Distribution function of hot electrons and nonlinear resistivity of a metal at low temperatures

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The distribution function of the hot electrons from a thin metallic film in a "heating" constant electric field E is obtained at a finite thermostat temperature T in the case when the temperature approximation [M. I. Kaganov, I. M. Lifshitz, and L. V. Tanatarov, Sov. Phys. JETP **4**, 173 (1957)] is not applicable. This function is used to calculate the phonon contribution to the resistivity increment $\delta\rho_{ph}{}^{T}(E)$ that is nonlinear in the field. It turns out that $\delta\rho_{ph}{}^{T}(E)$ agrees quite closely (within less than 1%) with the analogous quantity $\delta\tilde{\rho}_{ph}{}^{T}(E)$ calculated in the cited temperature approximation of Kaganov *et al*.

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1. In our preceding paper[1] we obtained the distribution function of the electrons from a thin metallic film in a constant and homogeneous electric "heating" field E for the case when the conditions of the temperature approximation of Kaganov, Lifshitz, and Tanatarov[1] are not satisfied, and the lattice temperature is $T=0$. The initial kinetic equation corresponding to this problem, for a distribution function that depends on the electron energy, was a nonlinear integro-differential equation without a small parameter. It is reduced here by a number of transformations to a nonlinear differential equation of fourth order with nonlocal boundary conditions. An investigation of the latter has made it possible to obtain the analytic form of the asymptotic distribution function $n(\xi)$ as $\xi\to\infty$ (ξ is the dimensionless energy of the electron reckoned from the Fermi level). It turned out that $n(\xi)\propto\exp(-\frac{2}{5}\xi^{5/2})$, i. e., it decreases much more rapidly than the function $n_0(\xi)$ with pure exponential asymptotic form $n_0(\xi)\propto e^{-\xi}$ ($\xi\to\infty$), which is characteristic of an equilibrium distribution. Nonetheless, as shown by a computer integration of this problem,[1] the distribution function $n(\xi)$ in the characteristic energy variation region $\xi<2$ differed little from an equilibrium Fermi function with some effective field-dependent temperature T_e.

The present paper is a direct continuation of our preceding paper[1] (see footnote 5 there), and generalized the results obtained there at $T=0$ to the case of an arbitrary temperature of the thermostat. It is important here to note that this generalization does not mean merely that the calculation method of Ref. 1 is made more complicated because of the appearance of the additional parameter $T\neq 0$. In fact, simple physical considerations show that the case $T=0$, in the sense of the study of the asymptotic behavior of $n(\xi)$ as $\xi\to\infty$, is special, wherein arbitrarily small $T\neq 0$ convert the asymptotic function obtained there into the exponential $e^{-a\xi}$, where a should depend on the parameter $\gamma=T_e/T$.[1)] This is particularly obvious in the case $E\to 0$, when the asymptotic form of the distribution function is "imposed" by the thermostat, i. e., $n(\xi)\sim e^{-\xi}$ ($\xi\to\infty$). The special character of the asymptotic form of $n(\xi)$ obtained in Ref. 1 at $n=0$ is apparently connected physically with the fact that at $T=0$ any arbitrarily small (but finite) electric field is "heating" in the sense that it is only the only "external force" that shapes the form of $n(\xi)$. On the other hand if $T\to 0$ this is no longer the case, since the form of $n(\xi)$ as $E\to 0$ is shaped in the zeroth approximation by the thermostat with a corresponding slowly decreasing exponential asymptotic behavior.

At the same time it is physically obvious that turning on a thermostat with $T\to 0$, while changing substantially the analytic character of the exact solution of the problem, cannot nevertheless alter $n(\xi)$ substantially in the characteristic region of energy variation (determined in this limiting case by the quantity $T_e\gg T$).

For a numerical solution of our problem we had to use methods different from those employed in Ref. 1. A brief description of these methods and the results obtained through their use (which duplicate, naturally the results of Ref. 1 in the case $\gamma\to\infty$ and are thus independent check on the validity of the numerical solution obtained there) are given below. In addition, we calculate in the present paper the field contribution $\delta\rho_{ph}^{T}(E)$ to the phonon-induced resistivity of the metal, for the purpose of estimating the extent to which the difference we found between the distribution function and the "temperature" Fermi function influence such as integral characteristic as $\delta\rho_{ph}^{T}(E)$. We note also that we shall not dwell in greater detail on the formulation of the problem, on the notation, and on the derivation of the kinetic equation, since everything with important bearing on these questions is contained in Ref. 1.

2. We begin the analysis of our problem with a remark that the use in Ref. 1 of a nondimensional kinetic equation with respect to T_e is not convenient for the case presently of interest to us, that of an arbitrary ratio of T_e and T, since it does not admit of a direct transition to the limiting cases of weak ($E\to 0, T=\text{const}$) and "strong" ($T\to 0, E=\text{const}$) superheating. Therefore, bearing in mind the desired analogy with the temperature approximation of Kaganov, Lifshitz, and Tanatarov,[2] we make the kinetic equation (2) of Ref. 1 dimensionless with respect to the quantity Θ, defined by the relation

$$\Theta^5\equiv T^5+T_e^5, \tag{1}$$

where T_e is defined as in Ref. 1 [Eq. (5)]. Thus, if

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now $\xi \equiv (\varepsilon - \varepsilon_F)/\Theta$ and $\gamma \equiv \Theta/T$, then the kinetic equation for $n(\xi)$ takes the form

$$\left(\frac{1}{\gamma^5} - 1\right)\frac{d^2n}{d\xi^2} = I^0(\xi) + I^T(\xi, \gamma), \tag{2}$$

$$I^0(\xi) \equiv [1-n(\xi)]\int_0^\infty dx\, x^2 n(\xi+x) - n(\xi)\int_0^\infty dx\, x^2[1-n(\xi-x)],$$

$$I^T(\xi, \gamma) \equiv \int_0^\infty dx\, x^2 N(\gamma x)[n(\xi+x) + n(\xi-x) - 2n(\xi)].$$

It is easily seen that in the limit as $T \to 0$ ($\gamma \to \infty$) Eq. (2) goes over into Eq. (6) of Ref. 1. At $E = 0$ ($\gamma = 1$) the left-hand side of (2) vanishes and the solution of (2) is, as it should be, an equilibrium Fermi distribution function with a lattice temperature T.

To calculate the field dependence of the phonon-induced resistivity of the hot electrons we obtain the corresponding correction $n_p^{(2)}$ from the second-approximation equation (see Ref. 1):

$$\hat{\nu}_i n_p^{(2)} = eE\frac{\partial n_p^{(1)}}{\partial p} - \hat{\nu}_p\{n_p^{(0)} + n_p^{(1)}\}.$$

It is easy to show that in the right-hand side of this expression the only term that contributes to the current is $\hat{\nu}_p^{(0)} n_p^{(1)}$, where $\hat{\nu}_p^{(0)}$ is the electron-phonon collision operator linearized with respect to a small increment $n^{(0)}$. Standard calculations of the increment of the phonon contribution to the resistivity of the sample on account of the electric field $\delta\rho_{ph}^T(E)$ (defined such that $\delta\rho_{ph}^T(E=0) = 0$) yield

$$\delta\rho_{ph}{}^T(E) = \left[\gamma^5\frac{|Q(\gamma)|}{10D_5} - \frac{4}{5}\right]\rho_{ph}(T), \tag{3}$$

$$Q(\gamma) \equiv \int_0^\infty dx\, x^4 \int_0^\infty d\xi \frac{dn_\gamma}{d\xi}[n_\gamma(x+\xi) + n_\gamma(x-\xi)], \tag{4}$$

$n_\gamma(\xi)$ is the hot-electron distribution function obtained by numerical solution of Eq. (2). Here

$$D_5 = \int_0^\infty \frac{z^4\, dz}{e^z - 1} \approx 24.89, \tag{5}$$

and $\rho_{ph}(T)$ is the temperature increment to the resistivity in the equilibrium case at the temperature T. It can also be shown that the connection between γ and $\mathscr{P}$ ($\mathscr{P}$ is the power dissipated in a unit volume of the sample) is

$$\gamma^5 - 1 = \frac{10D_5}{6j_0{}^2}\frac{\mathscr{P}}{\rho_{ph}(T)}, \tag{6}$$

where $j_0 \equiv nes$, n is the number of electrons per unit volume, e is the electron charge, and s is the speed of sound.

Thus, in the general case, when the thermalization criterion $\nu_{ep}^{\varepsilon} \ll \nu_{ee}$ is not satisfied, i. e., when the electron-phonon collisions predominate in the formation of the distribution function $n(\xi)$ of the hot electrons, the procedure of calculating the nonlinear in the field increment to the phonon contribution $\delta\rho_{ph}^T(E)$ to the resistivity is the following: the nonlinear integro-differential equation (2) at different values of the parameter γ is used to obtain a single-parameter family of functions $n_\gamma(\xi)$. With the aid of Eq. (4) we calculate $Q(\gamma)$. The sought $\delta\rho_{ph}^T(E)$ as a function of T and γ is given by Eq. (3), where γ is connected with the physically observable quantity $\mathscr{P}$ by relation (6).

To find the function $n_\gamma(\xi)$ we must solve Eq. (2). Introducing, as in Ref. 1, the function

$$\varphi(\xi) = 2n(\xi) - 1,$$

we obtain for it the equation

$$d^2\varphi/d\xi^2 - I_\varphi^0 - I_\varphi^T \equiv \hat{K}\varphi = 0, \tag{7}$$

which must be solved in the internal $0 \leqslant \xi < \infty$ with the conditions $\varphi(0) = 0$ and $\varphi(\infty) = -1$.

A direct numerical solution of (7) is difficult, and we used for this purpose an establishing method. Considering the solution of the nonstationary equation

$$\partial u(t, \xi)/\partial t = \hat{K}u(t, \xi) \tag{8}$$

with initial condition $u(0, \xi) = u_0(\xi)$ and with boundary conditions $u(t, 0) = 0, u(t, \infty) = -1$, we obtain next $u(\infty, \xi)$.

It is obvious from physical considerations that this value should coincide with the sought value of $\varphi(\xi)$ regardless of the initial distribution function $u_0(\xi)$. The limiting value of $u(t, \xi)$ is naturally chosen to be the same as for the limiting distribution $\varphi(\xi) = u(\infty, \xi)$. The initial distribution was chosen to be $u_0(\xi) = -\tanh(\xi/2)$, and Eq. (8) was replaced by an explicit difference scheme.

To calculate $Q(\gamma)$ it was convenient to use a representation that can be easily obtained with the aid of certain transformations from (4):

$$Q(\gamma) = -\int_0^\infty x^4(1+\varphi(x))\,dx - 4\int_0^\infty [1+\varphi(\xi)]\,d\xi\int_0^\xi y^3[1+\varphi(y)]\,dy$$
$$-12\int_0^\infty \xi^2[1+\varphi(\xi)]\,d\xi\int_0^\xi y[1+\varphi(y)]\,dy. \tag{9}$$

We now discuss the calculation results. The quantity $Q(\gamma)$ turns out to be a monotonically increasing function of γ (see Fig. 1), such that $Q(1) = 8D_5$. In the analysis of the results we were primarily interested in the extent to which the exact numerical values of $n_\gamma(\xi)$ and $\delta\rho_{ph}^T(\gamma)$ differed from the corresponding functions calculated analytically in the temperature approximation. The latter corresponds to the choice $n_\gamma(\xi) \to f(\xi/\tilde{\Theta})$, where f is the equilibrium Fermi function, $\Theta^5 \equiv T^5 + T_e^5$, and $\tilde{T}_e$ is defined by the relation $T_e^5 = D_5\tilde{T}_e^5$, so that $(\Theta/T)^5 = 1 + (\gamma^5 - 1)/D_5$ (the tilde marks here results pertaining to the temperature approximation). Thus, taking (6) into account, $\tilde{\Theta}(\mathscr{P}, T)$ is that effective temperature which yields for the equilibrium Fermi function $f(\xi/\tilde{\Theta})$, substituted in the heat-balance equation of Ref. 2, the same dissipated power $\mathscr{P}$ as the true distribution function $n_\gamma(\xi)$.

The physical basis for this choice of $\tilde{\Theta}$ may be the

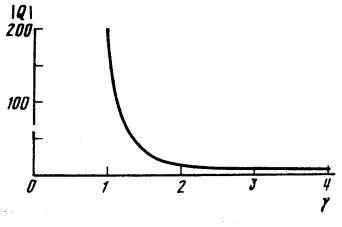

FIG. 1. Plot of $|Q|$ against γ.

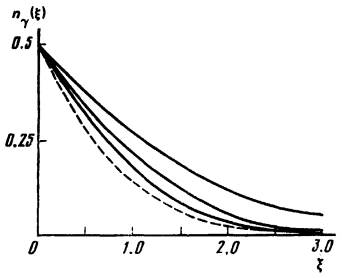

FIG. 2. Plot of the distribution $n_\gamma(\xi)$ at $\gamma=1$, 1.5, and ∞ (the solid lines, reading down); dashed—temperature approximation for $\gamma=\infty$.

consideration that $f(\xi/\tilde{\Theta})$ is that limit to which $n_\gamma(\xi)$ must tend in the case of a "gradual" turning on of the interactions between the electrons and $\mathscr{P}$ is constant, so that in final analysis the thermalization condition $\nu_{ee} \gg \nu_{ep}^{\varepsilon}$ is satisfied. Calculation of the field contribution to the resistivity in the temperature approximation leads to the equation

$$\delta\tilde{\rho}_{ph}{}^{T}(E)=\frac{4(\gamma^5-1)}{5D_5}\rho_{ph}(T). \tag{10}$$

Comparison of (3) and (10) as functions of the parameter γ shows that the difference between them in the entire investigated interval $1.1<\gamma<10$ is less than 1%.

Thus, for the considered simplest model of electrons with isotropic and quadratic dispersion law, and phonons with a Debye spectrum, in the absence of collisions between the electrons, the temperature approximation describes an integral characteristic [that depends on the distribution function $n_\gamma(\xi)$] such as $\delta\rho_{ph}^{T}(E)$ with very high accuracy. The possible cause of this accuracy, in our opinion, may be that both $\mathscr{P}$ and $\delta\rho_{ph}^{T}(E)$ are, in the model investigated by us, actually moments of the same order of the function $n_\gamma(\xi)$. Therefore, although the functions $n_\gamma(\xi)$ and $f[\xi/\tilde{\Theta}(\gamma)]$ differ insignificantly (but still noticeably, see Fig. 2), the proximity of $\delta\tilde{\rho}_{ph}(E)$ to $\delta\rho_{ph}(E)$ can be of quite high order, since $\tilde{\mathscr{P}}_\gamma = \mathscr{P}_\gamma$ by definition. This in turn leads to the assumption that if we consider in the temperature approximation defined above the physical responses corresponding to moments of $n_\gamma(\xi)$ of a different order than $\mathscr{P}_\gamma$, then the difference between these responses and the "true" ones should be more noticeable.

We remark also that by combining Eqs. (6) and (10) we see that from the connection between the experimentally observed quantities[3] $\delta\rho_{ph}^{T}(E)$ and $\mathscr{P}$, the electron-phonon interaction constant[1] α drops out in the temperature approximation. Namely, a linear connection exists of the type

$$\delta\tilde{\rho}_{ph}{}^{T}(E)=4\mathscr{P}/3j_0{}^2. \tag{11}$$

In conclusion, the authors thank A. A. Motornaya for great help with the computer calculations.

[1)]We note, however that we were unable to obtain the exact form of $a(\gamma)$, primarily because of the nonlocality and nonlinearity of the investigated equation.

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Infrared absorption of light by small-scale fluctuations in compensated semiconductors

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We calculate the absorption coefficient of light in transitions between energy levels of small-scale fluctuations of impurities in weakly doped compensated semiconductors. The absorption spectrum is hydrogenlike. The shape of the absorption line corresponding to a transition from the ground state to the first excited state of the fluctuations is calculated. It is shown that the line width is due to deviation of the fluctuation shape from spherical.

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The far tail of the state density in doped semiconductors is determined by the small-scale fluctuations whose potential coincides with the potential of the effective point charge $Z>1$.[1] Such a system has hydrogenlike energy levels. This paper discusses the possibility of direct optical observation of transitions between the energy levels of a small-scale fluctuation. The frequency range of the investigated transitions lies in the far infrared or in the submillimeter region.

To observe the transition between the levels of such a small-scale fluctuations it is necessary that the state

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