'l_xl_z^2)[\mathbf{l}\times\mathbf{e}_x]-(\beta_3l_z+\beta_3'l_z^3+{}^1/_2\beta_2'l_x^2l_z)[\mathbf{l}\times\mathbf{e}_z]; \tag{6}$$

$$\frac{2}{gM_0}\frac{\partial\mathbf{l}}{\partial t}=\delta[\mathbf{l}\times\mathbf{m}]+\alpha[\mathbf{m}\times\Delta\times\mathbf{l}]+\alpha'[\mathbf{l}\times\Delta\times\mathbf{m}]-d_1(l_z[\mathbf{l}\times\mathbf{e}_x]+m_x[\mathbf{m}\times\mathbf{e}_z])$$
$$+d_3(l_x[\mathbf{l}\times\mathbf{e}_z]+m_z[\mathbf{m}\times\mathbf{e}_x])-(\beta_1l_x+\beta_1'l_x^3+{}^1/_2\beta_2'l_xl_z^2)[\mathbf{m}\times\mathbf{e}_x]-$$
$$-(\beta_3l_z+\beta_3'l_z^3+{}^1/_2\beta_2'l_zl_x^2)[\mathbf{m}\times\mathbf{e}_z]. \tag{7}$$

Here $\mathbf{e}_x$, $\mathbf{e}_y$, and $\mathbf{e}_z$ are unit vectors along the corresponding coordinate axes.

In the solution of the system of equations (6)–(7), we shall use the fact that the constant δ is determined by exchange interaction, the Dzyaloshinskiĭ constants d_1 and d_3 by exchange-relativistic interactions, and the anisotropy constants β_i and β_i' by relativistic interactions,[12] by virtue of which the inequality $\beta\ll d\ll\delta$ is usually satisfied. Furthermore, the difference between the constants d_1 and d_3 is due to the anisotropy of the crystal and is therefore much smaller than the values of d_1 and d_3 themselves: that is, $|d_1-d_3|\sim\beta\ll d$. By using also the assumption that the weak-ferromagnetism vector remains much smaller than $\mathbf{l}$ in the nonlinear wave, as it is in static solutions of equations (6) and (7) (in statics, $|\mathbf{m}|\sim(d/\delta)|\mathbf{l}|\ll|\mathbf{l}|\approx 1$), we can considerably simplify the system of equations (6)–(7).[1)] On restricting ourselves in equations (6) and (7) to the leading approximation with respect to the small parameter (β/δ), we get

$$\frac{2}{gM_0}\frac{\partial\mathbf{m}}{\partial t}=\alpha[\mathbf{l}\times\Delta\times\mathbf{l}]+[\mathbf{d}\times[\mathbf{m}\times\mathbf{l}]]-(\beta_1+\beta_1'l_x^2+{}^1/_2\beta_2'l_z^2)l_x[\mathbf{l}\times\mathbf{e}_x]$$
$$-(\beta_3+\beta_3'l_z^2+{}^1/_2\beta_2'l_x^2)l_z[\mathbf{l}\times\mathbf{e}_z], \tag{8}$$

$$\frac{2}{gM_0}\frac{\partial\mathbf{l}}{\partial t}=\delta[\mathbf{m}\times\mathbf{l}]+[\mathbf{l}\times[\mathbf{d}\times\mathbf{l}]], \tag{9}$$

where we have introduced the vector $\mathbf{d}=d_y^{\mathbf{1}}$; $d=d_1\approx d_3$.

From (9), it is easy to express the vector $\mathbf{m}$ in terms of the vector $\mathbf{l}$:

$$\delta\mathbf{m}=\frac{2}{gM_0}\left[\mathbf{l}\times\frac{\partial\mathbf{l}}{\partial t}\right]+[\mathbf{d}\times\mathbf{l}]. \tag{10}$$

On substituting this relation in equation (8), we obtain an equation containing only the vector $\mathbf{l}$:

$$\alpha[\mathbf{l}\times\Delta\times\mathbf{l}]-\frac{4}{\delta(gM_0)^2}\left[\mathbf{l}\times\frac{\partial^2\mathbf{l}}{\partial t^2}\right]+\frac{1}{\delta}(\mathbf{d}\times\mathbf{l})[\mathbf{d}\times\mathbf{l}]$$
$$-(\beta_1+\beta_1'l_x^2+{}^1/_2\beta_2'l_z^2)l_x[\mathbf{l}\times\mathbf{e}_x]-(\beta_3+\beta_3'l_z^2+{}^1/_2\beta_2'l_x^2)l_z[\mathbf{l}\times\mathbf{e}_z]=0. \tag{11}$$

It is easy to show that among the nonlinear waves satisfying this equation there are two classes of solutions, in one of which $l_y=0$, and in the other $l_z=0$. Using the fact that in our approximation the length of the vector $\mathbf{l}$ is constant ($\mathbf{l}^2=1-\mathbf{m}^2\approx 1$), we transform to the angular variables θ and φ:

$$l_x=\cos\theta,\quad l_y=\sin\theta\sin\varphi,\quad l_z=\sin\theta\cos\varphi. \tag{12}$$

In these variables, the first class of solutions corresponds to $\varphi=0$, the second to $\varphi=\pi/2$. In both cases the angle θ is determined by the equation

$$\alpha\Delta\theta-\frac{4}{\delta(gM_0)^2}\frac{\partial^2\theta}{\partial t^2}+2(K_1+8K_2)\sin\theta\cos\theta-32K_2\sin^3\theta\cos\theta=0, \tag{13}$$

where K_1 and K_2 are the effective anisotropy constants ordinarily used to describe REO.[9,10] For the first class of solutions ($l_y=0$ or $\varphi=0$),

$$K_1^{(1)}={}^1/_2(\beta_1-\beta_3)+{}^1/_4(\beta_1'-\beta_3'),\quad K_2^{(1)}={}^1/_{32}(\beta_1'-\beta_2'+\beta_3'), \tag{14}$$

for the second class of solutions ($l_z=0$ or $\varphi=\pi/2$),

*$[\mathbf{m}\mathbf{H}_m]=\mathbf{m}\times\mathbf{H}_m$, etc.

$$K_1^{(2)}=-\frac{d^2}{2\delta}+\frac{\beta_1}{2}+\frac{\beta_1'}{4},\quad K_2^{(2)}=\frac{1}{32}\beta_1'. \tag{15}$$

To these two classes of solutions correspond two different types of nonlinear waves (in particular, DW), satisfying identical boundary conditions. As we shall show, depending on the relations between the effective anisotropy constants, one of these waves turns out to be absolutely unstable.

In concluding this section, we note that equations (11) can be obtained from the following expression for the Lagrangian of a WFM, written in terms of the vector $\mathbf{l}$ alone:

$$\mathscr{L}=M_0^2\left\{\frac{2}{\delta(gM_0)^2}\left[\left(\frac{\partial\mathbf{l}}{\partial t}\right)^2-c^2(\nabla\mathbf{l})^2\right]-W_a(\mathbf{l})\right\},$$

where $c=\frac{1}{2}gM_0(\alpha\delta)^{1/2}$ coincides with the minimum spin-wave velocity of the linear theory of WFM, and

$$W_a=-(K_1^{(2)}+8K_2^{(2)})l_y^2+8K_2^{(2)}l_y^4-(K_1^{(1)}+8K_2^{(1)})l_z^2$$
$$+8K_2^{(1)}l_z^4+{}^1/_4(2\beta_1'-\beta_2')l_y^2l_z^2.$$

The corresponding Hamiltonian function determines the expression for the energy density of the WFM, written in terms of the vector $\mathbf{l}$.

2. ISOLATED WAVES IN WEAK FERROMAGNETS AND THEIR INTERACTION

As a rule, the second-order anisotropy constants β_i are substantially larger than the fourth-order anisotropy constants β_i'. Since the parameter K_2, in contrast to K_1, is determined solely by the value of β_i', satisfaction of the inequality $K_2 \ll K_1$ is to be expected. But bearing in mind the use of the results for description of actual magnetic materials, we must allow for the possibility that at certain values of the external conditions K_1 changes sign, so that near this point the inequality $K_2 \ll K_1$ is violated. It is known that this occurs in a number of WFM (several REO, α-Fe_2O_3, etc.) and leads to the phenomenon of spin reorientation.

Far from the reorientation region, K_2 may be omitted; then equation (13) permits a considerably more complete analysis, which is carried out in this section. Near and within the reorientation region (which usually occupies 10–20 K), it is necessary to take account of K_2 along with K_1, and this leads to important singularities in the structure of isolated waves (see Section 3).

It is known that when $K_1 \gg K_2$, the equilibrium values of the vectors $\mathbf{m}$ and $\mathbf{l}$ are oriented along crystallographic axes. We shall suppose for definiteness that in the ground state, $\mathbf{l}\parallel\mathbf{e}_x$ and $\mathbf{m}\parallel\mathbf{e}_z$ (this corresponds to $K_1^{(1,2)}<0$); that is, the equilibrium value of the angle θ is 0 or π. We shall furthermore restrict ourselves to the case of plane waves, supposing for definiteness that $\theta=\theta(y,t)$. Since we are interested in isolated waves, we must take as boundary conditions

$$\theta=0,\pi;\quad \partial\theta/\partial y=0 \tag{16}$$

far from the wave.

When $K_2=0$, equation (13) takes the form of the well-studied sinusoidal Klein-Gordon equation (sine-Gordon equation; see, for example, Ref. 13) for value 2θ:

$$2\left(\frac{\partial^2\theta}{\partial y^2}-\frac{1}{c^2}\frac{\partial^2\theta}{\partial t^2}\right)-\varkappa_{1,2}^2(0)\sin 2\theta=0, \tag{17}$$

which permits complete integration by the method of the inverse problem of scattering theory.[14] Here the following symbols have been introduced:

$$c={}^1/_2gM_0(\alpha\delta)^{1/2},\quad \varkappa_{1,2}(0)=[-2K_1^{(1,2)}/\alpha]^{1/2}. \tag{18}$$

The quantity c, the same for both classes of solutions ($\varphi=0$ and $\varphi=\pi/2$), is the minimum phase velocity of spin waves in WFM, the same for both spin-wave branches. The parameter $\varkappa_{1,2}(0)$ has different values for the two classes of solutions.

We consider solutions of (17) of the simple-wave type, in which $\theta=\theta(\xi)$, $\xi=y-Vt$, V = velocity of the wave. On integrating equation (17) with use of (16), we get

$$\operatorname{tg}\theta=1/\operatorname{sh}[\varkappa_{1,2}(V)\xi]. \tag{19}$$

This solution describes a 180-degree DW moving along the y axis with velocity V; the quantities $\varkappa_{1,2}(V)$ have the meaning of inverse thickness of DW of the two types and are determined by the relations

$$\varkappa_{1,2}(V)=\frac{\varkappa_{1,2}(0)}{(1-V^2/c^2)^{1/2}}. \tag{20}$$

The variation (20) of the thickness of a DW with its velocity was obtained in Ref. 2 by another method.

Thus in REO under the prescribed boundary conditions (16), there can exist two types of moving DW. DWI corresponds to $\varphi=0$; that is, $\mathbf{l}$ rotates in the (XZ) plane,

$$l_x=\operatorname{th}[\varkappa_1(V)\xi],\quad l_y=0,\quad l_z=1/\operatorname{ch}[\varkappa_1(V)\xi], \tag{21}$$

and the vector $\mathbf{m}$ is determined by the relation

$$m_x=\frac{d}{\delta}\frac{1}{\operatorname{ch}[\varkappa_1(V)\xi]},\quad m_y=\frac{2V}{\delta gM_0}\frac{\varkappa_1(V)}{\operatorname{ch}[\varkappa_1(V)\xi]},\quad m_z=-\frac{d}{\delta}\operatorname{th}[\varkappa_1(V)\xi]. \tag{22}$$

In DWII ($\varphi=\pi/2$), $\mathbf{l}$ rotates in the (XY) plane,

$$l_x=\operatorname{th}[\varkappa_2(V)\xi],\quad l_y=1/\operatorname{ch}[\varkappa_2(V)\xi],\quad l_z=0. \tag{23}$$

and the vector $\mathbf{m}$ is directed along the z axis and varies only in magnitude:

$$m_x=m_y=0,\quad m_z=-\left[\frac{d}{\delta}\operatorname{th}[\varkappa_2(V)\xi]+\frac{2V}{\delta gM_0}\frac{\varkappa_2(V)}{\operatorname{ch}[\varkappa_2(V)\xi]}\right]. \tag{24}$$

When $V=0$, the expressions (21)–(24) reduce to the well-known relations that describe stationary DW of the two types in REO.[15,16] The energy of both walls varies with velocity in a "relativistic" manner[2]:

$$\sigma_{1,2}(V)=\frac{\sigma_{1,2}(0)}{(1-V^2/c^2)^{1/2}}, \tag{25}$$

where $\alpha_{2,1}(0)=2\alpha M_0^2\varkappa_{1,2}(0)$ is the energy of stationary DW of the two types[15,16] per unit area of the DW.

We shall discuss the approximations made in the derivation of equation (11) and then of (13). First, it was assumed that in the nonlinear wave the weak-ferromagnetism vector $\mathbf{m}$ remains small in magnitude: $|\mathbf{m}|\ll 1$. This corresponds (see (22) and (24) and also (20)) to the inequality

$$V\varkappa(V)/\delta g M_0 \ll 1 \quad \text{or} \quad (c-V)/c \gg K_1/\delta. \tag{26}$$

Thus the relations obtained above, describing a moving DW, lose their significance in a narrow ($\sim\beta/\delta \ll 1$) interval of DW velocities near the limiting velocity c. It must be noted that this limitation is one of principle, since the condition that the DW shall be macroscopic (that the DW thickness $1/\varkappa(V)$ shall be much larger than the lattice constant a) also leads to the condition (26). As was noted in Ref. 2, when the condition (26) is violated a description in terms of the macroscopic equations (4) is inapplicable, and when $(c-V)/c \lesssim \beta/\delta$ it is necessary to start from a quantum-mechanical description of a discrete lattice of spins of a magnetically ordered crystal, on the basis of a specific model of the exchange interaction.

Our second simplification consists in the fact that we considered only a limited class of solutions of the equations (11) of magnetization dynamics, in which the angle φ is constant and equal to 0 or $\pi/2$ (the vector **l** lies in the (XZ) or (XY) plane, respectively). It is therefore necessary to investigate the stability of these solutions with respect to departure of **l** from the corresponding plane. By starting from the general equation (11) and setting $\beta_i'=0$, it is easy to obtain equations for the angles $\theta(y,t)$ and $\varphi(y,t)$. To investigate the stability of moving DWI and DWII, we linearize these equations about the solutions found, writing

$$\theta(y,t)=\theta_0(\xi)+\vartheta(y,t), \quad \varphi(y,t)=\varphi_0+\psi(y,t); \quad \vartheta,\psi\ll 1,$$

where $\varphi_0=0$ and $\pi/2$ for DWI and DWII respectively, and where $\theta_0(\xi)$ is determined by the expression (19). For DWI it is easily found that $\vartheta\sim\psi^2$ and that ψ is determined by the equation

$$\frac{\partial^2\psi}{\partial y^2}-\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2}+2\varkappa_1(V)\,\mathrm{th}[\varkappa_1(V)\xi]\left(\frac{\partial\psi}{\partial y}+\frac{V}{c^2}\frac{\partial\psi}{\partial t}\right)+\frac{(K_1^{(1)}-K_1^{(2)})}{\alpha}\psi=0, \tag{27}$$

whose solution can be written in the form

$$\psi=\psi_0\exp\left\{\pm\lambda\left(t-\frac{yV}{c^2}\right)\right\}, \quad \lambda=c^2\left(\frac{K_1^{(2)}-K_1^{(1)}}{\alpha(c^2-V^2)}\right)^{1/2}. \tag{28}$$

Therefore DWI is stable if $K_1^{(1)}>K_1^{(2)}$ and is unstable in the contrary case. Taking into account also that for stability of the ground state with $\mathbf{l}\parallel\mathbf{e}_x$ it is necessary that $K_1^{(1)}<0$, we finally write the condition for existence of DWI in the form

$$K_1^{(2)}<K_1^{(1)}<0 \quad (\text{DW I stable}). \tag{29}$$

Investigation of the stability of DWII proceeds similarly and leads to the following condition:

$$K_1^{(1)}<K_1^{(2)}<0 \quad (\text{DW II stable}). \tag{30}$$

The conditions (29) and (30) are contradictory; that is, in WFM with a given relation between $K_1^{(1)}$ and $K_1^{(2)}$ only one of the DW can be stable. That DW is stable to which corresponds the smaller value of $\varkappa(V)$; that is, the smaller energy. The other DW is not metastable but absolutely unstable (this is valid, in particular, also for a stationary DW; see (28)). Thus in each actual specimen, only one of the two types of DW can exist. Since the parameters $K_1^{(1)}$ and $K_1^{(2)}$ in general vary differently with temperature, the difference $K_1^{(1)}-K_1^{(2)}$ may change sign at some value of the temperature; then there should be a hysteresisless[3)] transition DWI $\rightleftharpoons$ DWII. This phenomenon has been observed in dysprosium orthoferrite[17] far from the spin-reorientation region (which is located near 40 K). Below 150 K, DWII is observed; above, DWI (in our terminology).

Above, we considered solutions of equation (17) that have the form of a stationary-profile wave; they describe moving 180-degree DW. But as has been mentioned, the sine-Gordon equation (17) permits complete integration by the method of the inverse problem of scattering theory,[14] and this makes it possible to carry out a very complete description of the nonlinear dynamics of WFM.

In investigation of equation (17), it is convenient to use the transformation[13]

$$u=\mathrm{tg}(\theta/2), \tag{31}$$

since the equation for u permits separation of the variables, and its solution can be expressed in terms of elliptic functions. An important special case of these solutions is the solution of Perring and Skyrme,[13]

$$\mathrm{tg}\frac{\theta}{2}=\frac{V}{c}\frac{\mathrm{sh}[\varkappa(V)y]}{\mathrm{ch}[\varkappa(V)Vt]} \tag{32}$$

which describes the interaction of two DW moving toward each other with velocity V. Another known solution is the two-parameter solution (parameters ω and V)

$$\mathrm{tg}\frac{\theta}{2}=\frac{(\varkappa^2(0)-\omega^2/c^2)^{1/2}}{\omega/c}\frac{\cos[\omega(t-yV/c^2)/(1-V^2/c^2)^{1/2}]}{\mathrm{ch}[(y-Vt)(\varkappa^2(0)-\omega^2/c^2)^{1/2}/(1-V^2/c^2)^{1/2}]} \tag{33}$$

which exists when $V<c$ and $\omega<\varkappa(0)c$ and describes a moving localized magnetic soliton, in which the magnetization approaches the same value $\theta=0$ for $\xi\to+\infty$ and for $\xi\to-\infty$. A characteristic of this soliton is a periodic motion of the vectors **l** and **m** in a reference system moving with the soliton, at frequency $\omega'=\omega(1-V^2/c^2)^{1/2}$.

For equation (17) one can also obtain N-soliton solutions, describing the interaction of several nonlinear waves in the WFM. A general property of these many-soliton solutions is the satisfaction of an asymptotic superposition principle. The essence of this principle consists in the fact that if isolated waves enter into interaction, then in the course of time they emerge from the region of interaction, reestablishing for $t\to+\infty$ their original form and velocity.

3. NONLINEAR WAVES IN THE SPIN-REORIENTATION REGION

We turn now to investigation of the vicinity of the spin-reorientation region, in which one of the parameters K_1 ($K_1^{(1)}$ or $K_1^{(2)}$) becomes comparable with the corresponding parameter K_2. For concreteness we shall suppose that $K_1^{(2)}<K_1^{(1)}$; that is, the reorientation occurs by rotation of **l** in the (XZ) plane and is described by the parameters $K_1^{(1)}$ and $K_2^{(1)}$ (we shall hereafter omit the upper indices). Then the solutions of the

first class, $\varphi=0$, in particular DWI, are stable. The case of spin reorientation in the (YZ) plane, described by the parameters $K_1^{(2)}$ and $K_2^{(2)}$ and corresponding to stability of the solutions of the second class ($\varphi=\pi/2$, DWII), is treated analogously, and we shall not discuss it.

It is well known that the sign of the parameter K_2 determines the number of homogeneous phases that can be realized in the REO. When $K_2>0$, three ground states are possible: $\Phi_\parallel(\mathbf{l}\parallel\mathbf{e}_x)$, existing for $K_1<-8K_2$; $\Phi_\perp(\mathbf{l}\parallel\mathbf{e}_z)$, existing for $K_1>8K_2$; and also $\Phi_<$, in which the vector $\mathbf{l}$ lies in the (XZ) plane and makes an angle θ_0 with the x axis, where θ_0 is determined by the relation

$$\sin^2\theta_0=(K_1+8K_2)/16K_2. \tag{34}$$

Spin reorientation from phase $\Phi_\parallel$ to phase $\Phi_\perp$ is accomplished by means of two phase transitions of second order: $\Phi_\parallel\to\Phi_<$ and $\Phi_<\to\Phi_\perp$, at $K_1=-8K_2$ and $K_1=8K_2$ respectively.

When $K_2<0$, only two homogeneous phases, $\Phi_\parallel$ and $\Phi_\perp$, are possible in REO; their existence ranges ($K_1<-8K_2$ and $K_1>8K_2$) overlap. Spin reorientation occurs at $K_1=0$ as a phase transition of first order.

To describe nonlinear waves in the spin-reorientation region, we shall start from equation (13). If we restrict ourselves to the study solely of stationary-profile solutions, then it is convenient to rewrite equation (13) in the form

$$\left(1-\frac{V^2}{c^2}\right)\theta''-\frac{16K_2}{\alpha p}(1+2p\sin^2\theta)\sin\theta\cos\theta=0, \tag{35}$$

where we have introduced the parameter

$$p=-8K_2/(K_1+8K_2). \tag{36}$$

The form of the nonlinear waves of stationary profile is determined by the sign of K_2 and the relation between K_1 and K_2.

We shall first consider the case $K_2>0$.

a) Let $K_1<-8K_2$. Then the phase $\Phi_\parallel$, in which $\mathbf{l}\parallel\mathbf{e}_x$ and $\mathbf{m}\parallel\mathbf{e}_z$, is stable; that is, the boundary conditions have the form (16). The corresponding solution of equation (35) has the form

$$\operatorname{tg}\theta=\frac{1}{(1+p)^{1/2}\operatorname{sh}[\varkappa_\parallel(V)\xi]} \tag{37}$$

where

$$\varkappa_\parallel(V)=4\left[\frac{K_2}{\alpha p(1-V^2/c^2)}\right]^{1/2}=\left[-\frac{2(K_1+8K_2)c^2}{\alpha(c^2-V^2)}\right]^{1/2}. \tag{38}$$

For this solution $\theta=0$ for $\xi\to+\infty$ and $\theta=\pi$ for $\xi\to-\infty$; that is, it describes a moving 180-degree DW in the phase $\Phi_\parallel$. The energy of such a wall is determined by the expression

$$\sigma_\parallel(V)=4M_0^2\left[\frac{\alpha K_2}{p(1-V^2/c^2)}\right]^{1/2}\left[1+(1+p)\frac{\operatorname{arctg}p^{1/2}}{p^{1/2}}\right]. \tag{39}$$

Far from the spin-reorientation region, where $|K_1|\gg 8K_2$ and $p\ll 1$, the expression (37) and (39) reduce to the relations (19) and (25) obtained in the preceding section.

On approach to the phase-transition point ($K_1\to-8K_2$), $\varkappa_\parallel(V)\to 0$; that is, the DW "spreads." At the phase-transition point $\Phi_\parallel\to\Phi_<$ itself ($K_1=-8K_2$), the solution of equation (35) takes the form

$$\operatorname{tg}\theta=\frac{1}{4\xi}\left(\frac{\alpha(1-V^2/c^2)}{K_2}\right)^{1/2}. \tag{40}$$

This solution describes a so-called 180-degree "algebraic" DW, in which the vector $\mathbf{l}$ approaches its equilibrium value not exponentially, as is usual [see (37)], but according to a power law.[4)] The energy of such a DW is finite and is determined by the expression

$$\sigma=2\pi M_0^2\left[\frac{\alpha K_2}{1-V^2/c^2}\right]^{1/2}. \tag{41}$$

b) Let $-8K_2<K_1<8K_2$ ($-\infty<p<-\frac{1}{2}$). In this case the ground state of the REO is the phase $\Phi_<$, in which the vector $\mathbf{l}$ makes with the x axis an angle θ_0 determined by the relation (34). It should be noted that in the phase $\Phi_<$ there exist not two ground states, as in the phase $\Phi_\parallel(\theta=0,\pi)$, but four, to which correspond angles $\theta=\pm\theta_0$, $\theta=\pi\pm\theta_0$. Analysis shows that in this phase two different forms of DW can be realized that correspond to $\varphi=0$: namely DWA, separating states with $\theta=\theta_0$ and with $\theta=-\theta_0$, and DWB, separating states with $\theta=\theta_0$ and with $\theta=\pi-\theta_0$. The corresponding two solutions of equation (35) can be written in the form

$$\operatorname{tg}\theta=\operatorname{tg}\theta_0\operatorname{th}[\varkappa_<(V)\xi], \tag{42A}$$

$$\operatorname{tg}\theta=\operatorname{tg}\theta_0\operatorname{cth}[\varkappa_<(V)\xi], \tag{42B}$$

where

$$\varkappa_<(V)=\frac{1}{4}\left[\frac{(8K_2-K_1)(8K_2+K_1)}{\alpha K_2(1-V^2/c^2)}\right]^{1/2}. \tag{43}$$

The solution (42A) describes a moving $2\theta_0$-degree DW (DWA), whose energy is determined by the formula

$$\sigma_A(V)=\frac{M_0^2}{4}\left[\frac{\alpha}{K_2(1-V^2/c^2)}\right]^{1/2}\times\left\{2K_1\operatorname{arctg}\left(\frac{8K_2+K_1}{8K_2-K_1}\right)^{1/2}+[(8K_2+K_1)(8K_2-K_1)]^{1/2}\right\}. \tag{44}$$

The solution (42B) describes a moving $(180-2\theta_0)$-degree DW (DWB), whose energy is

$$\sigma_B(V)=\sigma_A(V)-\frac{\pi K_1M_0^2}{4K_2}\left(\frac{\alpha K_2}{1-V^2/c^2}\right)^{1/2}. \tag{45}$$

When $V=0$, the expressions (42) and (44) reduce to the corresponding expressions of the paper of Ivanov and Krasnov,[22] in which stationary DW in REO were considered in the spin-reorientation region.

On approach to the phase-transition points, the amplitude of one of the DW (DWA when $\Phi_<\to\Phi_\parallel$ and DWB when $\Phi_<\to\Phi_\perp$) approaches zero, and the energy of this DW also approaches zero. The other DW (DWB when $\Phi_<\to\Phi_\parallel$ and DWA when $\Phi_<\to\Phi_\perp$) has a finite amplitude at the transition, $[\theta(+\infty)-\theta(-\infty)]=\pi$; but at the transition point this DW becomes algebraic (cf. (40)). Its energy coincides with (41).

c) Let $K_1>8K_2$, The ground state of the REO is the phase $\Phi_\perp$, in which $\mathbf{l}\parallel\mathbf{e}_z$ and $\mathbf{m}\parallel\mathbf{e}_x$; that is, $\theta_0=\pm\pi/2$. The solution of equation (35) for such boundary conditions describes a 180-degree DW in the phase $\Phi_\perp$:

$$\operatorname{ctg}\theta=\{(1+p')^{1/2}\operatorname{sh}[\varkappa_\perp(V)\xi]\}^{-1},\tag{46}$$

where $p'=-p/(1+2p)=8K_2/(K_1-8K_2)$, while the quantity $\varkappa_\perp(V)$ and the DW energy $\sigma_\perp(V)$ are determined by the relations (38) and (39) with the parameter p replaced by the parameter p'.

Thus when $K_2>0$, isolated stationary-profile waves describe moving DW. In the phases $\Phi_\parallel$ and $\Phi_\perp$, 180-degree DW exist; within the reorientation region, there exist DW of two types [DWA and DWB; see (42)], which behave differently on approach to the phase-transition points: one of them becomes delocalized ($\varkappa_< \to 0$) and disappears, while the other becomes a 180-degree DW existing in the phases $\Phi_\parallel$ and $\Phi_\perp$ (see Fig. 1); at the phase-transition point, these walls become algebraic.

We turn now to study of the case $K_2<0$. Here it is sufficient to restrict ourselves to investigation of the region of existence of one of the two possible phases, for example $\Phi_\parallel$ (that is, the region $K_1<-8K_2=8|K_2|$), choosing as boundary conditions the relation (16). The solution in the phase $\Phi_\perp$ can be obtained from the solution in the phase $\Phi_\parallel$ by replacing the parameter p by the parameter p' [see (46)] and tan θ by cot θ.

When the inequalities $K_2<0$ and $K_1<-8K_2$ are satisfied, the parameter p is negative: $-\infty<p<0$. The form of the solution of equation (48) depends substantially on the value of this parameter:

$$\operatorname{tg}\theta=\begin{cases}[(1+p)^{1/2}\operatorname{sh}\varkappa_\parallel\xi]^{-1}, & -1<p<0\\ \exp(-\varkappa_\parallel\xi), & p=-1.\\ [(|1+p|)^{1/2}\operatorname{ch}\varkappa_\parallel\xi]^{-1}, & p<-1\end{cases}\tag{47}$$

The energies of these nonlinear waves are determined by a single expression:

$$\sigma(V)=4M_0^2\left[\frac{\alpha K_2}{p(1-V^2/c^2)}\right]^{1/2}\left[1+\frac{1+p}{2(-p)^{1/2}}\ln\left|\frac{1+(-p)^{1/2}}{1-(-p)^{1/2}}\right|\right].\tag{48}$$

It is easily seen that when $-1<p<0$ ($K_1<0$), the solution (47) describes a moving 180-degree DW; when $p=-1$ ($K_1=0$), it describes a 90-degree DW; and when $p<-1$ ($K_1>0$), the solution corresponds to a localized magnetic soliton, for which the values of the angle θ at $\xi\to+\infty$ and at $\xi\to-\infty$ coincide ($\theta\to 0$ when $\xi\to\pm\infty$).

This behavior of the solutions of the equations of motion becomes intelligible if one takes into account that when $K_1<0$ the phase $\Phi_\parallel$ is stable, but when $K_1>0$ it is metastable (the phase $\Phi_\perp$ is stable). At the point $K_1=0$ there occurs a phase transition of the first kind $\Phi_\parallel \rightleftarrows \Phi_\perp$; consequently, at this point there is a 90-degree DW—an interphase boundary separating $\Phi_\parallel$ and $\Phi_\perp$, considered for $V=0$ in Ref. 23. The soliton solution that exists when $K_1>0$ must be regarded as the dynamic analog of a nucleus of the stable phase $\Phi_\perp$ in the metastable phase (the question of the stability of such solitons remains open). Upon approach to the point of instability of $\Phi_\parallel(K_1\to 8K_2,\ p\to-\infty)$, the amplitude of the soliton decreases $[\theta(0)\sim 1/(|p|^{1/2})$ and its region of localization $(1/\varkappa_\parallel)\sim(|p|)^{1/2}$ (meanwhile its energy approaches zero). As $K_1\to+0$, that is as $p\to-1-0$, two 90-degree DW can be distinguished in the soliton, separating a broad ($\Delta\xi\sim\ln|1+p|$) region occupied by the homogeneous phase $\Phi_\perp$ from the remaining part of the magnet. Meanwhile the energy of the soliton approaches the value

$$4M_0^2\left(\frac{\alpha K_2}{1-V^2/c^2}\right)^{1/2}=2\sigma_{\pi/2}(V),\tag{49}$$

which is twice the energy $[\sigma_{\pi/2}(V)]$ of a moving 90-degree DW.

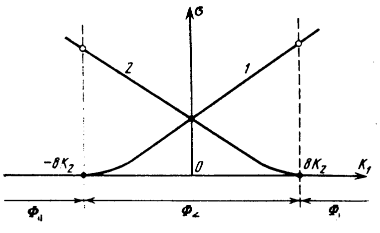

FIG. 1. Regions of existence and energies of moving DW for $K_2>0$ near the spin-reorientation region: 1, DWA; 2, DWB. The circles mark the points corresponding to algebraic DW.

4. MOTION OF DW IN WFM UNDER THE ACTION OF AN EXTERNAL MAGNETIC FIELD

The solutions obtained in the preceding sections describe the motion of DW and other nonlinear waves "inertially": that is, without allowance for dissipative processes and a driving force. As a driving force, one usually uses an external magnetic field H, applied in such a way that because of the Zeeman energy $w_H=-(\mathbf{M}_1+\mathbf{M}_2)\,\mathbf{H}=-2M_0(\mathbf{m}\cdot\mathbf{H})$, one of the homogeneous phases of the magnetic material that are separated by the DW [that in which $(\mathbf{M}\cdot\mathbf{H})>0$] becomes energetically advantageous as compared with the other (the metastable one, in which $(\mathbf{m}\cdot\mathbf{H})<0$]. Then there acts on the DW a force of magnetic pressure p_H that is directed toward the less advantageous phase. The moving DW is also subject to a retarding force $F(V)$, produced by various dissipative processes and dependent on the DW velocity. At a certain value of V equilibrium occurs, $p_H=F(V)$, and the DW motion becomes stationary. It is of interest to calculate the relation $V=V(H)$, since it is this relation that is usually determined experimentally.[4-6, 24]

The relation $V(H)$ for a magnetic with a single sublattice was obtained by Walker (see Ref. 1, Chap. II). The relation $V(H)$ in WFM was found for the case of DWII far from the spin-reorientation region by Zvezdin[25] and by us.[26] We note that this result, like the well-known result of Walker (see Ref. 1, Chap. II), was obtained without limitations on the value of the relaxation constant λ or of the driving field H; but its generalization to the case of other types of DW does not seem possible without certain approximations.

We shall treat the motion of DW in WFM on the supposition that both the relaxation constant λ and the driving field H are small in comparison with the characteristic quantities of the problem[5)] (in particular, $H\ll\beta M_0$, dM_0; $\lambda\ll\beta, d, \ldots$). In this case it may be supposed that the DW structure is the same as for $H=\lambda=0$ and is described by the formulas obtained above. Then the magnetic pressure p_H is described by the equilibrium values of the magnetization to the right and to the left of the DW:

$$p_H = 2M_0(\mathbf{H}(\mathbf{m}(+\infty) - \mathbf{m}(-\infty))). \tag{50}$$

For the magnon force of retardation $F_m(V)$, we shall use the expression obtained by introduction into the Landau-Lifshitz equation of a phenomenological relaxation term in Gilbert's form. Then one easily obtains

$$F_m(V) = -\frac{\lambda V}{2\alpha g M_0} \frac{\sigma(0)}{(1 - V^2/c^2)^{1/2}}, \tag{51}$$

where $\sigma(0)$ is the energy of a static DW and λ is a dimensionless relaxation constant. The velocity $V(H)$ of stationary motion of the DW is found from the condition $p_H = F_m(V)$. On supposing that $\mathbf{H} \parallel \mathbf{e}_z$ and restricting ourselves to the case of phases $\Phi_\parallel$ and $\Phi_<$, we obtain

$$V(H) = c\frac{H}{H_*}\left\{\left[\frac{c\sigma(0)}{2\alpha g M_0^3}\right]^2 + \left(\frac{H}{H_*}\right)^2\right\}^{-1/2}, \tag{52}$$

where $H_* = \lambda\delta M_0/4d\cos\theta_0$; $\cos\theta_0 = 1$ for $\Phi_\parallel$ and is determined by formula (34) for $\Phi_<$. Far from the spin-reorientation region, this formula simplifies and for both DW takes the form

$$V_{1,2}(H) = c\frac{H}{H_*}\left\{\left[\frac{c\varkappa_{1,2}(0)}{gM_0}\right]^2 + \left(\frac{H}{H_*}\right)^2\right\}^{-1/2}. \tag{53}$$

For values of the velocity not too close to c [see (26)], that is for $(c - V)/c \gg \beta/\delta$, (53) agrees with the result obtained in Refs. 25 and 26.

The $V(H)$ relation (52) is shown in Fig. 2. We note the substantial difference of the form of $V(H)$ for WFM from Walker's result for a single-sublattice ferromagnet. In the latter, stationary motion is possible only at values of the field less than a certain critical value H_c, where $H_c \sim \lambda$ and is small in proportion to the smallness of λ. The sublattice structure of WFM shows up in the fact that stationary motion of DW is possible at an arbitrary value of the field H (as compared with $H_* \sim \lambda$); the DW velocity V approaches c when $H \gg H_*$.

We shall first discuss the range of applicability of formulas (52) and (53). Formula (52) was obtained on the assumption that $H \ll \beta M_0$, $dM_0, \ldots$ and that $(c - V)/c \gg \beta/\delta$; that is, in the analysis the assumption $H_* \ll \beta M_0, dM_0$ is essential. Furthermore, in the derivation of these formulas we disregarded magnetoelastic interaction, which may make an important contribution to the retarding force at a DW velocity comparable with the velocity of sound s; that is, when $V \sim s$ one must add to the magnon force of retardation $F_m(V)$, (51), a phonon force $F_{ph}(V)$. The effect of $F_{ph}(V)$ was investigated in detail in Ref. 27; the corresponding section in Fig. 2 for $V \sim s$ has been constructed according to the results of this paper.

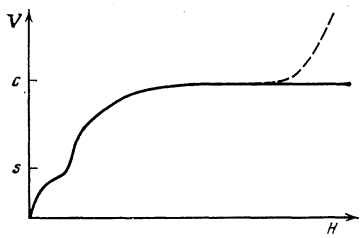

FIG. 2. Variation of the velocity of stationary motion of a 180-degree DW, with allowance for relaxation processes. The $V(H)$ relation at a DW velocity close to the speed of longitudinal or transverse sound, s_t or s_l, was plotted on the basis of the results of Ref. 27. The dotted line shows the nonstationary DW motion with above-limit velocity.[24,26]

It must also be remembered that the phenomenological treatment of relaxation is of approximate, essentially semiqualitative, character (Ref. 11, §31), and that calculation of the magnon retardation of a nonlinear wave requires a microscopic approach.[28] This leads to more complicated $F_m(V)$ and $V(H)$ relations, whose character furthermore is significantly determined by the temperature.[28] In addition, in an analysis of experimental data in REO, a substantial contribution to the dynamic retardation of DW may come from the presence of the rare-earth sublattices, whose spins are ordered only at low temperatures (of the order of helium temperatures). Detailed discussion of these questions, however, falls outside the scope of the present paper.

In conclusion, we note that our results indicate that the velocity of motion of stationary waves of various types in WFM does not exceed the minimum phase velocity of spin waves c. It can therefore be concluded that the above-limit ($V > c$) DW motion observed by Chetkin's group[24] cannot be stationary. Possible mechanisms of this nonstationarity have been discussed in Refs. 24 and 26.

[1]This assumption imposes definite limits on the velocity of motion of the nonlinear wave, as will be indicated below [see formula (26)].

[2]This law is valid for arbitrary stationary waves in WFM both far from and within the reorientation region; it is a consequence of the Lorentz invariance of equation (11). The role of characteristic velocity is played by the minimum phase velocity c of spin waves.

[3]Hysteresis, or "smearing" of the transition, may develop if, in the anisotropy energy, we allow for terms of the type $\beta_2' l_y^2 l_z^2$; but discussion of the details of the transition DWI $\rightleftharpoons$ DWII falls outside the scope of this paper.

[4]Algebraic solitons have been investigated intensively in recent years and have been obtained for the Korteveg-de Vries equation,[18] the non linear Schrödinger equation,[19] and several magnetic systems.[20,21] Algebraic DW have been obtained in an antiferromagnet in a strong magnetic field.[7] We note that algebraic localized solitons may prove unstable[18,19]; but the stability of algebraic DW in WFM is guaranteed by topological considerations.

[5]Smallness of λ means a small value of the damping of a WFM spin wave in comparison with its frequency; this condition may always be considered satisfied at temperatures less than the Néel temperature of the WFM. The driving field in the experiments[4-6,24] did not exceed 10^3 Oe, whereas the smallest characteristic field in REO is the anisotropy field $\beta M_0 \gtrsim 10^4$ Oe.

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Combined phonon resonance in semiconductors

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The absorption of electromagnetic radiation by band carriers whose motion is quantized by a magnetic field is considered. It is shown that the electronic transitions with spin flip that take place on photon absorption and *LO*-phonon emission result in resonance of the absorption coefficient (combined phonon resonance—CPR). The line shape of the CPR is investigated and the parameters of the resonance peaks are determined. The results are compared with the experimental data.

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1. INTRODUCTION

Absorption of electromagnetic radiation by band carriers under conditions in which their motion is quantized by a magnetic field leads to various resonance effects. Part of such effects is connected with the scattering of the carriers by optical phonons. A comparison of the existing experimental data with the results of theoretical researches in this region gives essentially good agreement; however, as is noted in the review of Ref. 1, a number of experimental results has not yet found theoretical explanation. In particular, the nature of the resonance found experimentally in n-InSb at the frequency $\omega_r = \omega_H + g\beta_0 H + \omega_{LO}$ for longitudinal polarization of the electromagnetic field[2] remains unclear.

As follows from the frequency condition of this resonance, it is obviously determined by the electronic transitions with spin flip upon absorption of a photon and emission of a longitudinal optical phonon. The spin flip in the electronic transition can be connected in this case either with the interaction of the band electron with the high-frequency electromagnetic field or with the spin-phonon interaction. We shall consider both cases. Resonance in the absorption of electromagnetic radiation on account of spin-phonon interaction was studied theoretically in Ref. 3, where it was shown that the resonance in the case of an isotropic energy spec-

