

- ³⁶J. M. Ziman, *Electrons and Phonons*, Oxford, 1960.
³⁷S. V. Maleev and B. P. Toperverg, *Zh. Eksp. Teor. Fiz.* 69, 1440 (1975) [*Sov. Phys. JETP* 42, 734 (1975)].
³⁸B. J. Alder and T. I. Wainwright, *Phys. Rev. Lett.* 18, 988 (1967).

- ³⁹Y. Pomiay and P. Resibois, *Phys. Rep.* 19C, 64 (1975).
⁴⁰M. A. Ernst and J. R. Jorfman, *J. Stat. Phys.* 12, 311 (1975).

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Static skin effect in metals with open Fermi surfaces

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Galvanomagnetic effects in thin metallic conductors with open Fermi surfaces are investigated. It is shown that in strong magnetic fields the direct current is concentrated near the conductor surface and the magnetoresistance is sensitive to the character of the conduction-electron reflection by the sample boundary. For specular reflection of the carriers by the sample surface, the skin effect is produced in a plate of thickness $d < l$ in a magnetic field parallel to the surface if the magnetic field satisfies the condition $r < \delta d$; for diffuse reflection the condition is $r < l\delta$ (l and r are the electron mean free path and trajectory curvature radius in a magnetic field H , and δ is the thickness of the layer of open electron orbits and is referred to the Fermi momentum). Under static skin-effect conditions the electric current is carried mainly by electrons that glide along the sample surface and belong to closed sections of the Fermi surface. Electrons with open orbits participate only in the formation of Sondheimer magnetoresistance oscillations that appear in specular reflection even in plates whose surfaces are symmetry planes of the crystal. In the case of a single group of carriers the transverse resistance ρ_{\perp} grows linearly with the magnetic field H in the case of specular reflection, and $\rho_{\perp} \sim H^2$ in diffuse reflection. In metals with several carrier groups and in magnetic fields for which $r \ll d$, saturation occurs in one and the same sample and the magnetoresistance grows quadratically or linearly with transverse-magnetoresistance field. From the dependence of ρ_{\perp} on H it is possible to determine the degree of imperfection of the crystal surface and the probability of charge recombination on the sample surface. The electroneutrality equation is analyzed for an arbitrary scattering indicatrix of the carriers by the sample boundary. It is shown that the electric field in the sample is appreciably inhomogeneous if the electron reflection is not specular.

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The unrestricted growth of the resistance of a number of metals with increasing field in strong magnetic fields is accompanied by a substantial redistribution of the electric current over the sample cross section. This is caused by the special role played by collisions of the electrons with the sample boundary, which are as a rule accompanied by a jump of the center of the orbit, leading to an increase of the mobility of the electrons near the surface over the mobility of the electrons moving in the center of the sample. For example, in a plate with specularly reflecting faces placed in a magnetic field H parallel to its surface, the effective mean free path of the near-surface electrons over the entire plane of the plate is comparable with the mean free path l for collisions inside the volume. At the same time, the electrons that do not collide with the sample boundary can drift only along the magnetic field. The result is a substantial difference between the contributions of the surface and interior electrons to the transverse electric conductivity (electric current density $\mathbf{j} \perp \mathbf{H}$), and the direct electric current can become concentrated near the surface of the sample in magnetic fields for which the electron-trajectory curvature radius $r \ll l$.

The static skin effect was predicted by Azbel¹ in

1962. He considered the electric conductivity of metallic plates of thickness d , whose surfaces reflect the conduction electrons diffusely, and showed that in a magnetic field parallel to the sample surface of the electric current flows mainly in a surface layer of thickness $\sim r$, and the transverse resistivity ρ_{\perp} increases linearly with the magnetic field at $r \ll l^2/d$, if the numbers of the electrons and holes n_1 and n_2 are equal. At $r > l^2/d$ the resistivity is proportional to H^2 , just as in bulky samples, in which surface effects are negligible. In uncompensated metals ($n_1 \neq n_2$) the electric conductivity of the core of the sample is high enough at $r \ll l$ (of the order of the electric conductivity σ_0 of the metal in the absence of the magnetic field) and the distribution of the electric field over the sample cross section remains practically unchanged when the magnetic field is increased.

The theory of the static skin effect was subsequently developed by Azbel' with one of us.²⁻⁴ It turns out that in a magnetic field inclined to the plate surface the static skin effect does not lead to new dependences of the resistivity ρ on the strong magnetic field compared with bulky samples. This makes it possible to use, in addition to bulky material, also thin conductors for the

investigation of the electron energy spectrum with the aid of galvanomagnetic measurements.⁵⁻⁷

In a magnetic field parallel to the surface of the plate and in compensated metals ($n_1=n_2$) the surface current is sensitive to the character of the reflection of the electrons by the sample boundary, i.e., to the state of the sample surface. The infinite character of the motion of the electrons specularly reflected by the sample boundary along the direction of the transverse current makes the conductivity of a surface layer of thickness $2r$ approximately equal to σ_0 . In the case of pure diffuse reflection, when all the states are equally probable for the reflected electrons, the electric conductivity of the same layer is smaller by a factor $1/r$ and the resistivity of the plate increases quadratically with the magnetic field ($\rho \sim H^2$). If the reflections do not completely diffuse and some correlation remains between the incident electrons and those reflected by the sample boundary, then the dependence of ρ on H has a complicated character.⁴

In semimetals, the linear growth of the plate resistance in the case of diffuse reflection of the carriers, obtained by Azbel,¹ is realized, but in a magnetic-field region somewhat different than predicted. The static skin effect in semimetals, which was investigated in detail by Babkin and Kravchenko,⁸ was quite unique because of the weak interaction between carriers belonging to different groups (valleys). If it is assumed that there is no interaction between the valleys and that the electron and hole states do not become intermixed upon reflection of the carriers, i.e., this reflection remains specular with respect to the number of the group, then regardless of the character of the intravalley scattering the electric conductivity of a surface layer of thickness $2r$ on the surface boundary turns out to be the same in order of magnitude. For example, in diffuse intravalley scattering, as shown by Babkin and Kravchenko, the electric-current density in the immediate vicinity of the sample boundary $\xi_z=0$ or d (the ξ axis is normal to the surface of the plate), just as in the case of completely diffuse reflection, turns out to be smaller by a factor $1/r$ than at $H=0$. However, with increasing distance from the boundary, the electric conductivity increases sharply and becomes comparable with σ_0 at $\xi \approx r$ and $d - \xi \approx r$. The relations between the numbers of the electrons and holes are in this case not so critical for the value of the surface current.

The electric conductivity in the core of the sample, on the contrary, is substantially different at different degrees of decompensation of the electrons and holes $\Delta n/n = (n_1 - n_2)/(n_1 + n_2)$. The electric conductivity of bulky samples is of the order of $\sigma_0(r/l)^2$ if $\Delta n/n \ll r/l \ll 1$ and of the order of σ_0 at $r/l \ll \Delta n/n$. In the latter case $\rho \approx \sigma_0^{-1}$ and the surface current introduces a small resistance increment proportional to $1/H$. At $\Delta n/n \ll r/l \ll 1$ one should expect a linear increase of the resistance with the magnetic field for any intravalley scattering. However, when the correlation between the incident and reflected carriers is violated, i.e., when the reflection is not purely specular, redistribution of the charges takes place in the conductor. An excess of

electrons accumulates on one of the boundaries and an excess of holes on the other.⁹

Babkin and Kravchenko⁸ considered the influence of intervalley scattering, and of recombination of charges inside the conductor and on its surface, on the distribution of the electric current over the sample cross section in strong magnetic fields, and have shown that the decompensation of the electrons and holes is so large that the resistance of a semimetallic plate whose thickness d is less than the intervalley diffusion length is independent of H in the entire range of magnetic fields satisfying the condition $r \ll l_1$ (l_1 is the carrier mean free path for intravalley scattering in the core of the sample). In sufficiently thick plates, whose thickness d exceeds the intervalley diffusion length L ($d \gg L$), the resistance increases quadratically with the magnetic field if $r < l(L/d)^{1/2}$, and a linear dependence of ρ on H should be observed at $l(L/d)^{1/2} < r \ll l$. The character of the reflection of the electrons by the sample boundary inside the valley likewise does not influence the dependence of the resistance on the magnetic field.

Metals are characterized by a substantial mixing of the electron and hole states in the case of diffuse scattering of the carriers by the sample boundary, and their mean free paths relative to intervalley and intravalley scattering are of the same order of magnitude.

In compensated metals, recombination of the carriers upon reflection from the sample boundary weakens the surface current, since the electron and hole drifts due to collisions with the metal surface have opposite signs. Their effective range in the direction of the transverse current, with account taken of recombination, is less than l even in the case of an ideally smooth sample surface. An experimental investigation of the dependence of ρ on H makes it possible to determine only the sum of the probabilities of the diffuse reflection (the characteristic of the sample surface) and the probability of charge recombination upon reflection. Panchenko, Karamov, and Ptushinskii¹⁰ have shown that the probability of charge recombination on the boundary of a metallic sample may turn out to be large enough and the plate resistance is not very sensitive to the state of the plate surface. They investigated galvanomagnetic effects in tungsten plates with atomically smooth surfaces and observed that the static skin effect manifests itself most strongly in those crystallographic orientations of the sample at which the transitions of the carriers after specular reflection from the electron state to the hole state and vice versa are hindered. Only in this case it is possible to determine from the dependence of ρ on H the degree of imperfection of the surface of the investigated crystal.

In the studies cited above, the role of the mechanism of the carrier interaction with the sample boundary was investigated for metals with closed Fermi surfaces. From the point of view of the theory of the static skin effect this is undoubtedly the most interesting case, since by specular reflections from the conductor surface it is possible to produce artificially in the surface layer open electron orbits, which alter substantially the asymptotic value of the resistance in a parallel magne-

tic field if $d < l^2/r$.

No less interesting, however, is an investigation of the role of electrons that glide along the sample surface in metals with open Fermi surfaces. In bulky samples, the asymptotic behavior of the resistance changes substantially if, at a given direction of the magnetic field, a small layer of open sections of the Fermi surface is produced, with thickness δ , referred to the Fermi momentum, exceeding $(\tau/l)^2$. In conductors with small dimensions the static skin effect can substantially interchange the roles of open and closed sections of the Fermi surface.

We consider below the role of open electron orbits in the electric conductivity of thin conductors at an arbitrary character of the reflection of the electrons from the sample boundary. For the sake of argument, we assume that the conductor is a plane-parallel plate whose thickness d is small compared with l , and that the current contacts are located on the end faces of the plate.

The complete system of equations of the problem consists of the Boltzmann kinetic equation for the electron distribution function $f(\mathbf{r}, \mathbf{p})$ and of Maxwell's equations, which in static fields reduce to the condition that the conductor be electrically neutral:

$$\int \{f(\mathbf{r}, \mathbf{p}) - f_0(\varepsilon)\} \frac{2d\mathbf{p}}{(2\pi\hbar)^3} = 0, \quad (1)$$

where $f_0(\varepsilon)$ is the equilibrium Fermi distribution function of the electrons, $\varepsilon(\mathbf{p})$ is the energy and \mathbf{p} the momentum of the electron, and \hbar is Planck's constant. We confine ourselves to the Ohm's-law approximation and seek the electron distribution function in the form

$$f(\mathbf{r}, \mathbf{p}) = f_0(\varepsilon) - \psi \partial f_0 / \partial \varepsilon. \quad (2)$$

The Boltzmann kinetic equation linearized over the weak electric field, in the τ approximation,

$$\frac{\partial \psi}{\partial t} + \mathbf{v} \frac{\partial \psi}{\partial \mathbf{r}} + \frac{\psi}{\tau} = e \mathbf{E}(\mathbf{r}) \mathbf{v}(t) \quad (3)$$

can be easily solved by the method of characteristics. Here t is the time of motion of the electron in the magnetic field, e and \mathbf{v} are the charge and velocity of the electron, and $\mathbf{E}(\mathbf{r})$ is the electric field intensity. A solution of the homogeneous equation corresponding to (3) must be chosen by using a boundary condition that relates the functions ψ for the incident electrons and those reflected by the sample boundary

$$\psi(\mathbf{r}_s, \mathbf{p}) = \int d\mathbf{p}' w(\mathbf{p}, \mathbf{p}'; \mathbf{r}_s) \psi(\mathbf{r}_s, \mathbf{p}'). \quad (4)$$

The integration in the right-hand side of (4) is over all the states of the electrons incident on the boundary of the sample at the point \mathbf{r}_s .

Electron scattering by not too rough a surface or by a surface containing defects was theoretically investigated by many workers, who found the connection between the kernel $w(\mathbf{p}, \mathbf{p}'; \mathbf{r}_s)$ and the properties of the surface. For example, it follows from the papers of Fal'kovskii¹¹ and of Okulov and Ustinov¹² that w is of the form

$$w(\mathbf{p}, \mathbf{p}'; \mathbf{r}_s) = \delta(\mathbf{p} - \tilde{\mathbf{p}}) \left[1 - \int d\mathbf{p}'' W(\mathbf{p}, \mathbf{p}''; \mathbf{r}_s) \right] + W(\mathbf{p}, \mathbf{p}'; \mathbf{r}_s), \quad (5)$$

where the function $W(\mathbf{p}, \mathbf{p}'; \mathbf{r}_s)$ is the probability of elec-

tron scattering from a state with momentum \mathbf{p}' into a state with momentum \mathbf{p} , while $\tilde{\mathbf{p}}$ is the momentum of the electron incident on the surface and reflected specularly with momentum \mathbf{p} .

Using the boundary condition (4) we obtain for the following expression (see, e.g., Refs. 13 and 14)

$$\begin{aligned} \psi(\mathbf{r}, \mathbf{p}) = & \int_{\lambda_1}^t dt' \exp\left(\frac{t'-t}{\tau}\right) \mathbf{v}(t') \mathbf{E}[\mathbf{r}_1 - \mathbf{r}(\lambda_1) + \mathbf{r}(t')] \\ & + \int d\mathbf{p}' w(\mathbf{p}, \mathbf{p}'; \mathbf{r}_1) \int_{\lambda_1}^{\lambda_2} dt' \exp\left(\frac{t'-t}{\tau}\right) \mathbf{v}(t') \mathbf{E}[\mathbf{r}_2 - \mathbf{r}(\lambda_2) + \mathbf{r}(t')] \\ & + \int d\mathbf{p}' w(\mathbf{p}, \mathbf{p}'; \mathbf{r}_1) \int w(\mathbf{p}_2, \mathbf{p}''; \mathbf{r}_2) d\mathbf{p}'' \int_{\lambda_2}^{\lambda_3} dt' \exp\left(\frac{t'-t}{\tau}\right) \\ & \times \mathbf{v}(t') \mathbf{E}[\mathbf{r}_3 - \mathbf{r}(\lambda_3) + \mathbf{r}(t')] + \dots, \end{aligned} \quad (6)$$

where λ_i are the instants of reflection of the electron by the sample boundary at the points \mathbf{r}_i , $\mathbf{p}_i = \mathbf{p}(\lambda_i)$ is the momentum of the electron after reflection, and $\lambda_{i+1} < \lambda_i$, $i = 1, 2, \dots$. This solution can be obtained also by the Chambers method.¹⁵ It is easy to note that the function ψ has the meaning of the energy acquired by the electrons in the electric field and averaged over all possible electron trajectories.

Knowing the electron distribution function, we can find the distribution of the electron current in the conductor:

$$\mathbf{j}(\mathbf{r}) = - \frac{2e^2}{(2\pi\hbar)^3} \int \mathbf{v}\psi(\mathbf{r}, \mathbf{p}) \frac{\partial f_0}{\partial \varepsilon} d\mathbf{p}. \quad (7)$$

If the dimensions of the roughnesses of the surface are small compared with τ , d , and l , we can use in expression (6) for ψ an averaged function $w(\mathbf{p}, \mathbf{p}')$, which must be assumed to be the same for the entire surface of the plate. Then the problem is homogeneous in the (η, ξ) plane of the plate (the ξ axis coincides with the projection of the direction of the magnetic field, taken to be the z axis, on the plane of the plate) and the electrostatic potential can be sought in the form

$$\varphi(\mathbf{r}) = -\zeta E_\zeta - \eta E_\eta + \varphi(\xi), \quad (8)$$

where E_ζ and E_η are constant quantities.

It is necessary to obtain from the electroneutrality condition the inhomogeneous electric field $E_\zeta(\xi) = -d\varphi(\xi)/d\xi$ and to determine the connection between the density of the electric current with the electric field components in the plane of the particle E_ζ and E_η :

$$j_\alpha(\xi) = \sigma_{\alpha\beta}(\xi) E_\beta, \quad \alpha, \beta = \zeta, \eta. \quad (9)$$

The tensor $\sigma_{\alpha\beta}(\xi)$ averaged over the depth of the sample determines the electric conductivity and consequently the resistance of the plate.

We assume that the Fermi surface is a surface of revolution of the corrugated-cylinder type, whose axis coincides with the p_x axis, and whose closed flat sections correspond to the electron states. The figure shows the trajectories of the electrons in the xy plane between collisions with the sample boundaries (trajectories of type I and II) and the trajectories of the electrons that do not collide with the sample boundary (III) in the case when $\xi = y$ and $\zeta = z$. The electrons moving along trajectories of type I and II can drift along the x

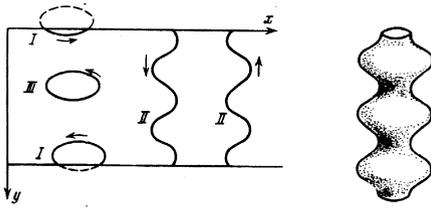


FIG. 1.

axis, and their contribution to the electric conductivity of the plate can alter substantially the asymptotic form of the resistance at $r \ll l$ compared with bulky samples.

We now obtain the distribution of the electric current and the resistance of the plate in a magnetic field parallel to the plate surface. Undoubtedly the most laborious part of the problem is the determination of the electric field $E_z(\xi)$. In the absence of open electron orbits inside the conductor far from its surface, the solution of the electroneutrality equation

$$\langle \psi(\xi, t, p_x) \rangle = 0 \quad (10)$$

is a homogeneous electric field $\varphi(\xi) = A\xi + C$, and the constant A is determined from the condition that the electric current be continuous

$$j_z(\xi) = \langle ev_z \psi(\xi, t, p_x) \rangle = j_z(\xi_s) = 0, \quad \xi_s = 0, d. \quad (11)$$

In a narrow surface layer of thickness of the order of the largest diameter of the electron orbit $2r$, however, the electric field is essentially inhomogeneous. Only in the case of a quadratic dispersion law of the carriers $\varepsilon = \alpha_{ik} p_i p_k$ and of pure specular reflection of the carriers by the sample boundary is the electric field E_z homogeneous everywhere, and the resistance of the metallic plate does not depend at all on its thickness.

The presence of open electron orbits leads to inhomogeneity of the field E_z over the entire sample. If the layer of open orbits is small, then the inhomogeneity of the field in the core of the sample is small and can be analyzed with the aid of the electroneutrality condition (10) by successive approximations. Under conditions of the static skin effect, when the electric current is concentrated near the surface of the sample, it is necessary to know the behavior of the electric field mainly at $|\xi - \xi_s| < 2r$. In this case the integral equation (10) for the electrostatic potential $\varphi(\xi)$ is quite complicated and an explicit solution for it can be obtained only in certain special cases, for example at $|\xi - \xi_s| \ll 2r$.

In the case of diffuse reflection of the conduction electrons by the sample boundary, when all the momentum directions of the reflected electrons are equally probable and their distribution function is $f_0(\varepsilon) - F^+ \partial f_0 / \partial \varepsilon$ on the surface $\xi = 0$ and $f_0(\varepsilon) - F^- \partial f_0 / \partial \varepsilon$ on the surface $\xi = d$, it depends only on the energy of the electron, Babkin and Kravechenko⁸ investigated in detail the solution of the electroneutrality equation for the case $\delta = 0$. They have shown that in the near-surface layer the electric field E_z is highly inhomogeneous and increases without limit as $\xi \rightarrow \xi_s$.

It is easy to verify that the presence of open electron orbits does not change the asymptotic behavior of $E_z(\xi)$ at $|\xi - \xi_s| \ll 2r$. The values of F^\pm , which are constant on

the Fermi surface, can be determined easily by using the condition that the charge cannot flow through the sample boundary. Substituting the expression for

$$\psi(\xi, t, p_x) = \exp\left(\frac{\lambda - t}{\tau}\right) F^\pm + \int \exp\left(\frac{t' - t}{\tau}\right) v_p(t') E_p dt' - \varphi(\xi) + \exp\left(\frac{\lambda - t}{\tau}\right) \varphi(\xi_s) + \frac{1}{\tau} \int \exp\left(\frac{t' - t}{\tau}\right) \varphi(\xi + \xi(t') - \xi(t)) dt' \quad (12)$$

in the expression Eq. (11), we get

$$F^+ = -F^- = u^{-1} \left\langle v_v(t) \int \exp\left(\frac{t' - t}{\tau}\right) v(t') E(\xi(t') - \xi(t)) dt' \right\rangle, \quad (13)$$

where

$$u = \langle v_z \rangle_- - \left\langle v_z(t) \exp\left[\frac{\lambda(0, t) - t}{\tau}\right] \right\rangle_- + \left\langle v_z(t) \exp\left[\frac{\lambda(0, t) - t}{\tau}\right] \right\rangle_+ \quad (14)$$

The angle brackets denote here integration over the Fermi surface with a corresponding weighting factor (see the definition of the current (7)), the symbols $\langle \dots \rangle_-$ and $\langle \dots \rangle_+$ denote integration over the states of the electrons with $v_z < 0$ for closed and open electron orbits $\varepsilon = \text{const}$ and $p_x = \text{const}$; the instant of reflection of the charge by the sample boundary $\lambda \equiv \lambda_1$ is the root, closest to t , of the equation

$$\xi(t, p_x) - \xi(\lambda, p_x) = \xi - \xi_s, \quad \xi_s = 0, d, \quad (15)$$

and

$$\langle f \rangle = \langle f \rangle_- + \langle f \rangle_+.$$

After eliminating the constants, the electroneutrality equation takes the form

$$\begin{aligned} \varphi(\xi) - \frac{1}{\langle 1 \rangle} \left\langle \frac{1}{\tau} \int \exp\left(\frac{t' - t}{\tau}\right) \varphi(\xi + \xi(t') - \xi(t)) dt' \right\rangle \\ - \frac{g(\xi)}{\langle 1 \rangle u} \left\langle \frac{1}{\tau} v_z(t) \int \exp\left(\frac{t' - t}{\tau}\right) \varphi(\xi(t') - \xi(t)) dt' \right\rangle = a_p(\xi) E_p; \\ a_p(\xi) = \frac{1}{\langle 1 \rangle} \left\langle \int \exp\left(\frac{t' - t}{\tau}\right) v_p(t') dt' \right\rangle \\ - \frac{g(\xi)}{\langle 1 \rangle u} \left\langle v_z(t) \int \exp\left(\frac{t' - t}{\tau}\right) v_p(t') dt' \right\rangle, \\ g(\xi) = \left\langle \exp\left[\frac{\lambda(\xi, t) - t}{\tau}\right] \right\rangle_+ - \left\langle \exp\left[\frac{\lambda(\xi, t) - t}{\tau}\right] \right\rangle_-. \end{aligned} \quad (16)$$

The superscripts + and - of the angle brackets indicate that the electron has arrived at the point (η, ξ, z) from the surface $\xi = 0$ or from the surface $\xi = d$, respectively.

It should be noted that the second term in the left-hand side of (16) receives contributions mainly from electrons that do not interact with the sample boundary, while the last term in the left-hand side, and the right-hand side of (16), are determined only by the electrons that collide with the surface of the conductor. Near the surface of the plate at $\xi \ll r$ and $(d - \xi) \ll r$ the electrons that do not collide with the sample boundary can be disregarded, and we have for $\varphi(\xi)$ with sufficient accuracy

$$\varphi(\xi) \approx a_p(\xi) E_p - B g(\xi). \quad (17)$$

Substituting (17) in (16) without the second term in the left-hand side, we easily determine the constant B . It is easy to note that

$$\varphi(\xi) - \varphi(\xi_s) \sim \left| \frac{\xi - \xi_s}{r} \right|^{1/2}. \quad (18)$$

If $|\xi - \xi_s| \ll r$, i.e., the electric field $E_\xi = -d\varphi/d\xi$ increases without limit at the surface of the sample also $\delta \neq 0$. The reason is that the function $\psi(\xi, t, p_x)$ for closed sections of the Fermi surface experiences a jump when ξ and t satisfy the conditions

$$\begin{aligned} \xi(t, p_x) - \xi_{\min}(p_x) &= \xi, \\ \xi(t, p_x) - \xi_{\max}(p_x) &= \xi - d. \end{aligned} \quad (19)$$

It is easy to verify that even at an arbitrary character of the carrier reflection from the sample boundary the solution of the electroneutrality equation

$$\begin{aligned} \varphi(\xi) \langle 1 \rangle &= \left\langle \frac{1}{\tau} \int_{\lambda_1}^t \exp\left(\frac{t'-t}{\tau}\right) \varphi(\xi + \xi(t') - \xi(t)) dt' \right\rangle \\ &- \left\langle \int dp' w(p', p_x) \frac{1}{\tau} \int_{\lambda_2}^{\lambda_1} \exp\left(\frac{t'-t}{\tau}\right) \varphi(\xi_i + \xi(t') - \xi(\lambda_i)) dt' \right\rangle \\ &- \left\langle \int dp' \int dp'' w(p', p_x) w(p'', p_x) \frac{1}{\tau} \int_{\lambda_3}^{\lambda_2} \exp\left(\frac{t'-t}{\tau}\right) \varphi(\xi_2 + \xi(t') \right. \\ &\quad \left. - \xi(\lambda_2)) dt' \right\rangle = \langle \psi_\alpha \rangle E_\alpha, \quad \alpha = \eta, \zeta, \end{aligned} \quad (20)$$

at $|\xi - \xi_s| \ll 2r$ will have a square-root dependence on $|\xi - \xi_s|$ if the reflection is not specular. Here $\psi_\alpha E_\alpha$ coincides formally with the function ψ if we put $E_\xi(\xi) = 0$ in (6).

In the Fuchs approximation of the specularly parameter the electrostatic potential $\varphi(\xi)$ takes the form

$$\varphi(\xi) = (1-q)D|\xi - \xi_s|^{1/2} + \mathcal{E}(\xi - \xi_s) + \varphi(\xi_s), \quad (21)$$

$$|\xi - \xi_s| \ll r,$$

where q is the probability of specular reflection of the electron from the sample boundary. In the case of specular reflection ($q=1$) the electric field tends to a finite limit when $\xi \rightarrow \xi_s$, but the field \mathcal{E} does not coincide with the electric field E_ξ at the center of the sample even in the absence of open electron orbits if the carrier dispersion law is anisotropic and nonquadratic. The quantity D does not depend on q and ξ .

An analysis of the electroneutrality equation at the core of the sample shows that the electric field far from the conductor boundaries undergoes almost periodic changes with a period $T\bar{v}_t$, where \bar{v}_t is the average electron velocity along the normal to the surface of the sample during the period T of its motion in the magnetic field.

We now obtain the resistance of a thin plate and the distribution of the electric current over the sample cross section, assuming that the thickness δ of the layer of open electron orbits is small.

A. Specular reflection. In specular reflection, the electrons belonging to the closed Fermi-surface sections can drift in the near-surface layer and their effective mean free path coincides with l . If the y axis coincides with the ξ axis, then the displacement of the electrons with the open orbits along the direction of the transverse current during the free-path time does not exceed rl/d , i.e., they are much less mobile than the electrons that do not leave the surface layer with thick-

ness on the order of r . In extremely strong magnetic fields ($r \rightarrow 0$) the electric current is concentrated in a narrow surface layer and the electric conductivity of the plate is determined mainly by the electrons belonging to the closed sections of the Fermi surface.

The role of the electrons with the open orbits reduces to formation of Sondheimer oscillations,¹⁶ since they do not drift along the η axis if they pass through the entire thickness d of the sample during a time that is a multiple of the period of motion T in the magnetic field. With changing magnetic field, the condition $d = nT\bar{v}_t$ will again be satisfied at a different value of H , and this in fact is the cause of the oscillatory dependence of the resistance on H . Since all the electrons with open orbits shift into the interior of the sample during the period T by an equal distance, then there are no preferred electrons among them and all contribute to the oscillatory effect. The possibility of Sondheimer oscillations in a parallel magnetic field was pointed out to Pippard.¹⁷ In many theoretical papers, including Pippard's, it is customarily assumed that the Sondheimer oscillations are due to the dissipative character of the collisions of the electrons with the sample boundary. This effect, however, can occur also in pure specular reflection,¹⁸⁻²⁰ as will be shown below.

In the case when the axes ξ and y coincide, the motion of the electrons specularly reflected from the sample boundary is strictly periodic (see the figure):

$$\begin{aligned} \psi(y, t, p_x) &= \int_{\lambda_1}^t \exp\left(\frac{t'-t}{\tau}\right) v(t') E(y+y(t') - y(t)) dt' \\ &+ \left[1 - \exp\left(-\frac{2T\lambda}{\tau}\right) \right]^{-1} \left[\int_{\lambda_1}^{\lambda_2} \exp\left(\frac{t'-t}{\tau}\right) v(t'+t_1) E(y_1+y(t') - y(\lambda_1)) dt' \right. \\ &\quad \left. + \int_{\lambda_2}^{\lambda_3} \exp\left(\frac{t'-t}{\tau}\right) v(t'+t_2) E(y_2+y(t') - y(\lambda_2)) dt' \right], \end{aligned} \quad (22)$$

where $2T\lambda = \lambda_1 - \lambda_3$ is the period of the motion of the charges on the open orbits and double the period for electrons belonging to closed sections of the Fermi surface; Ωt_1 and Ωt_2 are the phase discontinuities occurring upon reflection of the electron. For electrons that do not collide with the sample boundary, ψ is the first term in (22), in which we must put $\lambda_1 = -\infty$.

Electrons that move on open orbits retain upon reflection their velocity along the surface of the plate, while v_y reverses sign. The phase t of the electron changes to $T-t$, so that

$$v_x'(t) = v_x(T-t), \quad v_y'(t) = -v_y(T-t), \quad \bar{v}_y = cb/eHT > 0, \quad \bar{v}_y' = -\bar{v}_y, \quad (23)$$

where b is the period of the open orbit in momentum space:

$$b = p_x(T+t) - p_x(t). \quad (24)$$

By virtue of the symmetry of the Fermi surface, the time of motion of the charge from one sample boundary to the other does not depend on the direction of motion and is equal to

$$T_\lambda = \frac{c}{eH} \int_{p_1}^{p_2} \frac{dp_x}{v_y}, \quad p_1 = p_x(\lambda), \quad p_2 = p_1 + \frac{eHd}{c}. \quad (25)$$

Taking (23) into account, as well as the periodicity,

with period $\lambda_1 - \lambda_2$, of the motion of the charges belonging to closed sections of the Fermi surface, we obtain the following expression for ψ in the case of closed and open electron orbits

$$\begin{aligned} \psi(y, t, p_z) &= \int_{\lambda_1}^t \exp\left(-\frac{t'-t}{\tau}\right) v(t') E(y+y(t')-y(t)) dt' \\ &+ \int_{\lambda_2}^{\lambda_1} \exp\left(\frac{t'-t}{\tau}\right) v(t'+\lambda_1-\lambda_2) E(y_1+y(t')-y(\lambda_1)) dt' \left[1-\exp\left(-\frac{\lambda_2-\lambda_1}{\tau}\right)\right]^{-1} \\ &\times E_1(y_1+y(t')-y(\lambda_1)) dt' + \int_{\lambda_2}^{\lambda_1} \exp\left(\frac{t'-t}{\tau}\right) v(2T_\lambda+t') E(y_2+y(t')-y(\lambda_2)) dt' \}. \end{aligned} \quad (26)$$

$$\begin{aligned} \psi(y, t, p_z) &= \int_{\lambda_1}^t \exp\left(-\frac{t'-t}{\tau}\right) v(t') E(y+y(t')-y(t)) dt' \\ &+ \left[1-\exp\left(-\frac{2T_\lambda}{\tau}\right)\right]^{-1} \left\{ \int_{\lambda_2}^{\lambda_1} \exp\left(-\frac{t'-t}{\tau}\right) S_x v_x(2\lambda_1-t') \right. \\ &\times E_1(y_1+y(t')-y(\lambda_1)) dt' + \int_{\lambda_2}^{\lambda_1} \exp\left(-\frac{t'-t}{\tau}\right) v(2T_\lambda+t') E(y_2+y(t')-y(\lambda_2)) dt' \}. \end{aligned} \quad (27)$$

In (27), the electron velocities in all the integrals are reduced to the values on one edge of the open section of the Fermi surface, $S_x = 1$, $S_y = -1$.

With the aid of (26), (27), and the electroneutrality condition we obtain the distribution of electric current in the plate and determine its resistance. In the calculation of the magnetoresistance it is useful to substitute in the expression for the electric current the function ψ averaged over the depth of the sample. We can then replace the integration with respect to y to integration with respect to λ_1 , with the aid of the relation

$$dy = -v_y(\lambda_1) d\lambda_1.$$

Further calculations entail no special difficulty if the electron velocity is expanded in a Fourier series.

$$v_x(t) = \sum_k v_k^x e^{i k t}. \quad (28)$$

To determine the correct order of magnitude of that part of the plate resistance which varies smoothly with H , the approximation of the homogeneous electric field is perfectly satisfactory. Using expressions (26) and (17) for the function ψ , we can easily determine the transverse electric conductivity of the plate:

$$\sigma_{\perp} = \sigma_{nn} - \sigma_{nt} \sigma_{tn} / \sigma_{tt}. \quad (29)$$

Omitting numerical factors of the order of unity, which are not reliable in this approximation, we obtain for the transverse resistivity ρ_{\perp} the expression

$$\rho_{\perp} = \sigma_{\perp}^{-1} = \rho_0 \frac{1+\delta(d/r)^2}{1+\delta d/r}, \quad (30)$$

where $\rho_0 = \sigma_0^{-1}$ is the resistivity of the plate in the absence of a magnetic field.

In magnetic fields satisfying the condition $(r/d)^2 \approx \delta$, the saturation of the resistivity gives way to a quadratic growth, but the distribution of the electric current sample cross section is practically homogeneous. The role of the open electron orbits is quite appreciable so long as $\delta d < r < \delta^{1/2} d$, and then, with increasing magnetic field, the current gets crowded into the skin layer, and the electrons with open orbits take practically no part in the electric conductivity of the plate:

$$\rho = \begin{cases} \rho_0, & \delta^{1/2} d < r < d \\ \rho_0 \delta (d/r)^2, & \delta d < r < \delta^{1/2} d. \\ \rho_0 d/r, & r < \delta d \end{cases} \quad (31)$$

Under the conditions of the static skin effect, the Hall field

$$\frac{\varphi(0) - \varphi(d)}{d} = \rho_0 j \frac{l}{r} \left(1 + \delta \frac{d}{r}\right)^{-1} \quad (32)$$

does not depend at all on the strong magnetic field ($r \ll d\delta$). If there are several groups of carriers (for example, the Fermi surface is made up of a corrugated cylinder and a closed hole surface), then, under the condition that there is no recombination of the charges upon specular reflection, we obtain for the magnetoresistance

$$\rho_{\perp} = \rho_0 \frac{1+\delta(d/r)^2}{\Delta n/n + r/d + \delta d/r}. \quad (33)$$

If the deviation $\Delta n/n$ from the compensation of the electron and hole volumes, due for example to the presence of impurities in the metal, is small compared with $\delta^{1/4}$, then the resistance of the plate increases linearly with the magnetic field and the electric current is confined to the skin layer in the entire region of magnetic fields satisfying the condition $r \ll d$. At $\Delta n/n \gg \delta^{1/4}$ saturation of the resistivity is possible, which gives way to a quadratic growth, after which ρ_{\perp} again increases linearly with the magnetic field:

$$\rho_{\perp} = \begin{cases} \rho_0 d/r, & (\Delta n/n)^2 < r/d < 1 \\ \rho_0 (n/\Delta n)^2, & \delta^{1/2} < r/d < (\Delta n/n)^2 \\ \rho_0 (n/\Delta n)^2 \delta (d/r)^2, & \delta (n/\Delta n)^2 < r/d < \delta^{1/2} \\ \rho_0 d/r, & r/d < \delta (n/\Delta n)^2 \end{cases} \quad (34)$$

If the direction of the electron drift into the interior of the sample deviates from the normal to the surface of the plate by an angle ϑ , then Eqs. (30)–(33) turn out to be valid for the transverse resistivity, and ρ_{\perp} is independent of ϑ so long as $l \cos \vartheta > d$, even though the electric conductivity tensor components, naturally, depend strongly on the direction of the electron drift. For example,

$$\sigma_{nn} = \sigma_0 \{ r/d + \delta (d/l)^2 \operatorname{tg}^2 \vartheta + \delta (r/d) \Phi \sin^2 \vartheta + \delta (r^2/d) \Phi \}. \quad (35)$$

The last two terms in (35) oscillate with the magnetic field:

$$\Phi = 1 - e^{-d/l} \cos(d/r). \quad (36)$$

For the sake of brevity, only the first harmonics are written out in the oscillating terms.

The homogeneous-field approximation is too crude for a determination of the oscillatory dependence of ρ on H . Even under conditions of the static skin effect, when the part of the plate electric conductivity that varies smoothly with the magnetic field is determined mainly by the component σ_{nn} , the part of the second term in (29) oscillates with H and turns out to be larger than in σ_{nn} , or else of the same order. Therefore allowance for the inhomogeneous electric field $E_z(\xi)$ is very important. A specific case is when the closed orbits are ellipses, and the transition region to the open orbits, where the shape of the trajectories is far from elliptic, is of the order of δ . As $\delta \rightarrow 0$ the inhomogeneity of the electric field is small in the entire sample, and putting $E_z = \Omega \tau E_n$ we obtain for the oscillating part of the resis-

tivity ρ^{osc} the expression

$$\frac{\rho^{\text{osc}}}{\rho^{\text{mon}}} \approx \delta \frac{l}{d} \Phi. \quad (37)$$

Simple estimates show that allowance for the weak inhomogeneity of the field is of no importance for the calculation of ρ^{osc} , at least so long as $\delta \lesssim r^2/l$.

B. Diffuse reflection. In the case of diffuse reflection of the carriers, the concentration of the electric current in the skin does not change the dependence of the resistivity ρ_{\perp} on the magnetic field H . The transverse electric conductivity

$$\sigma_{\perp} = \sigma_0 \left\{ \frac{r^2/l}{r/l + \delta} + \frac{1}{1 + \delta(l/r^2)} \right\} \quad (38)$$

either saturates or is proportional to r^2 :

$$\rho_{\perp} = \begin{cases} \rho_0, & r \gg (\delta l d)^{1/2} \\ \rho_0 (l d / r^2) \delta, & r \ll (\delta l d)^{1/2} \end{cases} \quad (39)$$

At $r > l\delta$ the distribution of the electric current is homogeneous, and the quadratic growth of the resistivity with the magnetic field is due to the substantial contribution made to σ_{\perp} by the electrons with open orbits. At $r \ll l\delta$ the resistivity also increases quadratically with the field and does not depend on the mean free path, but the electric current is carried mainly by the electrons belonging to the closed sections of the Fermi surface and moving near the sample boundary. The concentration of the direct current in the skin layer at $r < l\delta$ is not accompanied by a change of the amplitude of the Sondheimer oscillations, which take the form

$$\frac{\rho^{\text{osc}}}{\rho^{\text{mon}}} = \begin{cases} \delta(l/d) \sin \theta + d^2/r l \sin(d/r), & r > (\delta l d)^{1/2} \\ \delta(l/d) \sin \theta + r/l \sin(d/r), & r < (\delta l d)^{1/2} \end{cases} \quad (40)$$

An investigation of the smooth and oscillatory dependences of the resistivity on the strong magnetic field does not entail any special difficulty also if the scattering indicatrix is arbitrary. If it is assumed that the reflection of the electrons by the sample boundary is elastic and the reflection changes only the value of p_x and the phase on the electron orbit, then to analyze $\rho^{\text{mon}}(H)$ and $\rho^{\text{osc}}(H)$ it is advisable to represent the scattering indicatrix of the electrons with Fermi energy in the form of a Fourier series:

$$w(p_{sj}, \alpha_j; p_{sj-1}, \Omega_{j-1}(\lambda_j - \lambda_{j-1})) = \sum_{n,m} w_{nm}(p_{sj}, p_{sj-1}) \times \exp[-in\Omega_{j-1}(\lambda_j - \lambda_{j-1}) + i(m-n)\alpha_j], \quad (41)$$

where α_j is the phase of the electron on the orbit prior to the collision with the sample surface at the point \mathbf{r}_j at the instant of time λ_j ; $(\lambda_j - \lambda_{j-1})$ is the time during which the electron traverses a path equal to $\mathbf{r}_j - \mathbf{r}_{j-1}$, i.e., the root of the equation

$$\xi(p_{sj-1}, \alpha_{j-1}) - \xi(p_{sj-1}, \alpha_{j-1} + \Omega_{j-1}(\lambda_j - \lambda_{j-1})) = \xi_{j-1} - \xi_j. \quad (42)$$

Here $\Omega_j \equiv \Omega(p_{sj})$, and ξ_j is the projection of the vector \mathbf{r}_j on the ξ axis, and is equal either to zero or to d .

Substituting (41) in (6) we can analyze with the aid of relation (7) and Eq. (20) the distribution of the electric current over the section of the sample and the dependence of the resistance on H in the case of arbitrary reflection of the carriers by the sample boundary. If this reflection differs substantially from specular, the

scattering indicatrix $W(\mathbf{p}, \mathbf{p}'; \mathbf{r}_j)$ is a smooth function of its arguments and the probability of electron drift from one surface of the plate to the opposite is proportional to the weight of the states of the electrons with open orbits, i.e., it is proportional to δ . The effective electron mean free path at the surface of the plate turns out to be

$$l_{\text{eff}} = \frac{r}{(1-w)r/l + w(1+e^{-d/l})}, \quad (43)$$

where w is the probability of the transition of the electrons from the closed orbit to any open orbit. At small δ the averaged probability is $w = A\delta$, where $A \sim 1$. We have left out of (43) numerical factors of the order of unity.

In a bulky sample ($d \gg l$) the effective mean free path of the surface electrons, as seen from (43), is

$$l_{\text{eff}} = \frac{r}{w + r/l}, \quad (44)$$

and in thin plates ($d \ll l$) it decreases because of the possible drift of the electrons in opposite directions at the surfaces $\xi=0$ and $\xi=d$.

The electric conductivity of the surface layer at $w \gg r/l$ turns out to be much larger than the conductivity of the sample core and is of the same order of magnitude as for pure diffuse reflection, i.e.,

$$\sigma_{\perp} = \sigma_0 \frac{r}{lw} \frac{r}{d} \approx \sigma_0 \frac{r^2}{ld\delta}. \quad (45)$$

The substantial deviation from the quadratic dependence of the resistivity on H begins to manifest itself when the electron reflection is close to specular. To describe the static skin effect in this case it is quite convenient to use an approximation that makes use of the Fuchs specularity parameter q . In the presence of several groups of carriers, the quantity $(1-q)$ that enters in the formulas for the resistivity (see Refs. 2-4 and 20) is the sum of the probabilities of diffuse reflection and the probability of charge recombination, i.e., the arguments advanced in Ref. 10 are fully justified.

In a magnetic field inclined to the surface of the plate, $\rho^{\text{mon}}(H)$ is not very sensitive to the surface state of the sample, and the amplitude of the Sondheimer oscillations decreases sharply, since the drift of the carriers into the interior of the sample in one period depends on the projection of the electron momentum p_x on the direction of the magnetic field. Therefore only selected electrons take part in the Sondheimer oscillations, just as in the absence of open electron orbits, namely the electrons with extremal displacement, over the period, along the magnetic-field direction, and the electrons at the limiting point of the Fermi surface.

The amplitude of the oscillations of ρ reaches a maximum value in the case of specular reflection of the carriers from the sample boundary; it is equal to

$$\frac{\rho^{\text{osc}}}{\rho^{\text{mon}}} \approx \frac{l}{d} \left(\frac{r_1}{d} \right)^{1/2} \cos \frac{d}{r_1} + \frac{l}{d} \left(\frac{r_0}{d} \right)^2 \cos \frac{d}{r_0}, \quad (46)$$

where r_0 is the displacement of the electrons at the limiting point of the Fermi surface along a normal to the surface of the plate during the period, and r_1 is the extremal displacement.

The simplest situation arises in a magnetic field normal to the surface of the plate. If the sample boundary coincides with the symmetry plane of the crystal, then the solution of the electroneutrality equation is a homogeneous electric field, and $E_z(\xi)=0$. In the case of specular reflection of the charges by the surface of the plate, which coincides with the symmetry plane of the crystal, there are no Sondheimer oscillations in a magnetic field normal to the plate, and the appearance of these oscillations is due to the imperfection of the sample surface. The dependence of the amplitude of the Sondheimer oscillations on the magnitude of the magnetic field turns out to be as a rule the same as in the specularity-parameter approximation, i.e., the amplitude is proportional either to $H^{-1/2}$ or to H^{-2} , and the proportionality coefficients are functionals of the scattering indicatrix. The linear dependence we obtained previously¹⁴ for the amplitude of Sondheimer oscillations on $1/H$ is more readily an exception than the rule, since the employed model of the scattering indicatrix, in which its width $\Delta\varphi$ in terms of the angular variables was assumed to be independent of p_x , up to the limiting point of the Fermi surface, does not describe the properties of real slightly rough surfaces of a conductor. (We are most grateful to V. I. Okulov and V. V. Ustinov for calling our attention to this circumstance.)

Thus, the specularity-parameter approximation is perfectly satisfactory for the description of galvanomagnetic phenomena in strong magnetic fields.

In thin metallic conductors, the role of open electron orbits is less important than in bulky samples. The electric conductivity of plates in a strong magnetic field parallel to the plate surface is determined mainly by the electrons belonging to closed sections of the Fermi surface, which glide along the sample surface. The static skin effect sets in at $r < \delta d$, if the carriers are specularly reflected by the sample boundary, and at $r < l\delta$ in the case of diffuse reflection. In specular reflection, the transverse resistivity ρ_{\perp} reaches saturation at $r < d$, then gives way to a quadratic growth, and under conditions of the static skin effect it increases linearly with the field. At arbitrary reflection of the carriers from the sample boundary, ρ_{\perp} has a complicated dependence on the magnetic field H and from the $\rho_{\perp}(H)$ dependence we can determine the degree of specularity of the plate surface.

In metals with compensated electron and hole volumes the static skin effect sets in fields $r < d$. An important role is played here by umklapp processes in specular reflection, which are capable of decreasing the effective mean free path of the carriers. The principal contribution to the electric conductivity of plates with ideally smooth faces is made by conduction electrons, for which such processes are impossible. The decrease in the number of effective electrons on account

of umklapp processes leads to an increase of the sample resistance, thereby hindering somewhat the determination of the specularity parameter q from the function $\rho_{\perp}(H)$.

The oscillatory dependence of the resistance of thin plates in a parallel magnetic field on H can take place also for purely specular reflection of the carriers from the sample boundary. The formulas presented for ρ^{osc} at $q=1$ were obtained in the approximation of a homogeneous electric field. The investigation of the role of umklapp processes in the electric conductivity of thin conductors, will be the subject of a separate communication.

- ¹M. Ya. Azbel', Zh. Eksp. Teor. Fiz. 44, 983 (1963) [Sov. Phys. JETP 17, 667 (1963)].
- ²M. Ya. Azbel' and V. G. Peschanskiĭ, Zh. Eksp. Teor. Fiz. 49, 572 (1965) [Sov. Phys. JETP 22, 399 (1965)].
- ³M. Ya. Azbel' and V. G. Peschanskiĭ, Zh. Eksp. Teor. Fiz. 52, 1003 (1967) [Sov. Phys. JETP 25, 665 (1967)].
- ⁴V. G. Peschanskiĭ and M. Ya. Azbel', Zh. Eksp. Teor. Fiz. 55, 1980 (1968) [Sov. Phys. JETP 28, 1045 (1969)].
- ⁵I. M. Lifshitz, M. Ya. Azbel', and M. I. Kaganov, Zh. Eksp. Teor. Fiz. 31, 63 (1956) [Sov. Phys. JETP 4, 41 (1957)].
- ⁶I. M. Lifshitz and V. G. Peschanskiĭ, Zh. Eksp. Teor. Fiz. 35, 1251 (1958) [Sov. Phys. JETP 8, 875 (1959)].
- ⁷I. M. Lifshitz, M. Ya. Azbel', and M. I. Kaganov, Elektronnaya teoriya metallov (Electron Theory of Metals), Nauka, 1971 [Plenum].
- ⁸G. I. Babkin and V. Ya. Kravchenko, Zh. Eksp. Teor. Fiz. 60, 695 (1971) [Sov. Phys. JETP 33, 378 (1971)].
- ⁹V. Ya. Kravchenko and É. I. Rashba, Zh. Eksp. Teor. Fiz. 56, 1713 (1969) [Sov. Phys. JETP 29, 918 (1969)].
- ¹⁰O. A. Panchenko, A. A. Kharlamov, and Yu. G. Ptushinskiĭ, Zh. Eksp. Teor. Fiz. 67, 780 (1974) [Sov. Phys. JETP 40, 386 (1975)].
- ¹¹L. A. Fal'kovskiĭ, Zh. Eksp. Teor. Fiz. 58, 1830 (1970) [Sov. Phys. JETP 31, 981 (1970)].
- ¹²V. I. Okulov and V. V. Ustinov, Zh. Eksp. Teor. Fiz. 67, 1176 (1974) [Sov. Phys. JETP 40, 584 (1975)].
- ¹³O. V. Kirichenko, V. G. Peschanskiĭ, and S. N. Savel'eva, Pis'ma Zh. Eksp. Teor. Fiz. 25, 187 (1977) [JETP Lett. 25, 171 (1977)].
- ¹⁴O. V. Kirichenko, V. G. Peschanskiĭ, and S. N. Savel'eva, Fiz. Nizk. Temp. 4, 180 (1978) [Sov. J. Low Temp. Phys. 4, 88 (1978)].
- ¹⁵R. G. Chambers, Proc. R. Soc. London Ser. A202, 378 (1950).
- ¹⁶E. H. Sondheimer, Phys. Rev. 80, 401 (1950).
- ¹⁷A. B. Pippard, Philos. Mag. 13, 1143 (1966).
- ¹⁸T. V. Emel'yanova, O. V. Kirichenko, and V. G. Peschanskiĭ, Fizika nizkikh temperatur (Low-Temperature Physics), Physicotech. Inst. Low Temp. Phys. Ukr. Acad. Sci. 18, 52 (1966).
- ¹⁹O. V. Kirichenko, Fizika kondensirovannogo sostoyaniya (Physics of the Condensed State), Physicotech. Inst. Low Temp. Phys. Ukr. Acad. Sci. 30, 51 (1974).
- ²⁰O. V. Kirichenko, V. G. Peschanskiĭ, and S. N. Savel'eva, Zh. Eksp. Teor. Fiz. 67, 1451 (1974) [Sov. Phys. JETP 40, 722 (1975)].

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