6. We can easily see that Eq. (14) is unsuitable for the calculation of the temperature dependence of the birefringence. It is found experimentally that $n_e - n_o$ decreases on increase in temperature. If the only reason for this were the thermal expansion, then—in accordance with Eq. (14)—the birefringence would have increased on increase in the lattice constants, because the coefficient of expansion along the z axis is an order of magnitude greater than the coefficient of expansion along the x axis. Thus, the thermooptic effect not associated with the change in the lattice parameters should predominate.

The authors are deeply grateful to P. L. Kapitsa for his continuous interest, to A. A. Pankov and V. G. Zhotikov for valuable discussions, and to E. M. Elagin for help in the experiments. They also thank R. V. Pisarev for discussing the results and making available the manuscript of his paper before publication.

1)Physics Institute, Czechoslovak Academy of Sciences, Prague.

2)The authors are grateful to S. V. Petrov for growing these single crystals.

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Translated by A. Tybulewicz

Scattering of light by parametric magnons and phonons in $CoCO_3$

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(Submitted 26 June 1979)
Zh. Eksp. Teor. Fiz. 77, 2486–2497 (December 1979)

Mandel'shtam-Brillouin light scattering is used in antiferromagnetic $CoCO_3$ at $T \lesssim 2$ K to detect and investigate magnons parametrically excited by microwave pumping, and propagating along the C_3 axis and in the basal plane of the crystal. The excitation thresholds for parametric magnons are determined, and the maximum number of parametric particles observed in the experiment is estimated. Disintegration of a single uniform-precession ($k = 0$) magnon into two phonons, with $k \neq 0$ and with a frequency equal to half the antiferromagnetic-resonance frequency, is also observed.

PACS numbers: 75.30.Ds, 75.50.Ee, 78.35.+c, 71.36.+c

I. INTRODUCTION

The study of parametrically excited spin waves PSW (or parametric magnons PM) in magnetically ordered materials has been the object of much research (see, for example, Refs. 1 and 2). Most of these investigations have been made by radio-frequency methods, involving absorption by the material of microwave power incident on it.

A comparatively small part of the work has been devoted to optical methods of investigating PM. In these papers, the magnetooptical Faraday effects[3-5] and magnetic birefringence[6] have been used to record the change of the component of the magnetization of the crystal parallel to the external magnetic field when PM were excited.

All these experiments give information about the relaxation times of PM, their total number, etc. But they do not permit a direct answer to the question: in what direction do the parametric excitations propagate, and to what wave vector **k** do they correspond? It is also difficult to determine, by the methods indicated above, which kind of quasiparticles, phonons or magnons, are excited parametrically in the experiment. Some of these questions, it seems, can be answered by investigation of inelastic scattering of light by elementary excitations. Since parametric excitation produces quasiparticles of the low-frequency branch of the spectrum of

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the magnetic material it is appropriate to use, in the study of inelastic scattering by them, the method of Mandel'shtam-Brillouin scattering (MBS) of light.[7] This method permits the recording of small frequency shifts, of order 1 to 3 cm^{-1}. The presence of parametric quasiparticles leads to the result that in the scattering spectrum there appear satellites whose frequency coincides with the frequency ν of these quasiparticles. The value of the wave vector and the direction of the propagation of the quasiparticles are uniquely determined by the geometry of the experiment and the orientation of the crystal. The intensity of the satellites in the scattering spectrum depends, when other conditions are the same, on the number of quasiparticles from which scattering occurs.[8] The appearance of parametric magnons, i.e., of an excess number of magnons with a given **k** and ν as compared with the thermal level, can be observed with MBS from the increase of the intensity of a satellite corresponding to thermal magnons with the same **k** and ν. Because of the presence of magnetoelastic interaction, microwave pumping may also produce parametric phonons in the crystal. The MBS method permits observation of these quasiparticles also.

Light scattering by parametrically excited quasiparticles–magnons and phonons–was first observed by us in the antiferromagnetic crystal $CoCO_3$.[9] At present such investigations are being carried out on still two other materials: yttrium-iron garnet (YIG), $Y_3Fe_5O_{12}$,[10] and $FeBO_3$.[11] YIG is a well investigated cubic ferrimagnet. Parametrically excited spin waves in it have been studied in detail by radiofrequency methods.[12] Reference 10 reports detection and investigation of MBS in YIG by magnons excited by parallel pumping, at frequency 10 GHz, over the wave-vector range $5\times10^2 < k < 2.6\cdot10^4$ cm^{-1}. In antiferromagnetic $FeBO_3$, quite recently, inelastic scattering of light has been detected from parametric phonons produced on excitation of antiferromagnetic resonance in the specimen. As was reported in Ref. 9, in $CoCO_3$ we succeeded in exciting parametric magnons and phonons and detecting inelastic light scattering from them. Till the present time, parametric excitation of spin waves in antiferromagnets has been observed only with parallel pumping and in compounds containing manganese ions.[2]

$CoCO_3$ is an antiferromagnet with weak ferromagnetism. Below the Néel temperature $T_N = 18.1$ K, magnetic order occurs in it, with anisotropy of the "easy plane" type. The magnetic and magnetooptic properties of $CoCO_3$ have been quite well studied. A comparatively detailed summary of data on its properties and appropriate references are given, for example, in Ref. 13. The same paper reports a study, by the MBS method, of the low-frequency part of the spin-wave spectrum and the phonon spectrum at $T \sim 300$ K in $CoCO_3$. Values are given for the exchange constants for the three principal directions of propagation of magnons, and for the velocities of sound along the C_3 axis and in the basal plane.

The present paper is devoted to a detailed description of experiments on parametrically excited quasiparticles–magnons and phonons–in $CoCO_3$, and to a study of them by the method of inelastic scattering of light.

II. METHOD AND SPECIMEN

In the experiments on scattering in $CoCO_3$, the spectrum investigated was that of light scattered at angle 90° to the incident beam. At wavelength $\lambda = 632.8$ nm of the incident light and with the geometry of the experiment mentioned above, the value of the wave vector of the scattering quasiparticles was $k = 2.5\cdot10^5$ cm^{-1}. As spectral apparatus, we used a high-contrast three-passage Fabry-Perot interferometer from the American firm "Burleigh." The control of the interferometer, the adjustment of it, and also the recording of the scattering spectrum obtained were carried out by means of the DAS-I system manufactured by the same firm. A detailed description of the optical scheme that we used in the study of scattering is given in Ref. 13.

In all the experiments, the specimen was located in a wave-guide at $T \leq 2$ K in a bath of superfluid helium. The magnetic field was produced by superconducting magnets of various constructions and could be applied along various directions. The light to the specimen was supplied through a window in the cryostat and special openings in the superconducting magnets and the waveguide. The general scheme of the experimental setup is shown in Fig. 1.

Parametric excitation of quasiparticles in the crystal was carried out by the usual radiofrequency method. To feed high-frequency power to the specimen, we used a simple straight-amplification microwave spectrometer (see Fig. 1). As sources of power we used a klystron, with generation frequency $\nu_1' = 36.2$ GHz, and two magnetrons, operating in the continuous mode at frequencies $\nu_1' = 35.4$ GHz and $\nu_2 = 51.02$ GHz. The maximum power used in the experiments was ~500 mW. The specimen under study was attached to a plunger and placed inside the shorted waveguide, at an antinode of the magnetic component of the microwave field. The reflected microwave signal was detected by a crystal detector and entered either an oscillograph or the Y coordinate of an XY recorder. The X coordinate of this recorder received a signal proportional to the magnetic

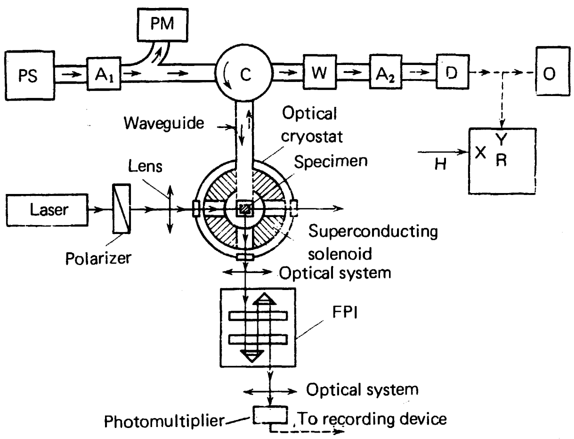

FIG. 1. Schematic drawing of the experimental setup for observation of light scattering when microwave power is fed to the specimen: FPI, Fabry-Perot interferometer; PS, microwave pumping source; A_1 and A_2, attenuators; PM, microwave power meter; C, circulator; W, wavemeter; D, detector; O, oscilloscope; R, XY recorder.

field. Control of the microwave power entering the waveguide was accomplished with an M3-21 power meter, connected into the second branch of a calibrated directional coupler. To calculate the amplitude h_{mic} of the microwave field in the waveguide, we used the following formula:[14]

$$h_{mic}=(4P/abZ)^{1/2}, \quad (1)$$

where P is the power in watts, a and b are the dimensions of the cross section of the waveguide in meters, and $Z=120\pi\lambda_g/\lambda$ is the impedance of the waveguide in ohms; λ_g is the wavelength in the waveguide, λ the wavelength in free space. Here attenuation of the wave in the waveguide is disregarded.

In all the experiments, the $CoCO_3$ crystal used[1)] was the same one as in Ref. 13. It was cut into the form of a rectangular parallelopiped with dimensions $1\times1.2\times1.8$ mm. The C_3 axis coincided with the direction of the diagonal of the smallest face. This specimen geometry made it possible, with 90° scattering, to investigate quasiparticles propagating along rational directions in the crystal. All faces of the specimen were optically polished.

III. EXPERIMENTAL RESULTS AND DISCUSSION

We introduce a system of coordinates, in accordance with Ref. 13, such that the z axis coincides with the third-order symmetry axis C_3 of the crystal, the x and y axes lie in the basal plane σ, and the x axis is directed along the external magnetic field **H**. Hereafter we shall call quasiparticles propagating along $\mathbf{x}(\mathbf{k}\parallel\mathbf{x})$ x particles, along $\mathbf{y}(\mathbf{k}\parallel\mathbf{y})$ y particles, and along $\mathbf{z}(\mathbf{k}\parallel\mathbf{z})$ z particles.

We studied light scattering by x, y, and z quasiparticles excited by microwave power (pumping). It should be noted that the construction of the experimental setup did not permit arbitrary realization of the conditions of parallel ($\mathbf{H}\parallel\mathbf{h}_{mic}$) or perpendicular ($\mathbf{H}\perp\mathbf{h}_{mic}$) pumping. In excitation of z and y particles, the conditions of perpendicular microwave pumping were always fulfilled. In excitation of x particles, conditions both of perpendicular and of parallel pumping were realized; that is, components $h^z_{mic}\parallel H^z, h^x_{mic}\parallel H^x$ and $h^z_{mic}\perp H^x, h^x_{mic}\perp H^z$ existed. In this last case we do not know by what method of excitation a quasiparticle actually was produced.

1. Parametric magnons

Here we shall discuss the investigation of spectra of scattering by z particles, at pumping frequencies $\nu_p=\nu_1\sim36$ GHz, and $\nu_p=\nu_2=51.02$ GHz, and by x particles, at $\nu_p=\nu_2$. The results of these investigations show that in a magnetic field corresponding (in accordance with the form of the magnon spectrum[13]) to the existence of magnons with $\nu=\nu_p/2$ at $k=2.5\cdot10^5$ cm^{-1}, where ν_p is the frequency of the microwave pump, switching on of the latter causes appearance in the scattered-light spectrum of intense peaks with $\nu=\nu_p/2$. In the absence of microwave pumping, there are in the light spectrum peaks of much smaller intensity, corresponding to thermal magnons with $\nu=\nu_p/2$. In accordance with what was said in the Introduction, the intensity of the peaks in the light-scattering spectrum is determined by the number of scattering particles. Hence it follows that the growth of intensity of the peaks on introduction of microwave pumping indicates production of a large number of quasiparticles with $\nu=\nu_p/2$ and $k=2.5\cdot10^5$ cm^{-1} at this value of the magnetic field.

An example of the results described is shown in Fig. 2 for z particles. The lower curve in the figure corresponds to the spectrum of scattering by thermal magnons with $\nu=25.5$ GHz and $H=0.572$ kOe. The upper curve, with the intense peaks, corresponds to scattering by the excited quasiparticles produced when microwave power with $\nu_p=51.02$ GHz was switched on, in the same magnetic field. It must be emphasized that the value of the magnetic field in which the increase of intensity of the peaks is observed is substantially smaller than the field for excitation of antiferromagnetic resonance at the corresponding frequency.

In the observation of scattering by generated quasiparticles, the following polarization conditions are always fulfilled: a) $\mathbf{E}_{inc}\perp\mathbf{E}_{scat}$, where $\mathbf{E}_{inc}$ and $\mathbf{E}_{scat}$ are the electric-field vectors of the incident and scattered light waves, respectively; b) one of the vectors, $\mathbf{E}_{inc}$ or $\mathbf{E}_{scat}$, lies in the plane of scattering, while the other is perpendicular to it. In our experiments the vector $\mathbf{E}_{inc}$ was always parallel to the external magnetic field.

Study of the influence of the value of the magnetic field on the position and intensity of the satellites, for values of the wave-vector and pumping frequency prescribed by the experimental conditions, shows that the magnetic-field range for existence of excited quasipar-

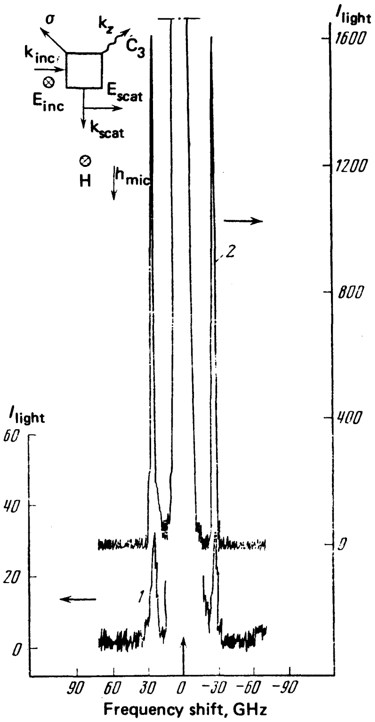

FIG. 2. Spectrum of scattered light ($\lambda=632.8$ nm) at 90° in $CoCO_3$ ($T\lesssim2K$) from z magnons: 1, thermal; 2, parametric, excited by microwave power with $\nu_p=51.02$ GHz. The light intensity I_{light} is given in number of counts. The accumulation time of spectrum 1 is 2000 sec; of spectrum 2, 100 sec.

ticles is ±10–20 Oe. Within this field interval, the frequency of the satellite corresponds, with accuracy ±0.5 GHz, to half the frequency of the pump. The variation of the satellite intensity with the value of the magnetic field, for x particles, is shown in Fig. 3.

We studied further the influence of the value of the microwave pumping power, with $\nu = \nu_p$, on the intensity of the satellites, with $\nu = \nu_p/2$ at constant magnetic field. It was observed that this dependence has a threshold character. This means that the increase of the intensity I of the satellites begins with a certain finite value of the microwave power, larger than zero. Figure 4 shows, as an example, the dependence of the intensity of the satellite with $\nu = \nu_p/2$ on the value of the pumping power P in a constant magnetic field, for x particles. The curve in Fig. 4 illustrates the presence of a threshold in this dependence. The experimental data obtained enable us to determine the threshold values of power P_c at which parametric excitation of quasiparticles begins. These values are given in Table I. The values of threshold power indicated in the table are in the nature of estimates, since the accuracy of determination of the absolute value of the microwave power supplied in our experiments is not great.

Excess of the pumping power above the threshold value leads at first to a linear growth of the satellite intensity (see Fig. 4); then the $I(P)$ curve saturates and drops. Deviation from a linear $I(P)$ relation begins for z particles at pumping powers ~300 mW (or ~0.1 Oe) for x particles at ~100 mW (or 0.06 Oe). Production of excited quasiparticles must, it seems, be accompanied by absorption of microwave power within a definite magnetic-field interval. But in our experiments such absorption could not be detected. Probably the value of the absorbed power is small, and our method of recording the absorption of this power is very crude.

The experimental results obtained indicate that under the influence of microwave power, an excess number of quasiparticles, as compared with the thermal level, are produced in the crystal. The production of quasiparticles occurs parametrically, since their frequency is

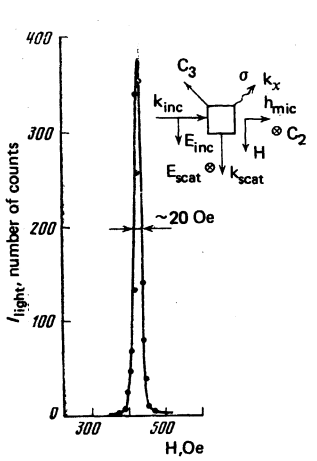

FIG. 3. Variation of the intensity of light scattering by parametric x magnons with the value of the external magnetic field H in $CoCO_3$ ($T \lesssim 2K$). Pumping frequency $\nu_p = 51.02$ GHz.

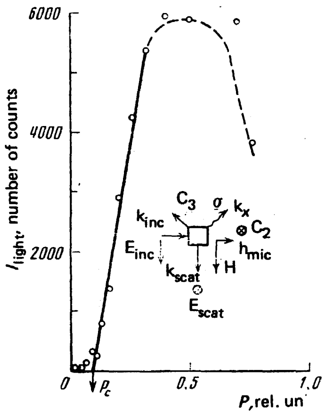

FIG. 4. Variation of the intensity of light scattering by parametric x magnons with microwave pumping power P in $CoCO_3$ ($T \lesssim 2K$). Pumping frequency $\nu_p = 51.02$ GHz.

half the pumping frequency, and the process by which they are produced is of threshold type. We assume that in this case we have to do with parametric x and z magnons. They originate as the result of disintegration of a single microwave photon, with $\nu_p = \nu_{mic}$, into two magnons, which have opposite values of the wave vector and a frequency equal to half the frequency of the microwave photon: $\nu_M = \nu_p/2$. The basis for this assumption is the experimentally determined conditions under which scattering of light by these quasiparticles is observed:

a) The polarization conditions of the observation coincide with the observation conditions for thermal magnons[13];

b) The value of the magnetic field at which these quasiparticles originate is determined by the magnon spectrum obtained in Ref. 13. In the absence of microwave pumping, thermal magnons with a frequency equal to $\nu_p/2$ are observed at this field (see Fig. 2);

c) The parametrically excited quasiparticles exist within a narrow magnetic-field interval (see Fig. 3).

We shall explain the last fact in more detail with the aid of Fig. 5. This figure shows the low-frequency part of the spectrum $\nu(k)$ for x, y, and z magnons (according to Ref. 13) for a fixed value of the magnetic field. It also shows schematically the process of disintegration of a single microwave photon with $\nu_p = \nu_{mic}$ into two z magnons with $\nu_M = \nu_{mic}/2$. This schematic drawing shows that at each value of the magnetic field, in a prescribed direction in the crystal, magnons with a fixed value of the wave vector are parametrically excited. As was indicated above, in our case the value of the wave vector and the direction of propagation of the quasiparticle, and also the frequency of the exciting photon, are

TABLE I. Threshold power for excitation of quasiparticles.

Quasiparticles	ν, GHz	P_c, mW	h_c, Oe
z-particles	36	30	0.03
»	51.02	60	0.045
x-particles	51.02	80	0.053

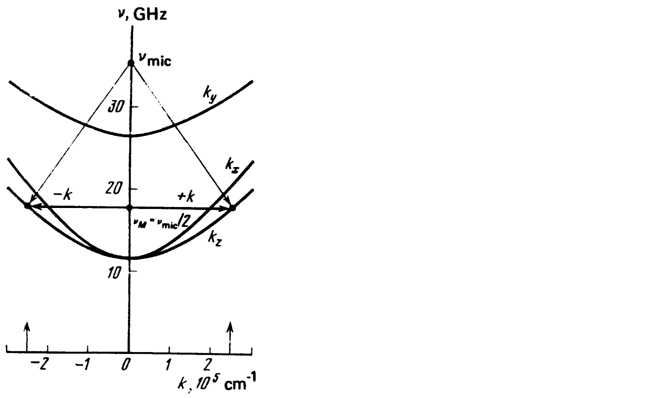

FIG. 5. Sketch illustrating parametric excitation of z magnons of half frequency in $CoCO_3$, at $\nu_p \approx 36$ GHz.

prescribed by the conditions of the experiment. The only free parameter is the magnetic field, which can vary arbitrarily in value. As it varies, the magnon spectrum must shift in the $\nu(k)$ plane along the axis of abscissas (ν). In order that scattering of light by parametric magnons may be perceived, it is necessary to select that value of the magnetic field for which the frequency of the magnon under study (x, y, or z), with $k = 2.5 \cdot 10^5$ cm^{-1}, amounts to half the pumping frequency, as is shown in Fig. 5 for a z magnon. It is evident from the sketch that departure from this value of the field must lead to violation of the conditions for observation. The theoretical range for satisfaction of the conditions must be determined by the intrinsic linewidth of the scattering by parametric magnons and must correspond to a very narrow magnetic-field interval. In the experiment, the range of observation of scattering is ~20–10 Oe (see Fig. 3). This value corresponds to the actually observed linewidth of the scattering, contributions to which, in addition to the intrinsic width, are made by the instrumental functioning of the interferometer, the laser linewidth, etc.

Thus our experiments show that parametric excitation of spin waves has been detected in $CoCO_3$. Parametric magnons propagating along the C_3 axis and in the basal plane of the crystal, along the magnetic field, havebeen directly observed.

As for y particles propagating in the basal plane perpendicular to the magnetic field, for all the pumping frequencies available to us, 35.4, 36.2, and 51.12 GHz, no parametric excitation of spin waves in this direction was detected. But this fact is fully explained by the form of the y-magnon spectrum obtained in Ref. 13. In zero magnetic field, the y-magnon spectrum has a gap of 23.7 GHz, which is caused by dipole-dipole interaction. As a result, at $k = 2.5 \cdot 10^5$ cm^{-1}, the value used in the experiment, the frequency of a y-magnon in zero magnetic field is already 29.5 GHz. In order to detect parametric y magnons in $CoCO_3$, it is necessary to use a still shorter-wavelength source of microwave pumping. On the basis of our experiments, one can evidently state that at a sufficiently high level of power, parametric magnons are produced in all directions.

As is evident from Fig. 2, the intensities of the Stokes and anti-Stokes components of the spectrum, corresponding to the parametric magnons, practically coincide. Since the experimental geometry was the same in all the experiments, the Stokes component of the spectrum is always related to particles with one direction of the wave vector **k**, the anti-Stokes to those with the opposite direction. When this remark is taken into account, it follows from the experimental fact mentioned above that in the above-threshold state, there are equal numbers of magnons with oppositely directed wave vectors. This deduction is in agreement with the generally accepted picture of the process of parametric excitation of magnons.

As was indicated above, we determined the values of the threshold fields at which parametric excitation of magnons begins. The values for x and y magnons (see Table I) differ by a factor 1.2. The z magnons have the lower threshold; this is, upon introduction of microwave pumping, magnons along the C_3 axis, perpendicular to the magnetic field, are first generated, and then ones in the basal plane along the field.

The dependence of the value of the threshold microwave field h_c on the pumping frequency ν_p, on the magnitude of the magnon damping $\propto \Delta\nu_k$, and on other parameters, for antiferromagnets of the "easy plane" type under conditions of parallel pumping, i.e., $h_{mic} \parallel H$, have been derived theoretically by Ozhogin.[15] It has the form

$$h_c = \frac{\nu_p \Delta\nu_k}{\gamma^2 (2H_0 + H_D)}, \qquad (2)$$

where H_D is the Dzyaloshinskii field, H_0 is the field in which parametric magnons are observed, and γ is the gyromagnetic ratio. For $CoCO_3$, $H_D = 27$ kOe and $\gamma = 5.8$ GHz/kOe.[13]

In our experiments, the parallel-pumping conditions can be satisfied only when x magnons are being excited. If we assume that parametric x magnons are produced as a result of parallel pumping, then by using formula (2) and the experimental value of the threshold (Table I) we can estimate the relaxation time $\tau = 1/2\pi\Delta\nu_k$ of x magnons in $CoCO_3$. This estimate shows that $\Delta\nu_k \approx 0.7$ MHz and $\tau \approx 0.2 \mu$sec. The value of the relaxation time of magnons in $CoCO_3$ agrees in order of magnitude with measurements for $MnCO_3$ and $CsMnF_3$.[16] It must be noted, however, that for these compounds the relaxation time of magnons depends strongly on temperature. We have not studied this dependence of $CoCO_3$. All our experiments were done at practically a single temperature, ~1.8–1.9 K.

Parametric z magnons in our experiments were excited under conditions of transverse pumping. At the present time, we know of no formula that describes the relation between the threshold field and the relaxation time for this case. Therefore the only conclusion that can be drawn from the present experiment, for the time being, is that the value of the threshold field is proportional to the pumping frequency (see Table I).

Comparison of the intensities of the satellites in the light-scattering spectrum corresponding to thermal and to parametric magnons enables us to determine the value F of the excess over the thermal level. Such an estimate is correct if the observed width of the scattering peak coincides with the characteristic width $\Delta\nu_k$ of a parametric magnon. But as the above estimates showed, $\Delta\nu_k \sim 0.7$ MHz. This value is about three orders of magnitude smaller than the observed scattering linewidth, which has an instrumental width ~1 GHz. This means that in order to estimate the excess of the number of parametric magnons over the thermal level, we must increase the value obtained from the intensity ratio by a factor ~1000. By taking this remark into account, we obtained for the maximum value of the excess $F \sim 6 \cdot 10^5$, within an interval Δk determined by the linewidth $\Delta\nu_k \sim 0.7$ MHz:

$$\Delta k = \frac{1}{2}\frac{\partial k}{\partial \nu_k}\Delta\nu_k \sim 20 \text{ cm}^{-1}. \tag{3}$$

The estimate given is correct for x magnons. In the case of z magnons, the ratio of the intensities of the satellites for thermal and for parametric magnons is ~10 times smaller. Therefore for them, $F \sim 5\times10^4$.

It is now easy to determine the maximum number N_k of magnons in 1 cm^3, with $k = 2.5\times10^5$ cm^{-1} and $\Delta k \sim 20$ cm^{-1}, that make a contribution to light scattering:

$$N_k = n_k F. \tag{4}$$

Here n_k is the number of thermal magnons collected from a k surface determined by the aperture of the light beam.

For occupation numbers[2)]

$$n_0 = \frac{1}{\exp(h\nu_k/kT) - 1} \sim 2 \quad (T \sim 2\text{K})$$

n_k amounts to $2\cdot10^8$ particles in 1 cm^3, and for N_k we get the value ~10^{13}–10^{14} particles in 1 cm^3.

The experimentally found relation between the intensity of the satellites corresponding to scattering by parametric magnons and the power supplied (Fig. 4) enables us to judge the change of the number of magnons N_k at large pumping powers. The initial section of the curve in Fig. 4 shows that above the threshold, N_k increases linearly with $P - P_c$, or

$$N_k = A[(h/h_c)^2 - 1]. \tag{5}$$

This relation holds for excess over the threshold within the interval $1 \leqslant h/h_c \leqslant 2$.

As is well know, excess of the pumping power over the threshold value leads to establishment of a stationary state in the magnon system. One of the basic problems that arise in the study of this state is explanation of the mechanism that limits the number or the amplitude of the parametric magnons.[17] Two limitation mechanisms are usually considered: a) the mechanism of nonlinear attenuation and b) the phase mechanism. As was shown in Ref. 17, each of the mechanisms is characterized by its own relation between the number of parametric magnons N_k and the value of the pumping field h:

$$\text{a)}\ N_k = \frac{V}{\eta}(h - h_c) = B\left(\frac{h}{h_c} - 1\right), \tag{6}$$

$$\text{b)}\ N_k = \frac{\gamma V}{S}\left[\left(\frac{h}{h_c}\right)^2 - 1\right]^{1/2} = C\left[\left(\frac{h}{h_c}\right)^2 - 1\right]^{1/2}. \tag{7}$$

Here V is the coefficient of coupling of the microwave magnetic field with the spin wave, η is the nonlinear attenuation, S is the amplitude of the four-particle process, and γ is the attenuation.

We have undertaken to compare the experimentally obtained result (5) with the theory (6)–(7). Figure 6 shows the dependence of the number of particles N_k (in the relative units) on the value of the excess of the microwave field above the threshold value, h/h_c. The coefficients A, B, and C were so chosen that the value of N_k at $h/h_c = 1.65$ agreed for all three curves. The experimental points are also plotted in Fig. 6. It is evident that Curve 1 [formula (6)] describes the experimental data better than does Curve 3 [formula (7)]. But the large scatter of the experimental data prevents an unambiguous choice between the relations (5) and (6). Further increase of the pumping power ($h/h_c \geqslant 2$) leads to saturation of the $N_k(P)$ curve and then to a drop in the number of magnons. Such behavior may be due to additional superheating of the specimen by the microwave power. Estimates made show that when the specimen absorbs power ~50 mW, it becomes overheated with respect to the helium bath by $\Delta T \sim 2$K. Rise of the specimen temperature leads to decrease of the value of H_D and consequently to increase of h_c. In consequence of this, the number of magnons produced at a given level of pumping drops. An increase of h_c with temperature may also be caused by an increase of the relaxation $\Delta\nu_k$. But all these questions require further investigations.

2. Parametric phonons

Here we shall briefly discuss results obtained in an investigation of excited y particles.[18]

When AFMR was excited in the specimen at frequency $\nu_{AFMR} = 36.2$ GHz, in the spectrum of light scattering by y particles, along with thermal magnons (Fig. 7, 1) whose frequency is in agreement with the y-magnon spectrum (in Fig. 7, $\nu_1 = 44.4$ GHz), additional peaks were detected at frequency $\nu_2 = \nu_{AFMR/2} = 18.1$ CHz. The polarization conditions for observation of these peaks correspond to the conditions for observation of transverse phonons. Their frequency is close to the frequen-

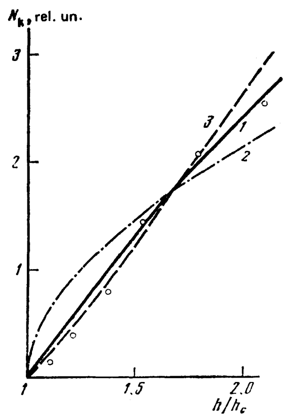

FIG. 6. Dependence of the number of parametric magnons N_k on the value of the excess of the microwave field above the threshold value, h/h_c. Curve 1 is plotted from formula (6), Curve 2 from formula (7), Curve 3 from formula (5). The experimental points correspond to Fig. 4.

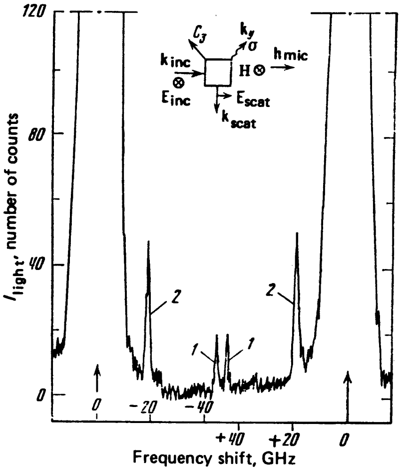

FIG. 7. Spectrum of scattering of light ($\lambda = 632.8$ nm) at 90° in $CoCO_3$ ($T \leq 2K$) by thermal y magnons (1) and by parametric y phonons (2), excited by microwave power with $\nu_p = 36.2$ GHz under AFMR conditions.

cy of transverse y phonons, measured at room temperature, which is $\nu_{ph} = 16.9$ GHz.[13] All these facts provide a basis for supposing that in this experiment we are observing disintegration of a uniform-precession magnon, with $k = 0$, into two parametric phonons with a frequency equal to half the AFMR frequency and with $k = \pm 2.5 \cdot 10^5$ cm^{-1}. A schematic representation of the disintegration is shown in Fig. 8. Represented here are the low-frequency part of the magnon spectrum, in a magnetic field corresponding to the conditions of the experiment (Fig. 7), and the spectrum of transverse y phonons at room temperature. Point 1 in this sketch corresponds to peak 1 in Fig. 7, point 2 to peak 2. The disintegration process indicated above is connected with saturation of the resonance and should be of threshold type. Experimentally, we observed saturation of the resonance absorption line at the pumping power used in the scattering experiment (Fig. 7).

A similar result was recently obtained by Wettling and Jantz[11] in antiferromagnetic $FeBO_3$, for quasiparticles propagating within a wide cone of directions about the z axis, at room temperature. The intensity of the phonon lines observed in this work was very large. The small intensity of the y phonons excited in our experiments is apparently due to the fact that our experiments were done at $T \leq 2$ K. Comparison of the intensities of the light scattering by thermal y phonons at $T \sim 300$ K and by parametric y phonons at $T \leq 2$K (Fig. 7) enables us to

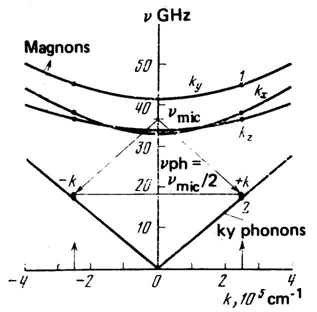

FIG. 8. Sketch illustrating parametric excitation of y phonons in $CoCO_3$ under AFMR conditions at $\nu_{AFMR} = 36$ GHz.

state that in the pumping experiment, we heat the phonons from 2 to ~500 K. We propose hereafter to study this phenomenon in more detail. The type of instability described above is possible only in the presence of magnetoelastic interaction in the crystal. The existence of the latter in $CoCO_3$ is substantiated also by our experiments on parametric excitation of x phonons at pumping frequency $\nu_p \sim 36$ GHz.[9]

CONCLUSION

We shall state briefly the principal results of the research.

1. The method of Mandel'shtam-Brillouin light scattering has been used to observe and to investigate in detail parametrically excited spin waves in $CoCO_3$ ($T \leq 2$ K).

2. Excitation thresholds of parametric magnons have been determined: at $\nu_p = 51.02$ GHz, $P_c = 80$ mW for x particles and $P_c = 60$ mW for z particles. For x particles, the relaxation time is estimated as $\tau \sim 0.2 \mu$sec and the intrinsic linewidth as $\Delta\nu_k \approx 0.7$ MHz.

3. An estimate of the maximum number of parametric particles observed in the experiment gives the value ~10^{13}–10^{14} cm^{-3} over an interval $\Delta k \sim 20$ cm^{-1}, when the pumping power exceeds the threshold value by a factor 3.

4. It is concluded that at a sufficiently high level of pumping, parametric magnons are propagated in all directions in the crystal.

5. It is shown that in the beyond-threshold state, the number of parametric magnons with equal but oppositely directed wave vectors are the same.

6. Disintegration of a single uniform-precession ($k = 0$) magnon into two parametric phonons has been detected experimentally.

7. The results obtained show that the MBS light method that we used makes it possible to obtain substantially new information in the investigation of parametrically excited quasiparticles.

In conclusion, we express our thanks to P. L. Kapitza for his interest in the research. We sincerely thank A. S. Borovik-Romanov for constant attention to the work and for helpful discussions. We thank V. I. Ozhogin, L. A. Prozorova, S. I. Smirnov, and V. S. Lutovinov for useful discussions and S. M. Elagin for help in carrying out the experiment.

1) The authors sincerely thank N. Yu. Ikornikova and V. M. Egorov for providing the $CoCO_3$ specimens.

2) For $n_0 = 2$, the intensities of the Stokes ($\sim n_0$) and anti-Stokes ($\sim n_0 + 1$) components in the spectrum of scattering by thermal magnons should differ by 30%. In the experiment we do not see this difference (Fig. 2). This fact can be explained by supposing that the temperature in the spot of the crystal ~0.1 mm^2 from which the scattered radiation comes is a few degrees above the temperature of the bath. Thus for $T = 6K$, $n_0 = 6$, and the intensities of the components differ by 15%. This difference lies beyond the limit of experimental error. Heating can occur because of focusing of the laser

beam. Verification of the presence of heating by direct experiment is difficult.

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Translated by W. F. Brown, Jr.

On the theory of electrons localized in the field of defects

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(Submitted 2 July 1979)
Zh. Eksp. Teor. Fiz. 77, 2498–2517 (December 1979)

We formulate a mathematical criterion for the localization of single-particle wave functions in a random field. The localization is characterized by the appearance of singular terms in the expressions for the correlation functions of various physical quantities as functions as the energy (independent of the dimensionality of the problem). We propose for one-dimensional problems a method which, in principle, allows one to evaluate directly correlators which contain products of several exact wave functions of electrons in a random field, pertaining to different energies and taken at different points.

PACS numbers: 71.70.Ch

The present paper does not pretend to give new physical results, but has as its aim the solution of some methodological problems. We discuss in section 1 the problem of particle localization in the random field of randomly distributed defects and we propose a mathematical formulation of a criterion for the localization of the wave functions of eigenstates. The applicability of the results of this section is not limited to the case of one-dimensional problems. The criterion formulated there refers equally well to two- and three-dimensional cases.

The second part of the paper refers solely to a one-dimensional conductor. It contains a general method which allows us—at least in principle—to write down and evaluate an arbitrary correlator of the product of any number of exact wave functions with different energies of an electron in the random field of static defects, taken at different points in space.

The necessity to evaluate such correlators arises in a whole set of problems, such as, for instance, the determination of the temperature dependence of the conductivity. It is well known that the localization of electrons in the field of defects is complete in the sense that all states with arbitrary energies are localized.[1] The static conductivity therefore vanishes at zero temperature. Mott and Twose[2] were the first to note this fact and later on this result was obtained exactly by one of the present authors.[3] At non-zero temperatures the conductivity behaves like that of a semiconductor. The finite magnitude of the conductivity is caused by jumps of the electrons along the localized states due to inelastic interaction processes between them and the thermal phonons or between themselves. This effect was considered in Ref. 4 for the electron-phonon inter-

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