pected in research at low temperatures. The point is that for liquid-helium temperatures and below, account must be taken, in the formation of echo signals, of the influence of the dynamic frequency shift and of the nonlinear effect due to the high intensity of the nuclear signals in $FeBO_3$, particularly in signals enriched with the magnetic isotope ^{57}Fe. The first experiments performed by us have indeed shown that parametric pumping takes place at $T = 4.2$ K, but no parametric echo is observed.

We wish to note in conclusion that the here-revealed possiblility of investigating magnetoelastic oscillations with the aid of nuclear echo may be quite useful for the determination of the magnetoelastic constants as well as to study the above-threshold state in parametric excitation.

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[1)]This statement is valid also when several normal modes are excited, since the phase of the excited oscillations is rigidly connected with the phase of the pump field.

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Hopping photoconductivity of disordered systems

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The action of electromagnetic radiation on the static hopping conductivity of disordered systems, due to the change of the hopping probability in the radiation field, is considered. The model of a weakly doped compensated semiconductor is used. Expressions are obtained for the relative change of the conductivity, with multiphoton processes taken into account.

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1. At sufficiently low temperatures the conductivity of certain disordered systems, such as doped compensated semiconductors, amorphous semiconductors, and others, is due to carrier transfer between localized states (called hopping conduction[1,2]) (see also Refs. 3 and 4). In view of the random scatter of the scatter of the energies of the localized states, the transfers of carriers between them are inevitably accompanied by absorption or emission of acoustic phonons. If, however, the system is the field of electromagnetic radiation, then photons participate in the carrier transfer rather than phonons, and this produces a photocurrent.[5] For a monochromatic wave, however, owing to the indicated scatter of the localized-state energies, the contribution to the conductivity from the hops in which only photons take part is small compared with the contribution of the hops stimulated simultaneously by photons and phonons. The influence of these processes on the static hopping conductivity in the region of relatively low frequencies of the radiation was investigated in Ref. 6, and the method proposed there makes it possible to take multiphoton processes into account.

Hopping conduction was observed experimentally in extremely purified and compensated n-InSb.[7] A detailed exposition of the results of that study is contained in Ref. 8. The authors describe their own interpretation of the observed photoconductivity, but they use some phenomenological parameters (e.g., the time that the electron participates in the photocurrent, the light absorption coefficients) whose connection with the characteristics of the semiconductor has not been established. In Refs. 9 and 10 was investigated hopping photoconductivity of strongly doped uncompensated semiconductors, and also of amorphous semiconductors in the band-bending model. The photoconductivity considered in Refs. 6, 9, and 10 is due to the change of the

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probability of charge hopping between localized states under the influence of the radiation field (i.e., with change of the off-diagonal elements of the density matrix in the node representation). This action of the radiation was named dynamic by the authors. But the radiation leads also to a change of the population of the localized states, i.e., to a change of the diagonal elements of the density matrix. This ("heating") action of the radiation on the hopping conductivity was considered in Ref. 11.

The present paper is devoted to a consistent analysis of the action of electromagnetic radiation on the hopping conductivity of disordered systems, an action connected with photostimulated hops. The model of a disordered system with hopping conductivity was chosen to be that of weakly doped compensated semiconductors. In contrast to our brief communication[6] we develop in this paper an essentially more general approach, wherein the results can be extended to the region of high frequencies (exceeding the reciprocal time of flight of the electrons between the localized states with "atomic" velocity). In this frequency region the hopping conductivity can depend exponentially on the radiation frequency and exceed appreciably the photoconductivity at relatively low frequencies. In addition, as shown below, for a correct determination of the magnitude of the effect of hopping photoconductivity it is important to take into account the energy dependence of the electron-phonon interaction.

2. We consider a weakly bound compensated semiconductor (for the sake of argument, n-type) situated in the field of a monochromatic electromagnetic wave. The weak-doping condition is the inequality $Na^3 \ll 1$, where N is the concentration of the localization centers (of the donor impurity), $a = \hbar(2mE_0)^{-1/2}$ is the distance over which the wave function of the electron falls off at the center, m is the effective mass of the electron, and E_0 is the ionization energy of the center ($E_0 \approx \varepsilon_1$, where ε_1 is the bandconductivity activation energy). When the foregoing condition is satisfied, the wave functions of the electrons located in neighboring localized states overlap weakly. Compensation (addition of an acceptor impurity) ensures free places on the donors, and also causes the energy scatter of the localized states (of the electrons on the donors) to exceed greatly, owing to the influence of the Coulomb field of the charged acceptors greatly exceeds the overlap integral of the localized states.

We confine ourselves to those emission frequencies

$$\Omega < E_0/\hbar, \tag{1}$$

that exclude the electron ejection from the localized state into the region of delocalized states. For deep centers and weakly doped or amorphous semiconductors this condition can be satisfied for emission frequencies down to $\Omega \lesssim 10^{15}$ sec^{-1}. On the other hand we assume that the emission frequency greatly exceeds the frequency ν_h of the critical hops in the percolation paths (it is these hops which determine the static hopping conductivity):

$$\Omega > \nu_h. \tag{2}$$

The quantity ν_h is connected with the hopping conductivity σ_0 by the relation[3] $\nu_h \sim \sigma_0 \zeta_c^2 aT/e^2$, where T is the system temperature (in energy units), $\exp(-\zeta_c)$ is the exponential factor of the hopping conductivity, and e is the electron charge. For example, for $\zeta_c = 10$, $\sigma_0 \sim 10^{-4}$ $(\Omega\text{-cm})^{-1}$, $T \sim 10^{-16}$ erg, and $a \sim 10^{-6}$ cm the frequency is $\nu_h \sim 5 \cdot 10^6$ sec^{-1}. If the radiation wavelength exceeds the characteristic length $N^{-1/3}$ of the electron hops between the centers, then the interaction of the radiation with the electrons can be taken into account in the dipole approximation, i.e., it can be treated as the action on the electrons by the homogeneous alternating electric field

$$\vec{\mathscr{E}}(t) = \vec{\mathscr{E}} \cos \Omega t. \tag{3}$$

This calls for satisfaction of the condition

$$\Omega \ll \varkappa^{1/2} c N^{1/3}, \tag{4}$$

where $\varkappa$ is the total dielectric constant and c is the speed of light.

3. We consider now the quasiclassical wave function of a bound electron situated in the alternating electric field (3) using the method proposed in Ref. 12. This wave function $\psi(\mathbf{r}, t)$ is, apart from a pre-exponential factor, of the form[1)]

$$\psi(\mathbf{r}, t) \sim \exp(iS(\mathbf{r}, t)), \tag{5}$$

where $S(\mathbf{r}, t)$ is the classical action calculated along a classical trajectory. By classical trajectory is meant one along which the quantities $\mathbf{r}$ and $\dot{\mathbf{r}}$ satisfy the classical equations of motion of the electron in the field of a center and in the external electric field (3). If the alternating electric field is weak enough, then the action in (5) can be calculated along the unperturbed trajectory, which is the trajectory along the radius for a state with zero angular momentum. We assume a short-range center potential $V(\mathbf{r}) = v\delta(\mathbf{r})$, and then the equations of motion for the indicated trajectories take the form

$$\dot{\mathbf{r}}(t') = iv_0 \mathbf{r}/r, \quad \mathbf{r}(t') = \mathbf{r} + iv_0(t - t')\mathbf{r}/r, \tag{6}$$

where $v_0 = (2E_0/m)^{1/2}$ is the "atomic" velocity of the electron, and $r = |\mathbf{r}|$. At the instant of time t the electron is at the point $\mathbf{r}$, and the initial instant of "time" t_0 (which is complex), at which the electron "started" from the center, is determined from the condition $\mathbf{r}(t_0) = 0$. Integration of the Lagrangian along the trajectory (6) leads to the following value of the action:

$$S(\mathbf{r}, t) = i\frac{r}{a} + \Delta S(\mathbf{r}, t), \tag{7}$$

$$\Delta S(\mathbf{r}, t) = \frac{e\vec{\mathscr{E}}\mathbf{r}}{\Omega}(a_1 \sin \Omega t - ia_2 \cos \Omega t), \tag{8}$$

where

$$a_1 = \operatorname{ch}\gamma - \operatorname{sh}\gamma/\gamma, \quad a_2 = \operatorname{sh}\gamma + (1 - \operatorname{ch}\gamma)/\gamma,$$

$\gamma(r) = \Omega r/v_0$ is the number of oscillations of the electrons in the alternating electric field during its free flight with velocity v_0 over a distance r. Thus, the sought quasiclassical function of the bound electron in the external electric field (3) is

$$\psi(\mathbf{r}, t) \sim \exp\left(-\frac{r}{a} + i\Delta S(\mathbf{r}, t)\right). \tag{9}$$

In the region $r \ll v_0/\Omega$ we have

$$i\Delta S(\mathbf{r}, t) = \frac{e\vec{\mathscr{E}}\mathbf{r}}{2\Omega}\gamma(r)\cos\Omega t,$$

and in the region $r > v_0/\Omega$

$$i\Delta S(\mathbf{r}, t) = \frac{e\vec{\mathscr{E}}\mathbf{r}}{2\Omega}\exp\{i\Omega t + \gamma(r)\}.$$

The condition for weak perturbation of the trajectory by an alternating electric field is the inequality

$$|\Delta S(\mathbf{r}, t)| \ll r/a.$$

The constructed wave functions will be assumed to be unperturbed relative to the interaction of the electrons with the acoustic phonons. The Hamiltonian of this interaction will be chosen in the usual form:

$$H_{ep} = \sum_{\mathbf{q}} (F_{\mathbf{q}} b_{\mathbf{q}}^{+} \exp\{i\mathbf{q}\mathbf{r} - i\omega_{\mathbf{q}} t\} + F_{\mathbf{q}}^{*} b_{\mathbf{q}} \exp\{i\mathbf{q}\mathbf{r} + i\omega_{\mathbf{q}} t\}),$$

where $b_{\mathbf{q}}^{+}$ and $b_{\mathbf{q}}$ are the operators for the creation and annihilation of phonons with momentum $\hbar\mathbf{q}$; $\omega_q = Sq$ is the speed of sound; F_q is the matrix element of the electron-phonon interaction; in the case of deformation acoustic phonons $F_q = E_1 q(\hbar/2M\omega_q)^{1/2}$, where E_1 is a constant on the order of 1–10 eV, and M is the mass of the crystal-lattice atom. The non-orthogonality (overlap) of the wave functions of the neighboring localized states ψ_i and ψ_j can be taken into account by introducing into the Hamiltonian an effective tunneling operator[13] (see also Ref. 14), whose off-diagonal matrix elements T_{ij} and T_{ji} are the integrals of the overlap of these states. Using the electron wave functions (9) we obtain the following value of the overlap integral of the states i and j (with allowance for the time factors):

$$T_{ij}(t) = \beta E_0 \exp\{-r_{ij}/a\} u_{ij}(t),$$
$$u_{ij}(t) = \exp\left\{i(\varepsilon_i - \varepsilon_j)t - i\frac{e\vec{\mathscr{E}}\mathbf{r}_{ij}}{\Omega}\sin\Omega t - i\Delta S(\mathbf{r}_{ij}, t)\right\}, \quad (10)$$

$T_{ji} = T_{ij}$. Here $\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j$; $\mathbf{r}_i$ and $\mathbf{r}_j$ are the coordinates of the centers i and j; ε_i and ε_j are the state energies reckoned from the edge of the region of the delocalized states, $\varepsilon_i \approx \varepsilon_j \approx E_0$, and β is a coefficient of the order of unity.

Taking into account the interaction of the electrons with the phonons in second-order perturbation theory, we obtain the following expression for the probability of the electron transition between states i and j per unit time in the presence of electromagnetic radiation:

$$\Gamma_{ij} = \beta^2 E_0^2 \exp\left\{-\frac{2r_{ij}}{a}\right\} \sum_{\omega_q, \mathbf{q}} |F_{\mathbf{q}}|^2 |M_{\mathbf{q}}|^2 \frac{1}{\omega_{\mathbf{q}}^2} f(\varepsilon_i)(1 - f(\varepsilon_j))(N(|\omega_{\mathbf{q}}|) + \Theta(\omega_{\mathbf{q}}))\left\{\lim_{t\to\infty}\left[\frac{1}{t}\int_0^t dt' \int_0^{t'} dt'' \exp\{i\omega_{\mathbf{q}}(t' - t'')\} u_{ij}(t') u_{ji}(t'')\right]\right\}, \quad (11)$$

where

$$M_{\mathbf{q}} = \int d^3\mathbf{r} |\varphi_0|^2 e^{i\mathbf{q}\mathbf{r}} \sim (1 + (aq/2)^2)^{-2}.$$

The expression for Γ_{ji} is obtained from (11) by the permutation $i \rightleftharpoons j$. Here $\varphi_0(r)$ is the spatial part of the wave function of the localized state i, j; $N(\varepsilon)$ is the phonon Planck distribution function; $\Theta(\varepsilon)$ is the Heaviside function; the summation is also over the negative phonon frequencies ω_q; this corresponds to allowance for processes with phonon emission; $f(\varepsilon_{i,j})$ is the function of the electron occupation of the state i, j.

In the calculation of the integrals with respect to time in (11) it is convenient to use the known relation

$$\exp\{ix\sin y\} = \sum_{s=-\infty}^{\infty} J_s(x) e^{isy},$$

where $J_s(x)$ is a Bessel function of order s. The expression in the curly bracket in (11) then reduces to

$$\{\ldots\} = 2\pi \sum_{s=-\infty} A_s(\mathbf{r}_{ij}) \delta(\varepsilon_i - \varepsilon_j + \omega_{\mathbf{q}} + s\Omega), \quad (12)$$

$$A_s(\mathbf{r}) = \sum_{s', s''=-\infty}^{\infty} J_{s'}(x) J_{s-s'}(iy) J_{s''}(x) J_{s''-s}(iy) \exp\left\{\frac{i\pi(s' - s'')}{2}\right\}, \quad (13)$$

where

$$x = \frac{e\vec{\mathscr{E}}\mathbf{r}}{\Omega}\left(\operatorname{ch}\gamma - 1 - \frac{\operatorname{sh}\gamma}{\gamma}\right), \quad y = \frac{e\vec{\mathscr{E}}\mathbf{r}}{\Omega}\left(\frac{\operatorname{ch}\gamma - 1}{\gamma} - \operatorname{sh}\gamma\right);$$

here $\gamma = \gamma(r)$. If the emission frequency and intensity are low enough, so that $\gamma(r_{ij}) \ll 1$ and $|e\vec{\mathscr{E}}\mathbf{r}_{ij}\gamma/\hbar\Omega| \ll 1$, then the expression for $A_s(\mathbf{r}_{ij})$ can be simplified:

$$A_s(\mathbf{r}_{ij}) = J_s^2(e\vec{\mathscr{E}}\mathbf{r}_{ij}/\hbar\Omega). \quad (14)$$

This is precisely the case considered in Ref. 6. The principal action of the electric field of the wave on the electron tunneling between two centers consists then in the formation of an alternating potential difference between these centers.

Expression (11), with (12) taken into account, is the sum of the probabilities of the humps of the electron between the centers i and j with absorption or emission of one phonon and s photons. A renormalization of the transition probability then takes place also without absorption or emission of photons (the term with $s = 0$). As $\vec{\mathscr{E}} \to 0$ the expression (11) goes over into the known expression for the probability of the hop[15] Γ_{ij}, which takes no account of the influence of the radiation field. Summing over the phonon wave functions, we obtain from (11) and (12):

$$\Gamma_{ij} = 2\pi f(\varepsilon_i)(1 - f(\varepsilon_j)) \exp\left(-\frac{2r_{ij}}{a}\right) \sum_{s=-\infty}^{\infty} A_s(\mathbf{r}_{ij}) \Phi(\varepsilon_i - \varepsilon_j + s\Omega) \times [N(|\varepsilon_i - \varepsilon_j + s\Omega|) + \Theta(\varepsilon_i - \varepsilon_j + s\Omega)]. \quad (15)$$

Here $\Phi(\varepsilon) = 4\pi\beta^2 E_0^2 |F_q|^2 b^3 |M_q|^2_{q = |\varepsilon|/\hbar S}$, and b is the lattice period. In the case of electron interaction with deformation acoustic phonons we have $\Phi(\varepsilon) \sim |\varepsilon|[1 + (\varepsilon/\varepsilon_0)^2]^{-4}$. At $|\varepsilon| \ll \varepsilon_0 \equiv 2\hbar S/a$ we have $\Phi(\varepsilon) \sim |\varepsilon|$; this function has a maximum at $|\varepsilon| \approx \varepsilon_0$ and falls off rapidly $[\Phi(\varepsilon) \sim \varepsilon^{-7}]$ at $|\varepsilon| > \varepsilon_0$. This means that the bound electrons interact most strongly with phonons having an energy $\varepsilon \approx \varepsilon_0$. For shallow impurities in a semiconductor we have $\varepsilon_0 \approx 0.5$–$1.5$ K.

4. We use now the expression derived for the probability of electron hopping between centers to calculate the macroscopic static conductivity of the system. It is known that the hopping conductivity of a disordered system is determined on the basis of the analogy between the conductivity of this system and the conductivity of a network of greatly differing resistors R_{ij}

which are connected between the centers i and j are directly related to the probability of the electron hopping between these centers[2-4]:

$$R_{ij}=2T/e^2(\Gamma_{ij}+\Gamma_{ji}). \tag{16}$$

The largest scatter in the values of R_{ij}, just as of Γ_{ij}, is connected with the exponential dependence of the overlap integral on the distance between the centers. According to percolation theory, the hopping conductivity is determined by the principal resistances, or by the critical hops in the percolation paths. If turning on the radiation alters the hopping probability little, in the sense of the condition

$$\ln(|\Gamma_{ij}+\Gamma_{ji}|/2\Gamma_{ij}^0)\ll\zeta_c=2r_c/a, \tag{17}$$

where r_c is the length on the critical hops[2] (we note that when condition (17) is satisfied the hopping probabilities can change by many orders of magnitude), then the conductivity σ of the system acted upon by the radiation can be determined by a perturbation theory[2] whose meaning is that the critical hops remain the same as before (only their probability changes), and to calculate the probability it is necessary to average (16) over these hops. In other words,

$$\sigma=\sigma_0\exp\left[\left\langle\ln\frac{\Gamma_{ij}+\Gamma_{ji}}{2\Gamma_{ij}^0}\right\rangle_c\right], \tag{18}$$

where $\langle\ldots\rangle_c$ denotes averaging over the critical hops. If the hopping probability changes little when the radiation is turned on, so that

$$|\Delta\Gamma_{ij}+\Delta\Gamma_{ji}|\ll\Gamma_{ij}^0$$

then expression (18) simplifies to

$$\frac{\sigma-\sigma_0}{\sigma_0}=\left\langle\frac{\Delta\Gamma_{ij}+\Delta\Gamma_{ji}}{2\Gamma_{ij}^0}\right\rangle_c. \tag{19}$$

Substituting (15) with the equilibrium distribution functions of the electrons and phonons in (19), we obtain the following expression for the relative change of the conductivity

$$\frac{\sigma-\sigma_0}{\sigma_0}=\sum_{s=-\infty}^{\infty}\bar{A}_s(r_c)F(\hbar s\Omega), \tag{20}$$

$$F(x)=\int_{-\Delta/2}^{\Delta/2}d\varepsilon'\int_{-\Delta/2}^{\Delta/2}d\varepsilon\,\rho(\varepsilon)\rho(\varepsilon'-\varepsilon)\frac{\Phi(\varepsilon+x)}{\Phi(\varepsilon)}\frac{1-e^{|\varepsilon|/T}}{1-e^{|\varepsilon+x|/T}}(1+e^{(|\varepsilon+x|-|\varepsilon|)/T}). \tag{21}$$

Here

$$\bar{A}_s(r_c)=\frac{1}{4\pi}\int A_s(\mathbf{r})\delta(r-r_c)\,d^3\mathbf{r}$$

is the value of $A_s(\mathbf{r})$ averaged over the directions of the critical hops, and $\rho(\varepsilon)$ is the state density.

We assume hereafter that $\rho(\varepsilon)=1/\Delta=\text{const}$, where Δ is the mean energy difference of the states between which the critical hops take place. In the Mott hopping conduction regime, Δ is equal to the hopping-conduction activation energy $\varepsilon_3=\zeta_cT\gg T$ and the conduction is over states in an energy band of width Δ near the Fermi surface. In the constant-activation-energy regime $\Delta<\zeta_cT$ and the conduction is over states in an energy band of width Δ near the peak of the state density. In a weakly doped semiconductor we have in this case $\Delta\sim e^2K^{2/3}/\varkappa N^{-1/3}$ where $K=N_A/N$ is the degree of compensation and N_A is the acceptor density.

We determine now the hopping conductivity in the field of an electromagnetic wave in two limiting cases.

1. Low frequencies: $\hbar\Omega\ll T$

The parameter for the critical hops is then $\gamma(r_c)\ll1$. If, in addition, the electric field intensity is low enough ($e\mathscr{E}r_c\ll T$), then, expanding (20) in powers of Ω and summing in (20) over s we obtain, taking (14) into account and accurate to $(\hbar\Omega/T)^3$ (Ref. 6)

$$\frac{\sigma-\sigma_0}{\sigma_0}\sim\left(\frac{e\mathscr{E}r_c}{T}\right)^2. \tag{22}$$

The main contribution to the change of the conductivity is made in this case by hops with absorption and emission of $s_0=[e\mathscr{E}r_c/\hbar\Omega]+1$ photons, where $[x]$ is the integral part of the number x. At sufficient intensity of the radiation, the number s_0 can be quite large. We note also that in the indicated frequency range the change of the conductivity does not depend on the frequency.

2. High frequencies: $\hbar\Omega>T$

We confine ourselves to relatively low radiation intensities, such that

$$(e\mathscr{E}r_c/\hbar\Omega)\exp(\zeta_c\hbar\Omega/2E_0)\ll1.$$

We need then retain in (20) only the terms with $s=0$ and ±1. Expanding the functions A_0, A_1, and A_{-1} in the small argument accurate to second order in $\mathscr{E}$, we obtain from (20) the following expression for the relative change of the conductivity:

$$\frac{\sigma-\sigma_0}{\sigma_0}\sim(F(\hbar\Omega)-1)\left(\frac{e\mathscr{E}r_c}{\hbar\Omega}\right)^2\exp\left\{\zeta_c\frac{\hbar\Omega}{E_0}\right\}. \tag{23}$$

In the case $T<\hbar\Omega<\Delta$, the main contribution to the change of the conductivity is made by the critical hops between states whose energy difference lies in a band of width ε_0 near the energy $\hbar\Omega$. Then

$$F(\hbar\Omega)\sim(T/\Delta)(\hbar\Omega/\varepsilon_0)^7\exp(\hbar\Omega/T).$$

On the other hand if $\hbar\Omega\gg\Delta>T$, then

$$F(\hbar\Omega)\sim(\Delta/\hbar\Omega)^7\exp(\Delta/T).$$

In this case phonons with energy of the order of $\hbar\Omega$ are emitted spontaneously in the hops with photon absorption, whereas in the absence of radiation there are absorbed in the hops phonons with energy of the order of Δ. If $F(\hbar\Omega)<1$, then a negative photoconductivity is produced by the radiation-induced decrease of the critical hops. This fact was already pointed out in Ref. 6, as well as in Ref. 11.

As seen from (22) and (23), the relative conductivity change $\Delta\sigma/\sigma_0$ under the influence of the radiation has the following temperature dependence:

$$\Delta\sigma/\sigma_0\infty T^{-2}\quad\text{if}\quad\hbar\Omega\ll T,$$
$$\Delta\sigma/\sigma_0\infty Te^{\hbar\Omega/T}\quad\text{if}\quad T<\hbar\Omega<\Delta;$$

at $\hbar\Omega>\Delta$ the negative photoconductivity does not depend on the temperature, while the positive one depends on the temperature like $\Delta\sigma/\sigma_0\propto e^{\Delta/T}$. These temperature dependences of the photoconductivity agree qualitatively

with the experimental data.[8]

We present now numerical estimates of the sensitivity of the radiation detector on the basis of the considered mechanism of hopping conductivity. The current-voltage sensitivity of a detector measuring $S=1\times1$ cm to which a voltage $U=1$ V is applied is equal to

$$R=\frac{U}{IS}\frac{\Delta\sigma}{\sigma_0},\quad I=\frac{4\pi\sqrt{\varkappa}}{c}\mathscr{E}^2,$$

where I is the intensity of the electromagnetic radiation incident on the detector. For radiation frequencies $\Omega\lesssim10^{11}$ sec^{-1} at $T=4$ K and at a doping $N\sim10^{14}$ cm^{-3} we have according to (22) $R\sim1$ V/W. The effect of higher-frequency radiation on the conductivity can be much stronger. Thus, for an n-InSb sample with parameters $N=(1-2)\times10^{14}$ cm^{-3}, $K=0.85$ and at $T=1.5$ K($\varepsilon_1=1$ MeV) (Ref. 8) we have according to (23) $R\sim10^3$ V/W at an emission wavelength $\lambda=8$ mm ($\hbar\Omega=0.15$ meV), and $R\sim10^4$ V/W at $\lambda=2$ mm ($\hbar\Omega=0.62$ meV). Account is taken in these estimates that the calculated value is $\zeta_c=7$, and that the value of Δ determined from the plot of $\Delta\sigma/\sigma_0$ against T (Ref. 8) is $\Delta=0.2$ meV. These estimates agree in order of magnitude with the experimental values of the sensitivity.[8]

As seen from (22) and (23), the relative change of the hopping conductivity under the influence of radiation is larger the smaller the "dark" conductivity (in the absence of radiation). Therefore the volt-watt sensitivity of a detector based on the considered effect is limited only by its permissible resistance. For example, for p-Ge (Ref. 16) with acceptor density $N_A=1.5\times10^{15}$ cm^{-3} and compensation $K=0.4$ at $T=4$ K ($\sigma_0\sim10^{-7}$) $(\Omega-\text{cm})^{-1}$, $E_0=0.01$ eV; calculated values $\zeta_c=21$ and $\Delta=0.6$ meV) we obtain $R\sim10^6$ V/W for emission frequencies close to $E_0/\hbar$, i.e., $\Omega\approx10^{13}$ sec^{-1}. This means that the resistance of the sample can vary by several times even at an incident radiation intensity 10^{-6} W/cm^2.

One of the distinguishing features of hopping conductivity is its anomalously strong (exponential) dependence on the external magnetic field H (see Ref. 8). However, the relative conductivity change $\Delta\sigma(H)/\sigma_0(H)$ can depend little on the magnetic field, since the length of the critical hops increases only in a strong magnetic field $H(H>N^{1/3}c\hbar/ea)$ when the conductivity has a Mott temperature dependence. In the opposite case $\Delta\sigma(H)/\Delta\sigma(0)\sim\sigma_0(H)/\sigma_0(0)$. It appears that a situation close to this is realized in Ref. 8.

The change of the hopping probability under the influence of the radiation can lead also to a redistribution of the electrons over the localized states, since the probabilities Γ_{ij} and Γ_{ji} of the hops are generally speaking not equal for an equilibrium distribution function of the electrons. If the deviations of the distribution function $f(\varepsilon_{i,j})$ from rquilibrium are small, then they can be easily determined from the stationarity condition $\Gamma_{ij}=\Gamma_{ji}$. Allowance for these deviations leads to a correction factor in the formulas for the relative change of the conductivity (20)–(23). This factor ranges from 2 at $T\ll\Delta$ to 1 at $T\gg\Delta$.

We note that the effect can be larger at emission frequencies such that transitions are possible not to the ground state of the neighboring center, but to excited states. This can lead to the appearance of peaks on the frequency dependence of (see Ref. 8). As to absorption of electromagnetic radiation, it takes place predominantly on the "easiest" hops, whereas the value of the conductivity is determined by the "most difficult" hops in the percolation paths. In addition, if the emission frequency is high enough, then the main absorption may be due to excitation of the centers themselves.

We note in conclusion that experimental observation of hopping photoconductivity makes it possible to determine the most important parameters of the states over which the conduction is effected, as well as to use this effect to develop sensitive receivers for electromagnetic radiation.

[1)]Here and below we assume $\hbar=1$ in the intermediate calculations.

[2)]At constant activation energy we have $r_c\approx N^{-1/3}$, and at a Mott temperature dependence $r_c>N^{-1/3}$. Therefore $\zeta_c\gg1$ in all cases of weak doping.

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