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Subradiative structure in the absorption spectrum of a two-level system in a biharmonic radiation field

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The changes of the absorption spectrum of a two-level atomic system (Cd^{113} vapor) under the influence of two quasiresonant radiation fields at arbitrary amplitudes of these fields are investigated. The frequency of one of the fields was at resonance with the atomic transition, while the frequency of the other, test field, was scanned. Within the Stark structure of the absorption spectrum, "ultranarrow" resonances that converged towards the center of the line were observed. The width of the resonances observed in the absorption was much less than the radiative line width. A theory is constructed which describes the shape of the resonances, the positions of the maxima, and the widths of the resonances. The results of the theory agree with the experimental data.

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1. INTRODUCTION

The interaction of powerful electromagnetic radiation with resonant atomic systems is the subject of numerous experimental and theoretical investigations in nonlinear optics. The variety of the energy spectrum of the atomic and molecular systems admits as a rule of a choice of a situation wherein a good approximation of a two-level system is possible provided that the radiation frequency is not at resonance with the atomic transition. The analysis of the fundamental questions of atomic spectroscopy and the investigation of the emission and absorption spectra of resonant systems are of great importance both for a better understanding of the "atom + field" quantum system, and for practical problems in laser spectroscopy, stabilization of laser frequency, diagnostics of a medium, etc.[1-3]

The spectral and energy characteristics of an atomic system in an intense monochromatic field were investigated in a number of theoretical and experimental studies,[4-9] in which the quasienergy structure of the atom in a radiation field (Fig. 1a) and its absorption and emission spectra (Figs. 1b and 1c) were established. The quasilevel concept turned out to be most useful for the interpretation of the spectral characteristics of atomic systems in strong fields. Indeed, the spectrum of the noncoherent part of the spontaneous scattering in an intense radiation field can be set in correspondence with transitions between quasienergy levels that determine the number and position of the maxima, and also the distribution of the intensity of the radiation over the spectrum (Fig. 1b).

The structure of the quasilevels manifests itself also in the investigation of the absorption spectrum of atoms placed in an intense monochromatic field ($E_0\cos\omega_0 t$), which can be obtained by scanning the frequency of a probing field ($E\cos\omega t$) near the frequency of the atomic transition. A theoretical analysis of the absorption line shape[2] has shown that at strong-field amplitudes of the order of $dE_0/\hbar \leqslant \Gamma$ (Γ is the width of the atomic line) the absorption spectrum is equivalent both to the quasienergy structure (Fig. 1c) and to the spontaneous-emission spectrum. In a more intense field ($dE_0/\hbar > \Gamma$, $f = E/E_0 \ll 1$) there appear in the vicinity of the absorption line regions of enhancement of the weak probing field in the absence of population inversion. By now there are available reliable experimental results obtained in the radio frequency and optical bands, which agree fully with the theoretical concepts.[7-9]

On going to interactions of resonant systems with intense nonmonochromatic radiation, the spectral characteristics of the absorbing media will be determined not only by the usual parameters but also by the statistics of the radiation. These phenomena are observed when the radiation source contains more than one field

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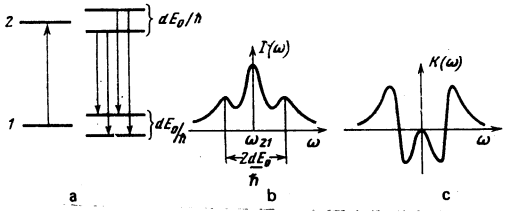

FIG. 1. a) Level scheme in radiation field and the spontaneous transitions between them; b) spontaneous-emission line contour; c) absorption line contour.

mode (natural light sources, multimode lasers, etc.).

Interest in the problem of the interaction of nonmonochromatic radiation with resonant media is both purely practical and connected with some interesting effects observed in experiment, the paradoxical nature of which is connected with the nonmonochromaticity of the radiation.[10,11] A theoretical analysis of most questions in this field of nonlinear optics is a complicated problem, for which there are at present no general methods of solution at arbitrary field statistics.

In particular, the problem of the absorption spectrum in two fields at an arbitrary ratio of the amplitudes ($f \lesssim 1$) cannot be solved within the framework of perturbation theory when the field intensity is sufficient to saturate an atomic transition. Problems with a similar formulation, such as those of the quasienergy states, of population dynamics, and of the polarization of a system have already been discussed in the literature.[12,13] We note, finally, that the problem of the behavior of a two-level system in two circularly polarized fields of equal intensity is formally equivalent to the problem of the behavior of a two-level system in one linearly polarized field, but outside the framework of the approximation of a rotating field.[14,15]

We report here a theoretical and experimental investigation of the absorption spectrum of a two-level system (Cd^{113} vapor) in a biharmonic radiation field at arbitrary amplitudes of both fields ($f \lesssim 1$), when the frequency of one field is fixed and the other is scanned.

2. THEORY

The dynamics of a two-level system in a biharmonic field of radiation will be described with the aid of the equations for the density matrix

$$\begin{aligned} i\dot{\rho}_{22} &= V_{21}\rho_{12} - \rho_{21}V_{12} - i\Gamma_1\rho_{22}, \\ i\dot{\rho}_{12} &= -\omega_0\rho_{12} + V_{12}(\rho_{22} - \rho_{11}) - i\Gamma_2\rho_{12}, \\ i\dot{\rho}_{21} &= \omega_0\rho_{21} + V_{21}(\rho_{11} - \rho_{22}) - i\Gamma_2\rho_{21}, \\ \rho_{11} &+ \rho_{22} = 1, \end{aligned} \tag{1}$$

where ω_0 is the transition frequency, Γ_1 and Γ_2 are the longitudinal and transverse relaxation constants, $V_{12} = V_{21}^*$ are the matrix elements of the interaction with the biharmonic field, which in the rotating-field approximation take the form

$$V_{12} = V_{21}^* = -V_0 e^{-i\omega_0 t} - Ve^{-i\omega t}, \tag{2}$$

where one of the fields ($V_0 e^{-i\omega_0 t}$) is at resonance with the transition frequency, while the frequency of the other field ($Ve^{-i\omega t}$) is scanned near the frequency ω_0; $V = dE/2\hbar$, d is the dipole moment of the transition, and $E(E_0)$ is the amplitude of the radiation field intensity.

We confine ourselves hereafter to a determination of the absorption spectrum of a two-level system in a biharmonic field, i.e., we calculate the experimentally recorded time-averaged power absorbed by the system from the quasiresonant field $Ve^{-i\omega t}$:

$$\bar{Q} = \overline{\frac{\partial}{\partial t}\mathrm{Sp}(\hat{\rho}\hat{V})}. \tag{3}$$

If the field $Ve^{-i\omega t}$ (the probing field) is assumed to be a small pertrubation ($f \ll 1$), then $\bar{Q}$ in a certain spectral band becomes a negative quantity regardless of the phases of the two waves, i.e., stationary amplification in a two-level system takes place.[2,7,9]

When V increases to a value comparable with V_0 ($f \sim 1$), as indicated by the available experimental data, absorption and amplification peaks appear in the spectrum of the regular structure; the width of the peaks can be much less than the natural line width.[16]

One of the real possibilities of obtaining the theoretical results in the region $f \sim 1$ (commensurate field amplitudes) is connected with the presence of a small parameter

$$|\varepsilon| = \frac{|\omega - \omega_0|}{2V_0} = \frac{|\Omega|}{2V_0} \ll 1.$$

To separate explicitly this parameter we change over in (1) to new sought functions

$$\sigma_{12} = \rho_{12}e^{-i\omega_0 t}, \quad \sigma_{21} = \rho_{21}e^{i\omega_0 t}, \quad z = {}^1/_2 - \rho_{22}. \tag{4}$$

Equation (1) then takes the form

$$\begin{aligned} i\dot{\sigma}_{12} &= -2(V_0 + Ve^{i\Omega t})z - i\Gamma_2\sigma_{12}, \\ i\dot{\sigma}_{21} &= 2(V_0 + Ve^{-i\Omega t})z - i\Gamma_2\sigma_{21}, \\ i\dot{z} &= -(V_0 + Ve^{-i\Omega t})\sigma_{12} + (V_0 + Ve^{i\Omega t})\sigma_{21} - i\Gamma_1 z + {}^1/_2 i\Gamma_1. \end{aligned} \tag{5}$$

Introducing the dimensionless variables $\tau = \Omega t$ and dividing both sides of the equation by $2iV_0$, we obtain

$$\begin{aligned} \varepsilon\dot{\sigma}_{12} &= i(1 + fe^{i\tau})z - \gamma_2\sigma_{12}, \\ \varepsilon\dot{\sigma}_{21} &= -i(1 + fe^{-i\tau})z - \gamma_2\sigma_{21}, \\ \varepsilon\dot{z} &= {}^1/_2 i(1 + fe^{-i\tau})\sigma_{12} - {}^1/_2 i(1 + fe^{i\tau})\sigma_{21} - \gamma_1 z + {}^1/_2\gamma_1, \end{aligned} \tag{6}$$

where $\gamma_{1,2} = \Gamma_{1,2}/2V_0$.

The obtained system of equations is inhomogeneous with periodic coefficients. Since (6) includes relaxation terms, the general solution of the system is damped and cannot contribute to the steady-state solution which is needed for the calculation of the stationary absorption (3). The steady state, on the other hand, will be a particular periodic solution of the inhomogeneous system, which according to Ref. 17 always exists. Substituting in (3) the periodic solution in the form of a Fourier series $\sigma_{12}(\tau) = \sum_n (\sigma_{12})_n e^{in\tau}$ etc. and averaging (3) over the period, we obtain

$$\bar{Q} = 2\omega V \,\mathrm{Im}\,(\sigma_{12})_{-1}. \tag{7}$$

Thus, the problem reduces to finding the first harmonic of a particular periodic solution of the system (6). This solution can be easily constructed if we know the fundamental matric $M(\tau)$ of the corresponding homogeneous equation, made up of three linearly indepen-

dent columns of solutions of a homogeneous solution. Then the general solution of the inhomogeneous system can be written in the form of the column

$$\begin{pmatrix}\sigma_{12}(\tau)\\ \sigma_{21}(\tau)\\ z(\tau)\end{pmatrix}=M(\tau)\left\{\int_0^\tau M^{-1}(\tau')\begin{pmatrix}0\\0\\\gamma_1/2\end{pmatrix}d\tau'+C\right\}, \tag{8}$$

where C is an arbitrary constant matrix. The particular periodic solution is identified by the condition

$$C=[M(0)-M(2\pi)]^{-1}M(2\pi)\int_0^{2\pi}M^{-1}(\tau)\begin{pmatrix}0\\0\\\gamma_1/2\end{pmatrix}d\tau. \tag{9}$$

For convenience in the subsequent analysis, we represent the homogeneous system corresponding to the inhomogeneous system (6) in matrix form

$$\varepsilon\frac{d}{d\tau}\begin{pmatrix}\sigma_{12}(\tau)\\ \sigma_{21}(\tau)\\ z(\tau)\end{pmatrix}=\begin{pmatrix}-\gamma_2 & 0 & i(1+fe^{i\tau})\\ 0 & -\gamma_2 & -i(1+fe^{-i\tau})\\ i(1+fe^{-i\tau})/2 & -i(1+fe^{i\tau})/2 & -\gamma_1\end{pmatrix}\begin{pmatrix}\sigma_{12}(\tau)\\ \sigma_{21}(\tau)\\ z(\tau)\end{pmatrix}. \tag{10}$$

Methods of solving systems of this kind, with a small parameter at the derivative, have been well developed.[18] The solution is expanded in an asymptotic series in the small parameter. The first step in obtaining the asymptotic expansion of the solution is diagonalization of the matrix of the coefficients of the system (10), after which the system breaks up, accurate to quantities of order ε, into separate first-order equations, which can be easily solved. The next terms of the asymptotic expansion can be obtained by taking into account the terms $\sim\varepsilon$ and again diagonalize with accuracy to ε^2. Leaving out the intermediate steps, we write down the principal term of the asymptotic form

$$M_0=\begin{pmatrix}e^{-i\varphi}v^+ & e^{-i\varphi}v^- & e^{-i\varphi}v^0\\ -e^{i\varphi}v^+ & e^{i\varphi}v^- & e^{i\varphi}v^0\\ v^+ & -v^- & 0\end{pmatrix}, \tag{11}$$

where

$$v^0=\exp\left\{-\frac{\gamma_2\tau}{\varepsilon}\right\},\quad v^\pm=\exp\left\{-\frac{\gamma\tau}{\varepsilon}\pm\frac{i}{\varepsilon}\int_0^\tau m(\tau')d\tau'\right\}. \tag{12}$$

Here $\gamma=(\gamma_1+\gamma_2)/2$, $m(\tau)$ and $\varphi(\tau)$ are the modulus and argument of $(1+fe^{-i\tau})$. As is clear from (12), at $\varepsilon\ll1$ the adiabatic functions $v^\pm$ oscillate rapidly over the interval of the periodicity of the coefficients $(-\pi,\pi)$, and this justifies the asymptotic approach. However, near the points $\tau_K=\pi(2K+1)$ the rate of change of the phase of the adiabatic function decreases and the ratio of the next terms of the asymptotic expansion to the principal term

$$\alpha=-\frac{\varepsilon}{m}\varphi'=\varepsilon f\frac{f+\cos\tau}{(1+f^2+2f\cos\tau)^{3/2}} \tag{13}$$

increases. Nevertheless, both at $f\ll1$ and $f\gg1$ there are wide regions of detunings $\varepsilon<1/f$ and $\varepsilon<f$, in which we have correspondingly at any instant of time $|\alpha|<1$.

The situation is different at $f\sim1$. In this case the rate of change of the phase of the adiabatic function decreases almost to zero,[1)] and the asymptotic series begins to diverge starting with the second term. A similar difficulty is encountered when a quasiclassical wave function is constructed near turning points. To overcome the difficulty it is necessary to join together the asymptotic solutions on opposite sides of the points τ_K with the aid of an exact solution of a model equation. The corresponding theory has not been developed for systems of third-order equations. One can hope, however, that in our problems there will be no need for detailed information on the behavior of the system in the vicinity of the instants of time τ_K. In fact, the mean value of the absorbed power (7) is determined apparently mainly by the large regions of the adiabatic development of the system, and not by the narrow vicinities of the points τ_K. We shall therefore use the principal term of the asymptotic expansion M_0, and regularize the diverging correction α by interpolation between $f\ll1$ and $f\gg1$:

$$\alpha=\varepsilon\frac{f^2}{(1+f^2)^{3/2}}. \tag{14}$$

Before we present the final results, we make the following remark. The functions $v^\pm$ in (12) are not convenient for substitution in (9). We therefore expand $m(\tau)=(1+f^2+2f\cos\tau)^{1/2}$ in a Fourier series

$$m(\tau)=\sum_n m_n\cos n\tau. \tag{15}$$

An estimate shows that the contribution of the sum of all the terms of this series, starting with the third, to the argument of the exponential (12) does not exceed 0.13 of the contribution of the first two terms. Therefore, confining ourselves in (15) to the constant M_0 and to the first harmonic $m_1\cos\tau$, we rewrite (12) in the form[2)]

$$\begin{aligned}v^\pm&=\exp\{-\gamma\tau/\varepsilon\pm im_0\tau/\varepsilon\pm im_1\sin\tau/\varepsilon\}\\&=\exp\{-\gamma\tau/\varepsilon\pm im_0\tau/\varepsilon\}\sum_{n=-\infty}^{\infty}(\pm1)^nJ_n(m_1/\varepsilon)e^{in\tau},\end{aligned} \tag{16}$$

where $J_n(m_1/\varepsilon)$ are Bessel functions, while m_0 and m_1 are expressed in terms of complete elliptic integrals ($m_0=1$ and $m_1=f$ at $f\ll1$ and $m_0=f$ and $m_1=1$ at $f\gg1$). Now all the integrals in (8) and (9) can be easily calculated and the periodic solution of (8) can be obtained immediately in the form of a Fourier series. Substituting the corresponding component in (7), we obtain the final expression for the absorbed power.

More interesting from our point of view, however, is the spectral characteristic of the atomic system, and in particular its absorption spectrum. An expression for this spectrum can be obtained after simple transformations from Eq. (7):

$$\begin{aligned}K(\varepsilon)=\tilde K_0\frac{\gamma^2+f^2}{\gamma^2+\varepsilon^2}\Bigg\{&(1-\alpha^2)\left[\frac{1}{\tilde m_0^2+\gamma^2}\left(\gamma^2J_0^2-\frac{\tilde m_0\varepsilon J_0J_0'}{f}\right)\right.\\&+\sum_{n\neq0}\left(\varepsilon\left(\varepsilon-\frac{\tilde m_0}{n}\right)\frac{J_nJ_n'}{fn}+\frac{\gamma^2}{n^2}J_n^2\left(1-\frac{n\varepsilon}{m_1f}\right)\right)\Big/\left(\left(\frac{\gamma}{n}\right)^2\right.\\&\left.\left.+\left(\varepsilon-\frac{\tilde m_0}{n}\right)^2\right)\right]+\alpha^2\Bigg\}.\end{aligned} \tag{17}$$

Here $\tilde m_0=m_0+\alpha^2(1+f^2)^{1/2}/4$ and $\tilde K_0=K_0\gamma^2/(\gamma^2+f^2)$; K_0 is the absorption coefficient at the center of the line in the absence of perturbing fields, K_0 is the absorption coefficient at the center of the line with allowance for saturation in the field of the probing wave. In this final form we used Eq. (17) for comparison with the experimental results.

3. EXPERIMENT

To investigate the absorption spectrum under conditions as close as possible to those described in Sec. 2, we chose the vapor of Cd^{113} atoms, which have in the ground state $^1S_0$ only a nuclear angular momentum $I = 1/2$. This state in a constant magnetic field splits into two Zeeman levels in accordance with the projections of the nuclear spin parallel and antiparallel to the direction of the magnetic field. The frequency of the Zeeman transition in a field $H_z = 5$ G was 5 kHz and was smaller by many orders of magnitude than the frequency of the optical transitions to the nearest levels. This circumstance enabled us to regard the chosen atomic system as a two-level system with respect to the interacting radio-frequency fields. The preferred population of one of the levels was produced by optical orientation of Cd^{113} vapor circularly polarized by resonant radiation of a 3261 Å lamp with the Cd^{114} isotope (the experimental setup is described in detail in Refs. 9 and 10). The population difference attained in the experiment ($\sim 10^{-2}$) was sufficient to register the absorption. The initial signal/noise ratio $\sim 10^3$ made it possible to perform experiments with intense fields capable of saturating the atomic transition.

The experimental setup is shown in Fig. 2. The cell with Cd^{113} saturated at 220 °C was placed in the center of a system consisting of three pairs of Helmholtz coils which produced mutually perpendicular fields: a constant field $H_z = 5$ G, and two radio frequency fields: $H_0(t) = H_0 \cos\omega_0 t$ (fixed in frequency and amplitude) and $H(t) = H\cos\omega t$ (frequency-scanned probing field). The orienting beam was directed in the YOZ plane at an angle 45° to the axes Y and Z and served simultaneously as the recording beam.

After passing through the cell, the light beam was modulated at the frequencies of the acting fields and their harmonics. With the aid of a synchronous detector with a reference voltage of frequency ω we measured the component of the light modulation at the frequency of the probing field $H(t)$, which was proportional to the absorption coefficient of the atomic system. Figure 3 shows the experimentally observed absorption spectrum[3)] at $H_0 = H(f = 1)$. Figure 4 shows the results of a comparison of the experimentally obtained absorption with the theoretical calculations by formula (17) for $f = 0.35, 0.64, 0.8$, and 1.0.

4. DISCUSSION OF RESULTS

It is simplest to interpret the experimental results if the field $H_0(t)$ is at resonance with the atomic transition, i.e., $\omega_0 = \omega_{21}$. This was precisely the circumstance that guided us when we obtained the experimental results and performed the theoretical calculation. In fact, in this case the absorption spectra are symmetrical in shape and it is easy to discern in them the following regularities.

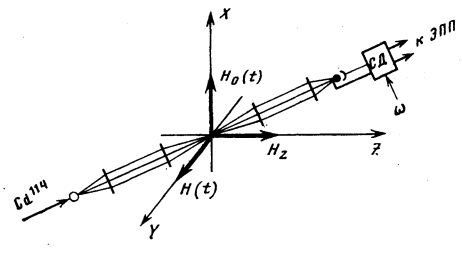

FIG. 2. Diagram of experimental setup.

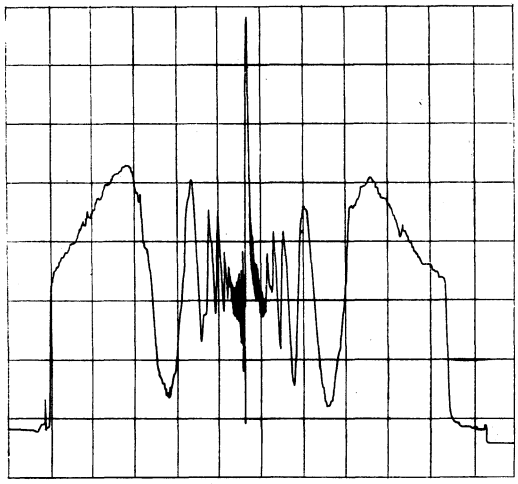

FIG. 3. Experimentally observed absorption spectrum at $H_0 = H$ ($f = 1$).

1. At $H_0 \gg H$ we obtain the previously observed[7,8] effect of reversal of the sign of the absorption coefficient. In addition, in the central region ($|\Omega| < 2V_0$) there appear additional absorption (amplification) peaks whose amplitude increases with increasing intensity of the probing field H (Fig. 4a).

2. The absorption (amplification) peaks "converge" towards the center of the line (ω_0), and their position

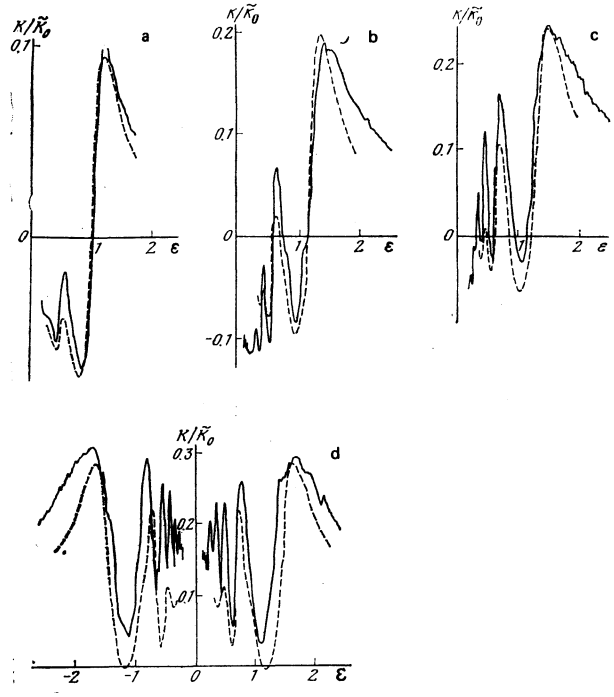

FIG. 4. Absorption spectra: a) at $f = 0.35$; b) $f = 0.64$; c) $f = 0.8$; d) $f = 1.0$; solid line—experiment, dashed—theory; $\varepsilon = (\omega - \omega_0)/2V_0$.

is described by the semi-empirical formula $\Omega_n = \Delta_S/n$, where Δ_S is the Stark splitting of the lines, $n = 1, 2, 3, \ldots$.

3. The half-width of the corresponding contours inside the Stark structure decreases approximately like $\Gamma/n (n = 1, 2, 3, \ldots)$, i.e., starting with the second maximum ($n = 2$) the half-width of the "excited" contours becomes less than the natural line width.

All the singularities noted above follow from Eq. (17) for the absorption coefficient. As seen from a comparison of the experimental and theoretical curves in Fig. 4, expression (17) describes sufficiently well the observed picture. Even in the case $H_0 = H(f = 1)$, when the interpolation (14) was used to calculate the correction to formula (17), quantitative agreement takes place within the limits of the first two maxima both with respect to the sizes of the absorption peak and with respect to the position in the frequency scale.

From an examination of the denominators of (17) we get both a relation for the positions of the maxima $\Omega_n = 2V_0\tilde{m}_0/n$, which is identical with the semi-empirical expression noted above if it is recognized that $\Delta_S = 2V_0\tilde{m}_0$, and an expression for the half-width of the resonances observed in the experiment.

The most interesting singularity of the absorption spectrum lies in the appearance of very narrow resonances, whose width can be much less than the natural width. The minimal width Γ_{min}, i.e., the number of the resonance still allowed spectrally, can be determined from the condition that the distances between the neighboring maxima $2V_0\tilde{m}_0/n^2$ be equal to the total width of the contour with number n

$$2\Gamma/n = 2V_0\tilde{m}_0/n^2, \quad n = \tilde{m}_0V_0/\Gamma.$$

Consequently, for the minimum width of the contour we obtain $\Gamma_{min} = \Gamma^2/\tilde{m}_0V_0$, i.e., Γ_{min} turns out to be less than the natural width in a ratio $\tilde{m}_0V_0/\Gamma$.

The relative intensity of the resonances is determined by the Bessel functions of corresponding order [see Eq. (17)]. At $f \ll 1$, when the argument of the Bessel functions is small, Eq. (17), as expected, goes over into the expression obtained previously[2] for a weak probing field in first-order perturbation theory. In this limiting case no infrastructure was observed in the absorption spectrum.

In the other limiting case $H_0 \ll H(f \gg 1)$, all the resonant terms in (17) are small compared with the first term and its asymptotic expression describes the central part of the contour near $\Omega < V$ under saturation conditions.

Finally, at $H_0 = H(f = 1)$ all the resonances have comparable amplitudes. From the experimental point of view, this case is of greatest interest, since it makes it possible to observe with sufficient reliability narrow resonance in the absorption, whose width is much less than the natural line width.

The appearance of a strongly enriched absorption spectrum of a two-level system in a biharmonic field of radiation can be interpreted in the language of multiphoton transitions. We consider the case $f \ll 1 (H_0 \gg H)$ and when the frequency of the strong field ω_0 is at resonance with the transition. This limiting situation is closest to the model of multiphoton transitions in the quasienergy structure of the level—see Fig. 5. This figure shows the scheme of formation of the high-frequency wing of the absorption spectrum at the frequency of the probing field ω as a result of multiphoton transitions. These resonances appear upon absorption of a single photon ω for the one-photon process, absorption of two photons ω and emission of one photon ω_0 for the three-photon process, etc., and finally, absorption of n photons ω and emission of $n-1$ photons ω_0 for the $(2n-1)$-photon process. Obviously, the resonances will "run down" towards the center of line like V_0/n (see Fig. 5). The probability of the multiphoton process will be maximal if the sum of all the photons (with allowance for the signs) is equal to the frequency of the corresponding transition. A decrease of the transition probability by a factor of 2 is obtained by tuning away from the frequency ω by an amount Γ for the single-photon process, by an amount $\Gamma/2$ for the three-photon process, etc., by Γ/n for the $(2n-1)$-photon process. Thus, the singularities of the absorption spectrum of the two-level system in a biharmonic field, which were experimentally observed and analytically described in Sec. 2, can be lucidly interpreted within a framework of multiphoton transitions.

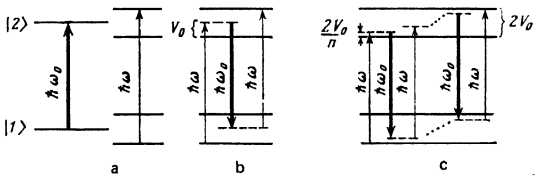

FIG. 5. Quasienergy level scheme in multiphoton transitions: a) one-photon, b) three-photon, c) $(2n-1)$-photon.

In conclusion, we note that it is possible in principle to employ the experimentally observed "ultranarrow" resonances in problems of stabilization of optical frequency standards.

The authors are deeply grateful to S. G. Przhibel'skiĭ for valuable advice and for a discussion of the results.

[1)]Near τ_K the amplitude of the biharmonic field

$$\left\{2V_0 \cos\frac{\tau}{2}\right\} \exp\left\{-i\frac{\omega_0+\omega}{2}t\right\}.$$

vanishes.

[2)]At very small ε, this approach leads to a substantial error, since the quantity discarded in the argument of the exponential, while small with the remaining quantities, is still comparable with unity. However, even in this case the expansion (16) remains meaningful, except that the Bessel functions must be replaced by the values of the integral

$$\frac{1}{\pi}\int_0^{\pi} \cos\left[\frac{1}{\varepsilon}\int_0^{\tau}(m(\tau') - m_0)\,d\tau' - n\tau\right] d\tau.$$

[3)]In the vicinity of the frequency ω_0 there were observed alter-

nating-sign parasitic signals at the frequency ω_0 and $2\omega_0 - \omega$, which were not integrated completely by the synchronous-detector filter and by the substraction circuit.

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Dispersion phenomena in the propagation of radiation in media with time-dependent refractive indices

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We consider the question of appearance of total internal reflection from a medium whose dielectric constant passes through zero and becomes negative within a time much longer than the period of the field. When the dielectric constant approaches zero the structure of the field inside the medium is determined mainly by dispersion effects. It is shown that the delay of the long-wave part of the radiation leads to formation of a slowly decreasing power-law tail in the radiation that had entered the medium in earlier instants of time.

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The propagation of radiation in nonstationary media has been extensively discussed recently (see e.g., the reviews[1,2]). Physical situations in which this question arises are quite varied. By way of examples we can cite measurements of the density of a nonstationary plasma by optical and microwave methods, the propagation of optical and infrared radiation in resonant media, passage of radio waves through the ionosphere and their reflection from it, propagation of microwaves in ferrite-filled waveguides, and others. The changes occurring in various parameters, such as the degree of plasma ionization, the resonant frequency of the medium, or the spin-wave frequency in a ferromagnet, cause the dielectric constant and the magnetic permeability of the medium to vary with time. In particular, changes in the refractive index can cause frequency shifts, reflection, changes of pulse waveforms, and others. These effects have found numerous applications in radio engineering and in optical and infrared laser technology.

To describe such phenomena theoretically we can use the fact that a rule the change of the parameters of the medium in one period of the electromagnetic field is small. This makes it possible to construct an adiabatic approximation[1,2] based on the inequality

$$\eta = \max(1/\Delta\omega T, 1/\omega T) \ll 1; \tag{1}$$

here T is the characteristic time of variation of the parameters of the medium, and $\Delta\omega$ is the characteristic scale of variation of the dielectric constant of the medium $\varepsilon(\omega)$. However, the adiabatic approximation developed in Refs. 1 and 2 has a rather limited range of validity. This is due to the use of perturbation theory and to expansion of the complex electric field in terms of the parameter η: $E = E^{(0)} + E^{(1)} + E^{(2)} + \ldots$, where

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