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Note added in proof (24 January 1980). We note that the tail of the distribution in region II [(3.8), Figs. 2 and 3] should decrease slowly with increasing energy, because of the dependence of the Coulomb logarithm on the ion velocity v, which exceeds v_{Te} in this region.

The authors have learned that the asymptotic forms of the ion distributions, given in Sec. 3 and in the preprint[24], were obtained also in Ref. 25.

[1)]We exclude in this case a solution with positive flux, for which $J(\infty)>0$ would assume the role of a sink. A flux of this kind can be formed by diffusion only at $\varepsilon \leqslant T$, and manifests itself in loss of particles from the corresponding section of the "thermal" distribution.

[2)]Neglect of diffusion is valid, of course, so long as $\Delta p \gg |D(p)/F(p)|$. If $\Delta p < |D(p_0)/F(p_0)|$, then the width and the structure of the front are determined by diffusion.

[3)]In the presence of a magnetic field, an important role is played by quasilinear relaxation of the fast ions (see the review[23] and the references therein).

[1]J. G. Cordy and M. J. Houghton, Nucl. Fus. 13, 215 (1973).

[2]A. V. Kats, V. M. Kontorovich, S. S. Moiseev, and V. E. Novikov, Pis'ma Zh. Eksp. Teor. Fiz. 21, 13 (1975) [JETP Lett. 21, 5 (1975)]; Zh. Eksp. Teor. Fiz. 71, 177 (1976) [Sov. Phys. JETP 44, 93 (1976)].

[3]Ya. I. Kolesnichenko and A. F. Fursa, Fiz. Plazmy 1, 806 (1975) [Sov. J. Plasma Phys. 1, 442 (1975)].

[4]N. V. Grishanov, Yu. N. Dnestrovskii, N. V. Karetkina, and D. P. Kostomarov, Fiz. Plazmy 2, 260 (1976) [Sov. J. Plasma Phys. 2, 142 (1976)].

[5]L. I. Gudzenko and S. I. Yakovlenko, Plazmennye lazery (Plasma Lasers), Atomizdat, 1978.

[6]I. A. Krinberg, Kinetika élektronov v ionosfere i plazmosfere Zemli (Electron Kinetics in the Earth's Ionosphere and Plasmosphere), Nauka, 214, 1978.

[7]B. A. Trubnikov, Voprosy teorii plazmy, Vol. 1, Atomizdat, 1963, p. 154.

[8]A. V. Kats, V. M. Kontorovich, and A. E. Kochanov, Preprint, Inst. Radio & Electr. Ukr. Acad. Sci., No. 93, 1978; Astrophys. Space Sci. 57, 347 (1978).

[9]A. V. Kats and V. M. Kontorovich, Izv. Vyssh. Uchebn. Zaved. Radiofiz. 20, 1112 (1977).

[10]Ya. B. Zel'dovich, Usp. Fiz. 115, 161 (1975) [Sov. Phys. Usp. 18, 79 (1975)].

[11]V. S. Kudryavtsev, Fizika plazmy i problemy UTR (Plasma Physics and Problems of Controlled Thermonuc. Fusion) 3, 114 (1958).

[12]B. B. Kadomtsev and V. M. Kontorovich, Izv. Vyssh. Uchebn. Zaved. Radiofiz. 17, 511 (1974).

[13]V. E. Zakharov and N. N. Filonenko, Dokl. Akad. Nauk SSSR 170, 1292 (1966) [Sov. Phys. Dokl. 11, 881 (1967)].

[14]E. Jahnke, F. Emde, and F. Loesch, Table of Higher Functions, McGraw, 1960 [Russ. Transl., Nauka, p. 308 (1968)].

[15]S. S. Moiseev, L. A. Nazarenko, and V. E. Novikov, Ukr. Fiz. Zh. 23, 1592 (1978).

[16]E. Kamke, Differenzial gleichungen, Vol. 1, Chelsea, 1971 (Russ. transl., Nauka, 1976, p. 153).

[17]G. I. Barenblatt, Podobie, avtomodel'nost', promeshutochnaya asimptotika (Similarity, Self-Similarity, Intermediate Asymptotics), Gidrometeoizdat, 1978, Chap. 9.

[18]R. Davis, D. S. Harmer, and K. C. Hoffman, Phys. Rev. Lett. 20, 1205 (1968).

[19]G. E. Kocharov, Izv. Akad. Nauk SSSR Ser. Fiz. 41, 1916 (1977).

[20]W. A. Fowler, G. R. Caughlan, and B. A. Zimmerman, Ann. Rev. Astron. and Astrophys. 5, 525 (1967).

[21]K. R. Lang, Astrophysical Formulae, Vol. 2, Springer, 1974.

[22]N. S. Kardashev, Astron. Zh. 39, 393 (1962) [Sov. Astron. 6, 317 (1962)].

[23]A. B. Mikhailovskiĭ, in: Voprosy teorii plazmy (Problems of Plasma Theory), Vol. 9, Atomizdat, 1979, p. 83.

[24]V. N. Mel'nik, A. V. Kats, and V. M. Kontorovich, Preprint, Inst. Radio & Electron., Ukr. Acad. Sci., No. 125, 1979.

[25]A. A. Husseiny, Z. A. Sabri, D. R. Harris, and D. E. Dei, Atomkernenergie 33, 108 (1979).

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The magnetic field in a conducting fluid in two-dimensional motion

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The evolution of a magnetic field in a fluid moving so that one of the velocity components vanishes is considered. A dependence of the other velocity components, of the electric conductivity, as well as of the magnetic field on the corresponding coordinate is assumed. Thus a significant generalization of the antidynamo theorem proved by one of the authors in 1956 is obtained. The result allows one to classify the dynamo solutions in terms of the magnitude of the magnetic Reynolds number.

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1. INTRODUCTION

The classical problem of the magnetic dynamo, which solves the problem whether the amplifcation or maintenance of a magnetic field is possible in a moving conducting fluid has, as is well known, recieved an affirmative solution. Many concrete examples of magnetic dynamos have been constructed and this has stimulated

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the applications of the theory to the explanation of the origin and maintenance of magnetic fields of planets, stars, and galaxies. On the other hand, even in its simplest kinematic form (i.e., when the velocity field is supposed known ahead of time) the theory of the magnetic dynamo remains incomplete, since until now no complete necessary and sufficient conditions have been found for its functioning. It is known that a dynamo is impossible when the magnetic field and the velocity field are independent of one of the space coordinates, for the case of planar[1,2] and axial[3,4] symmetry. In general all three velocity components may be different from zero in this case. However, if all components of the velocity are present these results do not extend to the spherical case (cf. infra) and to other geometries. For a detailed survey of anti-dynamo theorems see Refs. 5 and 6.

We recall the essence of the proofs, using as an example the plane case $\partial/\partial_z = 0$. The two-component part of the magnetic field in the xy-plane, $\mathbf{H}_2 = (H_x, H_y)$, separates from the z-component on account of the indicated symmetry, and can be expressed in terms of the single component A_z of the vector potential. It follows from the induction equation that the component A_z is subject to the same equation as the temperature in a moving heat-conducting medium. Consequently,[7] A_z can only decay. The two-dimensional field $\mathbf{H}_2$ can be amplified up to a certain point on account of the decrease of the scale of the field (entanglement of the field lines), however, in the long run it must also be damped out owing to ohmic losses.[1] The growth of this field and its subsequent decay was recently exhibited numerically in the paper of Pouque[8] for plane turbulent motion. For the remaining component H_z of the field, in the simplest case $v_z = 0$,[1] one also obtains an equation of the heat conduction type, leading to damping. If $v_z \neq 0$, but as before the motion does not depend on z, then in the equation for H_z there appears a source depending on $\mathbf{H}_2$ and v_z. However, the decay of $\mathbf{H}_2$ guarantees the impossibility of unbounded growth of the z-component of the field: if the electric resistivity is taken into account in the absence of fields at infinity, one is led to the damping of H_z.

A complete abandonment of translational symmetry ($\partial/\partial_z = 0$) does not yet mean that a dynamo is possible. In reality, as will be shown here, a dynamo effect may be absent even for fields depending on all three coordinates, if one of the components of the velocity of the fluid vanishes. The first indication that a dynamo is possible in the three-dimensional situation can be found in the paper[9] of Bullard and Gellman for the spherical case with $v_r = 0$ (cf. also Refs. 10, 14); the plane case has been discussed by Moffat.[6] The situation is simplest in a plane geometry for a conducting fluid moving with $v_z = 0$.

It is obvious that for such a motion the vertical distance $z_{12} = (\mathbf{r}_{12})_z$ between any pair of points 1, 2 remains constant. In the approximation of frozen-in magnetic fields it follows that H_z is conserved in an incompressible fluid (or H_z/ρ is conserved in a compressible fluid, where ρ is the density). Taking into account the finite conductivity, it follows that H_z is damped. For H_x, H_y, v_x, v_y depending not only on x, y, but also on z, the field H_z is an additional source of the components H_x and H_y, and the growth in time of these components, or even their appearance is quite possible. However, there is no feedback. A damped source cannot produce an undamped and all the more growing generation of fields H_x, H_y. Thus, one succeeds in obtaining an essential generalization of the anti-dynamo theorem for plane motion.

The result can be translated fully to the spherical case, replacing the condition $v_z = 0$ by $v_r = 0$ and H_z by rH_r. The plane and spherical cases are special because in these geometries the curvature of the surfaces along which the fluid flows is constant in all directions, therefore in the equations for H_z or rH_r the diffusion term $(\Delta H)_{z,r}$ can be expressed only in terms of the field H_z (or rH_r) and its derivatives, without the other components appearing. This allows one to obtain for H_z (or rH_r) a separate equation which is similar to the heat equation and leads to damping of these components.[1)] In an ideally conducting medium the assertion that H_z is conserved for $v_z = 0$ can be generalized to more complicated two-dimensional flows. Indeed, if the motion takes place in two dimensions, along non-intersecting and nondiverging unbounded surfaces, it is clear that the projection onto the normal of the distance between two points which move along neighboring surfaces will be bounded. Consequently, on account of the condition that the magnetic field is frozen-in, the magnetic field component normal to these surfaces must also be bounded. The two other field components, for which the normal component serves as a source, can increase; however, owing to the boundedness of the source this growth will be slow (not exponential). It will be similar, for instance, to the growth of the azimuthal component of a field for differential rotation in the presence of a radial field.

However, if the electric resistivity is taken into account, there appears in general a principally new circumstance: the normal component of the Laplacian of an arbitrary vector also depends on the tangential components of the vector. The magnetic viscosity leads to a feedback, owing to which the tangential components are capable of modifying (in particular, to amplify) the normal component. Therefore, in principle, an exponential coordinated amplification of all fields also becomes possible, i.e., a dynamo! But a peculiar kind of dynamo that depends on the magnetic viscosity.

The plane and spherical cases turn out to be degenerate and special.

It would be interesting to investigate (in both the flat and spherical cases) the rather difficult nonlinear problem of a fluid whose resistivity depends on the magnetic field. In this case the resistivity should be considered as a tensor relating the electric field vector to the current vector.

In connection with what was said above one can give a definite classification of dynamo-solutions as a function of the magnetic Reynolds number. For $R_m \gg 1$ and three-dimensional motion a fast dynamo is possible with a characteristic time $\tau \sim l/v$, where lv are the scales of length and velocity. A constructive example of a fast dynamo is the so-called figure-eight (cf. Ref. 5, p. 433): a torus with azimuthal field flux, which after increasing its length by a factor of two, with a corresponding thinning, is folded with a twist in such a way that the flux is doubled. The doubling time is of the order of l/v, so that the dynamo has an increment v/l (we neglect the difference between $\ln 2 = 0.7$ and unity). As was noted in Ref. 5, if the field is strictly frozen-in, the topology of the field lines changes during each doubling cycle. The introduction of a finite ν_m allows one to ontain a conservation of the whole field picture under a small change of the increment.

In two-dimensional motion in the flat and spherical cases with scalar resistivity, the dynamo is impossible, and in the general two-dimensional case, if it is possible, it will turn out to be slow (with a characteristic time tending to infinity for fixed l, v and for $R_m \to \infty$). For small R_m an effective dynamo is possible also for two-dimensional motion (excluding the flat and spherical cases).

In order to attain maximal clarity we carry through below a complete proof of the anti-dynamo theorem for the case of the plane motion with $v_z = 0$ in cartesian coordinates (cf. [6]). Then we discuss the general case more briefly.

2. PLANE INCOMPRESSIBLE FLOW

We consider the motion of a conducting fluid, subject to the conditions

$$v_z=0, \quad v_x=v_x(x,y,z), \quad v_y=v_y(x,y,z),$$
$$\operatorname{div} \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0, \tag{1}$$

and otherwise arbitrary. The intial field is arbitrary and may depend on all three coordinates. We note that the vorticity (helicity) of the velocity field (1), i,e., the scalar product

$$\mathbf{v}\operatorname{rot}\mathbf{v} = v_y\frac{\partial v_x}{\partial z} - v_x\frac{\partial v_y}{\partial z} = -v_x^2\frac{\partial}{\partial z}\left(\frac{v_y}{v_x}\right),$$

is generally nonzero (for $\partial/\partial z \neq 0$).

The general evolution equation of the magnetic field in the moving conducting medium has the form

$$\partial\mathbf{H}/\partial t = \operatorname{rot}\left([\mathbf{v}\times\mathbf{H}] - \nu_m \operatorname{rot}\mathbf{H}\right), \tag{2}$$

where $\nu_m = c^2/4\pi\sigma$ is the magnetic viscosity of the medium (σ is the conductivity). For simplicity we restrict our attention to an unbounded medium and assume that

$$\mathbf{H} \to 0, \quad |\mathbf{r}| \to \infty. \tag{3}$$

We write the z-component of Eq. (2) in the medium with isotropic conductivity moving according to Eq. (1):

$$\frac{dH_z}{dt} = \frac{\partial H_z}{\partial t} + \left(v_x\frac{\partial}{\partial x} + v_y\frac{\partial}{\partial y}\right)H_z = \nu_m \Delta H_z, \tag{4}$$

where d/dt is the substantional derivative and Δ is the Laplace operator. Thus H_z is subject to the same equation as a scalar quantity (the temperature or the concentration of an impurity) in a given velocity field if diffusion is taken into account. It follows from the equation that, if $\nu_m = 0$, H_z is conserved at each fluid particle, so that in this case

$$\int H_z^2\, dV = \text{const},$$

and for finite conductivity and the conditions at infinity (3)

$$\frac{d}{dt}\int H_z^2\, dV = -2\nu_m \int (\nabla H_z)^2\, dV, \tag{5}$$

i.e., the quantity H_z is damped out.

The behavior of the two-dimensional field $\mathbf{H}_2 = (H_x, H_y)$ becomes considerably more complicated if all the quantities depend on z. We recall that for $\partial H_z/\partial z = 0$ the condition $\operatorname{div}\mathbf{H} = 0$ would imply

$$\frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} = 0,$$

and consequently $\mathbf{H}_2$ could be represented as the curl of A_z. In the case under consideration, when H_z depends on z, this is impossible.

We note that even if at the initial instant the component H_z would depend only on x and y, and not on z, a dependence on z would appear later because the motion along x and y is accompanied by the deformation $\partial v_x/\partial z \neq 0$, $\partial v_y/\partial z \neq 0$. Therefore, for $\partial H_z/\partial z = 0$ at the intial time (in the absence of magnetic viscosity)

$$\frac{\partial}{\partial t}\frac{\partial H_z}{\partial z} = \frac{\partial v_x}{\partial z}\frac{\partial H_z}{\partial x} + \frac{\partial v_y}{\partial z}\frac{\partial H_z}{\partial y},$$

which is intuitively obvious, if one thinks of the transport of H_z by the fluid particles.

Thus, one obtains for H_z an equation which does not depend on the other components, describing the general damped character of H_z and defining the quantity $-\partial H_z/\partial z \equiv q(x, y, z, t)$. Consequently, the two-dimensional vector $\mathbf{H}_2$ can be represented only as the sum of a solenoidal and potential component:

$$H_x = \frac{\partial\Phi}{\partial y} + \frac{\partial\varphi}{\partial x}, \qquad H_y = -\frac{\partial\Phi}{\partial x} + \frac{\partial\varphi}{\partial y}. \tag{6}$$

We obtain from

$$\frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} = -\frac{\partial H_z}{\partial z}$$

that

$$\Delta_2\varphi = q, \qquad \Delta_2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}. \tag{7}$$

Taking account of the boundary conditions, this equation can be integrated by elementary methods at each instant and in a given layer. This determines the function φ. The damping of φ (caused by the damping of H_z) means that at a sufficiently late stage of development the plane field admits a representation of the form $\mathbf{H}_2 = \operatorname{curl}(\mathbf{n}\Phi)$, where $\mathbf{n} = (0, 0, 1)$. After that the function Φ (which now coincides with the vector potential component A_z) is also subject to an equation of the heat type. Consequently asymptotically $\mathbf{H}_2$ is damped out.

For the sake of clarity, and having in mind generalizations to the case of motion along curved surfaces, we write the equation for Φ before $\mathbf{H}_z$ and have decayed away. From the definition (6) it follows that

$$\Delta_2\Phi = -\operatorname{rot}_z\mathbf{H}. \tag{8}$$

Therefore, in order to obtain the equation for Φ it suffices to take the z-component of the curl of equation (2). We obtain thus

$$\partial\Phi/\partial t+\mathbf{v}\nabla\Phi=\nu_m\Delta\Phi+S, \tag{9}$$

where the source S has the form

$$S=[\mathbf{v}\times\nabla]_z\varphi+\Delta_2^{-1}\frac{\partial}{\partial z}\left(\frac{\partial v_x H_z}{\partial y}-\frac{\partial v_y H_z}{\partial x}\right). \tag{10}$$

Taking (7) into account, all terms in (10) depend linearly on H_z. The arbitrary function of z which may be present in the right-hand side of Eq. (9) is inessential in the calculation of the integral of Φ^2 since Φ can be selected in such a manner that its average over the planes z = const will vanish.

3. THE GENERAL CASE

It is clear that in the limit of complete freezing-in of the magnetic field ($\nu_m\to 0$) the results that have been obtained can be generalized to two-dimensional motions of a more general type. It is easy to check, for instance, that in motions along spherical or cylindrical surfaces the equation for the radial component separates, and this field component is conserved in each fluid particle. It is simplest and most intuitive to use the property that if the magnetic field is frozen into the fluid, H_r behaves like δr, which is conserved on account of the condition $v_r=0$.

If one considers motions along a family of stationary nonintersecting surfaces of a more general type[2)] one has to bear in mind that these surfaces are not parallel to one another, and therefore the normal projection of the infinitesimal distance, $(\delta r)_n$, between two points on neighboring surfaces varies and may increase in time. However, if the neighboring surfaces do not diverge in an unbounded fashion, which is true, e.g., for a family of closed surfaces, it is clear that $(\delta r)_n$, and consequently H_n will be bounded.[3)] In this case one can speak about the conservation of the product of H_n by a factor which measures the divergence of the surfaces (a Lamé coefficient).

When one considers a two-dimensional field on a surface, one can make use of the decomposition of an arbitrary solenoidal field into a "poloidal" and a "toroidal" part (cf., e.g.,[11,6]):

$$\mathbf{H}=\mathrm{rot}\,(\mathrm{rot}\,\mathbf{n}\Psi+\mathbf{n}\Phi)=\mathrm{rot}\,[\nabla\times\Psi\mathbf{n}]+[\nabla\times\Phi\mathbf{n}]. \tag{11}$$

This implies

$$H_n=-[\mathbf{n}\times\nabla]^2\Psi,\quad \mathrm{rot}_n\mathbf{H}=-[\mathbf{n}\times\nabla]^2\Phi\ . \tag{12}$$

For the function Φ one obtains again the equation (9) with source S determined by the quantity $\nabla_n H_n$ (which is bounded in time), its derivatives, and the velocity. This implies that a rapid (exponential) growth of the two-dimensional field component is impossible.

However, in the general case, the conclusion on the damping of H_n and the related source, when finite conductivity is taken into account, is no longer valid (see the Introduction). In the general case $(\Delta H)_n$ is expressed not only in terms of H_n and its derivatives, but also in term of the other field components.[4)] Thus, in cylindrical coordinates s, φ, z

$$(\Delta H)_s=-\frac{1}{s}\Delta(sH_s)+\frac{2}{s^2}\frac{\partial H_\varphi}{\partial\varphi}.$$

This allows one to draw two conclusions. Firstly, the theorem on the impossibility of a dynamo with $v_n=0$ for arbitrary magnetic Reynolds number R_m is valid only for flat and spherical geometries. Secondly, the increment of the dynamo, which is possible in other geometries, will be determined by the magnetic diffusion, and consequently such dynamos are effective only for small magnetic Reynolds numbers.

We note that the impossibility of a dynamo in the spherical and plane cases has already been noticed in Refs. 9, 10, 6. What we have show is that these cases are the only ones (for the stationary dynamo this was noticed in Ref. 14) and have generalized the statement on boundedness of one of the field components to the general case. As an illustration of the second conclusion we indicate the dynamo effect of a helical flow with $v_s=0$ in cylindrical coordinates.[12,13] The leading term in the maximal increment of this dynamo is proportional to $\nu_m^{1/3}$; for $\nu_m=0$ the dynamo effect is absent. Moreover here the most effective generation is for angular harmonics with $\partial/\partial\varphi\neq 0$, which is an additional indication of the decisive role played by the coupling term $2s^{-2}\partial H_\varphi/\partial\varphi$ in the equation for H_s.

4. THE ROLE OF INHOMOGENEITY AND ANISOTROPY OF THE CONDUCTIVITY AND COMPRESSIBILITY

We make some remarks on the cases when the magnetic viscosity and the density of the medium are functions of the coordinates. It is obvious that no anti-dynamo theorems are possible if ν_m depends on all three coordinates. In such a medium solutions are possible which simulate the functioning of dynamo-machines constructed by means of insulated conductors. However, in the simplest case $\nu_m=\nu_m(z)$ or $\nu_m=\nu_m(r)$ one can prove the impossibility of a dynamo. In this case the equation for H_z is easily verified to have again the form (4) but the relation (5) will no longer hold. In its place one must multiply Eq. (4) by H_z/ν_m, and then, making use of the relation $\mathbf{v}\cdot\nabla\nu_m=0$ we obtain

$$\frac{d}{dt}\int\frac{H_z^2}{\nu}dV=-2\int(\nabla H_z)^2dV. \tag{13}$$

Thus, a dynamo is impossible in the plane and spherical cases and will be slow in the general case.

It is easy to generalize the theorem also to the case when the fluid density is coordinate-dependent, $\rho=\rho(z)$ or $\rho=\rho(r)$. Indeed, in this case the continuity equation implies that for $v_z=0$ we have also $\mathrm{div}\mathbf{v}=0$, i.e., the problem ω reduces to the one already considered. We note that in an ideally conducting medium for arbitrary density the quantity H_z/ρ is conserved (in place of H_z in the case of an incompressible fluid), since in place of (4) we have in this case the equation

$$\frac{\partial}{\partial t}\frac{H_z}{\rho}+(\mathbf{v}\nabla)\frac{H_z}{\rho}=\frac{\nu_m}{\rho}\Delta H_z. \tag{14}$$

Of particular interest is the case of anisotropic tensorial conductivity, which couples together the various field and current components. It is then impossible to obtain a separate equation for H_z and, in principle a dynamo with an increment determined by the off-diagonal (in x, y, z coordinates) components of the conductivity tensor. Usually the direction of anisotropy is related to the magnetic field, but then the whole problem becomes nonlinear.

Finally, the most important problem is whether one can draw any conclusions for the general three-dimensional problem. Can one, for example, in the absence of helicity (vorticity) transpose directly to the case of isotropic three-dimensional turbulence of general form the concept of diamagnetism of a turbulent plasma and also the concepts of temporal growth of fields as the scale diminishes simutaneously. These notions can be applied not only to an infinite medium, but also to situations with boundary conditions at finite distances. But this problem is only posed here—its solution is a matter for the future.

We are grateful to T. Cowling who brought his paper[14] to our attention.

1) We note that the mean value of H_r over any close spherical surface vanishes, and consequently the volume average over any sphere will also vanish. Therefore the damping leads not to H_r= const. but to $H_r = 0$ as $t \to \infty$.

2) It is essential here to take into account the condition of stationarity of surfaces, allowing for a nonstationarity of the field of potential velocities on the surfaces. The possibility of having an instantaneous family of surfaces in terms of the instantaneous velocity field does not suffice for the validity of the assertions made in the present paper.

3) The boundedness of $(\delta \mathbf{r})_n$ is to be understood in the sense that this quantity remains infinitesimal of the same order of smallness. We underscore the fact that in the formulation of the freezing theorem for $\nu_m = 0$ the field or H/ρ change proportionally to the *infinitesimal* vector distance between two infinitely close fluid particles. Therefore the rapid dynamo is possible even in the case when the motion occurs in a finite volume.

4) The peculiarity of the plane and spherical cases is related to the fact that the curvature of the surface vanishes in the first case and is isotropic in the second.

1. Ya. B. Zel'dovich, Zh. Eksp. Teor. Fiz. **31**, 154 (1956) [Sov. Phys. JETP **4**, 460 (1957)].
2. D. Lortz, Phys. Fluids **11**, 913 (1968).
3. T. G. Cowling, Mon. Not. Roy. Astr. Soc. **94**, 39 (1934).
4. S. I. Braginskiĭ, Zh. Eksp. Teor. Fiz. **47**, 1084 (1964) [Sov. Phys. JETP **20**, 726 (1965)].
5. S. I. Vaĭnshteĭn and Ya. B. Zel'dovich, Usp. Fiz. Nauk **106**, 431 (1972) [Sov. Phys. Uspekhi **15**, 159 (1972)].
6. H. K. Moffat, Magnetic Field Generation in Electrically Conducting Fluids, Cambr. Univ. Press, 1978, pp. 108–117.
7. Ya. B. Zel'dovich, Zh. Eksp. Teor. Fiz. **7**, 1466 (1937).
8. A. Pouque, J. Fluid Mech. **88**, 1 (1978).
9. E. C. Bullard and H. Gellman, Phil. Trans. Roy. Soc. **A247**, 213 (1954).
10. G. E. Backus, Ann. Phys. (NY) **4**, 372 (1978).
11. K.-H. Rädler, Astron. Zs. Nachricht. **295**, 73 (1974).
12. Yu. B. Ponomarenko, PMTF **6**, 47 (1973).
13. A. K. Gaĭlitis and Ya. Zh. Freĭberg, Magnitnaya gidrodinamika **2**, 3 (1976).
14. T. G. Cowling, Quart. J. Mech. Appl. Math. **10**, 129 (1957).

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