Quasistationary states of a hydrogen atom in the field of a strong monochromatic wave

M. A. Preobrazhenskiĭ and L. P. Rapoport

Voronezh State University

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The method of complex coordinate rotation is used to find the shift and width of the ground state of a hydrogen atom in the field of a strong circularly polarized wave. It is shown that the action of the strong field leads to smoothing out of the intermediate resonance. In the formalism of quasistationary quasienergetic perturbation theory the interaction of the ground state of the atom with the field of an elliptically polarized wave is discussed. It is shown that the main mechanism of the smoothing out of the resonance is field broadening.

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1. INTRODUCTION

The purpose of the present paper is to extend the application of the method of quasistationary quasienergy states (QQES), as developed in the framework of perturbation theory in Ref. 1, to the calculation of quasienergy states of a hydrogen atom in the field of a strong light wave of field intensity $F_0 \lesssim 10^{-1}$ atomic unit, for which ordinary perturbation theory is no longer applicable. As was shown in Ref. 1, the problem of QQES in an external field which varies periodically with the time is analogous to the problem of quasistationary states in external static fields. One looks for solutions of the equation

$$H\Phi_E(\mathbf{r},t)=E\Phi_E(\mathbf{r},t),\quad H=H_0(\mathbf{r})+V(\mathbf{r},t)-i\partial/\partial t, \tag{1}$$

which satisfy the radiation boundary condition. Here the function Φ_E increases exponentially as $r\to\infty$, and the energy eigenvalue E is complex. The region of exponential decay is fixed by the condition that the inequality $\mathrm{Im}E/(\mathrm{Re}E+N\omega)<1$, where N is the threshold number of photons required for ionization, ω is the frequency of the field, and ReE determines the position and ImE the ionization probability of the quasistationary state that arises when the field is turned on.

The exact solution of the problem of the decay of a quasistationary state is at present known only for a short-range potential.[2,3] In the physically more interesting case of the hydrogen atom there is a perturbation-theory solution for QQES.[1] In fields $F_0 \lesssim 10^{-1}$, however, when the decay is still exponential, the perturbation theory for the QQES which does not take resonance effects into account can no longer properly describe the processes of interaction of atoms with a strong electromagnetic field for frequencies $\omega \lesssim I$, where I is the ionization potential.

In Sec. 2 of the present paper we use the method of complex rotation of coordinates[5,6] to work outside the framework of perturbation theory and deal with the decay of the ground state of a hydrogen atom in the field of a strong circularly polarized wave of a frequency ω for which the two-photon ionization channel is open. It is shown that the main effect of the strong field reduces to a smoothing-out of the resonance.

In Sec. 3 this same effect is taken into account in the first nonvanishing order of perturbation theory in the case of elliptical polarization, for closely spaced QQES. A comparison of the results obtained in this approximation with the exact calculation in Sec. 2 provides evidence that the field broadening of the level, proportional to the field intensity F_0, is the main mechanism by which smoothing-out of the intermediate resonance occurs.

2. EXACT SOLUTION OF THE PROBLEM OF THE DECAY OF THE QQES IN A CIRCULARLY POLARIZED FIELD

To calculate the complex quasienergy of an H atom in the field of a circularly polarized wave we use the method of complex rotation of coordinates, which has previously been successfully applied to various problems of quasistationary states in static fields[5] and to the ionization of an H atom in a linearly polarized wave.[6] The idea of the method is that the coordinate transformation $r'=\alpha re^{i\varphi}$ applied in the Hamiltonian, without any change of the eigenvalues corresponding to the quasistationary states will take their eigenfunctions into the space of functions that are quadratically integrable; thus the problem of finding E can be reduced to the search for the eigenvalues of a nonhermitian operator.

We consider a hydrogen atom in the field of a strong circularly polarized wave1)

$$\mathbf{F}(t)=2^{-1/2}F_0\{\cos\omega t,\ \sin\omega t,\ 0\},\quad V(r,t)=-e\mathbf{r}\mathbf{F}(t).$$

As is well known, in a coordinate system that rotates with the frequency of the wave the equation for the QQES is of the form[2]

$$(-{}^1/{}_2\nabla^2-r^{-1}+\omega L_z+F_0x)\psi_E(r)=E\psi_E(r).$$

Therefore after the complex coordinate transformation $r'=re^{i\varphi}$ we have the following equation for the eigenvalues and eigenfunctions:

$$(-{}^1/{}_2\nabla^2e^{-2i\varphi}-r^{-1}e^{-i\varphi}+\omega L_z+F_0xe^{i\varphi}-E_n{}^0)\psi(re^{i\varphi})=(E-E_n{}^0)\psi(re^{i\varphi}), \tag{2}$$

where $E_n^0=-1/2n^2$ are the unperturbed values of the energy. The quadratic integrability of the functions of the quasistationary states is assured for values of the complex-rotation angle that satisfy the inequalities

$$|\arg[-(E+N\omega)]^{1/2}|<\varphi<\pi/2-\arg[-[E+(N-1)\omega]]^{1/2},$$

where N is the threshold number of photons necessary for ionization. The lower bound ensures quadratic integrability of the components of the function of the QQES with $n > N$ absorbed photons, and the upper limit, for components with $n < N$. Over this range the exact solution of Eq. (2) does not depend on φ. When, however, Eq. (2) is diagonalized on a chosen finite basis, the complex eigenvalues depend on φ as a parameter, and have only a stationary point[5,6] on the given segment. When a definite accuracy has been prescribed for the calculation, the dimensionality of the basis is chosen such that varying it does not change the accuracy.

As a basis for the expansion we use the system of eigenfunctions of the Sturm-Liouville problem of the hydrogen atom:

$$\psi_{nlm}=\left[\frac{(n+l+1)!}{n!}\right]^{1/2}\frac{M_{n+l+1,l+1/2}(2r/\nu)}{r}Y_{lm}\left(\frac{\mathbf{r}}{r}\right), \tag{3}$$

where M is the Whittaker function. Since the functions (3) are orthonormalized to satisfy the condition

$$\langle nlm|r^{-1}|n_1l_1m_1\rangle=\delta_{n,n_1}\delta_{l,l_1}\delta_{m,m_1},$$

it is convenient to examine their representation not the Hamiltonian H but the operator H/r, which obviously has the same eigenvalues and functions. In this case the problem reduces to the stationary problem of the eigenvalues and eigenvectors of the matrix A:

$$A\mathbf{b}=(E_n{}^0-E_n)\mathbf{b},$$

where

$$A_{nlm,n_1l_1m_1}=\delta_{l,l_1}\delta_{m,m_1}\Bigg[\langle nlm|n_1l_1m_1\rangle\left(\frac{n_1+l_1+1}{\nu}e^{-2i\varphi}-e^{-i\varphi}\right)$$
$$+\delta_{n,n_1}\left(E_n{}^0-\frac{1}{2\nu^2}+\omega m\right)\Bigg]+F_0e^{i\varphi}\langle nlm|x|n_1l_1m_1\rangle, \tag{4}$$

and $|nlm\rangle$ is the radial part of the function (3).

The complete diagonalization of a many rowed matrix of the type of (4) is very complicated. Therefore we shall look for only one eigenvalue, which when the field is turned off goes over into the ground-state value E_1^0. For this purpose the inverse of the matrix A was taken and the eigenvalue corresponding to E_1^0 was then the largest one. The solution of the problem of finding the largest eigenvalue and the corresponding eigenvector was then constructed by standard iteration methods. In a case when there is a resonance between atomic levels with allowance for their shift in the field, the matrix has several eigenvalues closely spaced in absolute value. Their separation was accomplished by means of the first steps of the q,p algorithm.[7]

Let us further consider a field frequency ω for which the channel of two-photon annihilation is open. In this case the dimensionality of the basis for Eq. (4) that assures a fractional accuracy of 10^{-2} in the eigenvalues for fields up to $F_0 = 10^{-1}$ is given by $l_{\max} = 4$, $n_{\max} = 10$, and the rank R of the matrix A is then $R = \frac{1}{2}(l_{\max}+1)(l_{\max}+2)(n_{\max}+1) = 165$.

The stationary point of the complex eigenvalues in this case is in the interval $0.4 < \varphi < 0.7$, varying with the frequency and the field strength, and the linear parameter ν of the expansion basis was chosen from the condition that the eigenvalues be stationary under small variations of this parameter.[6,9]

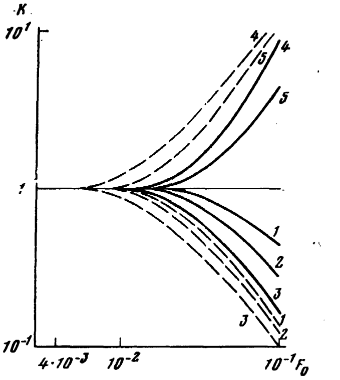

FIG. 1. Dependence of K on the field strength F_0, plotted in double logarithmic scales. Solid curves are K_{sh}, dashed curves, K_{ion}. Curves 1, 2, 3, 4, 5 correspond to frequencies $\omega = 0.3$, 0.3125, 0.35, 0.42, 0.43. The axis of abscissas corresponds to perturbation theory. The resonance frequency corresponding to the transition $1S-2P$ is $\omega_R = \frac{3}{8}$. Curves with $\omega < \omega_R$ go above the axis, those with $\omega > \omega_R$, below it. This indicates a smoothing out of the resonance.

Figure 1 shows a comparison of the results of the calculation for the shift $K_{\text{sh}} = \alpha F_0^2/4\,\text{Re}\Delta E$ and the probability of ionization $K_{\text{ion}} = \sigma F_0^4/2\,\text{Im}\Delta E$ of the ground state of the hydrogen atom, as found by the present method, with a calculation carried out in second-order perturbation theory.[1] The figure shows that the results from second-order perturbation theory agree with the exact results for fields $F_0 \lesssim 10^{-2}$. On the dispersion curve the points with $\omega = 0.3$, 0.3125, and 0.35 lie close to its maximum, and those with $\omega = 0.42$ and 0.43, close to the minimum; consequently the location of the curves with $\omega < \omega_R = 3/8$ below the axis of abscissas and of those with $\omega > \omega_R$ above the axis indicates that the resonance is flattened out. This effect has also been pointed out, in the case of a linearly polarized field, in Ref. 6.

Figure 1 shows that the perturbation theory breaks down for the width in weaker fields than for the shift of the level. The situation is similar in the case of short-range potentials, as was noted in Ref. 1.

3. PERTURBATION THEORY FOR QQES IN THE PRESENCE OF RESONANCE

The effect of the smoothing out of a resonance in a strong field can be taken into account in a natural way in the formalism of perturbation theory for the case of closely spaced quasienergy levels, which was developed in Ref. 1. In the quasienergy theory there is effectively close spacing of levels not only for nearly degenerate levels but also for resonance states $|n\rangle, |m\rangle$ for which $|E_n - E_m| \approx k\omega$, where $k = 1, 2, \ldots$. As was shown in Ref. 1, in the case of a system of closely spaced quasienergy levels $\{\Phi_i^0, E_i^0\}$, $i = 1, \ldots, N$, Eq. 1 can be represented in the form of an integral equation, which is convenient for iteration:

$$|\Phi\rangle\!\rangle=\sum_{i=1}^{N}a_i|\Phi_i{}^0\rangle\!\rangle+G_E{}^pV|\Phi\rangle\!\rangle,$$
$$G_E{}^p=(1-|\Phi_i{}^0\rangle\!\rangle\langle\!\langle\Phi_i{}^0|)\mathcal{G}_E \tag{5}$$

where G_E^p is the reduced quasienergetic Green's function, and $\mathcal{G}_E$ is the quasienergetic Green's function:

$$\mathcal{G}_E=1/(E-\mathbf{H}),$$

and the coefficients a_i and the quasienergies E_i satisfy the system of equations

$$a_i(E_k-E_k^0)=\sum_{j=1}^{N}\langle\langle\Phi_i|R(E)|\Phi_j\rangle\rangle a_j, \qquad (6)$$
$$R(E)=V(1-G_E^pV)^{-1}=V+VG_E^pV+\dots .$$

Let us consider the one-photon resonance $1S-2P$. In the field of a wave with elliptical polarization

$$\mathbf{F}(t)=[F_0/(1+\varepsilon^2)^{1/2}]\{\cos\omega t, \varepsilon\sin\omega t, 0\}$$

[ε is the degree of elliptical polarization, with $\varepsilon=\pm 1$ corresponding to right (left) circular polarization and $\varepsilon=0$ to linear polarization], a system of three closely spaced quasienergy states $|1S\rangle\rangle, |2P\pm 1\rangle\rangle$ is formed; these are intermixed already in the iterations of first order in V of the equations (5):

$$|\Phi_k^1\rangle\rangle=a_0|1S\rangle\rangle+a_1|2P1\rangle\rangle+a_2|2P-1\rangle\rangle,$$

where the a_i are determined from the system of equations

$$\begin{aligned}&a_0(E-E_0^0)-a_1V_1-a_2V_2=0,\\&-a_0V_1+a_1(E-E_1^0+\omega)=0,\\&-a_0V_2+a_2(E-E_1^0+\omega)=0,\\&V_{1,2}=\langle\langle 1S|V|2P\pm 1\rangle\rangle.\end{aligned} \qquad (7)$$

The solution of the secular equation corresponding to Eqs. (7) is

$$E^\pm=\frac{1}{2}\{E_0^0+E_1^0-\omega\pm(E_0^0-E_1^0+\omega)[1+(V_1^2+V_2^2)/(E_0^0-E_1^0+\omega)^2]^{1/2}\}, \qquad (8)$$

where E^+ is the solution that goes over adiabatically into the ground state of the unperturbed atom as the field is turned off. This is the case we shall be considering from now on. The ionization channel opens up in the second order in V in the iterations of Eq. (5):

$$E=E^++\langle\langle\Phi_k^1|VG_E^pV|\Phi_k^1\rangle\rangle.$$

The eigenvectors $|\Phi_k^1\rangle\rangle$ satisfy the orthornormalization conditions

$$\langle\langle\Phi_k^1|\Phi_m^1\rangle\rangle=\delta_{k,m},$$

which gives for the coefficients a_i the condition

$$\sum_{i=1}^{3}|a_i^+|^2=1.$$

The normalized eigenvectors are given by

$$a_0^+=\frac{E^+-E_1^0-\omega}{[(E^+-E_1^0-\omega)^2+V_1^2+V_2^2]^{1/2}}, \qquad a_{1,2}^+=\frac{V_{1,2}}{[(E^+-E^0-\omega)^2+V_1^2+V_2^2]^{1/2}}.$$

The energy E can be expressed in terms of radial composite matrix elements

$$D_{l_0l}^{n_0}(\omega)=\sum_n\frac{|V_{l_0l}^{n_0n}|^2}{E\ -E^++\omega}, \qquad V_{l_0l}^{n_0n}=\langle n_0l_00|r|nl0\rangle,$$

$$E=E^+-\frac{F_0^2}{4[(E^+-E_1^0+\omega)^2+2(F_0V_{10}^{10})^2]}\left\{\frac{1}{3}(E^+-E_1+\omega)^2\right.$$
$$\times\left[D_{01}^0(\omega)+D_{01}^0(-\omega)+\frac{(V_{01}^{01})^2}{(E^+-E_1^0+\omega)}\right]+\frac{(F_0V_{01}^{01})^2}{45}[(2A^2+4)D_{12}^1(-\omega)$$
$$+(4-3A^2)D_{12}^1(\omega)]+{}^1/_9(F_0V_{01}^{01})^2[l^2D_{10}^1(-\omega)$$
$$\left.+D_{10}^1(\omega)+3l^2(V_{01}^{01})^2/(E^+-E_0^0)]\right\}, \qquad (9)$$

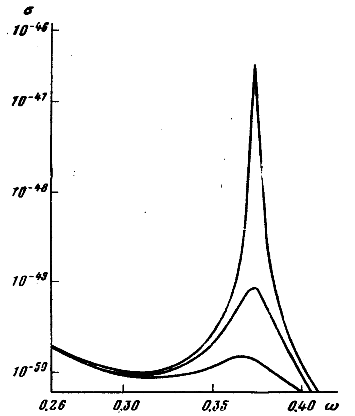

FIG. 2. Dispersion curves of the cross section [cm$^4\cdot$sec] for photoionization of the ground state of a hydrogen atom by a linearly polarized field, for field strengths $F_0=5\cdot 10^{-4}$, 10^{-2}, and $2.5\cdot 10^{-2}$ (from top down).

where $A=2\varepsilon/(1+\varepsilon^2)$ and $l=(1-\varepsilon^2)/(1+\varepsilon^2)$ are the degrees of linear and circular polarization. A comparison of the expansion of Eq. (8) in powers of the field, F_0^{2n}, with the results of nonresonance perturbation theory shows that they agree only in the coefficients of F_0^2. Equation (8) takes into account resonance terms of all orders in the field. By using the Coulomb Green's function the composite matrix elements $D_{l_0l}^{n_0}(\omega)$ can be expressed in terms of absolutely convergent hypergeometric series. These formulas are extraordinarily cumbersome, however, and will not be given here.

Figure 2 shows dispersion curves of the cross section for ionization of the ground state of the hydrogen atom by a field with linear polarization. Comparison with the results of Ref. 6 shows agreement for the resonance frequencies and a modest difference for frequencies $\omega<0.33$, which occurs because only the resonating $|1S\rangle\rangle$ and $|2P\rangle\rangle$ states were taken into account in Eq. (6). A similar situation occurs in comparing the results of the exact calculation of Sec. 2 with calculations made with Eq. (9) in the case of a circularly polarized field. Smoothing out of the resonance also occurs for the shift of the level, as is seen clearly from Fig. 3, where dis-

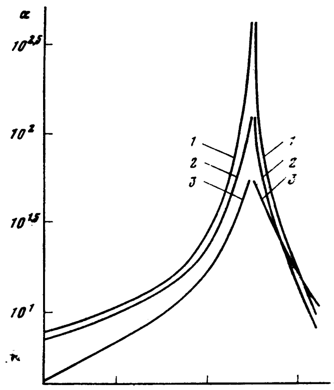

FIG. 3. Dispersion curves of the polarizability of the ground state of a hydrogen atom by a linearly polarized field. Curves 1, 2, 3 correspond to field strengths $F_0=5\cdot 10^{-4}$, 10^{-2}, and $2.5\cdot 10^{-2}$.

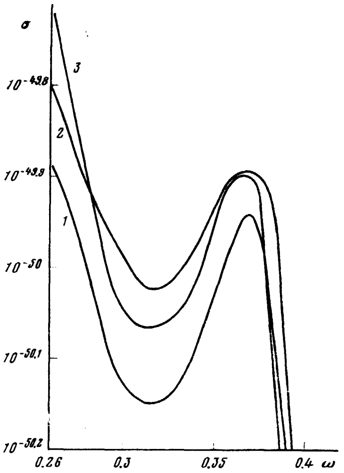

FIG. 4. Dispersion curves of the cross section for photoionization of the ground state of a hydrogen atom by a field of intensity $F_0 = 10^{-2}$ with three different degrees of ellipticity ε. Curves 1, 2, 3 correspond to values $\varepsilon = 0$, 0.5, and 1.

persion curves are given for the polarizability of the ground state by the field of a linearly polarized wave. Figure 4 shows the dependence of the ionization cross section on the frequency for three different values of the ellipticity of the polarization. The complicated character of the curves reflects the interference of the contributions of the *s* and *d* states in Eq. (9). For the shift the main contribution in Eq. (9) is from the term proportional to F_0^2 and independent of the parameter ε. Therefore the dependence of the shift of the ground state on the degree of ellipticity is extremely weak.

The approximation made in Sec. 3 corresponds to including only the "field" mechanism of broadening of a quasistationary level, in which the width is linear in the field strength and is proportional to the matrix element of the resonance transition. The calculations indicate that in fields $F_0 = 3 \cdot 10^{-2}$ this width is of the same order of magnitude as the maximum detuning of the resonance. Summarizing all of these statements, we can say that inclusion of only the field broadening is sufficient, and that in strong fields it is absolutely necessary to include it for a correct description of the interaction of a hydrogen atom with a monochromatic wave.

In conclusion the writers express their gratidude to A. G. Fainshtein for a helpful discussion.

[1)] An atomic system of units is used in this paper.

[1] N. L. Manakov, M. A. Preobrazhenskiĭ, L. P. Rapoport, and A. G. Faĭnshteĭn, Zh. Eksp. Teor. Fiz. 75, 1243 (1978) [Sov. Phys. JETP 48, 626 (1978)].

[2] N. L. Manakov and L. P. Rapoport, Zh. Eksp. Teor. Fiz. 69, 842 (1975) [Sov. Phys. JETP 42, 430 (1975)].

[3] N. L. Manakov and A. G. Faĭnshteĭn, Dokl. Akad. Nauk SSSR 244, 567 (1979) [Sov. Phys. Doklady 24, 41 (1979)].

[4] P. Lambropoulos, in Advances in Atomic and Molecular Physics, D. R. Bates and B. Bederson, Eds., 12, 87 (1976).

[5] G. D. Doolen, J. Phys. B8, 525 (1975). G. D. Doolen, J. Nutfall, and R. W. Stagat, Phys. Rev. A10, 1612 (1974). E. Brändas and P. Froelich, Phys. Rev. A16, 2207 (1977).

[6] Shin-I Chu and W. P. Reinhardt, Phys. Rev. Lett. 39, 1195 (1978).

[7] C. Lanczos, Applied Analysis, Englewood Cliffs N.J., Prentice-Hall, 1964.

[8] B. Simons, Ann. Math. Statistics 97, 247 (1973).

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Investigation of transition x-radiation in tin foils

A. L. Avakyan, R. A. Astabatyan, L. A. Vardanyan, K. R. Grigoryan, M. P. Lorikyan, and K. Zh. Makaryan

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The transition x-radiation of electrons of energy 1–1.35 GeV was investigated in a tin-foil radiator consisting of $N = 20$ plates each $a = 20\,\mu$m thick and separated by a distance $b = 1$ mm. The experimental spectra are compared with the theoretically calculated ones. The experimental results are in better agreement with the transition x-ray theory that takes into account the influence of multiple scattering.

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Transition radiation in the x-ray band (XTR) has by now been sufficiently well investigated both experimentally and theoretically.[1-3] Agreement between the experimental results and the XTR theory is observed in practically all the studies. Only in a few cases is a deviation from the theory observed and ascribed to the influence of multiple scattering.[4-6] The characteristics of the deviations observed in Refs. 4 and 5 were analyzed in Refs. 7–9. The results of Ref. 6, however, have not been explained to this day. Theoretical calculations by Garibyan's group[10] have shown that the experimental results of Ref. 6 cannot be attributed to the influence of multiple scattering.

An investigation[11] of XTR in a radiator of copper foils has shown that the XTR theory without allowance for multiple scattering also describes the experimental result satisfactorily in this case.

A theory of XTR with allowance for multiple scattering has by now been developed for both an isolated plate[12,13] and a stack of plates.[14,15] It has therefore become possible to analyze more correctly the experimental results in those cases when multiple scattering can influence the XTR. In particular, it follows from these theoretical papers that for a stack of N absorbing plates of thickness a and with vacuum gaps b, subject to satisfaction of the condition $b \gg Z_{\text{vac}}$, where Z_{vac} is the zone of formation of the transition radiation in vacuum, the frequency distribution of the intensity of the total radiation emerging from the stack, just as in the case where no allowance is made for multiple scattering, is equivalent to emission from a single plate, multiplied by the effective number of plates N_{eff}, where

$$N_{\text{eff}} = (1-e^{-N\mu a})/(1-e^{-\mu a}) \tag{1}$$

(μ is the linear absorption coefficient in the medium).

Yan Shi[15] has shown, in particular, that the total radiation from an absorbing plate can be divided into two parts: 1) a part due to the presence of boundaries (edge effect in the total radiation) and 2) a part corresponding to bremsstrahlung from a path length equal to the effective thickness of the plate, in other words,

$$\left\langle \frac{dW(\omega)}{d\omega} \right\rangle_{1,a}^{\text{tot}} = \left\langle \frac{dW(\omega)}{d\omega} \right\rangle_{1,a}^{\text{cr. eff}} + \frac{a_{\text{eff}}}{Z_{\text{br}}} \frac{dW_M(\omega)}{d\omega},$$

where $Z_{\text{br}}^{-1} dW_M(\omega)/d\omega$ is the bremsstrahlung emitted from a unit length in an unbounded medium, Z_{br} is the zone of bremsstrahlung formation, and $a_{\text{eff}} = (1-e^{-\mu a})/\mu$.

In experiment one always measures the total radiation, therefore to determine the expression for $\langle dW(\omega)/dW \rangle_{1,a}^{\text{cr. eff}}$ it is necessary to measure $(a_{\text{eff}}/Z_{\text{br}}) dW_M(\omega)/dW$ separately. Usually one measures for this purpose the radiation from a solid radiator with thickness equal to the summary thickness of the layered radiator, but it is clear that in this method of XTR separation no account is taken of the contribution of the two boundaries of the equivalent radiator. In the case of a large number N of plates and small $\mu(\omega)$, the resultant error is small, but for small N and large $\mu(\omega)$ the influence of the boundaries of the equivalent radiator may become substantial.

In this study we used tin as the radiator material. (This is precisely the material where a quadratic dependence of the total number of photons on the electron energy was observed[6] in contradiction to the theory.) We used a radiator consisting of twenty plates each $a = 20$ m thick and spaced $b = 1$ mm apart.

The measurements were made at electron energies 1–3.5 GeV, using the secondary electron beam of the Erevan accelerator. The experimental setup is described in detail in Ref. 8. In this installation, the dimensions of the apparatus counters were decreased to 20 × 20 × 15 mm. This made it possible to localize more accurately the trajectories of the electrons passing through the radiator and through the entire installation.

The XTR detector was a spectrometric scintillation counter based on an NaI (Tl) crystal. The crystal had a diameter $\Phi = 7$ cm, a thickness 2 cm, and an acceptance angle $\sim 6.5 \times 10^{-3}$. The detector was calibrated against radioactive sources in the gamma-quantum energy range 34–250 keV.

The experimental spectra described below were obtained after taking into account the corrections for the multiplicity of the photons and for the losses due to the characteristic radiation in the NaI(Tl), for the influence of the finite resolution of the spectrometer,[16,17] and for the absorption of the gamma quanta along the path from the radiator to the spectrometer. To take into account the bremsstrahlung as well as the background in the laboratory, we measured each time the radiation from an equivalent tin-plate radiator 400 μm thick. The contribution of the XTR produced on the two boundaries of the equivalent radiator was determined by theoretical calculation. For the radiator in question, at $E_e \geq 2.0$ GeV, this correction amounts to ~15% in the $\hbar\omega$ region 20–35 keV, and is negligible at $\hbar\omega > 40$ keV.

Since the condition $b \gg Z_{\text{vac}}$ for the independence of the radiation from individual plates are satisfied in our case for practically all the considered frequencies and energies of the electrons, the theoretical calculations were made for an isolated plate and recalculated for a stack by multiplying by N_{eff}. If we disregard the influence of the multiple scattering, then discrimination with respect to the emission angle leads in our case to a decrease in the number of photons by ~45%, ~10%, ~1% at photon energies $\hbar\omega$ 20–25, 30–35, and >50 keV respectively, regardless of the electron energy.

Figure 1 shows the XTR spectra calculated with (dashed curves) and without (solid curves) allowance for multiple scattering at an electron energy 1, 2, 3, and 3.5 GeV. The abrupt decrease in the region $\hbar\omega \sim 31$ keV is due to the presence of the K absorption edge. It follows from the figures that under our conditions the influence of multiple scattering leads to a ~25% lowering of the XTR spectra in the region of the right-hand maximum, but in the other frequency regions the change is negligible. The same figure shows the experimental XTR spectra for the same electron energies (points).

It follows from Fig. 1 that at an electron energy $E_e \geq 2$ GeV in the photon energy range 40 keV $\leq \hbar\omega \leq$ 100 keV the experimental results agree better with calculations that take multiple scattering into account. Unfortunately, the energy resolution of the γ spectrometer is insufficient to observe a narrow maximum in the region $\hbar\omega \sim 28$ keV. This, obviously, leads to a smoothing of the maximum and accordingly to an increase in the number of the observed photons in the vicinities of this maximum. Appreciable discrepancies are observed between the experimental results and the theory at $E_e = 1.0$ GeV. This fact can be qualitatively explained in the following manner. As already mentioned, the theoretical spectra were calculated for one plate, and the conversion for a stack of plates was with the aid of expression (1). It is clear that this method takes correct account of the photon absorption, but the additional radiation divergence angle, due to the multiple scattering of the

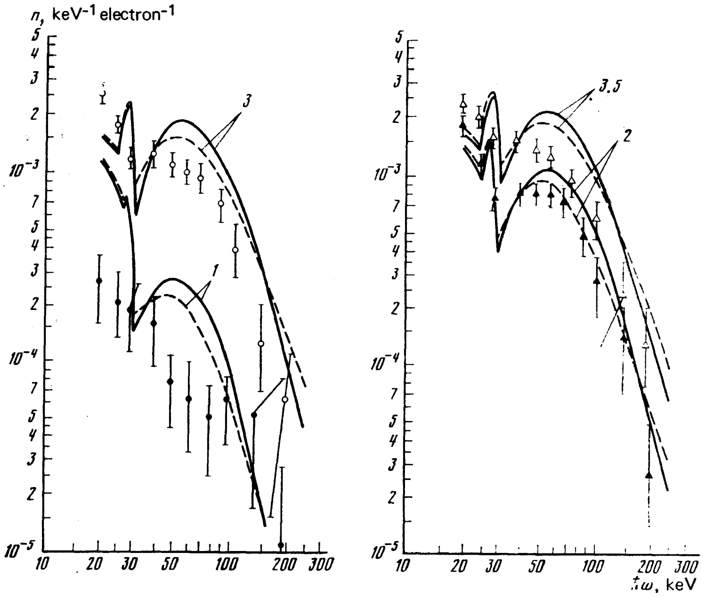

FIG. 1. XTR spectra produced in a stack of tin foils. The numbers on the curves correspond to different values of the electron energy E_a in GeV: ●—1; ▲—2, ○—3, △—3.5.

electrons in the stack, is not accounted for here. Since the multiple-scattering angle is inversely proportional to the particle energy, the photon loss for low electron energies, due to the finite dimensions of the gamma spectrometer, is larger than for high energies.

In addition, it appears that the angle spread of the radiation in the presence of multiple scattering is larger than in its absence. Therefore the finite dimensions of the gamma spectrometer lead in this case, too, to a stronger decrease of the fraction of the photons registered at small E_e than at large E_e. Obviously, this qualitative analysis is insufficient to explain the experimental results, and for a final solution of the problem it is necessary to have theoretical spectra integrated in the region of the angles within which the photons are registered in experiment.

Figure 2 shows the dependence of the average number of XTR photons on electron energy. The solid and dashed curves show the results of the theoretical calculations without and with allowance for multiple scattering, while the points show the corresponding experimental results. It is seen from the figure that the number of photons increases linearly with increasing energy up to $E_e = 2.5$ GeV, while at $E_e > 2.5$ GeV this dependence becomes weaker, whereas the theoretical dependence remains linear in the entire range of E_e. The experimental points lie somewhat lower in the entire interval than the corresponding theoretical values and agree better with the theory that takes multiple scattering into account.

Figure 3 shows the dependence of the total XTR energy release W on E_e. The solid and dashed curves correspond to theoretical calculations without and with allowance for multiple scattering, while the points show the experimental results. As seen from the figure, a linear dependence of W on E_e is expected here, too. A similar dependence is observed experimentally, but with a smaller slope than in the theory.

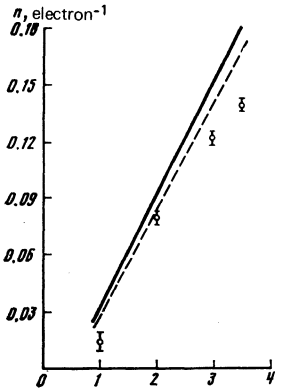

FIG. 2. Dependence of the total number of quanta n on the electron energy.

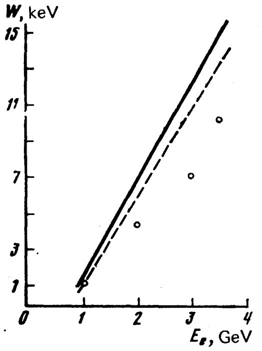

FIG. 3. Dependence of the total radiation energy on the electron energy.

Thus, the results of our investigations show that the quadratic dependence of W on E_e, observed in Ref. 6 for XTR in tin granules, does not appear in the case of a layered radiator made of the same material. It is possible that this disparity is due to the different methods of generation and registration of the XTR.

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[1] Proc. Internat. Symp. on Transition Radiation of High-Energy Particles, Erevan Phys. Inst., 1977.

[2] E. S. Belyakov, M. P. Lorikyan, K. Zh. Makaryan, M. M. Muradyan, and K. K. Shikhlyarov, Nauchnoe Soobshchenie EFI (Scient. Communication, Erevan Phys. Inst.) 44-(73), 1973.

[3] A. I. Alikhanyan, E. S. Belyakov, G. M. Garibyan, M. P. Lorikyan, K. Zh. Markaryan, and K. K. Shikhlyarov, Pis'ma Zh. Eksp. Teor Fiz. **16**, 315 (1972) [JETP Lett. **16**, 221 (1972)].

[4] F. R. Artyunyan, K. A. Isiryan, A. G. Oganesyan, and A. A. Frangyan, Pis'ma Zh. Eksp. Teor. Fiz. **4**, 277 (1966) [JETP Lett. **4**, 187 (1966)].

[5] F. R. Arutyunyan, K. A. Isiryan, and A. G. Oganesyan, Zh. Eksp. Teor. Fiz. **52**, 1121 (1967) [Sov. Phys. JETP **25**, 743 (1967)].

[6] A. K. Drukier, C. Valette, G. Waysand, L. C. L. Yuan, and F. Peters, Lett. Nuovo Cimento **14**, 300 (1975).

[7] M. P. Lorikyan, Nauchnoe soobshchenie EFI 58-(74), 1974.

[8] A. I. Alikhanyan, E. S. Belyakov, G. M. Garibyan, M. P. Lorikyan, K. Zh. Markaryan, and K. K. Shikhlyarov, Pis'ma Zh. Eksp. Teor. Fiz. **18**, 35 (1973) [JETP Lett. **18**, 18 (1973)].

[9] A. I. Alikhanyan, Ref. 1, p. 41.

[10] L. A. Vardanyan, G. M. Garibyan, and Yan Shi, *ibid*. p. 374.

[11] A. L. Avakyan, M. P. Lorikyan, Yu. L. Margaryan, and K. Zh. Markaryan, *ibid*., p. 287.

[12] G. M. Garibyan and Yan Shi, Zh. Eksp. Teor. Fiz. **70**, 1627 (1976) [Sov. Phys. JETP **43**, 848 (1976)].

[13] L. A. Vardanyan, G. M. Garibyan, and Yan Shi. Izv. Akad. Nauk Arm. SSR, Fizika, **11**, 329 (1976).

[14] G. M. Garibyan, and Yan Shi, Izv. Akad. Nauk Arm. SSR Fiz. **12**, 105 (1977).

[15] Yan Shi, Ref. 1, p. 102.

[16] K. Liden and N. Starfelt, Ark. Fys. **7**, 427 (1953).

[17] J. W. Motz, Phys. Rev. **100**, 1560 (1955).

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Collapse of 4*f*-electron in the configuration 3*d*^{9}4*f* in xenonlike ions

A. A. Maĭste, R. É. Ruus, S. A. Kuchas, R. I. Karaziya, and M. A. Élango

Physics Institute, Estonian Academy of Sciences
and Physics Institute, Lithuanian Academy of Sciences
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The $M_{4,5}(3d)$ absorption and emission spectra of the xenonlike ions I^-, Cs^+, Ba^{2+}, and La^{3+} were measured in ionic compounds (spectral range 600–1100 eV). It is shown that the form of the spectrum characterizes mainly the absorbing ion and depends little on its surrounding. The absorption spectra of Cs^+, Ba^{2+}, and La^{3+} contain an intensive doublet near the ionization threshold of the $3d$ shell, and their emission spectra contain bands that are at resonance with the absorption bands. It is concluded that in the Ba^{2+} and La^{3+} ions the $4f$ orbit in the configuration $3d^94f$ is collapsed and that its collapse takes place on going from Xe to Cs^+. This conclusion is confirmed by a calculation of the energies and cross sections of the $3d^{10}\rightarrow 3d^94f$ transitions in the Hartree-Fock-Pauli approximation.

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1. INTRODUCTION

The phenomenon of sudden compression of the radial wave function of an excited electron in a number of neutral atoms or in an isoelectronic sequence, which was named electron collapse, was predicted by Fermi in 1928.[1] Only in the last decade, however, in connection with the extensive studies of excited configurations of atoms and ions, did it attract considerable attention and made it possible to explain a number of interesting effects in atomic spectra and in the physics of atomic collisions.[2,3]

The effective potential in the centrosymmetric field in the Hartree-Fock equation for the electron nl consists of two terms:

$$V_{\text{eff}}(nl|r)=V(nl|r)+l(l+1)/r^2,$$

where $V(nl|r)$ is the potential of the Coulomb field of the nucleus and of the other electrons, while the second term is centrifugal. In the interval $r\approx 1$–5 a. u., $V(nl|r)$ varies approximately like r^{-2}, and in the case of an electron with $l\geqslant 2$ these two terms compete with each other in a certain interval of r.[2] This leads to a specific form of the effective potential, with two minima separated by a positive potential barrier. The localization of the electron in the field of this potential is highly sensitive to a change in the charge of the nucleus or of the states of the other electrons. Whereas in a certain configuration of the atom Z the wave function of the excited electron is localized predominantly in the region of the outer potential well, for the same configuration of

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