a characteristic scale $L \sim 10^{-2}$ cm. The time of its dispersion t_H is of the order of 10^{-9} s. In order that the field may be created by the mechanism considered in the present paper it is necessary that the condition $t_H \gg t_0$ be satisfied. It is clear from (17) and (20) that as to order of magnitude $t_0 \approx L^2 m_e / k_B T_0 \tau_0$. The temperature of the plasma in the drop $T_0 \approx 10^7$ K, and the density $n_0 \approx 10^{21}$ cm^{-3}. Under those conditions $\Lambda \approx 8$, $\tau_0 \approx 10^{-12}$ s. For the growth time of the instability we get $t_0 \approx 6 \times 10^{-11}$ s, i.e., the thermomagnetic instability can fully guarantee the generation of a strong magnetic field during the time the drop exists, provided condition (16) is satisfied. It is rather complicated to establish whether there exists a region in the laser plasma drop in which (16) is valid. To do this it is necessary to know the detailed coordinate dependence of the temperature and density. It follows from the calculations by the authors of Ref. 8 that near the front of the heat wave the temperature and density gradients are in the opposite direction. However, one sees easily from Fig. 1a of Ref. 8 that there exists a region near the front where $|d \ln \rho_0 / dz| < |d \ln T_0 / dz|$ and $d^2 T_0 / dz^2 > 0$. In that region condition (16) can fully be satisfied and the instability will develop.

We make similar estimates for a plasma with parameters which are typical for the solar corona. Here $n_0 \sim 10^{10}$ cm^{-3}, $T_0 \sim 10^6$ K so that $\Lambda \approx 18$ and $\tau_0 \approx 1.5 \times 10^{-3}$ s. Assuming that the scale of the inhomogeneities is of the order of 10^{10} cm we get for the time of the development of the instability about 5 days. If, however, the instability develops in a region with a characteristic scale of 10^9 cm, the growth time will be of the order of one hour. Inequality (14) is satisfied with a large margin. We see thus that the thermomagnetic instability may occur also in the solar corona leading to the formation of fine-scaled magnetic fields and motions.

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Parametric absorption of electromagnetic radiation in a magnetized plasma

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A new mechanism of saturation of parametric instabilities in a magnetized plasma is revealed. The saturation level and the spectrum of the Langmuir turbulence excited by the pump wave in a homogeneous magnetoactive plasma are obtained. The question of the efficiency of parametric absorption of microwave energy in installations with magnetic containment of plasma is investigated.

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1. Much attention is being paid presently to the problem of additional heating of the plasma in magnetic-containment installations. One of the most promising is the use of electromagnetic radiation in the centimeter and millimeter bands for this purpose. The electric field intensities needed to attain the necessary heating rate are such that the plasma in the electromagnetic-radiation field turns out to be parametrically unstable.[1-6] Thus, Alikaev *et al.*[2] observed experimentally ion heating under decay-instability conditions. Grek and Porkolab[3,4] observed, besides anomalous absorption, a rise in the level of the turbulent pulsations, which likewise points to the presence of a correlation between the anomalous absorption and the development of parametric instabilities. Theoretical estimates of the efficiency of the parametric absorption were made by a number of workers.[5,6] Rubenchik *et al.*[5] have shown that parametric effects can impede the penetration of the electromagnetic radiation into the central regions of the plasma. Ramazashvili and Starodub proposed a new method

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of heating plasma by millimeter waves of frequency larger than the maximum Langmuir frequency, via two-plasmon decay, and estimated by parametric-absorption length.

The analysis presented here of the saturation mechanisms of parametric instabilities of a magnetized plasma has revealed the nonlinear processes that are most important from the point of view of saturation instability. We determine the instability saturation level and construct a qualitative picture of the parametric turbulence of a plasma in the range from the electron Langmuir frequency to the lower hybrid frequency. Expressions are obtained for the characteristic length of the parametric absorption of electromagnetic-radiation energy under conditions of weak coupling of the waves. These equations are used to investigate the efficiency of plasma heating in a model in which the influence of the plasma inhomogeneity on the turbulence level is neglected. It is shown that estimates of the rate of parametric pump-energy absorption in a magnetized plasma, obtained elsewhere[5,6] within the framework of such a model, are too high because the instability saturation mechanisms used there are less effective than the two-plasmon decay of parametrically excited oblique Langmuir waves, considered in the present paper.[1)]

2. We dwell first on the question of the efficiency of parametric absorption of the pump energy at a frequency on the order of the electron Langmuir frequency. We assume a homogeneous plasma placed in a magnetic field $\mathbf{B}_0 = B_0\mathbf{b}$, with the electron cyclotron frequency Ω_e higher than the electron Langmuir frequency ω_{Le} ($\Omega_e > \omega_{Le}$). An electromagnetic wave of frequency $\omega_0 \sim \omega_{Le}$ propagating in such a plasma is unstable to parametric excitation of oblique Langmuir waves. The parametric instabilities can be diverse in this case[7,8] and constitute decay of the pump wave into an oblique Langmuir wave and an ion–sound wave, decay of the pump wave into two plasma waves, parametric instability of the type of pump-wave scattering induced in oblique Langmuir waves by plasma ions, or a periodic parametric instability.

Without specifying for the time being the instability excitation mechanism, we turn to a determination of the saturation level of oblique Langmuir waves that increase in the linear approximation exponentially with a certain growth rate γ, and have a frequency ω_l and a wave number k_l. To determine the maximum amplitude of the Langmuir oscillations it is necessary to find, among the possible secondary instabilities Langmuir waves with specified amplitude E_l, the wave with the maximum growth rate.[9] It is precisely this secondary instability which leads to the most effective transfer of energy from the parametrically excited waves to the other Langmuir waves, and by the same token to a limitation of the turbulence level.

Among the possible instabilities of oblique Langmuir waves in a magnetized non-isothermal plasma, the largest growth rates are possessed by two: decay into an oblique Langmuir wave and an ion-sound oscillation, and decay into two oblique Langmuir waves with approximately equal frequencies. In the former case the frequency of the oblique Langmuir wave with dispersion law

$$\omega_l(\mathbf{k}_l) \approx \omega_{Le}(\mathbf{k}_l\mathbf{b})/k_l$$

changes little: $\omega_l(\mathbf{k}_l) - \omega_l(\mathbf{k}_l') \approx \omega_{Li}|\mathbf{k}_l - \mathbf{k}_l'|r_D$ (here ω_{Li} is the ion Langmuir frequency and r_D is the electron Debye radius). In two-plasmon decay, two oblique Langmuir waves are excited with frequencies half the frequency of the initial wave. The growth rate γ^{ls} of the first instability, with account taken of the finite length of the Langmuir pump wave, was determined in Ref. 10 and is given by

$$\gamma^{ls} = {}^1/_4(\omega_l\omega_{Li}|\mathbf{k}_l - \mathbf{k}_l'|r_D)^{1/2}E_l/(4\pi n_e \varkappa T_e)^{1/2}. \tag{1}$$

Here E_l is the amplitude of the Langmuir wave, ω_l is its frequency, n_e and T_e are the density and temperature of the electrons, and $\varkappa$ is Boltzmann's constant.

The growth rate of the second process, in the particular case of long-wave decaying Langmuir waves, was obtained by Gusev.[11] Our present investigation of the growth rate of this instability at an arbitrary wave number k_l of the decaying wave, has shown that the maximum growth rate

$$\gamma^{ll} = {}^3/_8\omega_l k_l r_D E_l/(4\pi n_e \varkappa T_e)^{1/2} \tag{2}$$

is reached in the case when the frequencies of the excited oblique Langmuir waves are equal to each other (and consequently are equal to half the frequency of the decaying wave), and the sum of their wave numbers is double the wave number of the decaying wave. Equations (1) and (2) are given for the case when the spectral width $\delta\omega$ of the initial Langmuir wave and the damping decrements of the excited waves are less than the growth rates (1) and (2). At the same time, in the derivation of (1) we used the approximation of weak parametric coupling of the waves, which is valid when

$$\gamma^{ls} < \omega_{Li}|\mathbf{k}_l - \mathbf{k}_l'|r_D \equiv \omega_s.$$

The growth rate (1) constitutes a growing function of the wave number k_l'. In ion–sound instability it is therefore the short-wave oscillations with $k_l' \approx k_{\max}$ which increase at the maximum rate. The value of $k_{\max}$ is determined from the condition that the Cerenkov absorption of the waves by the electrons be small

$$k_{max} \approx 2^{-1/2} r_D^{-1} \ln^{-1/2}[2^{5/2}(\omega_0/\gamma)\ln^{5/2}(2^{5/2}\omega_0/\gamma)]. \tag{3}$$

From a comparison of expressions (1) and (2) it follows that so long as the frequency of the oblique Langmuir waves is not too small

$$\omega_l > {}^8/_9\omega_{Li}k_{max}/k_l^2 r_D, \tag{4}$$

the growth rate of the two-plasmon decay is larger, and consequently it is precisely this process which determines the saturation of the primary instability. To determine the quasistationary amplitude E_{l1} of the Langmuir waves in the pump region it is necessary, as shown by Silin and Tikhonchuk,[9] to equate the growth rates of the primary and secondary instabilities $\gamma(E_0) = \gamma^{ll}(E_{l1})$. Knowing E_{l1}, we can determine the power density pumped into the plasma, $Q = \gamma E_{l1}^2/4\pi$. Using Eq. (2) for γ^{ll}, we find that the density of the power pumped into the plasma is given by

$$Q = {}^{64}/_{9} n_e \varkappa T_e \gamma^3 / \omega_l^2 k_l^2 r_D^2, \tag{5}$$

a relation valid if condition (4) is satisfied. On the contrary, at lower frequencies of the excited Langmuir waves, the limitation of the turbulence level is due to secondary ion-sound instability. In this case in a non-isothermal plasma ($T_e \gg T_i$), when

$$\gamma > \omega_{Li}^2 k_{max} r_D / \omega_0, \tag{6}$$

the power pumped into the plasma is given by (cf. Ref. 12)

$$Q = 8 n_e \varkappa T_e \gamma^3 / \omega_l \omega_{Li} k_{max} r_D. \tag{7}$$

At lower primary-instability growth rates, on the contrary, when inequality (6) is not satisfied and the growth rate γ is less than the damping decrement of the ion-sound waves

$$\gamma_s = (\pi/8)^{1/2} (\omega_{Li}/\omega_{Le}) \omega_s^4 / k^2 v_{Te}^3 |\mathbf{kb}|,$$

the growth rate of the secondary instability is given by[10]

$$\gamma^{ls} = {}^1/_8 \omega_l \omega_s E_l^2 / 4 \gamma_s \pi n_e \varkappa T_e$$

The power pumped into the plasma, just as in a plasma without a magnetic field, is proportional to the square of the growth rate:

$$Q = 8 n_e \varkappa T_e \gamma^2 \gamma_s / \omega_l \omega_{Li} k_{max} r_D. \tag{8}$$

In an isothermal plasma, when $\gamma_s \approx \omega_s$, the principal nonlinear processes that limit the growth of the high-frequency Langmuir fluctuations is the induced scattering of oblique Langmuir waves by ions, and again, decay into two other oblique Langmuir waves. The growth rate of the first process is given by

$$\gamma^{li} = {}^1/_8 \omega_l E_l^2 / 4 \pi n_e \varkappa T_e. \tag{1a}$$

The power pumped into the plasma in this case, as well as into the plasma without the magnetic field, is proportional to the square of the growth rate[5,13]

$$Q = 8 n_e \varkappa T_e \gamma^2 / \omega_l. \tag{8a}$$

Decay into two oblique Langmuir waves does not depend on the degree of non-isothermy of the plasma. For this process, the pumped-in power is therefore given by (5). From a comparison of (1a) and (2) it follows that the saturation of the instability due to the two-plasmon decay in an isothermal plasma takes place if

$$\gamma^{ll} < (k_l r_D)^2 \omega_l, \tag{4a}$$

or

$$E_l^2 < 36 \pi n_e \varkappa T_e (k_l r_D)^2.$$

Recognizing that the primary instability results in excitation of predominantly short-wave oscillations with $k_l \approx k_{max}$, it is easy to verify that the inequality (4a) is satisfied in practically the entire range of parameters of interest for microwave heating of a plasma.

3. The possibility of secondary two-plasmon decay in a magnetized plasma leads to a situation wherein the saturation of the parametric instability differs from the previously considered[9,12] instability saturation in a plasma without a magnetic field. This difference manifests itself in Eq. (5), in particular, in that the power pumped into the plasma is determined by the parameters of the electron subsystem only and does not depend on the ionic parameters. The possibility of two-plasmon instability causes the power pumped into the plasma, under conditions (4) and (4a), to be less than that determined by the secondary ion-sound instability or by scattering from ions. For this reason, the estimates given in Refs. 5 and 6 for the saturation levels of the parametric instabilities in a magnetoactive plasma turned out to be too high.

Another important consequence of the two-plasmon mechanism of the instability saturation is that in the frequency region

$$\omega_l > \omega_{Li} (k_{max} r_D)^{-1}$$

the spectrum of the Langmuir turbulence is discrete and there can be present in the spectrum waves with frequencies

$$\omega_{l,n} = 2^{1-n} \omega_l,$$

where ω_l is the frequency of the waves excited as the result of the primary instability. To find the intensities of the individual spectral lines in the turbulence spectrum, we use the energy-balance relation proposed in Ref. 12. Namely, in the quasistationary state the energy pumped into the peak with number $n (\gamma_{n-1} E_{l,n}^2$ where γ_{n-1} is the growth rate of the decay of the $(n-1)$-st peak and $E_{l,n}^2$ is the intensity of the n-th peak) dissipates in the peak at a rate determined by the decrement $\tilde{\gamma}$, and is pumped over into the $(n+1)$-st peak, i.e.,

$$\gamma_{n-1} E_{l,n}^2 \approx \tilde{\gamma} E_{l,n}^2 + \gamma_n E_{l,n+1}^2.$$

Solving this equation with the growth rate (2), we get

$$E_{l,n} \approx \frac{8}{3} (4 \pi n_e \varkappa T_e)^{1/2} \frac{2^{(n-1)/3} \gamma}{\omega_0 k_{max} r_D} \left\{ 1 - 2^{(2n-7)/3} \frac{\tilde{\gamma}}{\gamma} \frac{1 - 2^{-2(n-1)/3}}{\ln 2} \right\}. \tag{9}$$

The intensities of the satellites increase exponentially with increasing n at $n < [3.5 + 1.5 \ln(\gamma/\tilde{\gamma})]$. The growth rate of the secondary instability decreases with increasing n:

$$\gamma_n \approx 2^{-2(n-1)/3} \gamma.$$

To find the limit of the two-plasmon spectral transfer we compare, for the lines with intensities (9), the growth rates γ_n^{ll} and γ_n^{ls} of the two-plasmon and ion-sound instabilities, respectively. At

$$n > N = 1 + [\log_2 (\omega_0 k_{max} r_D / \omega_{Li})] \tag{10}$$

the growth rate of the ion-sound instability turns out to be larger. Therefore the spectrum (9) is produced in a non-isothermal plasma at $n \leqslant N$. When $n > N$ the distance between the satellites becomes of the order of the ion-sound frequency:

$$\omega_{l,n} = \omega_{l,N} - 2(n-N) k_{max} r_D \omega_{Li}.$$

The energy transfer takes place qualitatively in the same manner as in a plasma without a magnetic field.[12] We note that in the region of the lower-hybrid frequency ($\omega_l \sim \omega_{LH}$) the character of the spectral transfer calls for a description with greater detail. In particular, the increase of the growth rate γ_s of the ion-sound waves with increasing angle θ between the wave vector $\mathbf{k}$ and the magnetic field ($\gamma_s \propto |\cos\theta|^{-1}$) leads to suppression of the ion-sound instability.

In an isothermal plasma, two-plasmon decay ensures a transfer to the region of lower frequencies: $\omega \sim \omega_{LH}$. In fact, comparing the growth rates (2) and (1a) for the spectrum (9), we get

$$N=\min[1+3\log_2(\omega_0 k_{max}^2 r_D^2/\gamma),\quad 1+\log_2(\omega_0/\omega_{Li})]. \qquad (11)$$

Thus, independently of the pump radiation frequency, the level of the turbulence increases with decreasing frequency of the oblique Langmuir waves and the maximum energy is possessed by waves in the lower-hybrid frequency region, i.e., propagating at large angles to the magnetic field:

$$|\cos\theta| \leqslant \omega_{Li}/\omega_{Le}.$$

This allows us to state that the character of the interaction of the pump wave with the plasma depends little on the pump frequency. The presence of intense oscillations in the region of lower-hybrid frequency can lead, in particular, to a transfer of a noticable fraction of the absorbed energy directly to the ionic component even in the case of relatively high frequency pumping ($\omega_0 \sim \omega_{Le}$). The question of the redistribution of the pump energy absorbed in the plasma is beyond the scope of the present paper and calls for further research.

4. The equations obtained for Q in Sec. 2 can answer the question of the efficiency of parametric pump-energy absorption in a plasma. The absorption efficiency will be characterized by the length

$$l_{eff}=v_{gr}E_0^2/4\pi Q,$$

where v_{gr} is the group velocity of the pump wave. From relations (5), (7), and (8) it follows that the efficiency of the absorption of the heating-radiation energy increases with increasing growth rate γ of the primary parametric instability and with decreasing frequency ω_l of the excited oblique Langmuir waves. Thus, under conditions (4) and (4a), when the nonlinear mechanism of saturation of the instability is two-plasmon decay [see (5)], the effective absorption length is

$$l_{eff}=\frac{9}{64}\frac{v_{gr}}{\omega_l}\left(\frac{\omega_l}{\gamma}\right)^3 (k_l r_D)^2 \frac{E_0^2}{4\pi n_e \varkappa T_e}. \qquad (12)$$

For ion-sound instability that develops in a non-isothermal plasma, the frequency $\omega_l \approx \omega_0$, and the maximum growth rate is[7]:

$$\gamma^{tis}=\frac{1}{4}(\omega_0\omega_{Li}k_{max}^2 r_D)^{1/2}\left[v_{E_\parallel}^2+\frac{\omega_{Le}^2}{\Omega_e^2}\left(1-\frac{\omega_0^2}{\omega_{Le}^2}\right)v_{E_\perp}^2\right]^{1/2}\frac{1}{v_{Te}}. \qquad (13)$$

Here $\mathbf{v}_E = e\mathbf{E}_0/m\omega_0$, $\mathbf{E}_\parallel = (\mathbf{Eb})\mathbf{b}$, $\mathbf{E}_\perp = \mathbf{E}_0 - \mathbf{E}_\parallel$, $\mathbf{b} = \mathbf{B}_0/B_0$, v_{Te} is the thermal velocity of the electrons. The quantity k_{max}, just as in the case of ion-sound decay of a potential wave, is determined by Eq. (3).

In an isothermal plasma, the maximum growth rate is possessed by parametric instability of the type of induced scattering of the pump wave into oblique Langmuir waves:

$$\gamma^{tli}=\frac{1}{16}\omega_0\left[v_{E_\parallel}^2+\frac{\omega_{Le}^2}{\Omega_e^2}\left(1-\frac{\omega_0^2}{\omega_{Le}^2}\right)v_{E_\perp}^2\right]\frac{1}{v_{Te}^2}, \qquad (14)$$

where $\omega_l \approx \omega_0$ just as in the non-isothermal plasma.

For a two-plasmon decay of an electromagnetic pump wave into two oblique Langmuir waves, which takes place both in a non-isothermal and in an isothermal plasma, the frequency is $\omega_l \approx \omega_0/2$, and the maximum growth rate is

$$\gamma^{tll}=\frac{1}{8}\left\{9(\mathbf{k}_0\mathbf{b})^2(\mathbf{v}_E\mathbf{b})^2+\frac{\omega_0^2}{\Omega_e^2}\frac{([\mathbf{b}\times\mathbf{v}_E][(\mathbf{k}_l\mathbf{b})^2(\mathbf{k}_0-\mathbf{k}_l)+(\mathbf{k}_0-\mathbf{k}_l,\mathbf{b})^2\mathbf{k}_l])^2}{(\mathbf{k}_l\mathbf{b})^2(\mathbf{k}_0-\mathbf{k}_l,\mathbf{b})^2}\right\}^{1/2} \qquad (15)$$

where k_0 is the wave vector of the pump wave in the plasma. Depending on the actual conditions (degree of non-isothermy of the plasma, frequency and density of the heating-radiation energy flux) it is necessary to substitute into (12) the particular growth rate (13), (14), or (15) which corresponds to the most rapidly developing instability.

We note that from the point of view of using parametric instabilities for plasma heating it is desirable to realize a situation wherein the energy release is sufficiently uniform over the entire plasma cross section. This can be attained if the effective absorption length is comparable with the plasma dimension in the pump-wave propagation direction. To ascertain the possibility of realizing this situation, we present below estimates of l_{eff} in the homogeneous-plasma model, neglecting the influence of the inhomogeneities of the plasma parameters on the turbulence saturation level.

5. When dealing with plasma heating by parametric excitation of oblique Langmuir waves, a distinction must be made between three different pump wave frequency bands.

At sufficiently high frequencies

$$\omega_{Le}<\omega_0<2\omega_{Le}$$

the only possible parametric instability that leads to excitation of oblique Langmuir waves, in either an isothermal or a non-isothermal plasma, is two-plasmon decay. If we use for the pumping an extraordinary wave propagating across $\mathbf{B}_0$, with a polarization vector oriented across the constant magnetic field, the effective length of the parametric absorption can be represented in the form ($v_{gr} \approx c$):

$$l_{eff}=18r_D\frac{B_0}{E_0}\left(\frac{c}{v_{Te}}\right)^3\frac{\Omega_e^2}{\omega_{Le}\omega_0}(k_l r_D)^2=\frac{18}{\pi}(k_l r_D)^2\frac{\lambda_0}{\beta}\frac{B_0}{E_0}, \qquad (16)$$

where $\beta = 8\pi n_e \varkappa T_e/B_0^2$. The wave number k_l in (16) must be $k_l \approx k_{max}$ (3), for although the growth rate of the two-plasmon decay does not depend on the wave number, the bulk of the energy of the Langmuir oscillations is concentrated in the short-wave region of the spectrum because of the growth of the phase volume with increasing k_l in the wave-number space.

For typical plasma parameters, k_{max} amounts to $\sim 0.2 r_D^{-1}$. The absorption length (16) for reasonable parameters turns out to be quite large. Thus, if we use as the pump radiation with wavelength $\lambda_0 \approx 0.3$ cm and with energy flux density 50 kW/cm², then in a plasma with density $n_e = 10^4$ cm^{-3} and temperature $T_e = 5$ keV, situated in a magnetic field $B \approx 50$ kG, the effective absorption length is $l_{eff} \approx 300$ m. With decreasing pump power and pump frequency, and with decreasing plasma density and temperature, the absorption length increases, but can be decreased somewhat by using the ordinary wave for pumping. Then, however, it is impossible to decrease l_{eff} to the dimensions of present-day or projected thermonuclear installations. In our opinion, in contrast to the viewpoint of Ramazashvili and Starodub,[6] it is difficult to attain noticable parametric absorption in a magnetized plasma at frequencies $\omega_0 \gtrsim \omega_{Le}$.

We consider now the region of lower pump frequency: $\omega_0 \lesssim \omega_{Le}$. In a plasma with such a density, no ordinary electromagnetic wave is produced, and the parametric instabilities can be excited only by the extraordinary wave. At normal incidence of the wave ($\mathbf{k}_0\mathbf{b}=0$), its reflection point is given by[14]

$$\omega_{Le}^2=(\Omega_e+\omega_0)\,\omega_0\approx\omega_0\Omega_e.$$

Therefore in our estimates of the parametric absorption length we confine ourselves to a frequency range

$$\omega_{Le}>\omega_0>\omega_{Le}^2/\Omega_e \quad (\omega_{Le}\ll\Omega_e), \tag{17}$$

wherein the plasma is transparent to the extraordinary wave. In this region it is possible to excite in a non-isothermal plasma both ion-sound instabilities and two-plasmon decay. However, their growth-rate ratio $\gamma^{tls}/\gamma^{tll}$ exceeds unity for temperatures $T_e < 10$ keV, so that in the temperature region of practical interest ($T_e < 10$ keV) parametric absorption in the frequency range (17) is connected with ion-sound instability. For the effective absorption length in the case of normal incidence of the wave on the plasma ($\mathbf{k}_0 \perp \mathbf{B}_0$) we have ($v_{gr} \approx c$)

$$l_{eff}=9r_D\frac{B_0}{E_0}\left(\frac{\Omega_e}{\omega_{Le}}\right)^2\left(\frac{\omega_{Le}}{\omega_{Li}}\right)^{3/4}\left(\frac{\omega_0}{\omega_{Le}}\right)^{7/2}\left(1-\frac{\omega_0^2}{\omega_{Le}^2}\right)^{-3/2}(k_{max}r_D)^{1/4}. \tag{18}$$

The effective absorption length (18) increases with increasing pump frequency. To estimate the minimal absorption length we therefore choose the minimal value of the frequency, defined by the condition (17). Then

$$l_{eff,min}\approx 9r_D\frac{B_0}{E_0}\frac{\omega_{Le}^3}{(\Omega_e\omega_{Li})^{3/4}}(k_{max}r_D)^{1/4}. \tag{19}$$

For the parameters presented above (with the only difference that the wavelength is $\lambda_0 = 0.52$ cm) the parametric absorption length turns out to be $l_{eff,min} \approx 200$.

In an isothermal plasma in the frequency region (17) the growth rate of the produced scattering in the parameter region of interest from the experimental point of view is larger than the growth rate of the two-plasmon decay. Therefore the parametric-absorption length in an isothermal plasma takes according to (12) and (14) the form

$$l_{eff}=5{,}8\cdot10^2\,r_D\frac{B_0}{E_0}\left(\frac{\Omega_e\omega_0}{\omega_{Le}^2}\right)^5(k_{max}r_D)^2\frac{(4\pi n_e\varkappa T_e)^{3/2}}{E_0^3}\left(1-\frac{\omega_0^2}{\omega_{Le}^2}\right)^{-3}. \tag{20}$$

With decreasing pump frequency, the absorption efficiency, just as in the case of a non-isothermal plasma, increases and for the minimum frequency, defined by the condition (17), the effective absorption length is

$$l_{eff,min}\approx 8{,}2\cdot10^2\,r_D\frac{(k_{max}r_D)^2}{\beta^{1/2}}\frac{(4\pi n_e\varkappa T_e)^2}{E_0^4}.$$

For the same parameters of a plasma with $T_i \approx T_e$ the absorption length amounts to ~10^9 m.

The estimates (18) and (20) obtained for l_{eff} indicate that in the frequency range (17), just as at $\omega_0 > \omega_{Le}$, it is difficult to obtain noticable parametric absorption at the sizes of the existing plasma-containment installations. At the same time, bearing in mind the decrease of l_{eff} with decreasing pump frequency, one should expect an effective parametric interaction between the plasma and radiation of frequency $\omega_0 < \omega_{Le}^2/\Omega_e$.

For an electromagnetic wave to be able to penetrate into a plasma of density $\omega_{Le}^2 > \omega_0\Omega_e$, it is necessary to slow it down longitudinally, i.e., to introduce into the plasma an electromagnetic wave whose refractive index along the magnetic field satisfies the inequality[15]

$$n_\parallel^2\equiv c^2(\mathbf{k}_0\mathbf{b})^2/\omega_0^2>1+\omega_{Le}^2/\Omega_e^2.$$

In this way it is possible to introduce into the plasma both an extraordinary and an ordinary wave. By way of example let us estimate the efficiency of parametric absorption of an ordinary wave for which $E_\parallel \sim (\omega_0/\omega_{Le})E_0$. In a sufficiently dense plasma ($\omega_0\Omega_e < \omega_{Le}^2$) the contribution $E_\parallel$ to the growth rates (13) and (14) can be neglected. Taking into account the expression for the group velocity of the wave[15] $v_{gr} \approx \omega_0 c/\omega_{Le}$, we obtain for l_{eff} in a non-isothermal plasma in accord with (12) and (13)

$$l_{eff}\approx 9r_D\frac{B_0}{E_0}\left(\frac{\Omega_e}{\omega_{Le}}\right)^2\left(\frac{\omega_{Le}}{\omega_{Li}}\right)^{3/4}\left(\frac{\omega_0}{\omega_{Le}}\right)^{9/2}(k_{max}r_D)^{1/4}. \tag{21}$$

This equation holds at $(\omega_{Le}^2/\Omega_e) > \omega_0 > \omega_{Li}/k_{max}r_D$. We note the quite strong dependence of the absorption length on the plasma density: $l_{eff} \propto n_e^{-15/4}$. This enables us, by choosing the pump frequency, to obtain energy release from a region comparable in size with the plasma. Thus, for example, to realize parametric absorption of radiation with a flux density 50 kW/cm^2 over a length of one meter in a plasma with thermonuclear parameters $n_e = 10^{14}$ cm^{-1}, $T_e = 5$ keV, and $B_0 = 50$ kG, it is necessary to use radiation at a wavelength $\lambda_0 \approx 1.7$ cm$(\omega_0 \approx 9\omega_{Li})$. For lower frequencies

$$\omega_{Li}(1+\omega_{Le}^2/\Omega_e^2)^{-1/2}<\omega_0<\omega_{Li}(k_{max}r_D)$$

the saturation of the instability is due to secondary ion-sound instabilities. Using (17) and (13) we obtain for l_{eff} in this case

$$l_{eff}\approx 8\frac{c}{\omega_{Le}}\frac{(4\pi n_e\varkappa T_e)^{1/2}}{E_0}\left(\frac{\omega_0}{\omega_{Li}k_{max}r_D}\right)^{1/2}\left(\frac{\omega_0\Omega_e}{\omega_{Le}^2}\right)^3.$$

At reasonable parameters, the absorption length that follows from this formula turns out to be quite small ($\lesssim 1$ cm). Therefore when radiation with approximately lower-hybrid frequency ($\omega_0 \approx \omega_{LH}$) is used the parametric absorption takes place in the rarefied regions, and this can impede the penetration of the radiation into the dense layers of the plasma.

In an isothermal plasma, Eq. (12) for l_{eff} holds up to a pump frequency approximately double the lower hybrid frequency $\omega_{LH} = \omega_{Li}(1 + \omega_{Le}^2/\Omega_e^2)$. (Secondary two-plasmon decay is impossible at $\omega_0 < 2\omega_{LH}$.) In this case we have for l_{eff}, according to (12) and (14),

$$l_{eff}=5{,}8\cdot10^2\frac{c}{\omega_{Le}}(k_{max}r_D)^2\left(\frac{\omega_0\Omega_e}{\omega_{Le}^2}\right)^6\left(\frac{4\pi n_e\varkappa T_e}{E_0^2}\right)^2.$$

Comparison with the corresponding formula of Rubenchik[5] shows that the length of parametric absorption in this case is underestimated by a factor $5.8\cdot10^2(k_{max}r_D)^2\times(\omega_0\Omega_e/\omega_{Le}^2)^2(4\pi n_e\varkappa T_e/E_0^2)$. For the plasma parameters given above this value is $\sim10^3$.

CONCLUSION

Thus, our analysis has shown that saturation of parametric turbulence of oblique Langmuir waves in a magnetized plasma, in a wide range of parameters, is due to two-plasmon decay processes. In the homogeneous-plasma model this leads to a decrease of the efficiency of the parametric absorption of microwave radiation compared with the earlier estimates. As a result, the realization of noticable absorption at $\omega_0\sim\omega_{Le}$ becomes difficult. At the same time, in the intermediate frequency region $\omega_{Li}<\omega_0<\omega_{Le}$, in view of the strong dependence of l_{eff} on ω_0, it is possible to make the absorption length comparable with the plasma dimension.

Another important consequence of the two-plasmon mechanism of saturation of parametric instabilities of a magnetized plasma is that, regardless of the pump frequency, the most effective transfer of the energy of the turbulent fluctuations to the plasma particles is realized in the plasma-wave frequency region close to the frequency of the lower-hybrid resonance. This, in particular, can cause a noticeable fraction of the absorbed plasma energy to be transferred directly to the ionic component.

An experimental proof of the theoretical model proposed here for the saturation of instability might be the observation of subharmonics $\omega_{l,n}\sim2^{1-n}\omega_0$ of the pump frequency in the spectrum of the plasma turbulence.

When considering the question of the absorption of the pump energy we have used a homogeneous-plasma model. This imposes certain limitations on the region of applicability of the results to plasma with inhomogeneous density. Thus, the formulas used in the present paper for the growth increment are valid only in the case when absolute instability develops in the inhomogeneous plasma. To this end, the intensity of the pump field must exceed the threshold value, which in an inhomogeneous plasma depends on the inhomogeneity dimension $L=|\nabla\ln n_e|^{-1}$. For two-plasmon decay of a transverse pump wave, this threshold was obtained in Ref. 6:

$$E_{0,\text{thr}}^2/B_0^2=\frac{16\nu_{ei}^2}{\omega_0^2}+\frac{8r_D^2}{L^2}\ln\frac{2L}{r_D}.$$

The analogous formula for the threshold of the $l\to l+l$ instability can be written in the form

$$\frac{E_{l,\text{thr}}^2}{4\pi n_e\varkappa T_e}=\frac{16}{9}\frac{\nu_{ei}^2}{k_l^2r_D^2\omega_l^2}+\frac{8}{9k_l^2L^2}\ln\frac{2L}{r_D}.$$

For the plasma parameters discussed above ($n_e\approx10^{14}$ cm^{-3}, $T_e=5$ keV, $L\approx1$ m, and $\omega_{Le}/\omega_l=5$) it follows from this formula that the threshold field for the $l\to l+l$ instability is ~800 V/cm. Such plasma-wave field intensities are easy to realize even in present-day experiments.

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[1)]That such a mechanism of saturating parametric instability is possible was pointed out to us by R. R. Ramazashvili.

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