

On the theory of the electric conductivity of a thin metallic plate in a strong magnetic field

A. I. Kopeliovich

Physicotechnical Institute of Low Temperatures, Ukrainian Academy of Sciences
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Exact expressions for the conductivity of a plate are obtained for an arbitrary character of the electron reflection from its surface and under conditions when volume collisions of the electrons can be neglected. The static skin effect and oscillations of the conductivity in a strong magnetic field are considered on the basis of these expressions. The analysis leads to some conclusions that differ from those of the well-known theory for the static skin effect. [Azbel' and Peschanskii, Sov. Phys. JETP 17, 451 (1963); 22, 399 (1966); 28, 1045 (1969).] It turns out that the value of the current in the near-surface layer is not determined by the degree of diffuseness of the surface scattering, but mainly by recombination upon reflection. In a magnetic field inclined to the plate surface, skinning of the dissipative current component occurs not only in compensated, but also in noncompensated metals. It is shown that the electron mean free path in the near-surface layer is much smaller than the bulk value, provided that the surface scattering satisfies a condition formulated in the paper, that the scattering be relaxational in character. If this condition is not satisfied or if the relaxation is the result of a large number of collisions with the surface, the conductivity of the plate exhibits a considerable anisotropy. The found dependence of the plate resistance on the angle of inclination of the magnetic field to the plate surface can be used, in particular, to determine the surface recombination probability. The behavior of the conductivity and the nature of the skin effect for purely specular electron reflection are discussed. It is shown that if the surface is described by any scattering indicatrix which depends smoothly on the incident and reflected electron momenta, the Sondheimer oscillations take about the same form as in diffuse scattering. For the case of scattering containing a specular component, some new nonperiodic conductivity oscillations in a magnetic field inclined to the plate surface are found. Generally, speaking, the amplitudes of these oscillations are greater than the amplitude of the Sondheimer oscillations, which are defined by the limiting points of the Fermi surface.

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INTRODUCTION

The effect of the crowding out of a direct current towards the surface of a plate in a strong magnetic field (the static skin effect) was predicted by Azbel'.¹ The theory of the electric conductivity of a thin plate, taking this effect into account, was constructed by Azbel' and Peschanskii.^{2,3} A new method of calculation of the electric conductivity of a thin plate is proposed in the present work. This method, in the case in which the volume collisions are unimportant ($d \ll \mathcal{L}$, d is the plate thickness, \mathcal{L} is the volume free path length, ϑ the angle between the direction of the magnetic field and the surface of the plate) leads to an exact expression for the conductivity of the plate, averaged over the thickness, for an arbitrary character of the electron reflection from its surface. This makes it possible, first, to remove a number of obscurities in the principles of the theory of the static skin effect and, second, to obtain a detailed description of this and other size effects.

It is known⁴ that in a strong magnetic field ($r \ll l$, r is the mean radius of the electron orbit) in the case of closed electron orbits, the transverse components of the conductivity of the bulk sample are small in comparison with their value σ_0 at $H=0$: the transverse conductivity $\sigma_{\perp} \approx \sigma_0 (r/l)^2$ is due to diffusion of the centers of the orbits as a result of infrequent volume collisions, and the displacement per collision is of the order $r \propto H^{-1}$. In a layer of the same thickness $\sim r$, close to the surface of the sample, the centers of the orbits are displaced more frequently because of collisions with the surface: falling into this layer, the electron inevitably experiences a collision within a time of the order of r/v_F . This is in

fact the reason for the static skin effect; the density of near-surface current significantly exceeds the volume current; it is also possible that the current flowing along the plate concentrates essentially at the surface. However, it must be kept in mind that in a plate of a non-compensated metal ($n_e \neq n_h$; n_e, n_h are respectively the densities of electrons and holes) in a magnetic field that is parallel to the surface, large Hall currents develop; this leads to a high volume current density: $j_{\nu 01} \approx \sigma_0 E$. Therefore, at $\vartheta=0$, the skin effect is possible only in compensated metals.

As is asserted in the work of Peschanskii and Azbel',³ the value of the skin effect differs substantially for the cases of specular and diffuse reflection from the surface. The physical meaning of this difference, according to Lifshitz *et al.*,⁴ consists of the following. In specular scattering and at $\vartheta=0$, the electrons from the near-surface layer move along periodic trajectories in the direction of the electric field (see Fig. 1a) until they no longer experience volume collisions; therefore, the effective free path length in the near surface layer $l_{eff} \approx l$ and the conductivity $\sigma_{sur xx} \approx \sigma_0$, while the contribution of this layer to the conductivity, averaged over the thickness of the plate, is $s_{sur xx} \approx \sigma_0 r/d$. Thus, the skin effect

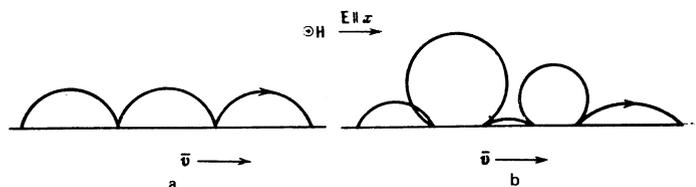


FIG. 1.

in this case leads to a linear field dependence of the average resistivity $\rho_{xx} \propto H$ in the case of plate thickness $d \ll l^2/r$ and $n_e = n_h$. In diffuse reflection, the electron forgets the result of the action of the electric field and therefore $l_{eff} \approx r$ in the near-surface layer. (In other words, the diffuse scattering leads to relaxation in the absence of specular scattering.) Thus, $s_{sur xx} \approx \sigma_0 r^2 / ld$ and, consequently, the skin effect in diffuse scattering does not change the dependence of the mean resistance on H .

It is asserted in the present work that the indicated difference between the cases of specular and diffuse reflection does not occur. In support of this, we advance the following qualitative considerations. It is not difficult to understand (see Fig. 1b) that at $\vartheta = 0$ diffuse scattering, just as specular, leads to infinite motion along the surface with mean velocity of the order of v_F . Although the value of the reflection angle is random, the electron is displaced by a distance of the order of r on the same side of the x axis between successive collisions with the surface, while the departure of the electron from the surface is impossible without volume collisions. If an electric field is turned on in the x direction, the energy of the electron increases without limit in the absence of volume collisions¹⁾; therefore, $\bar{l}_{eff} \approx l$ in the near-surface layer.²⁾ If the magnetic field is not parallel to the surface, then the electron leaves the surface after an average of ϑ^{-1} collisions with it. Such a mechanism of departure from the near-surface layer is more probable at $\vartheta > r/l$ and in this case, $l_{eff} \approx r/\vartheta$.

It is obvious from the considerations just advanced that an arbitrary internal surface scattering does not lead to relaxation. The situation changes, however, if the scattering by the surface is accompanied by recombination—transitions of quasiparticles between the electron and hole groups of the Fermi surface. It is not difficult to understand the latter by noting that the transition from an electron to a hole orbit changes the direction of motion of the quasiparticle along the surface. The general condition for relaxation surface scattering is established in Sec. 3b of the present work. In order that $l_{eff} \approx r$ at $\vartheta \ll 1$, it is at least necessary that $n_e = n_h$.

We now give a brief account of the sections of the work and the basic results. The complete set of equations of the problem is set forth in Sec. 1 and an exact solution is found for the distribution function under the condition that volume collisions can be neglected. Exact conveniently analyzed formulas are obtained in Sec. 2 for the mean conductivity of the plate \hat{s} , expressed in terms of the value of the displacements of the electron in the path between two successive collisions with the surface. These exact expressions contain, in particular, the contribution of trajectories beginning and ending on the same surface, not taken into account earlier.²⁾ This contribution turns out to be very significant. Specific questions of the electrical conductivity of the plate in an oblique field are considered in parts A and B of Sec. 2 on the basis of the expressions that have been obtained. In part A, the mean conductivity in general form is divided into "surface" and "volume" parts. This enables us to draw the conclusion that, in the absence of volume

collisions, every dissipative component of the current lies entirely in the skin layer, regardless of the relation between n_e and n_h . (It is maintained in Ref. 2 that the skin effect takes place in an oblique field only at $n_e = n_h$.) In part B, the behavior of the conductivity at small angles of inclination of the field is studied: $1 \gg \vartheta \gg d/l$. The dependence of the conductivity tensor on ϑ is first established here; for a number of cases, simple accurate results are obtained. It is shown that we can extract definite information on the character of the surface scattering from the form of the $s(\vartheta)$ dependence. The result $s_{xx} \propto \vartheta^{-1}$ obtained by direct calculation confirms the qualitative conclusion drawn above on the non-relaxational character of the surface scattering at $n_e \neq n_h$. In part B, the oscillatory dependence of the conductivity on the magnetic field is considered. It is shown that in the case of an arbitrary surface scattering indicatrix that depends smoothly on the angles of incidence and reflection, the oscillations of the conductivity do not differ qualitatively from the conductivity oscillations in purely diffuse scattering (this conclusion contradicts the results of Ref. 6). It is also shown that if the surface scattering contains a significant admixture of specular scattering, and $\vartheta = \pi/2$, the oscillations of the conductivity have a complicated nonperiodic character, and their amplitude is proportional to $(r/d)^3$ and, generally speaking, greater than the amplitude of the Sondheimer oscillations. (It is known that, in correspondence with quantum considerations scattering by the surface necessarily contains a definite, generally not small, specular scattering component; see, for example, Ref. 7.)

The conductivity of a plate is considered in Sec. 3 for the range of angles $\vartheta \ll d/l$, when account of volume collisions is necessary. The results of Sec. 3 are applicable also to thick, $d > l$, plates. In Sec. 3a, expressions are obtained for the mean conductivity, and the character of the skin effect is discussed under the assumption that the surface scattering establishes complete equilibrium not too slowly. The opposite situation is considered in Sec. 3b. Here, from analysis of the solvability of the equation for the mean path, general surface-scattering conditions of a relaxational character are established; in particular, at $\vartheta \ll r/l$, they determine the conditions of linear growth of the resistance. Interpolation formulas (44), (45) and (48) for the mean resistance of a plate, which are convenient for comparison with experiment, are cited in Sec. 3; these apply to a broad range of angles ϑ .

The conductivity of the plate under conditions of purely specular scattering by the surface is discussed in a brief final section. The exact value of the conductivity in the absence of volume collisions is found. The results are given of a semi-qualitative analysis of the mean conductivity and of the character of the skin effect in the presence of volume collisions that lead to recombinations. It is interesting that skinning is enhanced for a thin plate at $n_e = n_h$ with decrease in the angle ϑ from values $\vartheta > d/t$ to $\vartheta < d/t$, not only due to the increase in the surface current, but also due to the strong decrease in the volume current.

The orbits of the electrons in a magnetic field are assumed to be closed throughout.

1. BASIC EQUATIONS

As is known, the complete set of equations for the electron distribution function in a metallic plate consists of the kinetic equation

$$\frac{\partial \chi}{\partial t} + v_i \frac{\partial \chi}{\partial \xi} + \hat{v} \chi = e E v, \tag{1}$$

the condition of electrical neutrality

$$\frac{2}{h^3} \int \chi(\mathbf{p}, \xi) \frac{dS_p}{v} = \langle \langle \chi \rangle \rangle = 0 \tag{2}$$

and the boundary conditions, which, in the case of an arbitrary character of the scattering of the electrons by the surface, have the form

$$\chi(\mathbf{p}, \xi = \pm d/2) = \int Q(\mathbf{p}, \mathbf{p}') \chi(\mathbf{p}', \xi = \pm d/2) dp', \tag{3}$$

$$\int \dots dp = \frac{2}{h^3} \int \dots |v_i(\mathbf{p})| \frac{dS_p}{v}.$$

In Eqs. (1)–(3), the electron distribution function is represented in the form $f = f_0 - \chi \partial f_0 / \partial \varepsilon$ (f_0 is the equilibrium distribution function); t is the time of motion of the electron along the orbit in a magnetic field; \mathbf{v} is the velocity of the electron, the ξ axis is directed perpendicular to the plate (see Fig. 2); $\hat{\nu}$ is the operator of volume collisions of the electrons; the electric field intensity in the plate is $\mathbf{E} = \mathbf{E}_{||} + \mathbf{E}_{\perp}(\xi)$, $\mathbf{E}_{||} = \text{const}$ is the component of the field in the plane of the plate, dS_p is the element or area of the Fermi surface; the matrix (indicatrix) of the surface scattering is $Q(\mathbf{p}, \mathbf{p}') = 0$ at $v_i(\mathbf{p})v_i(\mathbf{p}') > 0$, the scattering of both surfaces of the plate is assumed to be identical: $Q(\mathbf{p}, \mathbf{p}') = Q(-\mathbf{p}, -\mathbf{p}')$; the upper sign in (3) holds at $v_i(\mathbf{p}) < 0$, the lower, at $v_i(\mathbf{p}) > 0$.

The scattering matrix satisfies the following relations:

$$Q(\mathbf{p}, \mathbf{p}') = Q(-\mathbf{p}', -\mathbf{p}) = Q(\mathbf{p}', \mathbf{p}), \tag{4}$$

$$\int Q(\mathbf{p}, \mathbf{p}') dp = 1. \tag{5}$$

The first of these is the well-known reciprocity relation, the second expresses the conservation of the number of particles in the scattering.

It is obvious that the volume collisions are unimportant if the relaxation due to surface scattering is achieved for all electrons before a collision takes place in the volume. The corresponding condition in the case of strong magnetic fields $r \ll d$ can be written in the form

$$d/\vartheta + Nr \ll l. \tag{6}$$

Here d/ϑ is the path traversed by the electron in the motion between the plate surfaces, and N is the number of collisions with the surface necessary to establish complete equilibrium in the electron system (for example, $N = \infty$ in the case of purely specular scattering).

If we neglect the term $\hat{\nu} \chi$ in (1), the exact solution of

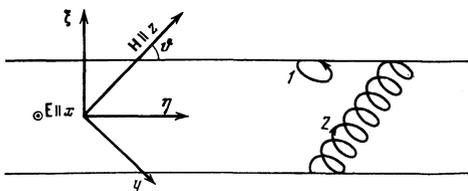


FIG. 2. Two types of trajectories (1, 2) of electrons in a plate.

the set (1), (2) can be written in the form

$$\chi(\mathbf{p}, \xi) = e E_{||} \langle \langle \mathbf{r} \rangle \rangle / \langle \langle 1 \rangle \rangle, \tag{7}$$

$$\langle \langle \mathbf{r} \rangle \rangle = \mathbf{r} - \mathbf{r}_s(\mathbf{p}, \mathbf{r}) + \mathbf{r}_s(\mathbf{p}_s), \quad \mathbf{E}_{\perp} = \frac{d}{d\xi} \mathbf{E}_{||} \langle \langle \mathbf{r} \rangle \rangle.$$

The quantity $\mathbf{r}(\mathbf{p}, \xi)$ has the following sense: this is the average path traversed by the electron in the infinite interval of time up to the moment at which it appears at the given point of phase space (we have in mind averaging over all possible realizations of the reflection of the electron by the surface), $\mathbf{r}_s(\mathbf{p}, \mathbf{r})$ and $\mathbf{p}_s(\mathbf{p}, \mathbf{r})$ are the coordinate and momentum of the given electron at the moment of its start from the surface after the last collision with it (we note that the quantity $\mathbf{r} - \mathbf{r}_s$ does not depend on the coordinates x and y , their relative position is clear from Fig. 2); $\mathbf{r}_s(\mathbf{p}_s)$ is the mean path for an electron starting from the surface.

The boundary condition (3) leads to the following integral equation for the mean path:

$$\bar{\mathbf{r}}(\mathbf{p}) = \int Q(\mathbf{p}_s, \mathbf{p}_s) \bar{\mathbf{r}}(\mathbf{p}_s) dp_s + \Delta \mathbf{r}(\mathbf{p}), \tag{8}$$

$$\bar{\mathbf{r}}_s(\mathbf{p}_s) = \bar{\mathbf{r}}(\mathbf{p}) - \Delta \mathbf{r}(\mathbf{p}).$$

Here we have introduced the quantity $\bar{\mathbf{r}}(\mathbf{p})$ —the mean path for an electron reaching the surface with momentum \mathbf{p} [$\mathbf{p}_s(\mathbf{p})$ is the momentum of this electron at the moment of the next start from the surface]; $\Delta \mathbf{r}(\mathbf{p})$ is the path traversed by this electron from the moment of start.

For further consideration, it is convenient to introduce also the Green's function $L(\mathbf{p}, \mathbf{p}')$, which satisfies the equation

$$L(\mathbf{p}, \mathbf{p}') = \int Q(\mathbf{p}_s, \mathbf{p}_s) L(\mathbf{p}_s, \mathbf{p}') dp_s + \frac{1}{2} [\delta(\mathbf{p} - \mathbf{p}') - \delta(\mathbf{p} + \mathbf{p}')]; \tag{9}$$

here the δ function is defined in the following fashion:

$$\int \delta(\mathbf{p} - \mathbf{p}_s) f(\mathbf{p}) dp = f(\mathbf{p}_s).$$

It is seen from a comparison of Eqs. (9) and (8) that the quantity $\bar{\mathbf{r}}$ is connected with L by the following relation [it is also necessary to take it into account that $\Delta \mathbf{r}(-\mathbf{p}) = -\Delta \mathbf{r}(\mathbf{p})$]:

$$\bar{\mathbf{r}}(\mathbf{p}) = \int L(\mathbf{p}, \mathbf{p}') \Delta \mathbf{r}(\mathbf{p}') dp'. \tag{10}$$

The quantity $L(\mathbf{p}, \mathbf{p}')$ also satisfies the following equation:

$$L(\mathbf{p}, \mathbf{p}') = \int Q(\mathbf{p}_s, \mathbf{p}') L(\mathbf{p}_s, \mathbf{p}') dp_s + \frac{1}{2} [\delta(\mathbf{p} - \mathbf{p}') - \delta(\mathbf{p} + \mathbf{p}')], \tag{11}$$

which we can verify by noting that identical iteration series follow for L from (9) and (11).

We also write out two relations for L that will be useful later. These can easily be obtained by integrating (9) over dp_s in the region where $v_i(\mathbf{p}_s) > 0$, and (11) over dp' in the region where $v_i(\mathbf{p}') < 0$, taking into account (4), (5) and also the relations $dp_s = dp$ and $L(\mathbf{p}, -\mathbf{p}') = -L(\mathbf{p}, \mathbf{p}') = L(-\mathbf{p}, \mathbf{p}')$:

$$\int L(\mathbf{p}, \mathbf{p}') dp \uparrow = -\frac{1}{2} \text{sign } v_i(\mathbf{p}_s'), \tag{12}$$

$$\int L(\mathbf{p}, \mathbf{p}') dp' \uparrow = -\frac{1}{2} \text{sign } v_i(\mathbf{p}); \tag{13}$$

the arrows here and below mean that the integration is carried out over those \mathbf{p} which correspond to trajectories that terminate on the upper surface or on the lower

surface (i.e., $v_z(\mathbf{p}) > 0$ and $v_z(\mathbf{p}_s) > 0$; these are trajectories of type 2 in Fig. 2).

For the simplest form of scattering in which $Q(\mathbf{p}, \mathbf{p}') = \text{const}$ at $v_z(\mathbf{p})v_z(\mathbf{p}') < 0$ and therefore $N=1$ (complete equilibrium is achieved as a result of a single collision with the surface), Eq. (9) can easily be solved:

$$L_z(\mathbf{p}, \mathbf{p}') = \frac{1}{2} [\delta(\mathbf{p}-\mathbf{p}') - \delta(\mathbf{p}+\mathbf{p}')] - \frac{\text{sign}[v_z(\mathbf{p}_s) \times v_z(\mathbf{p}')] }{4f d p \uparrow} \quad (14)$$

We define as diffuse the scattering that establishes equilibrium between each of the isolated groups of carriers within a single collision, but without transitions between them: $Q(\mathbf{p}_a, \mathbf{p}_b) = \text{const}_a \delta_{ab}$; a and b are the numbers of the groups. We emphasize the extreme important (for what follows) difference of the scatterings with $N=1$ and diffuse scattering in the presence of several groups of carriers³⁾: for diffuse scattering, $N=\infty$. For the quantity $L_d(\mathbf{p}_a, \mathbf{p}'_b)$ in the case of diffuse scattering, Eq. (14) is valid, in which we must replace $\int d\mathbf{p} \uparrow$ by $\int d\mathbf{p}_a \uparrow$ and add δ_{ab} as a common factor.

In the next section, we consider the conductivity of a plate under the conditions in which the surface scattering establishes the equilibrium not too slowly: $Nr \ll l$ and $\vartheta \gg d/l$; therefore we can neglect the volume collisions [see (6)].

2. RESISTANCE OF A PLATE IN A MAGNETIC FIELD THAT IS OBLIQUE TO ITS SURFACE

Using Eqs. (7) and (8), we obtain the exact expressions for the tensor $\hat{\xi}$ of the electrical conductivity of the plate:

$$\begin{aligned} \hat{j}_\alpha = d^{-1} \int_{-d/2}^{d/2} j_\alpha(\xi) d\xi = s_{\alpha\beta} E_{\beta\beta}; \quad \alpha, \beta = x, y, \\ j(\xi) = e \langle \chi v \rangle. \end{aligned} \quad (15)$$

We first note that in the calculation of the mean conductivity it is not convenient to use the expression (15) directly, substituting the distribution function (7) in it. First, these formulas contain redundant information—the dependence of the current density on ξ . Second, the expressions for $j(\xi)$ contain terms that are oscillatory in ξ and whose contribution of which to s is substantially cancelled.

In order to avoid these difficulties, we carry out the integration over ξ in (15) in general form. It is convenient here to choose the origin of the coordinates for each electron trajectory at the center of the trajectory; in the initial set, this point has the coordinate \mathbf{r}_{cen} such that $\xi_{\text{cen}} = 0$, and the component perpendicular to \mathbf{H} of the new coordinate of the electron $\mathbf{R} = \mathbf{r} - \mathbf{r}_{\text{cen}}$ is connected with its momentum by the relation

$$\mathbf{R}_\perp = \frac{c}{eH^2} [\mathbf{H} \times \mathbf{p}], \quad (16)$$

and the origin for \mathbf{p} is chosen at the center of inversion of the Brillouin zone. The coordinate origin \mathbf{r}_{cen} does not depend on the time of motion along the orbit, as can be shown by differentiating (16) with respect to t and comparing it with the equation of motion $\dot{\mathbf{p}} = (e/c)[\mathbf{v} \times \mathbf{H}]$. The new coordinate of the electron \mathbf{R} can be expressed in the following way in terms of \mathbf{R}_\perp and ξ :

$$\mathbf{R} = d\mathbf{R}_\perp + \xi \frac{d\mathbf{H}}{dH}, \quad D_{ik} = \delta_{ik} - \frac{H_i d_k}{Hd}; \quad (17)$$

here we have introduced the vector $d \parallel \xi$, $|d| = d; i, k = x, y, \xi$. Substituting (7) in (15) and using (16) and (17), we obtain, in the new coordinates,

$$\begin{aligned} s_{\alpha\beta} = \frac{e^2}{d} \int_{-d/2}^{d/2} \langle v_\alpha (\bar{r}_\beta(\mathbf{p}_s) - R_{\alpha\beta}(\mathbf{p}_s)) \rangle d\xi - b_{\alpha\beta}, \\ b_{\alpha\beta} = \frac{ec(n_e - n_h)}{H^2} D_{\beta i} e_{i\alpha k} H_k, \end{aligned} \quad (18)$$

where e_{ikh} is a unitary antisymmetric tensor, and repeated subscripts indicate summation. In the calculation of the quantity $\langle v_\alpha R_\beta \rangle$ we have used the identities

$$\langle v_\alpha \rangle = 0, \quad \langle v_\alpha p_i \rangle = (n_e - n_h) \delta_{\alpha i}.$$

We now transform in (18) from integration over the momentum of the electron at the instant when it reaches the point with coordinate ξ to integration over the momentum of this same electron at the instant of its start from the surface of the plate [the connection between the corresponding differentials has the form $|v_z(\mathbf{p})| d\mathbf{p} = |v_z(\mathbf{p}_s)| d\mathbf{p}_s$]:

$$s_{\alpha\beta} = \frac{2e^2}{dH^3} \int_{\nu(\mathbf{p}_s)} dS_{\nu s} |v_z(\mathbf{p}_s)| [\bar{r}_\beta(\mathbf{p}_s) - R_{\beta\alpha}(\mathbf{p}_s)] \int d\xi \sum_k \left(\frac{v_\alpha(\mathbf{p})}{|v_z(\mathbf{p})|} \right)_k - b_{\alpha\beta}; \quad (19)$$

since several points on the trajectory correspond, generally speaking, to fixed ξ and summation is carried out over all these points.

Changing in (19) the integration variable $d\xi = |v_z(\mathbf{p})| dt$ and taking it into account that

$$\int_{t_0}^{t_f} v_\alpha(\mathbf{p}) dt = \Delta R_\alpha(\mathbf{p}_f) = R_\alpha(\mathbf{p}_f) - R_\alpha(\mathbf{p}_s)$$

(\mathbf{p}_f is the momentum of the electron at the moment t_f of reaching the surface), we obtain

$$\begin{aligned} s_{\alpha\beta} = 2 \langle (\bar{r}_\beta - R_{f\beta}) \Delta R_\alpha \rangle - b_{\alpha\beta}, \\ \langle \dots \rangle = \frac{e^2}{d} \int_{v_z > 0} \dots d\mathbf{p}_f. \end{aligned} \quad (20)$$

This is in fact the desired exact expression for the mean conductivity; here $\hat{\xi}$ is represented in the form of a sum over all trajectories that terminate in the upper surface of the plate (the coefficient 2 takes into account the contribution to the same quantity from the trajectories that terminate on the lower surface).

In order to make the expression for the conductivity more convenient and graphic, we separate the symmetric s^s and the antisymmetric s^a (relative to the magnetic field) parts of (20):

$$\hat{s} = \hat{s}^s + \hat{s}^a, \quad \hat{s}^s(-H) = \hat{s}^s(H), \quad \hat{s}^a(-H) = -\hat{s}^a(H),$$

and also separate the parts in the electron coordinate \mathbf{R} that depend in different fashion on H :

$$\begin{aligned} \mathbf{R} = \mathbf{X} \pm \mathbf{T} \quad \xi = \pm d/2, \quad \mathbf{T} = d^2 \mathbf{H} / (dH), \\ \mathbf{X} = d\mathbf{R}_\perp; \end{aligned}$$

the vector $\mathbf{X} (|\mathbf{X}| \approx \nu \alpha H^{-1})$ is parallel to the surface of the plate. It is necessary to take into account in the transformations the fact that replacement of H by $-H$ is equivalent to reversal of the time, and therefore to the exchange $\mathbf{R}_s \rightleftharpoons \mathbf{R}_f$, while the quantity $\bar{\mathbf{r}}$ [see (10)] is replaced by

$$- \int L(\mathbf{p}, \mathbf{p}') \Delta \mathbf{r}(\mathbf{p}') d\mathbf{p}'.$$

Using the relations (12) and (13), we obtain

$$s_{\alpha\beta} = -\frac{e^2}{d} \int \mathcal{L}(\mathbf{p}, \mathbf{p}') \Delta X_\alpha(\mathbf{p}) \Delta X_\beta(\mathbf{p}') d\mathbf{p} d\mathbf{p}' \approx \frac{e^2 n r^2}{d p_F} \alpha H^{-2}, \quad (21)$$

$$\hat{s} = \hat{s}^{(1)} + \hat{s}^{(2)}, \quad s_{\alpha\beta}^{(1)} = \frac{ced(n_e - n_h)}{dH} e_{\alpha\beta} \alpha H^{-1}, \quad (22)$$

$$s_{\alpha\beta}^{(2)} = \langle X_{c\alpha} X_{f\beta} - X_{c\beta} X_{f\alpha} \rangle + \frac{e^2}{d} \int A(\mathbf{p}, \mathbf{p}') \Delta X_\alpha(\mathbf{p}) \Delta X_\beta(\mathbf{p}') d\mathbf{p} d\mathbf{p}' \approx \frac{e^2 n r^2}{d p_F} \alpha H^{-2},$$

$$\mathcal{L}(\mathbf{p}, \mathbf{p}') = \frac{1}{2} [L(\mathbf{p}, \mathbf{p}') + L(\mathbf{p}', \mathbf{p}) + \delta(\mathbf{p} + \mathbf{p}')],$$

$$A(\mathbf{p}, \mathbf{p}') = \frac{1}{2} [L(\mathbf{p}, \mathbf{p}') - L(\mathbf{p}', \mathbf{p})].$$

We note that the expression (21), which contains the product of the displacements $\Delta X_\alpha \Delta X_\beta$, has an explicit physical meaning—it shows the well-known connection between the dissipative part of the conductivity and the diffusion of the carriers at $\mathbf{E} = 0$. In formulas (21) and (22), the orders of the quantities at $\nu \approx 1$ are shown, as well as the smooth dependences on H at $\nu \ll d$ [the change in the quantity $L(\mathbf{p}, \mathbf{p}')$ with changing H does not affect the smooth $\hat{s}(H)$ dependence; this is proved in part C, see formula (32)]. As is seen from (22), the principal part of the conductivity $\hat{s}^{(1)} \propto H^{-1}$ does not depend on the character of the scattering by the surface (at $N = 1$, a similar expression for $\hat{s}^{(1)}$ was obtained in Ref. 2).

For the scattering with $N = 1$, we get from (14), (21) and (22):

$$s_{1\alpha\beta}^* = \langle \Delta X_\alpha \Delta X_\beta \rangle + [\langle \Delta X_\alpha \rangle \langle \Delta X_\beta \rangle + \langle \Delta X_\alpha \rangle \langle \Delta X_\beta \rangle] / \langle 1 \rangle, \quad (23)$$

$$s_{1\alpha\beta}^{(2)} = e_{\alpha\beta} \hat{s}_1^{(2)}, \quad s^{(2)} = \langle [X_s X_f] \rangle + [\langle \Delta X \rangle \langle \Delta X \rangle] / \langle 1 \rangle. \quad (24)$$

Formulas (23) and (24) do not coincide with the expressions for the mean conductivity in the case of scattering with $N = 1$, given in Ref. 2. The fact is that the approximate method employed in Ref. 2 does not make it possible to determine with the necessary accuracy the contribution of regions of order ν near the plate surface. In particular, the contribution of trajectories of type 1 in Fig. 2 is neglected; the integration over states \mathbf{p} corresponding to these trajectories is denoted in (23) and (24) by the symbol \downarrow . As will be seen below, precisely these regions determine the dissipative part of the conductivity and $\hat{s}^{(2)}$.

It is obvious that in the case of diffuse scattering each group of carriers makes an independent contribution to the conductivity, which is determined for each group by the formulas (23) and (24).

We now proceed to the analysis of the physical consequences of the expressions for the conductivity obtained above.

A. Static skin effect

Although the method developed above makes it possible to determine only the conductivity averaged over the thickness of the plate, it is not difficult to understand which of the trajectories make a "surface" contribution to \hat{s} from (21), (22) (i.e., are connected with currents concentrated at distances ν close to the surface), and which make a "volume" contribution. We turn for this purpose to (20). It is evident first that the surface con-

tribution is connected with trajectories of type 1 (Fig. 2), while the term $b_{\alpha\beta}$ has a volume origin [see Eq. (18)]. It can be thought that trajectories of type 2 make a volume contribution to (20); however, this is not entirely valid.

As is seen from (19), each trajectory of type 2 adds to the current density, a contribution whose dependence on ξ is determined by the factor

$$A(\xi) = \sum_k \left(\frac{v_\alpha(\mathbf{p})}{|v_\xi(\mathbf{p})|} \right)_k, \quad \mathbf{p} = \mathbf{p}(\mathbf{p}_s, \xi, k),$$

i.e., it is a periodic function of ξ with a period

$$\xi_0 = \sin \theta \oint v_\xi(\mathbf{p}_s, t) dt$$

(generally speaking, the periodicity is destroyed at distances of the order ν from the surface, where the number of points with fixed ξ on the trajectory changes as a function of ξ). We divide the quantity $A(\xi)$ into two parts: $A(\xi) = \bar{A} + A_{\text{osc}}(\xi)$, such that

$$\int_0^{\xi_0} A_{\text{osc}}(\xi) d\xi = 0.$$

It is not difficult to understand that the surface contribution to the conductivity is connected with the quantity A_{osc} , since the oscillations introduced by trajectories with different \mathbf{p}_s have different periods ξ_0 , and cancel one another at distances greater than ν from the surface; on the other hand, the contribution proportional to \bar{A} is purely volume. Taking it into account that $\int \bar{A} d\xi \propto d$, and $\int A_{\text{osc}} d\xi$ is a periodic function of d , both these contributions are easily separated in (20):

$$2 \langle (\bar{r}_\beta - R_{f\beta}) \Delta R_\alpha \rangle \uparrow = -4 T_\alpha \langle \bar{r}_\beta - R_{f\beta} \rangle \uparrow + \langle \Delta X_\alpha (\bar{r}_\beta - R_{f\beta}) \rangle \uparrow;$$

the first term on the right side is of volume origin ($T_\alpha \propto d$), and the second is of surface origin (ΔX_α is a periodic function of d with a period ξ_0).

If we now trace the contributions of different origin in the derivation of (21), (22) from (20), it is easy to establish the fact that the mean conductivity divides into volume and surface parts in the following way:

$$\hat{s} = \hat{s}_{\text{vol}} + \hat{s}_{\text{sur}}, \quad \hat{s}_{\text{vol}} = \hat{s}^{(1)}, \quad \hat{s}_{\text{sur}} = \hat{s}^* + \hat{s}^{(2)}. \quad (25)$$

Thus, the entire part of the conductivity that is dependent on the surface scattering is of the surface type. In particular, all the dissipative current that flows in the direction of the electric field lies entirely in the skin layer. We emphasize that such a skin effect exists independent of the relation between the numbers of electrons and holes. If $n_e = n_h$, however, then the non-dissipative, Hall part of the current also flows entirely in the skin layer.

We note that under the usual experimental conditions, when the specified quantity is not the electric field but the current flowing over the plate $I = \int \mathbf{j} d\xi$, the current density distribution at $n_e \neq n_h$ takes in the following form: volume current flows mainly in the direction of \mathbf{I} ; in the perpendicular direction, a current of density I/d flows over the surface, and is cancelled by a volume current of opposite sign and of density $I\nu/d^2$. We note also that it is easy to realize conditions under which the direction of the electric field is specified; for example, one can

interconnect the opposite ends of the plate (directly or with the help of low-resistance conductors).

B. Conductivity at small angles

In this part, we determine the electric conductivity tensor and its dependence on ϑ in the region of angles of inclination of the field $d/l \ll \vartheta \ll N^{-1}$. First of all we note the following circumstance: at $\vartheta \ll N^{-1}$, any surface scattering is in some sense equivalent to scattering with $N=1$. Speaking more precisely, the function $L(\mathbf{p}, \mathbf{p}')$ in the lowest approximation in ϑ has the form (14). This is not difficult to understand from physical considerations: at small ϑ the electron collides with one surface of the plate repeatedly, before going to the other and, as a result of these collisions, equilibrium manages to be established in the system of electrons lying near a given surface.

For a formal justification of the assertion just made, we must represent the integral operator in (11) in the following form:

$$\begin{aligned} \int Q(\mathbf{p}_1, \mathbf{p}') L(\mathbf{p}, \mathbf{p}_1) d\mathbf{p}_1 &= \hat{Q}_0 L + \hat{Q}_1 L, \\ \hat{Q}_0 L &= \int Q(\mathbf{p}_1, \mathbf{p}') L(\mathbf{p}, \mathbf{p}_1) d\mathbf{p}_1 + \int Q(\mathbf{p}_1, \mathbf{p}') L(\mathbf{p}, -\mathbf{p}_1) d\mathbf{p}_1, \\ \hat{Q}_1 L &= -2 \int Q(\mathbf{p}_1, \mathbf{p}') L(\mathbf{p}, -\mathbf{p}_1) d\mathbf{p}_1 \end{aligned} \quad (26)$$

(here we have made use of the fact that $L(\mathbf{p}, -\mathbf{p}') = L(\mathbf{p}, \mathbf{p}_1)$ and assume $v_z(\mathbf{p}') < 0$ for definiteness), separate out the smooth function

$$\tilde{L}(\mathbf{p}, \mathbf{p}') = L(\mathbf{p}, \mathbf{p}')^{-1/2} [\delta(\mathbf{p}-\mathbf{p}') - \delta(\mathbf{p}+\mathbf{p}')]$$

and apply the method of successive approximations in the small term $\hat{Q}_1 \tilde{L}$ to equation (11). This term takes into account the transition between the surfaces. The equation of lowest approximation $\tilde{L}^{(0)} = \hat{Q}_0 \tilde{L}^{(0)}$ has the solution $\tilde{L}^{(0)}(\mathbf{p}, \mathbf{p}') = C(\mathbf{p}) \text{sign} v_z(\mathbf{p}')$, the quantity $C(\mathbf{p})$ should be determined from the condition of solvability of the equation of the next approximation—this leads to the result (14) for $L^{(0)}$.

Analysis shows that the assertion just made is valid also with account of the fact that the surface scattering contains a certain fraction of specular scattering:

$$Q(\mathbf{p}, \mathbf{p}') = Q_{sm}(\mathbf{p}, \mathbf{p}') + \alpha(\mathbf{p}) \delta(\mathbf{p}' - \mathbf{p}) \quad (27)$$

[Q_{sm} is a smooth function of its variables, $\varepsilon(\mathbf{p}^*) = \varepsilon(\mathbf{p})$, $p_x^* = p_x$, $p_y^* = p_y$], while at sufficiently small angles of reflection the scattering is predominantly specular: $1 - \alpha(\mathbf{p}) \propto [v_z(\mathbf{p})]^2$.

The expression (14) for $L(\mathbf{p}, \mathbf{p}')$ leads to formulas (23) and (24) for the conductivity. However, we note that at $n_e = n_h$ of the conductivity in the calculation it is impossible to limit oneself to the principal approximation for the quantity $L(\mathbf{p}, \mathbf{p}')$. This is due to the fact that $\langle \Delta X_x \rangle \rightarrow 0$ at $\vartheta \rightarrow 0$ and therefore the components of the conductivity s_{xx} and s_{xy} from (23) and (24) do not contain terms proportional to ϑ^{-1} . In order to simplify the consideration of the next approximation and the calculation of the corresponding increment $\Delta s_{\alpha\beta}$ to the conductivity (23), (24) at $n_e = n_h$, we assume that the recombination in the surface scattering is much less probable than transitions within groups. Then we get

$$\begin{aligned} \Delta s_{xx} &= (ecn_e/dH)^2 \langle 1 \rangle_e^{-1} (N_{eq} - 1), \\ \Delta s_{xy} &= \frac{ecn_e \langle 1 \rangle_h \langle \Delta X_x \rangle_e - \langle 1 \rangle_e \langle \Delta X_x \rangle_h}{dH \langle 1 \rangle \langle 1 \rangle \uparrow} (N_{eq} - 1), \end{aligned} \quad (28)$$

where N_{eq} is the number of collisions necessary for establishment of equilibrium between electrons and holes (if $N=1$, then $N_{eq}=1$, $\Delta s_{\alpha\beta}=0$ also):

$$(N_{eq} - 1)^{-1} = \int Q(p_e, p_h) dp_e dp_h / \langle 1 \rangle.$$

From (23), (24) and (28), we find the following at $n_e \neq n_h$ by direct calculation, with account of the inequalities $\vartheta \ll 1$ and $r \ll d$:

$$\hat{s} = \begin{pmatrix} \frac{h^3 (n_e - n_h)^2 c^2}{2\theta H^2 \sum (S_{max} - S_{min})} & \frac{e(n_e - n_h)c}{H\theta} \\ -\frac{e(n_e - n_h)c}{H\theta} & \frac{ae^2 nr^2}{p_F d\theta} \end{pmatrix} \quad (29)$$

and at $n_e = n_h$,

$$\hat{s} = \frac{e^2 nr^2}{p_F d} \begin{pmatrix} N_{eq} b & \varphi + \varphi_1 + (N_{eq} - 1)(f + C) \\ \varphi - \varphi_1 + (N_{eq} - 1)(f - C) & g/\theta \end{pmatrix}. \quad (30)$$

In these formulas, the tensor of mean conductivity is written down in the x, η axes; S_{max} and S_{min} are the extremal cross sectional areas of the Fermi surface $p_x = \text{const}$, and the summation is carried out over all extremal cross sections; in the calculation, use is made of

$$\langle 1 \rangle \uparrow = \frac{e^2 \sin \theta}{dh^3} \sum (S_{max} - S_{min}), \quad \langle \Delta X_x \rangle = \frac{ec}{dH} (n_e - n_h)$$

at $\vartheta = 0$; $n \approx (p_F/h)^3$; the quantities $a, b, C, g, \varphi, \varphi_1, \bar{f}$ are smooth functions of the direction of \mathbf{H} relative to the crystallographic axes; $a, b, C, g \approx 1$; $|\varphi|, |\varphi_1|, |\bar{f}| \leq 1$.

The functions $\bar{f}, \varphi, \varphi_1 = 0$ if the direction of \mathbf{H} corresponds to certain elements symmetry of the crystal; $f = 0$ requires the same symmetry elements, which ensure the vanishing of σ_{bxz}^a and ρ_{bxz}^a ($\hat{\sigma}_b$ and $\hat{\rho}_b$ are the conductivity and resistivity tensors of the bulk sample); for $\varphi_1, \varphi_2 = 0$ this is generally not sufficient (in any case, $\varphi = \varphi_1 = \bar{f} = 0$ for spherical Fermi surfaces). The given difference from the bulk sample is explained by the fact that the symmetry of a sample with a surface in a magnetic field is lower than the unbounded case even at small $\vartheta \neq 0$; the terms $\varphi_1 \varphi_1$ and $(N_{eq} - 1)C$ in (30) are connected with trajectories of type 2 of Fig. 2, for which this violation of symmetry is extremely important.

We note that the dependences $\sigma_{xx} \propto \vartheta^{-1}$ at $n_e \neq n_h$ and $\sigma_{xx} \propto \vartheta^0$ at $n_e = n_h$ are in complete accord with the statements made in the Introduction on the effective length of the free path in the near-surface layer. Since complete equilibrium can be established at $\vartheta \ll N^{-1}$ near each surface (including intergroup equilibrium), it follows that $l_{eff} \approx r N_{eq}$ at $n_e = n_h$ and $l_{eff} \approx r/\vartheta$ at $n_e \neq n_h$.

If the transitions between groups are difficult ($N_{eq} \gg N_{in}, N_{in}$ is the number of collisions which establish equilibrium inside the group) then a different situation is possible: $N_{in} \ll \vartheta^{-1} \ll N_{eq}$, intragroup equilibrium is established but not intergroup equilibrium. Here $l_{eff} \approx r/\vartheta$, the scattering can be regarded as diffuse, and at $n_e = n_h$ a result is obtained that is essentially different from that of (30):

$$s_{xx} = \frac{h^3 c^2}{2\theta H^2 d} \sum_i \frac{n_i^2}{\sum (S_{max} - S_{min})_i}, \quad s_{xy} \approx -s_{yx} \approx \frac{e^2 nr^2}{p_F d\theta}, \quad (31)$$

where i is the number of the group, and the extremal cross sections are summed over the extrema of the i -th

group. Thus, one can extract information on the character of the scattering by the surface from the form of the $\xi(\vartheta)$ dependence.

The growth of the conductivity component $s_{\eta\eta}$ with decrease in ϑ assures trajectories of type 2. The electron departing from the surface has, generally speaking, a different radius of orbit than it had upon approaching it. This change in the radius leads to a shift in the center of the orbit along the η axis by an amount $\sim r/\vartheta$.

It is of interest that although the conductivity of the plate is ensured by all the electrons on the Fermi surface, the measurement of the resistivity $\rho_{\eta\eta}$ makes it possible to determine the extremal cross sections at different directions of \mathbf{H} .

If the Fermi surface is a set of spheres, the calculation of the conductivity by Eqs. (23), (24) and (28) at $\vartheta \ll 1$ and $r \ll d$ can be carried out to the end:

$$s_{\eta\eta} = \frac{4\pi c^2}{dh^2 H^2 \vartheta} \left[\sum p_F^4 - \frac{8}{9} \frac{(\sum p_F^3)^2}{\sum p_F^2} \right]$$

independently of the relation between n_e and n_h , and

$$s_{xx} = \frac{4\pi c^2}{dh^2 H^2} \left[\sum p_F^4 + (N_p - 1) \frac{(\sum p_F^3)^2}{9 \sum p_F^2} \right],$$

$$s_{yz} = -s_{zy} = \frac{4\pi c^2}{dh^2 H^2} \left[\frac{3}{4} (2\pi\vartheta)^{1/2} \sum \sigma p_F^4 + \frac{(\sum p_F^3)^2 \sum \sigma p_F^2}{9 (\sum p_F^2)^2} (N_{eq} - 1) \right]$$

at $n_e = n_h$. In these formulas, summation is carried out over all spheres, $\sigma = 1$ for the electron sphere and $\sigma = -1$ for the hole sphere.

C. Oscillations of the conductivity

It is known that in strong magnetic fields $r \ll d$ the conductivity of a thin plate, except for the basic part that is monotonically dependent on H and d , contains a small increment that is oscillatory in these variables. For scattering with $N = 1$ there take place Sondheimer oscillations^{8,9} due to the nonperiodic dependence on d of the fraction of turn of the helical trajectory of the electron included inside the plate.

In the case of an arbitrary surface scattering, the character of the oscillations depends significantly on whether the scattering matrix is a smooth function of its variables or whether it contains singularities^{10,6} (by smoothness, we mean a change in \mathbf{p} -space over distances that are large in comparison with $p_F r/d$).

We first consider the case of scattering without singularities. We first determine the dependence of the smooth part of the function $L(\mathbf{p}, \mathbf{p}')$ (for its definition, see part B) on d and H . We note that in Eqs. (9) and (11), the dependence on these quantities is connected with $\mathbf{p}_s(\mathbf{p}, d, H)$ for trajectories of type 2, Fig. 2: \mathbf{p}_s depends periodically on d with period $\xi_0(p_s)$ and almost periodically on H with period $H_0(p_s) = H \xi_0/d$. Since the momentum \mathbf{p}_s is not affected by the integration in (9) [in contrast to the momentum \mathbf{p}_{1s} in (11)], the function $\bar{L}(\mathbf{p}, \mathbf{p}')$ is naturally sought in the form

$$\bar{L}(\mathbf{p}, \mathbf{p}') = M(\mathbf{p}_s, \mathbf{p}') + M_{osc}(\mathbf{p}_s, \mathbf{p}'), \quad (32)$$

where the function $M(\mathbf{p}, \mathbf{p}')$ does not depend on d and H , and the oscillating increment of M_{osc} is small.

The assumption of the smallness of M_{osc} is easily confirmed by developing the procedure of the method of successive approximations in M_{osc} for Eq. (11). The equation of lowest approximation is

$$M(\mathbf{p}, \mathbf{p}') = \int W(\mathbf{p}_1, \mathbf{p}') M(\mathbf{p}, \mathbf{p}_1) dp_1 + \frac{1}{2} [Q(\mathbf{p}, \mathbf{p}') - Q(\mathbf{p}, -\mathbf{p}')], \quad (33)$$

$$W(\mathbf{p}, \mathbf{p}') = \overline{Q(\mathbf{p}_s, \mathbf{p}')}.$$

Here the quantity $Q(\mathbf{p}_s, \mathbf{p}')$ is averaged (indicated by the bar) over the change in d within the limits of a period; from the relation

$$\int_{t_s}^{t_f} v_i(t) dt = d$$

it follows that the indicated averaging reduces to

$$\overline{f(\mathbf{p}_s, \mathbf{p}')} = \frac{1}{\xi_0(p_s)} \int_2 f(\mathbf{p}_s, t) v_i(t, \mathbf{p}_s) dt_{s,f}, \quad (34)$$

if the trajectories with momentum at the start \mathbf{p}_s (or finish, \mathbf{p}_f , and we shall frequently omit the indices s and f) refer to type 2, Fig. 2 and $\overline{f(\mathbf{p}_s, \mathbf{p}')} = f(\mathbf{p}_s, \mathbf{p}')$ in the case of trajectory of type 1. Integration in (34) is carried out over that interval $t_{s,f}$ which corresponds to trajectories of type 2. We emphasize that, as a result of the indicated averaging, the kernel of Eq. (33) does not depend on d and H .

The equation of the next approximation determines the function M_{osc} :

$$M_{osc}(\mathbf{p}, \mathbf{p}') - \int W(\mathbf{p}_1, \mathbf{p}') M_{osc}(\mathbf{p}, \mathbf{p}_1) dp_1 = Y(\mathbf{p}, \mathbf{p}'), \quad (35)$$

$$Y(\mathbf{p}, \mathbf{p}') = \int \delta Q(\underline{\mathbf{p}}_1, \mathbf{p}') \delta M(\mathbf{p}, \underline{\mathbf{p}}_1) dp_1, \quad \delta f(\mathbf{p}) = f(\mathbf{p}) - \overline{f(\mathbf{p})} \quad (36)$$

(in the presence of several variables corresponding to the symbol δ , the variable is underlined). Since $\delta M(\mathbf{p}, \underline{\mathbf{p}}_1)$ is a smooth function of p_{1s} and $\delta Q(\underline{\mathbf{p}}_1, \mathbf{p}')$ oscillates rapidly with p_{1s} [the average of $\delta f(\mathbf{p}_s)$ over the small interval p_s is proportional in the principal approximation to the quantity $\overline{\delta f(\mathbf{p}_s)} = 0$], the function Y , and with it M_{osc} also, is small: $M_{osc} \ll M$.

Averaging the expressions (21), (22) over d (by the method shown above), it is not difficult to divide the conductivity into smooth and oscillating parts. For the oscillating part, we obtain the following asymptotically exact expression in the parameter r/d :

$$s_{osc \alpha\beta} = \frac{e^2}{d} \left\{ - \int \delta X_\alpha(\mathbf{p}) \delta X_\beta(\mathbf{p}_s) dp + \int M_{osc}(\mathbf{p}, \mathbf{p}') \right.$$

$$\times \overline{\Delta X_\alpha(\mathbf{p}_f) \Delta X_\beta(\mathbf{p}_f')} dp dp' + \int [\delta M(\mathbf{p}, \mathbf{p}') \delta X_\alpha(\mathbf{p}_f) \Delta X_\beta(\mathbf{p}_f') + \delta M(\mathbf{p}, \underline{\mathbf{p}}_f)$$

$$\times \overline{\Delta X_\alpha(\mathbf{p}_f) \delta X_\beta(\mathbf{p}_f')}] dp dp' \left. \right\}, \quad (37)$$

$$\overline{\Delta X(\mathbf{p}_f)} = \overline{X(\mathbf{p}_f)} - X(\mathbf{p}).$$

For scattering with $N = 1$ and diffusion, only the first term in (37) differs from zero.

As is seen from expressions (35) and (37), the Sondheimer oscillations, in arbitrary smooth surface scattering, are practically indistinct from the oscillations in the case of diffuse scattering (or $N = 1$). Actually,

each of the three terms in (37) leads to oscillations of the same frequency and are comparable in magnitude with the value of the amplitude. The oscillating contribution to each of the terms is connected with the electrons near the cross section $p_x = p_{x0}$, in which the quantity $\xi_0(p_x)$ has an extremum; if the extremum is lacking, the oscillations are connected with the reference points of the Fermi surface.⁹ We note that near the reference point, the quantities δQ and δM , which enter into the second and third terms in (37), are small: $\delta Q/Q, \delta M/M \approx (|p_x - p_{x0}|/p_F)^{1/2}$, i.e., they have the same order of magnitude as the displacement relative to the axis of the helical line $\delta X/r$ in the first term. The latter fact was not taken into account in Ref. 6; this led to an incorrect conclusion that in scattering that is different from diffuse, the amplitude of the oscillations is d/r times greater than in the diffuse case.

In the case of scattering with $N=1$ (or diffuse) the Eq. (37) makes it possible to carry out the calculations to the end and to obtain an exact formula for the Sondheimer oscillations at arbitrary ϑ . We only give the angular dependences of the amplitude of oscillations:

$$s_{\text{osc},\alpha} \propto (\sin \vartheta)^3, \quad s_{\text{osc},\alpha} \propto (\sin \vartheta)^2, \quad s_{\text{osc},\alpha} \propto \sin \vartheta. \quad (38)$$

Simple analysis shows that even for an arbitrary smooth scattering, the angular dependence (38) is valid at $\vartheta \ll 1$.

As has been pointed out above, the scattering matrix necessarily contains a singularity connected with a definite fraction of specularity in the reflection. In the work of Goland,¹⁰ oscillations of the conductivity were considered under conditions of partially specular scattering in a magnetic field perpendicular to the surface of the plate; it was assumed that the surface of the plate was a symmetry plane of the crystal. In this work it is shown that under the stated conditions, new "specular" oscillations develop with an amplitude of the same order of magnitude as in the "nonspecular," but with double, triple, and so forth, frequencies. It will be shown here that the position is significantly different in the case of an inclined magnetic field $\pi/2 - \vartheta \gg r/d$ and in a perpendicular field, if the surface of the plate is not a plane of symmetry.

In the determination of the contribution of the specular part of the scattering, we shall assume that the specular fraction is small¹¹; $\alpha(p) \ll 1$ [see formula (27)], we carry out iteration with respect to α in Eqs. (9) and (11) and keep only the lowest term in α in the function $L(p, p')$. (Analysis shows that consideration of the next orders of iteration does not change the character of the oscillations.) Then the contribution to the conductivity due to the singular part L_{spec} has the form

$$s_{\text{spec},\alpha} = \frac{e^2}{d} \int \alpha(p_s) \Delta X_\alpha(p) \Delta X_\alpha(p_s^*) dp. \quad (39)$$

It is not difficult to establish the fact that the products $X_\alpha(p)X_\beta(p_s^*)$ and $X_\alpha(p_s)X_\beta(p_s^*)$ in (39) lead to oscillations of the same type as in smooth scattering. Therefore, the specific specular oscillations are determined by the following expression:

$$s_{\text{spec},\alpha} = \frac{e^2}{d} \alpha(p) \delta X_\alpha(p_f) \delta X_\alpha(p_s^*) dp. \quad (40)$$

Here we have for convenience proceeded to integration over p_s and relabeled $p_s \rightarrow p, p \rightarrow p_f$. We note an important feature of Eq. (40): whereas in the smooth scattering each trajectory makes a contribution to s_{osc} that is periodic in d and H , the trajectory that consists of two (and more in higher orders) runs between the surfaces, connected by specular reflection, changes aperiodically with change in these parameters (the period for the quantities $\delta X(p_f)$ and $\delta X(p_s^*)$ is different).⁵ As will be seen below, this leads to an aperiodic oscillatory dependence.

To estimate $s_{\text{spec},\alpha}$ it suffices to take into account only the most rapid dependences of p , which arise from the dependence on p_x of the time of flight between the surfaces:

$$\delta X_\alpha(p_f) \approx r f[\Phi(p_i)], \quad \delta X_\alpha(p_s^*) \approx r \varphi[\Phi(p_i^*)], \\ \Phi(p_i) = d/\xi_0(p_i),$$

where $f(\Phi)$ and $\varphi(\Phi)$ are periodic functions with period 1, and

$$\int_0^1 f d\Phi = \int_0^1 \varphi d\Phi = 0.$$

Initially, we integrate over t in (40) at fixed p_x . In this integral, the principal contribution is made by points of stationary phase, which are determined by the condition $\partial \Phi(p_x^*)/\partial t = 0$, which is equivalent to

$$\partial p_x^*/\partial t = 0. \quad (41)$$

At arbitrary p_x , as a consequence of the periodicity of $p_x^*(t)$, there are at least two such points. The contribution of each point is proportional to the following:

$$s_{\text{spec},\alpha} \propto r^2 \left| \frac{eH}{c} \right| \int \left(\frac{\partial^2 \Phi(p_i^*)}{\partial t^2} \right)^{-1/2} \alpha f[\Phi(p_i)] \varphi[\Phi(p_i^*)] dp_i;$$

here $\partial^2 \Phi/\partial t^2 \propto H^3$, the quantity p_x^* is taken at the point defined by Eq. (41). In the case of changes in p_x that are small in comparison with p_F , the functions f and φ behave as almost periodic in p_x , but with different periods. The condition of stationarity for this integral over p_x is that the periods of the functions f and φ should be commensurate:

$$l \frac{\partial}{\partial p_x} \left[\int v_x(p_i, t) dt \right]^{-1} = n \frac{d}{dp_x} \left[\int v_x(p_i^*, t) dt \right]^{-1}, \quad (42)$$

where l and n are arbitrary integers. It is obvious that this equation inevitably has many solutions p_{xln} relative to p_x , corresponding to different l and n . These points of stationary phase give the following contribution:

$$s_{\text{spec},\alpha} \propto \sum \alpha r^2 \left| \frac{eH}{c} \right| \left[\frac{\partial^2 \Phi_{ln}}{\partial t^2} \right]^{-1/2} \\ \times \left[l \frac{\partial^2 \Phi_{ln}}{\partial p_x^2} - n \frac{\partial^2 \Phi_{ln}^*}{\partial p_x^2} \right]^{-1/2} \{ f_i \varphi_n \exp(2\pi i [l\Phi_{ln} - n\Phi_{ln}^*]) + \text{c.c.} \}; \\ \Phi_{ln} = \Phi(p_{xln}), \quad \Phi_{ln}^* = \Phi(p_{xln}^*); \quad (43)$$

here f_i and φ_n are respectively the Fourier components of the functions $f(\Phi)$ and $\varphi(\Phi)$; summation is performed over all solutions of Eqs. (41) and (42).

As follows from the expression (43), the specular oscillations of the conductivity under the considered conditions have a complicated nonperiodic character, and their amplitude is proportional to αH^3 , i.e., generally speaking, it exceeds the amplitude of the Sondheimer

oscillations at the reference points of the Fermi surface (which, as is known, is proportional to H^{-4}).

3. CONDUCTIVITY IN A MAGNETIC FIELD ALMOST PARALLEL TO THE SURFACE OF THE PLATE

In this section, we consider the conductivity in the case $\vartheta \ll d/l$, when a significant role is played by the volume collisions of the electrons. A rigorous quantitative approach, connected with the solution of the set of equations (1)–(3), is not realizable; however, the behavior of the mean conductivity and the character of the skin effect can be established even in this case. For simplicity, we shall assume that the volume oscillations achieve complete equilibrium (in the absence of an electric field) over the transport free path length l . Here, in particular, diffusion size effects are lacking,⁵ the appearance of which is connected with the low probability of volume recombination in comparison with the probability of intragroup transitions.

It is obvious that as a distance from the surface $d/2 - |\xi| > \vartheta l, r$ the electron departing from it experiences several volume collisions, and therefore the effect of the surface here is unimportant: the distribution function and the conductivity in this region are the same as in the bulk conductor. Thus the problem reduces to the determination of the contribution to the mean conductivity of the near-surface layer of the plate. We note that this contribution is not sensitive to the relation between l and d , since a distribution is established in the larger part of the plate that is characteristic for the bulk sample. Therefore, all the results obtained below are applicable also to thick plates, and at $d \gg l$ the condition $\vartheta \ll d/l$ does not place any limitation on the value of the angle ϑ . However, the surface contribution depends materially on the relation between the quantity N and the other parameters of the problem.

a) In this part, we consider the situation in which the surface scattering does not establish equilibrium too slowly $N \ll l/r$, and the angle $\vartheta \ll N^{-1}, d/l$. If the more rigorous condition $N - 1 \ll l/d$ is satisfied, then the indicated region of angles is joined to that considered in part B of Sec. 2 (in particular, in scattering with $N = 1$, we obtain a description at arbitrary ϑ and d/l). In the considered situation in a system of electrons colliding with a given surface, local equilibrium is achieved (see part B of Sec. 2).

It is not difficult to understand that, since the electron colliding with the surface at $\vartheta \gg r/l$, leaves it before experiencing a volume collision, the conductivity of the near-surface layer of width r is described approximately by the expression (25) for \hat{s}_{sur} . In the region $\vartheta l \geq d/2 - |\xi| \geq r$ the distribution function depends on ξ but its order of magnitude does not change. Thus at $d/l \gg \vartheta \gg r/l$, the quantity $\hat{s} \approx \hat{\sigma}_b + \hat{s}_{\text{sur}}$, where $\hat{\sigma}_b$ is the conductivity of the bulk sample and \hat{s}_{sur} is determined by the expression (25).

At very small angles $\vartheta \ll r/l$, an electron colliding with the surface leaves it, traversing a path of order l ; therefore, the components of the conductivity $s_{\text{sur}xx}$ at $n_e \neq n_h$ and $s_{\text{sur}\eta\eta}$ are determined at large angles by the path

length r/ϑ are established to be of the order of $e^2 n r l / dp_F$. The conductivity $s_{\text{sur}xx}$ at $n_e = n_h$ is determined by a path length of the order of $r N_{\text{eq}}$ —relaxation takes place sooner than emergence from the near-surface layer; therefore, the expression (30) for $s_{\text{sur}xx}$ is valid also at $\vartheta \ll r/l$. Further, since at $\vartheta = 0$ the presence of the surface does not change the symmetry of the sample in the magnetic field, the components $s_{x\eta}$ should vanish along with the xz component of the conductivity of the bulk sample; it can be shown that this is achieved by the appearance at $\vartheta \ll r/l$ of the factor $l\vartheta/r$ in front of these components in (30), which are due to the trajectories of type 2 of Fig. 2.

Summing up what has been said above, we write out the following interpolation formulas for the mean resistivity tensor $\hat{\rho} = (\hat{s})^{-1}$, which is directly measured in the experiment; they are valid at all $\vartheta \ll N^{-1}$. At $n_e \neq n_h$

$$\hat{\rho} \approx \frac{p_F}{e^2 n} \begin{pmatrix} l^{-1} & f_1 l^{-1} + \vartheta/r \\ f_1 l^{-1} - \vartheta/r & l^{-1} \end{pmatrix}, \quad l^{-1} = l^{-1} + \left(\frac{d}{\vartheta}\right)^{-1}; \quad (44)$$

at $n_e = n_h$

$$\hat{\rho} \approx \begin{pmatrix} s_{xx}^{-1} & s_{x\eta}/s_{xx}s_{\eta\eta} \\ s_{\eta x}/s_{xx}s_{\eta\eta} & s_{\eta\eta}^{-1} \end{pmatrix};$$

$$s \approx \frac{e^2 n}{p_F} \begin{pmatrix} \frac{r^2}{l} + \frac{N_p r^2}{d} & A^+ \frac{r^3}{r^2 + \vartheta^2 l^2} + B^+ \frac{r^2}{d} \\ A^- \frac{r^3}{r^2 + \vartheta^2 l^2} + B^- \frac{r^2}{d} & \frac{r^2 l (d + \vartheta l)}{d (r^2 + \vartheta^2 l^2)} \end{pmatrix}, \quad (45)$$

$$A^\pm = \pm f - \vartheta \psi \frac{l}{r} \pm \vartheta, \quad B^\pm = [\varphi \pm \varphi_1 \pm (N_p - 1)] \frac{l\vartheta}{l\vartheta + r} + (N_{\text{eq}} - 1) \tilde{f}.$$

At $d \rightarrow \infty$ these formulas transform into known⁴ expressions for the resistance of the bulk sample $\hat{\rho}_b$; the quantities $|f|$, $|f_1|$, $|\psi| \leq 1$ and vanish along with the components σ_{bx} , σ_{by} and σ_{bx}^2 , respectively.

It is seen from Eq. (44) that at $\vartheta \ll d/l$, the sample with $n_e \neq n_h$ behaves like a bulk sample in regard to the mean conductivity. However, at all $\vartheta \gg r/l$ a significant skin effect occurs: the current density in the near-surface layer of thickness r is significantly larger than the volume density (this follows from a comparison of the quantities σ_b and $d s_{\text{sur}}/r$). In a compensated metal, the size effect is different for different components of the mean conductivity (and resistance). In particular, the relation between the volume and surface parts of $s_{x\eta}$ depends on which elements of symmetry correspond to the direction of H .

b) Here we discuss the limiting case that is the opposite of the preceding point, when the surface scattering establishes the equilibrium very slowly: $N \gg l/r$. As was shown in the Introduction, it is in just this case that the dependence $\rho_{xx} \propto H$ occurs. We determine the necessary conditions for this at $\vartheta = 0$.

It is not difficult to understand that the desired conditions are connected with the conditions of solvability of Eq. (8) for the mean path. Actually, if Eq. (8) is unsolvable at $\vartheta = 0$, then this means that the stationary state in the near-surface case cannot be established by single collisions with the surface only [we recall that Eq. (8) does not take into account volume collisions] and,

consequently, the effective free path length here is of order l . This also leads to the dependence $\rho_{xx} \propto H$.

It is obvious that Eq. (8) can be unsolvable only in the case in which surface scattering does not lead to complete mixing, in other words, if the momentum space cannot be divided into regions between which transitions of electrons as a result of surface scattering and motion around an orbit are impossible. The absence of complete mixing can be connected, for example, with surface scattering that is close to specular in character, or with a low probability of intergroup transitions. We emphasize that exact prohibition of the corresponding transitions is not required (when $N = \infty$); it suffices to have so small a probability that the mixing takes place more rapidly in the case of volume scattering (this condition is expressed by the inequality $N \gg l/r$).

In order to establish sufficient conditions for the solvability of Eq. (8), we integrate both its parts with respect to V_i —one of the regions that does not mix the others on the Fermi surface. Using (5) and the nonmixing condition $Q(\mathbf{p}, \mathbf{p}') = 0$, if \mathbf{p} and \mathbf{p}' refer to different regions, we obtain

$$\int_{V_i} \Delta \mathbf{r}(\mathbf{p}) d\mathbf{p} = 0. \quad (46)$$

Condition (46) means that the mean displacement of an electron colliding with the surface is equal to zero, i.e., the mixing has the character of diffusion at large distances. This is in fact the condition of relaxation of the surface scattering. We note that the condition (46) cannot be satisfied for all regions V_i in an uncompensated metal; we verify this by summing (46) over i and taking it into account that $\int \Delta \mathbf{r}(\mathbf{p}) d\mathbf{p} \propto (n_e - n_h)$.

If the region V_i consists of one or several closed parts of the Fermi surface (carrier groups) the condition (46) takes the form

$$(n_e - n_h)_i = 0, \quad (47)$$

the quantity in the parentheses is the difference in the densities of electrons and holes belonging to the region V_i . Thus, the presence of any recombination processes is insufficient for recombination—this circumstance can be very important if, for example, the transitions between the electron and hole groups are effected by umklapp processes in the specular scattering.

We note that the law $\rho_{xx} \propto H$ can hold only in a certain intermediate region of magnetic fields, since the inequality $N \gg l/r$ is violated upon increase in H , and then $\rho_{xx} \propto H^2$.

If the condition (46) is satisfied, then the results of the previous part are obviously valid for the mean resistance at $\vartheta \ll d/l$. If (46) is not satisfied, then the character of the dependence of the surface conductivity on ϑ at $\vartheta \ll d/l$ is determined basically by the angular dependence of the free path length in the near-surface layer: $l_{\text{eff}}^{-1} \approx l^{-1} + \vartheta/r$. Uncomplicated analysis leads to the following result ($n_e = n_h$):

$$\begin{aligned} \hat{s} &\approx \hat{s}_b + \hat{s}_{\text{sur}}, & s_{\text{sur } xx} &\approx s_{\text{sur } \eta\eta} \approx \frac{e^2 n r^2 l}{p_F d (\vartheta l + r)}, \\ s_{\text{sur } xy} &\approx -s_{\text{sur } yx} \approx \frac{e^2 n r^2 l \vartheta}{p_F d (\vartheta^2 l^2 + r^2)}, \end{aligned}$$

whence

$$\rho_{xx} \approx \frac{p_F l}{e^2 n r^2} \left[1 + \frac{l^2}{d(\vartheta l + r)} \right]^{-1}, \quad \rho_{\eta\eta} \approx \frac{p_F d (\vartheta^2 l^2 + r^2)}{e^2 n r^2 l (d + \vartheta l)}. \quad (48)$$

These formulas are valid also in a very wide range of angles, $\vartheta \ll N_{\text{in}}^{-1}$, for scattering with $N_{\text{in}} \ll l/d$, $N_{\text{eq}} \gg l/r$ [they include the result (31)] and are valid for all ϑ for scattering that is close to being diffuse ($N_{\text{in}} \approx 1$).

If the condition (47) is not satisfied, then the skin effect at $d/l \gg \vartheta \gg r/l$ possesses the following feature: in addition to the current which is concentrated at a distance r from the surface, there is also a current of the same order, which is concentrated at a distance ϑl from it (its density is naturally $r/\vartheta l$ times smaller). The origin of this current can be explained in the following way: on the surface, there arises an excess concentration of electrons and holes $\Delta C_e = \Delta C_h \approx (p_F m / \hbar^3) e E l_{\text{eff}}$, which, thanks to the volume recombination, falls off in the interior of the sample at distances $\sim \vartheta l$. The presence of a gradient of the concentration leads, as is well known, to a current whose density in a strong magnetic field is $j_x \approx e v_F r d C / d \xi$.

In conclusion, we consider briefly the case of purely specular reflection of electrons from the surface of the plate (without umklapp processes). The motion of the electron in the absence of volume collisions is here completely determined and, as is not difficult to understand, the entire trajectory is included in some finite region of space. Actually, in the case of collisions with the surface, the changes in the y and z components of the momentum of the electron are connected by the relation $\Delta p_x = -\Delta p_y \tan \vartheta$ and p_x does not change. But since the change in p_x as a result of an arbitrary number of collisions cannot exceed the value determined by the size of the Fermi surface, it is obvious that the range of motion is limited in the x direction to the larger of the quantities $(c/eH \tan \vartheta)(p_{x \text{ max}} - p_{x \text{ min}})$ and $(c/eH)(p_{y \text{ max}} - p_{y \text{ min}})$, and in the y direction, to the quantity $(c/eH)(p_{x \text{ max}} - p_{x \text{ min}})$. It also follows from the finiteness of the motion that the symmetric part of the conductivity tensor is equal to zero.

It is easy to obtain the general solution of the set (1) and (2) (without the term with ϑ) by the method of characteristics, taking the trajectory to mean the entire path of the electron with account of reflections from the surface:

$$\begin{aligned} \chi &= u p + C \{ \text{Tr} \} - \langle C \{ \text{Tr} \} \rangle / \langle 1 \rangle, \\ E_{\perp} &= \frac{d}{d\xi} \frac{\langle C \{ \text{Tr} \} \rangle}{\langle 1 \rangle} - \frac{E_{\parallel} \mathbf{H}}{H \sin \vartheta}, \quad \mathbf{u} = \frac{c}{H d \sin \vartheta} [E_{\parallel} \times \mathbf{d}], \end{aligned} \quad (49)$$

where $C \{ \text{Tr} \}$ is an arbitrary constant for each trajectory.

It is easy to understand that the contribution from the second term in (49) to the mean conductivity is equal to zero. This can be shown with the help of expressions of the type (20), but even without this it is clear that the corresponding contribution of each trajectory is proportional to the time average of the velocity of the electron on it, which is equal to zero because of the finiteness of the motion. Thus we obtain the following exact result for the case of arbitrary $H \neq 0$:

$$s_{xx} = s_{\eta\eta} = 0, \quad s_{xy} = -s_{yx} = e c (n_e - n_h) / H \sin \vartheta.$$

In the presence of volume collisions, the diagonal components of the conductivity are different from zero. It is known⁵ that for a metal with a single group of carriers and a spherical Fermi surface, the size effect is lacking in the case of specular scattering. In the case of an arbitrary Fermi surface and angles $\vartheta \gg d/l$ the volume scattering can be taken into account by the method of successive approximations in ϑ . Analysis shows that the volume contribution to the conductivity is of the same order as for a spherical surface: $s_{vol \alpha\alpha} \approx (e^2 n / p_F) (r^2 / \vartheta^2 l)$, but in the general case there is also a current concentrated at a distance r from the surface: $s_{sur \alpha\alpha} \approx (e^2 n / p_F) (r^2 / \vartheta l)$. We note that in the case of the presence of several groups of carriers, and, in particular, at $n_e = n_h$, the comparatively large volume current at $\vartheta \ll 1$ is due to the presence in the volume of the plate of a nonequilibrium concentration of carriers $C_e = C_h$ with the gradient

$$dC/d\xi \approx (p_F m / \hbar^3) (e E r / l \theta^2).$$

In the range of angles $\vartheta \ll d/l$ at $n_e = n_h$ the result (48) is valid for the conductivity, since specular scattering is a special case of scattering for which the condition (46) is not satisfied. Thus, as has been shown, in Sec. 3b, the character of the skin effect is the same. It is of interest that with decrease in ϑ from the value $\vartheta > d/l$ to $\vartheta < d/l$, the skin effect increases, not only due to the growth in the surface current, but also due to the sharp drop in the volume conductivity to the value $\sigma_v \approx (e^2 n / p_F) (r^2 / l)$. The latter is connected with the fact that at $\vartheta > d/l$ recombination in the case of frequent collisions in the volume does not bring about the presence of a nonequilibrium concentration of carriers.

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¹In connection with this explanation, it can be shown that inelastic collisions with the surface would guarantee relaxation. Actually, the possible scattering inelasticity is unimportant, since the energy of the electrons colliding with the surface increases as a result of growth in the chemical potential and

not the temperature.

²We note that Babkin and Kravchenko's result⁵ $\rho \propto H$ was obtained for the case of diffuse surface scattering. However, they did not take into account the contradictions with the conclusions of Ref. 3, since they were not considering the specific situation of diffuse size effect.

³No difference was indicated between these types of scattering in Refs. 2 and 3.

⁴This inequality is certainly not satisfied at sufficiently small angles of incidence; therefore the case of too small angles ϑ should be excluded from consideration.

⁵The case considered in Ref. 10 of $\vartheta = \pi/2$ under the condition that the surface of the plate is a plane of symmetry of the crystal is an exception.

⁶To justify this assertion, it must also be verified that the distribution function of volume electrons arriving at the surface does not depend in order of magnitude on the presence or absence of volume collisions. This is seen from the comparison of (7) with the known distribution function for a bulky conductor.

¹M. Ya. Azbel', Zh. Eksp. Teor. Fiz. 44, 983 (1963) [Sov. Phys. JETP 17, 451 (1963)].

²M. Ya. Azbel' and V. G. Peschanskiĭ, Zh. Eksp. Teor. Fiz. 49, 572 (1965) [Sov. Phys. JETP 22, 399 (1966)].

³V. G. Peschanskiĭ and M. Ya. Azbel', Zh. Eksp. Teor. Fiz. 55, 1980 (1968) [Sov. Phys. JETP 28, 1045 (1969)].

⁴I. M. Lifshitz, M. Ya. Azbel', and M. I. Kaganov, Elektronnaya teoriya metallov (Electron Theory of Metals), Nauka, 1971.

⁵G. I. Babkin and V. Ya. Kravchenko, Zh. Eksp. Teor. Fiz. 60, 695 (1971) [Sov. Phys. JETP 33, 378 (1971)].

⁶O. V. Kirichenko, V. G. Peschanskiĭ, and S. N. Savel'eva, Pis'ma Zh. Eksp. Teor. Fiz. 25, 187 (1977) [JETP Lett. 25, 171 (1977)]; Fiz. Nizk. Temp. 4, 180 (1978) [Sov. J. Low Temp. Phys. 4, 88 (1978)].

⁷F. G. Bass and I. M. Fuks, Rasseyaniye voln na statisticheski nerovnoi poverkhnosti (Wave Scattering from a Statistically Rough Surface), Nauka, 1972.

⁸E. H. Sondheimer, Phys. Rev. 80, 401 (1950).

⁹V. L. Gurevich, Zh. Eksp. Teor. Fiz. 35, 668 (1958) [Sov. Phys. JETP 8, 464 (1959)].

¹⁰Yu. M. Golland, Fiz. Tverd. Tela (Leningrad) 10, 81 (1968) [Sov. Phys. Solid State 10, 58 (1968)].

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