Contribution to the theory of disordered metals in strongly doped semiconductors

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We investigate the influence of interference of the interaction between conduction electrons and their elastic scattering by static inhomogeneities in metals and in degenerate semiconductors. It is shown that this interference leads to a square-root dependence of the Fermi-level electron state density on the energy reckoned from this level. This singularity in the density of state leads to anomalous dependences of various thermodynamic quantities on, for example, the temperature. The temperature and frequency dependences of the electric resistance are also obtained.

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I. INTRODUCTION AND PRINCIPAL RESULTS

An ever increasing number of studies, mainly experimental, have been made in recent years of amorphous and disordered metals. These substances include metallic films, strongly doped semiconductors, and others.

Disordered metals have a number of properties that have attracted principal attention:

1. A negative temperature coefficient of resistivity is observed, sometimes up to the recrystallization temperature.[2] A minimum is frequently observed in the temperature dependence of the resistance, and this minimum can be located in a very wide range of temperatures.[4]

2. The resistance minimum does not depend on the magnetic field.[5,6] This minimum appears frequently far below the magnetic-ordering temperature.[5,7] In addition in the vicinity of this minimum there are no anomalies, typical of the Kondo effect, in the thermoelectric-power, which depends linearly on the temperature.[8] All this makes it impossible to attribute this minimum to the Kondo effect.

3. In a number of experiments, for example on the external photoeffect in amorphous metals, a minimum is observed in the state density on the Fermi surface.[9,10]

4. In investigations of M-I-M tunnel diodes or tunnel diodes made of degenerate semiconductors, the well known zero-bias anomaly is observed.[10] This anomaly sometimes takes the form of a minimum in the resistance of the tunnel junction, depends on the magnetic field, and can be attributed within the framework of the Appelbaum–Anderson theory[11,12] to tunneling with simultaneous scattering of the electrons by magnetic atoms. This anomaly takes sometimes the form of a maximum, for which there was no satisfactory explanation up to now.[10]

In connection with the experimental investigations of disordered metals, notice should be taken of the work by Bronevoĭ and Sharvin,[13] who have shown that when bismuth is plastically deformed and as a result its resistance increases strongly and the mean free path decreases to 10^{-5}–10^{-6} cm, a negative temperature dependence of the resistance is observed, and accordingly an anomaly in the resistance of a microjunction at zero bias. The authors explained the observed anomaly of the resistance of the microjunction and the dependence on the voltage as being due to heating of the electrons (and not of the lattice), i.e., by taking into account the temperature dependence of the resistance. This has led to the important conclusion that the resistance depends on the electron temperature Θ and not on the lattice temperature T.

Experiments on bismuth are of great interest, since all its parameters are well known (see, e.g., Ref. 14), so that the theory can be quantitatively compared with experiment.

Various theories were used to explain the anomalies observed in disordered metals. These are primarily the Ziman theory of the resistance of liquid metals[15-17] and the Anderson–Halperin–Varma theory[18,4,5] of electron scattering by tunnel states. The Ziman theory has explained the negative temperature dependence of the resistance in a wide range of temperatures. At the same time, attempts to explain the minimum in the resistance as being due to scattering of electrons by tunnel states was apparently unsuccessful.

In the present paper we investigate the properties of disordered metals from a somewhat different point of view. It is well known[19,20] that a metal can exist only in the case when the electron wavelength is smaller than its mean free path l. In the opposite case, a transition to the dielectric state (the Anderson transition) is observed. However, if the condition $p_F l \gg 1$ is satisfied,[1)] then the ladder approximation is sufficient for the description of the interaction of the electrons with the defects in the disordered metal, since all the diagrams containing intersections of impurity lines are small in the parameter $1/p_F l$ (Ref. 21), where p_F is the Fermi momentum.

On the other hand, an investigation of the temperature dependence of the kinetic effects calls for allowance for the inelastic interaction of the electrons with one another, or with phonons, or with other dynamic degrees of freedom of the metal. Scattering by these degrees of freedom is characterized by a momentum

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transfer q and by an energy transfer ω. If $ql \ll 1$ and $\omega\tau \ll 1$, where τ is the electron free path time, then the character of the interaction of the electron with the scatterer changes qualitatively, inasmuch as during the time of interaction the electron manages to collide many times with the impurities or with the defects. This means that the effective interaction vertex begins to depend substantially on the energy transfer ω, and it is this which leads to nontrivial dependences of the observed quantities, say on the temperature.

In 1957, Pippard[22] has noted that to analyze the lattice thermal conductivity in metals it is necessary to take into account the fact that the phonon mean free path depends on the electron mean free path. This leads to an additional contribution to the lattice thermal conductivity, proportional to T^2, thus increasing the apparent Lorentz number.

Bir, Pikus, and one of us[23] constructed a theory of spin relaxation of electrons on holes, when the latter collide frequently with impurities. It was shown that allowance for the diffusion motion of the holes through the interaction region leads to a substantial change of the temperature and concentration dependences of the spin-relaxation time compared with those given by perturbation theory, since the effective time of interaction of the electron with the hole is increased by a factor $(ql)^{-1} \gg 1$.

Keck and Schmidt[24] have shown that allowance for the scattering of electrons by impurities changes the Éliashberg function, which determines the effective electron-phonon interaction that leads to pairing of the electrons and to the appearance of superconductivity. This renormalization of the Éliashberg function leads to an increase of the gap and of the superconducting transition temperature.

Kozlov and Flerov[25] attempted later, to use the idea of diffusion renormalization of the interaction to explain the minimum in the temperature dependence of the resistance in beryllium,[26] which subsequently turned out to be due to the Kondo effect.[27] Their results, however, are incorrect because of an algebraic error and incorrect allowance for the electroneutrality. The point is that the vertex of the electron–phonon interaction, when averaged over the Fermi surface in the single-band approximation, vanishes at $q \to 0$ (Ref. 28) and therefore no diffusion renormalization takes place at all. Such a renormalization of the electron–phonon interaction exists only for multiply connected Fermi surfaces, when the umklapp processes are suppressed for some reason.

One of us[29] derived a kinetic equation with account taken of interference of the electron scattering by impurities and by any Bose excitation that preserves electroneutrality, and obtained the temperature dependence of the electron–phonon contribution to the resistance for multiply connected Fermi surfaces. The same equation is valid also for the description of the interference of the electron–electron interaction with impurity scattering and for the influence of this interference on the kinetic phenomena.

The present paper is devoted to a study of the influence of the electron–electron interaction on the properties of disordered metals.[2)] The principal results reduce to the following.

1. When account is taken of the interaction between the electrons, a singularity appears in the state density of the electrons near the Fermi level:

$$\delta\nu(\varepsilon, T) = \frac{\lambda}{2^{1/2}\pi^2} \frac{T^{1/2}\varphi(\varepsilon/2T)}{(D_1D_2D_3)^{1/2}}, \quad \varepsilon \ll \frac{1}{\tau}; \tag{1}$$

here τ is the electron departure relaxation time, ε is the energy reckoned from the Fermi level, λ is the effective electron interaction constant ($\lambda > 0$ in the case of repulsion), and D_i are the principal values of the diffusion-coefficient tensor. In the isotropic case $D_i = D = v_F^2\tau_{tr}/3$, where τ_{tr} is the electron transport relaxation time and v_F is the Fermi velocity. The function $\varphi(x) \to \text{const} \approx 1.07$ as $x \to 0$, and $\varphi(x) = \sqrt{2x}$ at $x \gg 1$. The state density on the Fermi level has a minimum in the case of electron repulsion and a maximum in the case of attraction.

The square-root dependence of the state density on the temperature leads to a corresponding temperature dependence of the thermodynamic quantities. Thus, the electronic specific heat acquires an increment

$$\delta c = \frac{\lambda T^{1/2}}{2^{1/2}\pi^2} \frac{c_0(T)}{D^{3/2}\nu_0} \propto T^{3/2}. \tag{2}$$

The temperature dependence of both the spin and diamagnetic susceptibility of the metals acquires a square-root dependence on the temperature. This dependence manifests itself most strongly in almost ferromagnetic metals, where the Stoner enhancement causes the effect to increase by a factor $1/\gamma$ ($\gamma \ll 1$ is the Stoner factor):

$$\delta\chi = \chi_0 \frac{\lambda}{2^{1/2}\pi^2} \frac{T^{1/2}}{D^{3/2}\gamma^2\nu_0}. \tag{3}$$

The dependence of the state density on the energy leads to the appearance of an anomaly in the resistance of tunnel contacts at zero bias, which takes the form of a maximum in the case of electron repulsion and a minimum in the case of attraction. The magnitude of this anomaly decreases with increasing temperature in proportion to $T^{1/2}$. The additional resistance of the tunnel contact takes at $T = 0$ the form

$$\frac{\delta R}{R} \propto -\frac{\delta\nu(eV)}{\nu_0} = -\frac{\lambda[eV]^{1/2}}{2^{1/2}\pi^2 D^{3/2}\nu_0} \tag{4}$$

and the indicated anomaly becomes smeared out at $T \neq 0$ over scales of the order of T. In the zeroth approximation $\delta R/R \propto -T^{1/2}\tau^{-5/2}$. Figure 1 shows the additional tunnel resistance as a function of $V^{1/2}$ for different tunnel junctions in accordance with the data of Bermon and So.[32] One of the electrodes of the tunnel junction was a gold film sputtered on the substrate at helium temperature in a residual oxygen atmosphere. It is seen that the experimental points fit straight lines well.

It seems to us also that the minimum of the state density observed in experiments on the external photoeffect can also be partially connected with the correction (1).

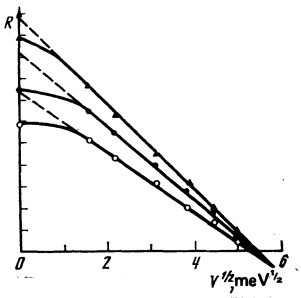

FIG. 1. Resistance of Al–I(O_2)–Au tunnel junction as a function of the square root of the bias voltage in accordance with the data of Ref. 32. The slope of the straight lines increases with increasing oxygen concentration in the gold.

2. Allowance for the electron–electron correlations leads to a decrease of the resistivity of the metal with increasing temperature:

$$\frac{\delta\sigma_{ik}}{\sigma_{ik}}=2.5\frac{\sqrt{2}}{6\pi^2}\frac{\lambda T^{1/2}}{\nu_0(D_1D_2D_3)^{1/2}}\sim\frac{\lambda(T\tau)^{1/2}}{(p_Fl)^2}. \tag{5}$$

Expression (5) is valid at $T\ll 1/\tau$. With further increase of the temperature, the resistivity begins to increase. The temperature dependence of the resistivity acquires therefore a minimum. It would be simplest to say that the correction (5) to the resistivity is a consequence of the increase of the state density on the Fermi surface with increasing temperature, in accord with (1). In the calculation of the conductivity, however, this correction to the state density is canceled out by the corresponding corrections to the relaxation time, as will be shown below. The true case of this dependence is much more complicated: it is the result of interference between the inelastic and multiple elastic scattering acts.

The characteristic value of the correction (5) to the conductivity in metallic glasses can be estimated in the following manner: it is presently assumed that these systems contain one or two conduction electrons per atom,[33] and therefore the concentration of the electrons is of the order of 10^{22} cm^{-3}, just as in an ordinary metal. This means that the Fermi momentum is of the order of 7×10^{-20} g-cm/sec. A typical value of the resistivity of metallic glass is $\rho_0\sim2\times10^{-2}$ Ω-cm. Therefore

$$p_Fl=3\pi^2/e^2\rho_0p_F\sim10.$$

If the Fermi velocity is of the order of 10^8 cm/sec, then $\tau^{-1}\sim10^{-4}$ K. A typical temperature at which the resistance reaches the minimum is $T_{min}\sim20$ K. Substitution of these numbers in (5) yields

$$\delta\sigma(T_{min})/\sigma\sim10^{-3},$$

as is in fact observed in experiment.[6,34]

Rayne and Levy[34] investigated the change of the temperature dependence of the resistance of the metallic glass $Fe_{40}Ni_{40}P_{14}B_6$ upon crystallization. The conductivity at $T=0$ increased by a factor of 1.6, and T_{min} decreased by an approximate factor of 9. According to (5)

$$\rho_0-\rho(T_{min})\propto T_{min}^{1/2}\rho_0^{3/2}$$

and this quantity should decrease by a factor of 10, as was in fact observed in experiment.

We present also a comparison of (5) with the already mentioned experiments of Bronevoĭ and Sharvin,[13] who investigated, in particular, the temperature dependence of the conductivity of pure but strongly deformed bismuth. Prior to the deformation, the resistance increased quadratically with temperature, as is customary for semimetals. After plastic deformation, the resistivity of the wire increased to 2×10^{-3} Ω-cm and decreased with increasing temperature approximately in accord with a square-root law. The Fermi surface of bismuth consists of three electron and one hole bands, the parameters of which are well known.[14] The total density of states is 2.48×10^{31} $(erg\text{-}cm)^{-1}$. Assume that the electrons and holes have equal diffusion coefficients. Then the diffusion coefficient can be easily determined from the Einstein relation, knowing the conductivity and the density of states. We obtain $D=82$ cm/sec^2. This corresponds to $\tau\approx5\cdot10^{-14}$. Substituting ν_0 and D in (5), we have at $\lambda=1$

$$[\rho_0-\rho(T)]/\rho_0=1.1\cdot10^{-3}\sqrt{T[\mathrm{K}]}.$$

This is approximately one third the experimentally observed value. For such a rough estimate, the agreement with experiment is good enough.

An important consequence of (5) is the existence of an anomaly in the resistance of the microjunction, a fact, as already noted, observed in bismuth[13] and in other metals,[35] inasmuch as the mechanical stresses produce a region very rich in defects near the junction.

In a magnetic field, the tensor of the diffusion coefficient becomes nondiagonal even in an isotropic metal:

$$D_\parallel=D,\quad D_\perp=D/[1+(\Omega_c\tau)^2],$$

where Ω_c is the cyclotron frequency. As a result we get

$$\frac{\rho(H,T)-\rho(H,0)}{\rho(0,T)-\rho(0,0)}=1+(\Omega_c\tau)^2,\quad \mathbf{H}\|\mathbf{j}, \tag{6}$$

i.e., the longitudinal magnetoresistance has the same temperature dependence as the resistance itself, and the minimum of the resistance becomes stronger following application of a magnetic field, rather than weaker as in the Kondo effect.

We note here that whereas in amorphous metals $\Omega_c\tau\ll1$, in bismuth in fields on the order of 100 kOe this quantity becomes of the order of unity at a mean free path $l\sim10^{-6}$.

3. In addition to the temperature dependence of the resistance, when account is taken of the interaction between the electrons, a frequency dependence of the high-frequency conductivity takes place:

$$\frac{\delta\sigma(0,\omega)}{\sigma}=\frac{4\sqrt{2}}{45\pi^2}\frac{\lambda\omega^{1/2}}{\nu_0D^{3/2}}[(2+4\sqrt{2})+i(1-4\sqrt{2})] \tag{7}$$

at $\omega\ll1/\tau$. At $\omega\gg1/\tau$ the real part of the conductivity, as usual, begins to decrease like ω^{-2}. Therefore the frequency dependence of the conductivity should also have a maximum at

$$\omega_m\sim\lambda^{3/2}/\tau_{tr}(p_Fl)^{-4/3}\ll1/\tau.$$

To obtain all these results we used only the fact that

the scattering of the electrons by the defects is elastic. On the other hand, it is known that in pure metals at $T \gg \Theta_D$, where Θ_D is the Debye temperature, scattering by phonons is quasielastic. This raises the question of whether the phonons can play at high temperatures the same role as defects at low temperatures,[3)] i.e., whether allowance for interference of the electron–electron or electron–phonon interaction can lead to a substantial change of the character of the temperature dependence of, for example, the resistance.

For the considered phenomena to take place, it is necessary, as already mentioned, to satisfy the conditions $\omega\tau \ll 1$ and $ql \ll 1$. The important values are $\omega \sim T$ and $q \sim (\omega/D)^{1/2} \sim (T\tau)^{1/2}/l$. Therefore if $T\tau \ll 1$ then the interference will be appreciable. At high temperatures[36] $T\tau = \alpha$ and is a temperature-independent constant for a given metal. Unfortunately, we do not have any data concerning the parameter α for different metals. A rough estimate for monovalent metals yields[36]

$$\alpha = 25\pi T_m/\mu \leqslant 1,$$

where T_m is the melting temperature. We can attempt, however, to assess the contribution of the electron–electron correlations by assuming $\alpha \ll 1$.

The first that should be noted is the growth of the deviation of the temperature dependence of the resistivity from linearity, i.e., the growth of the quantity $\rho(T)/\rho_0(T) - 1$ with increasing resistivity:

$$\rho(T)/\rho_0(T) - 1 \infty -\rho_0(T)^{1/2}T^{1/2} \underset{T\to\infty}{\infty} -T^2,$$

which is frequently observed in experiment [$\rho_0(T)$ is the resistivity of the metal without allowance for the electron–electron correlations].

Thermodynamic quantities, such as for example the increment to the specific heat of the electron gas, should increase like

$$c(T) - c_0(T) \infty T^{1/2}\rho_0^{1/2}(T)c_0(T) \underset{T\to\infty}{\infty} T^3$$

and depend substantially on the resistivity.

To conclude this section, we wish to point out that the considered phenomena should take place in degenerate semiconductors and should lead, for example, to anomalies in tunnel diodes. to negative magneto-resistance, and to other phenomena.

2. STATE DENSITY

The state density of electrons is defined by the expression

$$\nu(\varepsilon) = \frac{2}{\pi}\int \frac{d^3\mathbf{p}}{(2\pi)^3} G^r(\varepsilon, p), \tag{8}$$

where $G^r(\varepsilon, p)$ is the electron retarded Green's function.

We consider first the lowest approximation in the electron–electron interaction. The correction to the zeroth-approximation Green's function[21] in the temperature technique

$$G_0(\varepsilon_n, p) = \left(i\varepsilon_n - \xi_p + \frac{i}{2\tau}\operatorname{sign}\varepsilon_n\right)^{-1},$$
$$\varepsilon_n = \pi T(2n+1), \quad \xi_p = p^2/2m - \mu \tag{9}$$

is of the form

$$\delta G(\varepsilon_n, p) = G_0^2(\varepsilon_n, p)\Sigma_{ee}(\varepsilon_n, p), \tag{10}$$

where the diagram for $\Sigma_{ee}(\varepsilon_n, p)$ is shown in Fig. 2.

In accord with the foregoing, the vertex part is renormalized by the impurity lines; the wavy line represents the screened Coulomb potential

$$V(\omega_m, q) = 4\pi e^2/q^2\varepsilon(\omega_m, q), \tag{11}$$

where

$$\varepsilon(\omega_m, q) = 1 + \frac{D\varkappa^2}{\omega_m + Dq^2}\theta(\omega_m) + \frac{D\varkappa^2}{-\omega_m + Dq^2}\theta(-\omega_m), \qquad \varkappa^2 = 4\pi e^2\nu_0, \tag{12}$$

at $\omega_m\tau \ll 1$ and $ql \ll 1$;

$$\omega_m = 2\pi mT, \tag{13}$$

$\theta(x) = 1$ at $x > 0$ and $\theta(x) = 0$ at $x < 0$.

Accordingly, the vertex part at $\omega_m\tau \ll 1$ and $ql \ll 1$ is of the form

$$\gamma(\omega_m, q, \varepsilon_n) = \theta(\varepsilon_n)\theta(\varepsilon_n + \omega_m) + \theta(-\varepsilon_n)\theta(-\varepsilon_n - \omega_m) + \frac{\theta(\varepsilon_n)\theta(-\varepsilon_n - \omega_m)}{\tau(-\omega_m + Dq^2)} + \frac{\theta(-\varepsilon_n)\theta(\varepsilon_n + \omega_m)}{\tau(\omega_m + Dq^2)}. \tag{14}$$

We use throughout the point-defect model, wherein $\tau_{tr} = \tau$. It can be shown, however, that Eqs. (12) and (14) remain in force also in the case of anisotropy of the impurity scattering.

It is well known that the temperature Green's function coincides with the retarded one at discrete points on the positive imaginary semiaxis, i.e., $G(\varepsilon_n) = G^r(i\varepsilon_n)$ at $\varepsilon_n > 0$. We put

$$\delta\nu(\varepsilon_n) = -\frac{2}{\pi}\int \frac{d^3p}{(2\pi)^3}\delta G(\varepsilon_n, p). \tag{15}$$

Then the state density is obtained from (15) by analytic continuation to the real axis and taking the imaginary part:

$$\delta\nu(\varepsilon_n) = \frac{2}{\pi}\int \frac{d^3q}{(2\pi)^3} T\sum_m \frac{4\pi e^2}{q^2\varepsilon(\omega_m, q)}\gamma^2(\omega_m, q) \times \int \frac{d^3p}{(2\pi)^3} G_0^2(\varepsilon_n, p)G_0(\varepsilon_n + \omega_m, p+q). \tag{16}$$

At $\omega_m\tau \ll 1$, $ql \ll 1$ the integration of the three Green's functions with respect to d^3p yields

$$-\pi i\nu_0\tau^2\{\theta(\varepsilon_n)\theta(-\varepsilon_n - \omega_m) - \theta(-\varepsilon_n)\theta(\varepsilon_n + \omega_m)\}.$$

The remaining calculations are elementary. Let $\varepsilon_n > 0$, then

$$\delta\nu(\varepsilon_n) = -iT\sum_{\omega_m=-\infty}^{-\varepsilon_n} \frac{\lambda}{\pi^2 D^{3/2}}\int_0^\infty \frac{dx}{x^2 - \omega_m}, \tag{17}$$

where $\lambda = 4\pi e^2\nu_0/\varkappa^2 = 1$.

After subtraction of an inessential constant, the ana-

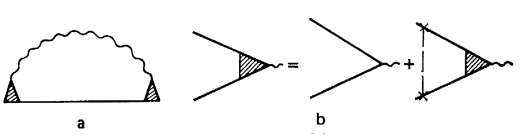

FIG. 2. a) Diagram of first-order perturbation theory in the interelectron interaction for the mass operator of the electron. A straight line corresponds to an electron propagator, and a wavy line to a renormalized Coulomb potential. b) Renormalization of the vertex part by impurity centers.

lytic continuation leads to the expression

$$\delta\nu(\varepsilon)=\frac{\lambda}{2^{3/2}\pi^2 D^{3/2}}\int_0^\infty \frac{d\omega}{\omega^{1/2}}\{n(\omega-\varepsilon)+n(\omega+\varepsilon)\},$$

where $n(\varepsilon)$ is the Fermi distribution function. Integrating by parts, we obtain

$$\delta\nu(\varepsilon)=\frac{\lambda T^{1/2}}{2^{3/2}\pi^2 D^{3/2}}\varphi\left(\frac{\varepsilon}{2T}\right); \tag{18}$$

here

$$\varphi(x)=\frac{1}{2^{1/2}}\int_0^\infty y^{1/2}dy\left[\frac{1}{\mathrm{ch}^2(x-y)}+\frac{1}{\mathrm{ch}^2(x+y)}\right].$$

As $x\to 0$

$$\varphi(x)=\sqrt{\pi}(1-\sqrt{2})\zeta(1/2)\approx 1.07,$$

where $\zeta(x)$ is the Riemann zeta function. At $x\gg 1$ we have $\varphi(x)=(2x)^{1/2}$.

In the sense of the derivation of (14), the pole in $\gamma(\omega_m, q)$ is a diffusion pole that determines the diffusion spreading of the fluctuations of the occupation numbers of the states with definite energy ε_n. In the anisotropic case this pole acquires, naturally, the form $(-\omega_m+D_{ik}q_iq_k)^{-1}$, where D_{ik} is the diffusion tensor of an electron with a given energy ε_n. A procedure similar to the one used in the isotropic case shows directly that the state density contains the quantity $(\det D_{ik})^{1/2}$ in place of the $D^{3/2}$ in the isotropic approximation.

We turn now to calculation of the tunnel current. If the tunnel junction is made up of two different metals, then the expression for the tunnel current is

$$j(V)=\frac{1}{eR}\int_{-\infty}^{\infty}d\varepsilon[n(\varepsilon-eV)-n(\varepsilon)]\nu_1(\varepsilon)\nu_2(\varepsilon-eV). \tag{19}$$

Here R is the resistance of the tunnel junction without allowance for the interaction between the electrons and V is the voltage on the junction. Assume for the sake of argument that the state density in the second metal is constant, and in the first metal $\nu_1(\varepsilon)=\nu_0+\delta\nu(\varepsilon)$. Then the additional resistance of the junction is

$$\frac{\delta R}{R}=\int_{-\infty}^{\infty}\frac{\partial n(\varepsilon-eV)}{\partial\varepsilon}\frac{\delta\nu(\varepsilon)}{\nu_0}d\varepsilon. \tag{20}$$

From this we obtain directly at $eV\gg T$, using (18), the expression (4) and at $V=0$ we have

$$\frac{\delta R}{R}=-C\frac{\lambda T^{1/2}}{2^{3/2}\pi^2 D^{3/2}}, \tag{21}$$

where

$$C=\frac{1}{2^{1/2}}\int_0^\infty y^{1/2}dy\int_{-\infty}^{\infty}\frac{dx}{\mathrm{ch}^2 x\,\mathrm{ch}^2(x+y)}.$$

The foregoing analysis was carried out in first-order perturbation theory in the electron–electron interaction. We show now that all the final expressions contain the exact vertex part of the electron–electron interaction with small energy and momentum transfers. We consider for simplicity the case $T=0$. We note first that frequent scattering of the electrons by defects does not change, if the distance from the Fermi surface is large enough, the fact that the probability of inelastic scattering of two quasiparticles is proportional to ε^2. Therefore the Green's function has the same form as in the Landau Fermi-liquid theory, but

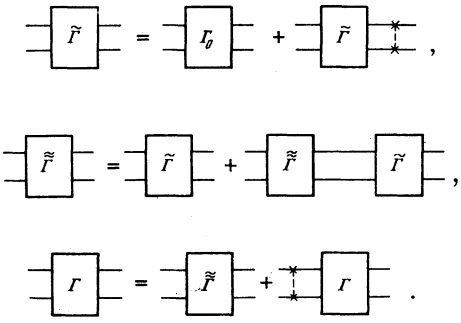

FIG. 3. Renormalization of electron—electron part by impurity centers. Γ_0 and $\tilde{\Gamma}$ are respectively nonrenormalized and renormalized vertex parts containing no two-particle sections.

with account taken of the impurity damping, i.e., expression (9) but with a residue different from unity at the pole.[21] The total vertex part can be represented in a form that separates all the two-particle sections and blocks that do not contain them. These two-particle sections with $ql\ll 1$ and $\omega\tau\ll 1$ must be "screened" by impurity lines without intersection, as shown in Fig. 3. It is easy to verify that the only combination of ω and q on which the vertex part depends is $i\omega/(i\omega+Dq^2)$. The self-energy part can be obtained from the vertex part

$$\Gamma_{\alpha\alpha,\beta\beta}\left(\mathbf{p},\mathbf{p},\frac{i\omega}{i\omega+Dq^2}\right).$$

As shown in Fig. 4, in the calculation, say, of the density of states we can neglect the dependence of $\Gamma_{\alpha\alpha,\beta\beta}$ on p, i.e., we can assume that $p=p_F$. As a result, instead of the Born amplitude of the Coulomb scattering $\lambda_0=4\pi e^2\nu_0/\varkappa^2=1$ all the expressions contain the quantity

$$\lambda=\mathrm{Im}\frac{2\sqrt{2}}{\pi}\int_0^\infty\frac{x^2dx}{(x^2-i)^2}\Gamma_{\alpha\alpha,\beta\beta}\left(\frac{1}{1-ix^2}\right). \tag{22}$$

We shall not stop here to express $\Gamma_{\alpha\alpha,\beta\beta}$ in terms of a vertex part that contains no singular sections, as is done in Fermi-liquid theory. We note only that the final expressions contain the true values (renormalized with account taken of the Fermi-liquid corrections) of the diffusion coefficient, of the time of relaxation on the impurities, and of the density of states.

In the derivation of (22) we have assumed that the essential role in all nonsingular elements is played by large momentum transfers. In other words, we took into account in (22) the renormalization of the interaction between the electrons on account of the region of large momenta. We consider now the case when several Coulomb lines carry small momenta and energies. To this end we note first that the first-order diagram can be estimated by redrawing it in the form shown in Fig. 5a, where the dash-dot line corresponds to $\theta(\varepsilon)\theta(-\varepsilon-\omega)/\tau(i\omega+Dq^2)$ and the cross corresponds to λ/ν_0. Then, if we separate in second order the region of small momenta $q_1l\ll 1$ and $q_2l\ll 1$, then this diagram

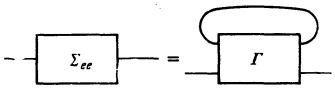

FIG. 4.

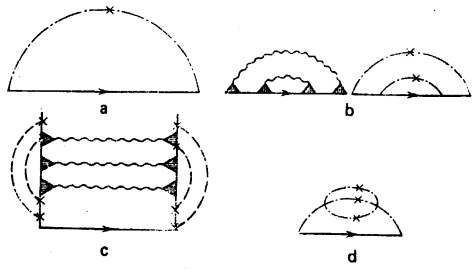

FIG. 5.

can be represented in the form shown in Fig. 5b. It is seen now that it is small relative to the first-order term in terms of the parameter

$$\eta_0 = \frac{\lambda\varepsilon^{1/2}}{\nu_0 D^{1/2}} \sim \lambda \frac{(\varepsilon\tau)^{1/2}}{(p_F l)^2} \ll 1.$$

We consider now the third-order term (Fig. 5c). If each of the momenta q_1, q_2, or q_3 is small, then their sum is also small. Therefore it is necessary to "screen" with impurities not only each vertex part, but also all three Coulomb-interaction lines, as shown in Fig. 5c. There is no need for "screening" two Coulomb lines because of the θ functions. Therefore the diagrams containing an even number of Coulomb lines can be disregarded: they are small in terms of the parameter $\eta_0 \ll 1$.

However, even diagrams with an odd number of Coulomb lines are also small in terms of this parameter. The point is that the vertex of a transition of one diffusion pole into three is of the order of $\tau^3\varepsilon$. A simple estimate shows that allowance for this diagram leads to a correction to the state density of the order of

$$\frac{\lambda\varepsilon^{1/2}}{D^{1/2}} \frac{\lambda^2\varepsilon}{\nu_0^2 D^3} = \eta_0^3 \nu_0 \tag{23}$$

η_0 is thus a true perturbation-theory parameter, and the theory is valid for all $\varepsilon \ll 1/\tau$.

3. TEMPERATURE AND FREQUENCY DEPENDENCES OF THE RESISTANCE OF DISORDERED METALS

We investigate the temperature and frequency dependences of the resistance in the temperature region $T \ll 1/\tau$ and at external-field frequencies $\omega \ll 1/\tau$. To calculate the contribution of the electron–electron interaction to the conductivity it is necessary to calculate the sum of the diagrams shown in Fig. 6.

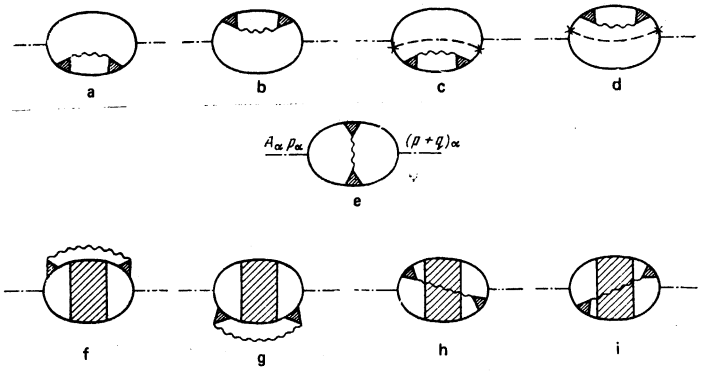

FIG. 6.

If the first two diagrams (a, b) describe the influence of the interaction between the electrons on the state density, then the two others (c, d) yield renormalization of the departure part of the relaxation time, and diagram 6e shows the renormalization of its arrival part. Since the vertex of the interaction of the electron with the external field is vectorlike and the potential of the impurities is pointlike, the field vertices need not be "screened" with impurities.

In the calculation of the corrections to the electric conductivity it must be remembered that the main contribution is made by the terms containing the maximum of diffusion poles. For this purpose it is necessary that the imaginary parts of the Green's functions, joined in each vertex of the electron-electron interaction have opposite signs. This condition limits the region of summation over ε_n and ω_m in each expression. Thus, in diagrams a and b the regions of summation over the frequencies can be either $-\omega_m < \varepsilon_n < 0$ at $0 < \omega_m < \Omega_k$, or $-\Omega_k < \varepsilon_n < 0$ at $\omega_m > \Omega_k$, or finally $-\omega_m < \varepsilon_n < -\Omega_k (\Omega_k > 0)$.

At the same time, the arrangement of the signs of the imaginary parts in diagrams e, f, g, h, and i is determined by the choice of the sign of ε_n. (As usual, we have here $\omega_m = 2\pi m T$, $\varepsilon_n = \pi T(2n+1)$, $\Omega_k = 2\pi k T$. We assume that $\Omega_k > 0$.)

We note immediately that at $\Omega_k\tau \ll 1$ and $T\tau \ll 1$ the sum of the first five terms (a–e) makes a small contribution compared with the sum of diagrams f–i, and it is the latter which we shall now calculate. To this end we must first calculate the element shown in Fig. 7.

As usual, it is necessary to take into account only the latter approximation. By standard calculations we find that at $\omega_m\tau \ll 1$, $ql \ll 1$

$$T(\mathbf{p}, \mathbf{p}_1 | \mathbf{p}+\mathbf{q}, \mathbf{p}_1+\mathbf{q}) = \frac{\gamma(\omega_m, q, \varepsilon_n)}{\pi\nu_0\tau}$$
$$= \frac{1}{\pi\nu_0\tau}\left\{\frac{\theta(\varepsilon_n)\theta(-\varepsilon_n-\omega_m)}{\tau(-\omega_m+Dq^2)} + \frac{\theta(-\varepsilon_n)\theta(\varepsilon_n+\omega_m)}{\tau(\omega_m+Dq^2)}\right.$$
$$\left. + \theta(\varepsilon_n)\theta(\varepsilon_n+\omega_m) + \theta(-\varepsilon_n)\theta(-\varepsilon_n-\omega_m)\right\}. \tag{24}$$

Since the field vertices are proportional to p_α and $p_{1\alpha}$, each diagram f–i vanishes if the Green's functions to the left and to the right of T are taken in the zero approximation in $q \cdot V$. On the other hand, allowance for the next terms of the expansion of the Green's functions in $q \cdot V$ leads to the appearance of the factor Dq_α^2, and this extra q^2 in the numerator cancels out one of

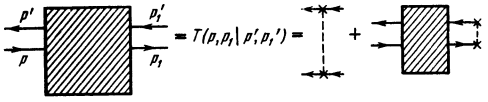

FIG. 7.

the diffusion poles, so that as a result each of the diagrams contains only two such poles. Integrating with respect to ξ_p and summing over ε_n, we obtain after cumbersome but quite straightforward transformations the following expression for the current:

$$j_\alpha=\sigma A_\alpha T\Big\{\sum_{\omega_m=0}^{\Omega_k}F(\Omega_k,\omega_m)\omega_m+\sum_{\omega_m=\Omega_k}^{\infty}\Omega_k F(\Omega_k,\omega_m)\Big\}, \tag{25}$$

where at $\Omega_k\tau\ll 1$, $\omega_m\tau\ll 1$ we have

$$F(\Omega_k,\omega_m)=8\frac{4\pi e^2}{\varkappa^2}\int\frac{d^3q}{(2\pi)^3}\frac{q_\alpha^2}{q^2}\frac{1}{(\omega_m+Dq^2)^2-\Omega_k^2}. \tag{26}$$

It is seen from (25) that as $\Omega_k\to 0$ the current contribution proportional to the vector potential A_α vanishes, as it should. To obtain the frequency and temperature dependence of the conductivity it is necessary to continue analytically the expression for the current to the real axis.

The expression in the curly brackets of (25) can be represented in the form

$$\Lambda=T\sum_0^{\Omega_k}\omega_m F(\Omega_k,\omega_m)+T\sum_{\Omega_k}^{\infty}\Omega_k F(\Omega_k,\omega_m)$$
$$=\frac{1}{2\pi i}\int_{C_1}d\Omega\,\Omega_k F(\Omega_k,-i\Omega)N(\Omega)+\frac{1}{2\pi i}\int_{C_2}d\Omega\,(-i\Omega)F(\Omega_k,-i\Omega)N(\Omega), \tag{27}$$

where the contours C_1 and C_2 are shown in Fig. 8a, and $N(\Omega)=(e^{\Omega/T}-1)^{-1}$. Since $F(\Omega_k,-i\Omega)$ has no singularities with respect to Ω in the upper half-plane, with the possible exception of a point on the imaginary axis $i\Omega_k-iDq^2$, the integration contours C_1 and C_2 can be deformed into the contours C_1', C_2' and C_2'', as shown in Fig. 8b. As a result we get

$$\Lambda=\frac{1}{2\pi i}\Big\{\int_{-\infty+i\Omega_k}^{\infty+i\Omega_k}d\Omega\,\Omega_k F(\Omega_k,-i\Omega)N(\Omega)$$
$$+\int_{-\infty}^{\infty}d\Omega(-i\Omega)F(\Omega_k,-i\Omega)N(\Omega)-\int_{-\infty+i\Omega_k}^{\infty+i\Omega_k}d\Omega\,(-i\Omega)F(\Omega_k,-i\Omega)N(\Omega)\Big\}. \tag{28}$$

Making the substitution $\Omega\to\Omega+i\Omega_k$ in the first and third integrals and using the fact that $N(\Omega+i\Omega_k)=N(\Omega)$, we obtain

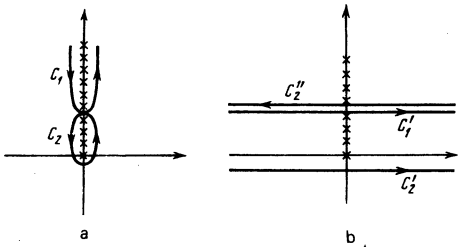

FIG. 8.

$$\Lambda=\frac{1}{2\pi}\int_{-\infty}^{\infty}d\Omega\,\Omega N(\Omega)\,[F(\Omega_k,-i\Omega+\Omega_k)-F(\Omega_k,-i\Omega)\,]. \tag{29}$$

Expression (29) can be easily continued downward to real frequencies by simply making the substitution $i\Omega_k\to\omega$ where ω is the frequency of the external field. The expression for the electric field takes the form

$$j_\alpha=\frac{\sigma A_\alpha}{2\pi}\int_{-\infty}^{\infty}\Omega\,d\Omega\,N(\Omega)\,[F(-i\omega,-i\Omega-i\omega)-F(-i\omega,-i\Omega)\,]. \tag{30}$$

We consider first the temperature dependence of the conductivity as $\omega\to 0$:

$$j_\alpha=\frac{\sigma A_\alpha\omega}{2\pi}\int_{-\infty}^{\infty}\Omega\,d\Omega\,N(\Omega)\frac{\partial}{\partial\Omega}F(0,-i\Omega). \tag{31}$$

Integrating by parts, we obtain

$$j_\alpha=-\frac{\sigma A_\alpha\omega}{2\pi}\int_{-\infty}^{\infty}d\Omega\,F(0,-i\Omega)\frac{\partial}{\partial\Omega}[\Omega N(\Omega)\,]. \tag{32}$$

In the calculation of the temperature-dependent part of the conductivity, an important role is played by $\Omega\sim T\ll 1/\tau$, so that we can use Eq. (26). Substituting (26) in (32) and integrating over the angle variables q, we obtain

$$\frac{\delta\sigma(T)}{\sigma}=-\frac{16}{3\pi}\frac{\lambda}{\nu_0}\operatorname{Im}\int_0^{\infty}d\Omega\frac{\partial}{\partial\Omega}[\Omega N(\Omega)\,]\int\frac{q^2\,dq}{2\pi^2(-i\Omega+Dq^2)^2}. \tag{33}$$

Integration with respect to dq yields

$$\frac{\delta\sigma(T)}{\sigma}=C\frac{\sqrt{2}}{6\pi^2}\frac{\lambda T^{1/2}}{D^{1/2}\nu_0}, \tag{34}$$

where

$$C=\int_0^{\infty}\frac{dx}{x^{1/2}}\Big[1-\frac{\operatorname{sh}x-x}{\operatorname{ch}x-x}\Big]\approx 2.5. \tag{35}$$

This result can be obtained also in a perfectly different manner,[31] namely, by using the kinetic equation derived in Ref. 30.

We now calculate the frequency dependence of the conductivity at $T=0$. To this end we substitute in (30) the expression (26) for $F(-i\omega,-i\Omega)$.

Separating the temperature-independent term, we get

$$\frac{\delta\sigma(\omega)}{\sigma}=\frac{4}{3\pi i\omega}\frac{\lambda}{\nu_0}\frac{1}{2\pi^2}\int_0^{\infty}\Omega\,d\Omega\int_0^{\infty}q^2\,dq\Big[\frac{1}{i(\Omega-\omega)+Dq^2}\frac{1}{i(\Omega-2\omega)+Dq^2}$$
$$-\frac{1}{i\omega+Dq^2}\frac{1}{i(\Omega+\omega)+Dq^2}\Big]. \tag{36}$$

Since we are interested only in the frequency-dependent part of the conductivity, we subtract from the integrand its limit as $\omega\to 0$, which is equal to $4i\omega/(i\Omega+Dq^2)^3$, i.e., that part of the conductivity which does not depend on the frequency of the external field. After this subtraction all the integrals in (36) can be calculated to conclusion. As a result,

$$\frac{\delta\sigma(\omega)}{\sigma}=\frac{4\sqrt{2}}{45\pi^2}\frac{\lambda\omega^{1/2}}{\nu_0 D^{1/2}}\{(2+4\sqrt{2})+i(1-4\sqrt{2})\}. \tag{37}$$

It must be emphasized here that it is precisely the interference of the inelastic and elastic scattering which leads to this dependence of the conductivity on the frequency. As shown by Maleev and Toperverg,[37] the interference of the elastic scattering of electrons by different centers introduced into the high-frequency con-

ductivity corrections that are proportional only to $\omega^{3/2}$.[1)]

In connection with the square-root dependence of the conductivity on the frequency, we note that according to the Einstein relation the increment to the diffusion coefficient also has a similar frequency dependence. According to Ref. 37, the change of the mean squared distance between particles with time is determined by the expression

$$\langle r^2(t)\rangle = 6\left\{Dt + \frac{1}{2\pi}\int \frac{d\omega}{\omega^2} e^{-i\omega t}\,\delta D(\omega)\right\}. \tag{38}$$

Since $\delta D(\omega) \sim \omega^{1/2}$, it follows from (38) that

$$\langle r^2(t)\rangle = 6(Dt + \alpha t^{1/2}), \tag{39}$$

and as $t \to \infty$ the law governing the spreading of the density has a non-diffusion character, i.e., a long time tail is produced in the diffusion.[38-40]

CONCLUSION

In conclusion, we summarize the principal physical results obtained in the present paper.

Allowance for the interference of inelastic electron-electron interaction and of the elastic impurity scattering of the electrons in disordered metals, semimetals, and degenerate semiconductors leads to the onset of singularities in the behavior of a large number of thermodynamic and kinetic quantities:

1. The state density on the Fermi level has a square-root singularity which is responsible for the anomalies of the resistivity of tunnel junctions at zero voltage.

2. Non-analytic corrections to the specific heat and to the magnetic susceptibility are obtained.

3. A minimum of non-Kondo type appears in the temperature dependence of the resistivity.

4. This effect becomes stronger when a longitudinal magnetic field is applied.

5. The anomalous temperature dependence of the resistivity leads to the onset of anomalies in the resistivity of the microjunctions.

6. The conductivity first increases with increasing frequency, and then begins to decrease.

7. Even in pure metals, at high temperatures the interference between the quasielastic scattering by phonons and the inelastic scattering of the electrons by one another leads to a deviation of the temperature dependence of the resistivity from linearity. This deviation is larger the larger the resistivity itself.

8. All these results are valid at an arbitrary strength of the electron–electron interaction, when the substance has metallic conductivity, i.e., above the Anderson localization limit so long as there is no restructuring of the ground state.

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[1)]Here and below Planck's constant is set equal to unity.

[2)]The results of the present paper we published in part in Refs. 30 and 31.

[3)]We are grateful to M. Vedernikov for calling our attention to this question.

[4)]Recently we have learned of the results of L. P. Gor'kov, A. I. Larkin, and D. E. Khmel'nitskiĭ [JETP Lett. 30, 000 (1979)], where it is shown that allowance for the quantum corrections and elastic scattering of electrons also leads to a conductivity increment proportional to $\omega^{1/2}$.

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Static skin effect in metals with open Fermi surfaces

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Galvanomagnetic effects in thin metallic conductors with open Fermi surfaces are investigated. It is shown that in strong magnetic fields the direct current is concentrated near the conductor surface and the magnetoresistance is sensitive to the character of the conduction-electron reflection by the sample boundary. For specular reflection of the carriers by the sample surface, the skin effect is produced in a plate of thickness $d < l$ in a magnetic field parallel to the surface if the magnetic field satisfies the condition $r < \delta d$; for diffuse reflection the condition is $r < l\delta$ (l and r are the electron mean free path and trajectory curvature radius in a magnetic field **H**, and δ is the thickness of the layer of open electron orbits and is referred to the Fermi momentum). Under static skin-effect conditions the electric current is carried mainly by electrons that glide along the sample surface and belong to closed sections of the Fermi surface. Electrons with open orbits participate only in the formation of Sondheimer magnetoresistance oscillations that appear in specular reflection even in plates whose surfaces are symmetry planes of the crystal. In the case of a single group of carriers the transverse resistance $\rho_\perp$ grows linearly with the magnetic field H in the case of specular reflection, and $\rho_\perp \sim H^2$ in diffuse reflection. In metals with several carrier groups and in magnetic fields for which $r \ll d$, saturation occurs in one and the same sample and the magnetoresistance grows quadratically or linearly with transverse-magnetoresistance field. From the dependence of $\rho_\perp$ on H it is possible to determine the degree of imperfection of the crystal surface and the probability of charge recombination on the sample surface. The electroneutrality equation is analyzed for an arbitrary scattering indicatrix of the carriers by the sample boundary. It is shown that the electric field in the sample is appreciably inhomogeneous if the electron reflection is not specular.

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The unrestricted growth of the resistance of a number of metals with increasing field in strong magnetic fields is accompanied by a substantial redistribution of the electric current over the sample cross section. This is caused by the special role played by collisions of the electrons with the sample boundary, which are as a rule accompanied by a jump of the center of the orbit, leading to an increase of the mobility of the electrons near the surface over the mobility of the electrons moving in the center of the sample. For example, in a plate with specularly reflecting faces placed in a magnetic field **H** parallel to its surface, the effective mean free path of the near-surface electrons over the entire plane of the plate is comparable with the mean free path l for collisions inside the volume. At the same time, the electrons that do not collide with the sample boundary can drift only along the magnetic field. The result is a substantial difference between the contributions of the surface and interior electrons to the transverse electric conductivity (electric current density $\mathbf{j} \perp \mathbf{H}$), and the direct electric current can become centrated near the surface of the sample in magnetic fields for which the electron-trajectory curvature radius $r \ll l$.

The static skin effect was predicted by Azbel'[1] in 1962. He considered the electric conductivity of metallic plates of thickness d, whose surfaces reflect the conduction electrons diffusely, and showed that in a magnetic field parallel to the sample surface the electric current flows mainly in a surface layer of thickness $\sim r$, and the transverse resistivity $\rho_\perp$ increases linearly with the magnetic field at $r \ll l^2/d$, if the numbers of the electrons and holes n_1 and n_2 are equal. At $r > l^2/d$ the resistivity is proportional to H^2, just as in bulky samples, in which surface effects are negligible. In uncompensated metals ($n_1 \neq n_2$) the electric conductivity of the core of the sample is high enough at $r \ll l$ (of the order of the electric conductivity σ_0 of the metal in the absence of the magnetic field) and the distribution of the electric field over the sample cross section remains practically unchanged when the magnetic field is increased.

The theory of the static skin effect was subsequently developed by Azbel' with one of us.[2-4] It turns out that in a magnetic field inclined to the plate surface the static skin effect does not lead to new dependences of the resistivity ρ on the strong magnetic field compared with bulky samples. This makes it possible to use, in addition to bulky material, also thin conductors for the

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