Renormalization group equations and thermodynamic anomalies near the tricritical point

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The behavior of matter in the vicinity of the tricritical point is studied by the renormalization group technique. Independent variables and effective coupling constants appropriate for the given problem are found. Their evolution in the fluctuation region can be described by equations of the Gell-Mann–Low type. The equations are derived in the lowest order of perturbation theory and are solved in three sectors of the phase diagram corresponding to the tricritical, crossover, and critical regimes. The susceptibility and heat capacity of the system are found as function of experimentally controllable parameters, of distance to the line of second-order phase transitions, and of the distance from the tricritical point measured along the line. The effect of higher nonrenormalized vertices on the critical thermodynamics of three-dimensional systems is considered briefly. An equation of state valid in the vicinity of the tricritical point is presented.

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1. INTRODUCTION

The purpose of the present paper is to derive the renormalization-group (RG) equations that describe the behavior of matter in the vicinity of a tricritical point (TCP) and to clarify the character of the anomalies of the thermodynamic quantities in this region.

It is known that this is not the first time that the question of the influence of fluctuations of the order parameter on the thermodynamics of a system near the TCP has been raised. In 1970, Migdal has considered the phase transition in a system in which, owing to the absence (screening) of paired forces, the principal role is played by three-particle interaction, i.e., a situation of the TCP type is realized.[1] He has summed for a six-point graph the graphs that contain the maximum number, in each order, of triply logarithmic cross sections (parquet graph), and obtained the RG equation for the effective three-particle coupling constant. It has turned out that this equation has a zero-charge solution, and one could therefore expect to obtain in thermodynamics the multiplicative logarithmic corrections for the results of the Landau theory. In explicit form these were obtained by Wegner and Riedel[2] with the aid of the Wilson recursion relations. It was observed that really the logarithmic factors appeared only in the expression for the order parameter and in the equation for the line of the second-order phase transitions, while the susceptibility, the heat capacity, the singular part of the entropy, etc. as functions of the reciprocal correlation radius $\varkappa$ were exactly as predicted by the phenomenological theory. Later Stephen, Abrahams, and Straley[3] attempted to find the thermodynamic characteristics of the system near the TCP as functions of the experimentally controllable quantities— the distance τ to the phase-transition line and the distance τ to the TCP measured along the tangent to this line. In Ref. 3 they obtained expressions for the susceptibility, heat capacity, and entropy, from which it follows that the logarithmic corrections to these quantities arise only at nonzero pair-interaction constant λ_4; on the other hand the corrections connected with the three-particle coupling constant λ_6 vanish asymptotically as the TCP is approached.

The cited papers have one common feature: the two-particle interaction $\lambda_4\varphi^4$, which usually plays the principal role, is either completely neglected in or is regarded as small enough for the leading term to become the three-particle vertex (six-point graph) γ_6. The corresponding limitation on the value of λ_4, given in Ref. 3, neglecting the logarithmic factor, takes the form

$$\lambda_4^2/\varkappa^2 \ll \lambda_6. \qquad (1)$$

Since $\varkappa = 0$ on the phase-transition line, the theory cannot be used in the vicinity of this line. Actually, its results correspond to a regime wherein we approach the TCP along a normal to the phase-transition line or along a trajectory very close to this normal. Beyond the limits of the applicability of this theory, there remains a very important and interesting case $\lambda_4 \sim \lambda_6$, to which there corresponds, in particular, the crossover of the system from the tricritical asymptotic form to the usual critical form $\varkappa \to 0$. And it is precisely this case which is realized as a rule in experiment, since it is hardly possible to end up exactly in the TCP, even when moving strictly along a normal to the phase-separation boundary.[1)]

On the other hand, if we confine ourselves to small unrenormalized λ_4 and λ_6 the variant $\lambda_4 \to \lambda_6$ should lend itself readily to a theoretical analysis in terms of the RG. The point is that here we do not encounter the traditional problem of phase transitions, namely the absence of a small parameter. In fact, the effective coupling constant λ_6, with a value $\lambda_6 \ll 1$ far from the TCP, decreases without limit as $\varkappa \to 0$, by virtue of the zero-charge character of the theory, and the dimensionless quantity $g_4 \sim \gamma_4/\varkappa$, even if it will be shown to increase with decreasing $\varkappa$, still remains small all the way to the region of the crossover, since the asymptotic form changes at $g_4 \sim \gamma_6 \ll 1$. This enables us to obtain reliable quantitative results for a model with the physical dimensionality of space $d = 3$, by operating in lowest

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order of the renormalized perturbation theory. The problem reduces only to a derivation of the corresponding RG equations.

We consider a theory of the type $\lambda_4\varphi^4+\lambda_6\varphi^6$ in the present article also for another purpose. We have in mind the study of the influence of multiparticle interactions (higher unrenormalized vertices λ_8, λ_8, ...) on the critical thermodynamics of three-dimensional systems. The RG equations derived below will enable us to obtain definite information also on this question. It should be noted that the RG equations for the vertices λ_4 and λ_6 have already been derived in Ref. 4. That paper, unfortunately, is not free of errors of both principle and computational character. We therefore consider this question here anew. Our results will differ quite substantially from the results obtained in Ref. 4.

The plan of the article is the following. In Sec. 2 we discuss graphic expansions for the four-point and six-point diagrams, introduce independent variables that are appropriate for this problem, as well as effective constants, and also derive the RG equations. Section 3 is devoted to a solution of these equations. Besides the general case, we consider here limiting regimes corresponding to three regions: tricritical, crossover, and critical. In Sec. 4 we investigate the behavior of the susceptibility x of the system and of the singular part of its heat capacity C in the three regions indicated above. It is shown here, in particular, that fluctuations of the order parameter lead to the appearance of additive corrections, containing logarithmic factors, to x and C in the tricritical region. These corrections differ from zero only at $\lambda_4\neq 0$, but have a somewhat different form than those obtained in Ref. 3. Finally Sec. 5 is devoted to an analysis of the influence of the upper vertices on the thermodynamics of the three-dimensional system near the critical point and to a discussion of the form of the equation of state in the vicinity of TCP.

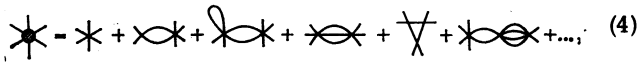
2. ULTRAVIOLET GRAPHS, AUXILIARY SIX-POINT DIAGRAM, AND RG EQUATIONS

Thus, we consider a system with an effective fluctuation Hamiltonian of the form

$$H=\int dx\left\{\frac{1}{2}(\nabla\varphi_\alpha)^2+\frac{\varkappa_0^2}{2}\varphi_\alpha^2+\frac{\bar\lambda_4}{8}(\varphi_\alpha^2)^2+\frac{\lambda_6}{48}(\varphi_\alpha^2)^3\right\}. \tag{2}$$

Here $\varphi_\alpha(\mathbf{x})$ is the n-component field of fluctuations of the order parameter, and $\varkappa_0^2$, $\bar\lambda_4$, and λ_6 are assumed to be linear functions of the temperature T and, say, of the pressure P. The thermodynamics of our system is determined in the fluctuation region by the character of the dependences on T and P of the total 1-irreducible vertices $\Gamma_{\alpha\beta\gamma\delta}(\mathbf{q},\mathbf{q}',\mathbf{q}'')$ and $\Gamma_{\alpha\beta\gamma\delta\mu\nu}(\mathbf{q},\ldots,\mathbf{q}'''')$, taken at zero momenta. For these vertices we can write the standard graphic expansions:

(3)



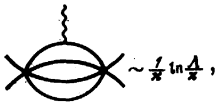
(4)

where the internal lines correspond to exact Green's functions $G_{\alpha\beta}(\mathbf{q})=\langle\varphi_\alpha(\mathbf{q})\varphi_\beta(-\mathbf{q})\rangle$. Since the anomalous dimensionality of the field of the fluctuations η in the problem with the Hamiltonian (2) is small, we shall assume henceforth that $G_{\alpha\beta}(\mathbf{q})$ is given by the simple pole equation:

$$G_{\alpha\beta}(\mathbf{q})=\delta_{\alpha\beta}/(\varkappa^2+q^2). \tag{5}$$

We note that in the lowest-order and the following approximations in the physical charges this formula is valid also at values of the index η which are not small.

The evolution of the vertices in the course of a variation of T and P is described by the RG equations. The technique of their derivation for problems in which there is one essential vertex or several vertices of one and the same dimensionality is described in detail in Refs. 5–8. Near the TCP, however, two vertices with different scale dimensionalities are essential, so that a direct application of the methods of Refs. 5–8 does not yield in our case the desired results. To obtain an idea of the character of the difficulties that arise here, let us consider, for example, that term which is generated in the expansion of the Gell-Mann–Low (GML) function for the four-point diagram, after suitable renormalizations by the sixth graph in (3). It is easy to show that[2)]



(6)

where Λ is the cutoff momentum. If we now change over, as is usually done, to dimensionless vertices, then the first factor in (6) vanishes, but $\ln(\Lambda/\varkappa)$ remains, and the GML function turns out to be dependent not only on the dimensionless physical charges g_4 and g_6, but also on the renormalization-group variable $t=-\ln\varkappa$. This case is not unique and it is possible to point out other skeleton graphs in the expansions of GML functions, which contain "extra" logarithms.

The second problem consists in the following. We know that in ordinary theory of critical phenomena there is only one essential vertex—the total four-point diagram. The evolution of this vertex is described by the RG equation, and the multiparticle correlations are determined by the effective paired constant and do not play an independent role in the theory. It follows therefore in our case, where generally speaking two physical charges γ_4 and γ_6 are essential and accordingly there are two equations of the GML type, that in the limit $\lambda_6\to 0$ there should remain only one independent GML equation for γ_4. At the same time, the total six-point diagram that characterizes triple correlations in the system at $\lambda_6=0$ is on the one hand different from zero and on the other hand is not expressed in finite form in terms of the two-particle vertex. As a result we cannot trace distinctly the manner in which the RG equation for γ_6 reduces as $\lambda_6\to 0$ to an equation containing actually only γ_4. This complicates greatly the interpretation of the results and, furthermore, makes a theory that involves a complete six-point diagram very inconvenient from the computational point of view.

We turn first to the first of the indicated difficulties. It can be eliminated by choosing in suitable fashion the independent variables in the problem under consideration. We note to this end that all the diagrams in the expansions of the vertex parts break up into two categories—ultraviolet and infrared. By ultraviolet we mean diagrams whose analytic expressions contain integrals that diverge like some finite power of the cutoff momentum Λ as $\Lambda \to \infty$. In the expansion (3), for example, the ultraviolet graphs are the second, fourth, and sixth, while in expression (4) they are the third and sixth. All the remaining graphs, including those containing logarithmic integrals as the most strongly diverging factors, will be called infrared. Obviously, the fluctuation effects and the thermodynamic anomalies associated with them are due precisely to the infared skeleton diagrams for the vertices, while the ultraviolet diagrams or, more accurately their ultraviolet internal blocks, describe trivial renormalizations of the coupling constants, which do not take us beyond the scope of the Landau theory. In such a situation it is natural to attempt to reconstruct the theory in such a way so as to cause the ultraviolet diagrams to vanish, and to retain in the expansions of $\Gamma_{\alpha\beta\gamma\delta}$, $\Gamma_{\alpha\beta\gamma\delta\mu\nu}$, etc. only the infrared diagrams. This can be done by excluding all the ultraviolet blocks in the bare vertices, i.e., by simply redefining the unrenormalized coupling constants. In a theory with a non-polynomial Hamiltonian such a transformation would affect the unrenormalized vertices of all orders. In our case, however, all that has to be redefined is the constant $\tilde{\lambda}_4$, since the ultraviolet vertex blocks can have here not more than four ends each. The new unrenormalized vertex λ_4 will be specified by the following expansion:

(7)

We shall take it to be the independent variable. We note immediately that the functional connection between the parameters λ_4 and $\varkappa$, expressed by formula (7), does not prevent from assuming them to be independent. To this end it suffices to assume that with decreasing $\varkappa$ we approach the phase-transition line along trajectories on which $d\lambda_4/d\varkappa = 0$. This question will be discussed in greater detail below.[3)]

The choice of λ_4 as the independent variable automatically solves the problem of the "extra" logarithms. The point is that the sources of these logarithms are graphs containing internal blocks with large numbers of equivalent lines. But it is precisely these blocks which turn out to be ultraviolet. As a result, replacement of $\tilde{\lambda}_4$ by λ_4 leads to a vanishing of all the dangerous diagrams. The following should be noted in this connection. It is easily seen that the transition from the variables $\varkappa_0^2$ and $\tilde{\lambda}_4$ to the variables $\varkappa^2$ and λ_4 does not reduce by far to only a transfer of the origin and to a rotation of the coordinate axes on the phase diagram of this system. The parameters $\varkappa^2$ and λ_4 are complicated nonlinear functions of $\varkappa_0^2$ and $\tilde{\lambda}_4$: in our field-theoretical approach they play the same role as the "nonlinear scaling fields" of Wegner and Riedel[9-11] in the method of the Wilson renormalization semigroup. It is therefore not surprising that the described change of variables affects quite substantially the properties of the graphic expansions for the GML functions.[4)]

We consider now the second problem, i.e., which field-theoretical object characterizing the effect of three-particle interaction might be the most appropriate for our problem. It is known that we have no exact relations connecting the complete six-point diagram with the complete four-point diagram at $\lambda_6 = 0$. It is therefore convenient to choose as a basis a three-particle vertex that simply vanishes in this limit. We define it in the following manner. Let $\Gamma_6(q, q', q'', q''', q'''')$ be the complete six-point diagram from which we separate the tensor factor

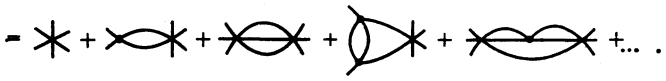
$$\Gamma_{\alpha\beta\gamma\delta\mu\nu}(q, \ldots, q'''') = \Gamma_6(q, \ldots, q'''')\,(\delta_{\alpha\beta}\delta_{\gamma\delta}\delta_{\mu\nu} + 14 \text{ permutations}). \qquad (8)$$

We introduce into consideration the vertex $\Gamma^{(4)}_{\alpha\beta\gamma\delta\mu\nu}(q, \ldots, q'''')$ (denoting the corresponding scalar function by $\Gamma_6^{(4)}(q, \ldots, q''')$), which constitutes the sum of all the six-point diagrams made up of the exact propagators $G_{\alpha\beta}(q)$, using only the unrenormalized vertices λ_4:

(9)

Then the object of interest to us is by definition

$$\gamma_6(q, \ldots, q'''') = \Gamma_6(q, \ldots, q'''') - \Gamma_6^{(4)}(q, \ldots, q'''')$$



(10)

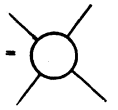
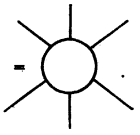
We shall name $\gamma_6(q, \ldots, q'''')$ the auxiliary six-point diagram. It is easily seen, any of the diagrams in the expansion of $\gamma_6(q, \ldots, q'''')$ contains at least one unrenormalized vertex λ_6. Therefore in a theory of the $\lambda_4\varphi^4$ type, i.e., at $\lambda_6 = 0$, the auxiliary six-point diagram is identically equal to zero. At the same time, as $\lambda_4 \to 0$ the total vertex $\Gamma_6(q, \ldots, q'''') \to \gamma_6(q, \ldots, q'''')$, and the auxiliary six-point diagram turns out to be precisely the object that determines the thermodynamics of the system in the tricritical region.[5)]

A deeper insight into the nature of the vertex $\gamma_6(q, \ldots, q'''')$ can be gained by turning to the language of operator algebra.[13,14] We note that in the fluctuation region any thermodynamic quantity can be represented in the form of Gibbs mean values of certain combinations of field operators φ_α and $\nabla_i\varphi_\alpha$, each of which has its own scale dimensionality. If the theory involves only the unrenormalized vertex λ_4, then the behavior of all the vertex parts in the region of strongly developed fluctuations is determined by a single nontrivial dimensionality—the dimensionality of the operator of the type[6)] φ^4. On the other hand, if the Hamiltonian of the interaction contains besides $\lambda_4\varphi^4$ also a term of the type $\lambda_6\varphi^6$, then the expressions for the

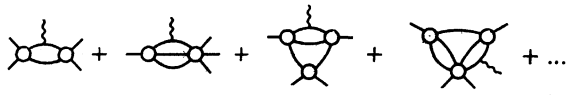
total vertices acquire additional terms that are characterized by their own independent dimensionality ω. This dimensionality, calculated on a Gaussian basis, determines the tricritical behavior of the system and the same dimensionality, but calculated on the basis of the type $\varkappa_0^2\varphi^2+\lambda_4\varphi^4$, describes the temperature dependence of the scaling corrections necessitated by the three-particle interaction. It is to the operator of the φ^6 type, with dimensionality ω, to which the additional six-point diagram introduced above in terms of Feynman diagrams obviously corresponds.

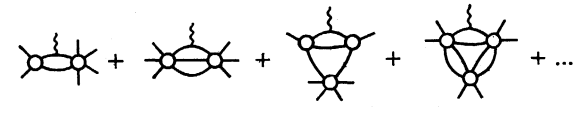
Thus, we now have everything necessary for the derivation of the RG equations. The physical charges γ_4 and γ_6 in our problem are defined by the following relations:

$$\Gamma_{\alpha\beta\gamma\delta}(\mathbf{q},\mathbf{q}',\mathbf{q}'')=\Gamma_4(\mathbf{q},\mathbf{q}',\mathbf{q}'')(\delta_{\alpha\beta}\delta_{\gamma\delta}+\delta_{\alpha\gamma}\delta_{\beta\delta}+\delta_{\alpha\delta}\delta_{\beta\gamma}), \tag{11}$$

$\gamma_4=\Gamma_4(0,0,0)$ =  , $\gamma_6=\gamma_6(0,\dots 0)$ =  . (12)

Differentiating the expansions (3) and (10) with respect to $\varkappa^2$ and taking (7) into account, and dressing the bare coupling constants, we obtain

$\frac{\partial\gamma_4}{\partial\varkappa^2}$ =  (13)

$\frac{\partial\gamma_6}{\partial\varkappa^2}$ =  , (14)

where subtraction at zero momentum transfer is implied in the internal 1-irreducible six-point block of the fourth diagram in (13). In expansion (14) there are no diagrams containing only the charge γ_4, since the effective coupling constant γ_6 was generated not by the total but the auxiliary six-point diagram. Bearing in mind the study of the TCP and its vicinity, where the charges γ_4 and γ_6 are small, we retain in (13) and (14) only terms of lowest (second) order in the charges. The corresponding RG equations can be expressed analytically in the form

$$\frac{\partial\gamma_4}{\partial\varkappa^2}=\frac{n+8}{32\pi\varkappa^3}\gamma_4^2+\frac{n+4}{16\pi^2\varkappa^2}\gamma_4\gamma_6, \tag{15}$$

and

$$\frac{\partial\gamma_6}{\partial\varkappa^2}=\frac{3(n+14)}{32\pi\varkappa^3}\gamma_4\gamma_6+\frac{3n+22}{32\pi^2\varkappa^2}\gamma_6^2. \tag{16}$$

It is convenient next to go over to the dimensionless physical charges

$$g_4=\frac{n+8}{16\pi\varkappa}\gamma_4, \quad g_6=\frac{3n+22}{16\pi^2}\gamma_6 \tag{17}$$

and to the logarithmic derivative $t=-\ln\varkappa$. The final forms of the RG equations of our problem are

$$\partial g_4/\partial t=g_4-g_4^2-a(n)g_4g_6, \qquad \partial g_6/\partial t=-b(n)g_4g_6-g_6^2, \tag{18}$$

where

$$a(n)=2(n+4)/(3n+22), \qquad b(n)=3(n+14)/(n+8). \tag{19}$$

The solution of Eqs. (18) will be dealt with in the next section. For the time being we call attention to one very important property of these equations. We have in mind the absence from these equations of generation terms. In fact, the right-hand side of the equation for g_4 does not contain a term proportional to g_6^2, while the right-hand side of the equation for g_6 has no terms of the type g_4^2. This property is not a consequence of the employed approximation; it can be shown, by analyzing the graphic expansions for the GML functions, that it is preserved in any order of perturbation theory. Thus, in the absence of some particular bare vertex in the problem, the corresponding physical charge is identically equal to zero, and the remaining RG equation describes directly, i.e., without any modifications, the proper "pure"—critical or tricritical—regime. This property of the theory makes it very convenient both technically and from the point of view of the interpretation of the results.

We note that the RG equations obtained by Gorodetskiĭ and Zaprudskiĭ[4] contain generation terms. The presence of such a term in their equation for g_4 is due to the inappropriate choice of the independent variables. (The extent to which this choice is inappropriable is clearly illustrated by the fact that the system of the RG equations in terms of these variables turns out to be nonconservative, provided, of course, that all the diagrams for the GML functions are calculated. This means in fact that there simply are no RG equations for the physical charges as functions of the variables of Ref. 4.) On the other hand, the presence of a generation term in the equation for the charge g_6, which in Ref. 4 is taken to mean not the auxiliary but the total three-particle coupling constant, might have been regarded as perfectly valid were it not for one circumstance. The point is that this term is proportional to g_4^3, and allowance for this term within the framework of the assumed approximation scheme is inconsistent.

3. SOLUTION OF THE RG EQUATIONS

We proceed to solve our RG equations. The system (18) can be integrated exactly. By making several variable changes it reduces to a general Riccati equation with a zero sum of the coefficients $f(x)$, $g(x)$ and $h(x)$, and with an integral that is known. The final result is

$$t=\int dy\left[1-\frac{b(n)}{b(n)-1}\frac{e^y y^{1-a(n)}}{C'-e^{-i\pi a(n)}\gamma(1-a(n),-y)}\right]^{-1}+C'', \tag{20}$$

for $y=1/g_6$, C' and C'' are integration constants, while $\gamma(\alpha,x)$ is the incomplete gamma function. Equation (20), being in principle the solution of the problem, has in fact a complicated structure that yields very little information. Therefore instead of analyzing the obtained solution in the general form, it is more convenient to turn directly to consideration of the most characteristic particular cases. We begin with the region where $g_4 \ll g_6$. It is natural to call it tricritical.

Tricritical region. The RG equations take here the form

$$\partial g_4/\partial t = g_4 - a(n) g_4 g_6, \quad \partial g_6/\partial t = -g_6^2. \tag{21}$$

Solving this system and returning to the variable $\varkappa$, we get

$$g_6 = \frac{g_6^{(0)}}{1-g_6^{(0)}\ln\varkappa}, \quad g_4 = \frac{g_4^{(0)}}{\varkappa(1-g_6^{(0)}\ln\varkappa)^{a(n)}}. \tag{22}$$

The cutoff momentum was taken here for simplicity equal to unity, while $g_4^{(0)}$ and $g_6^{(0)}$ are the values of g_4 and g_6 at the point $\varkappa = 1$. At this point we join together the formulas of the fluctuation theory and the results of the Landau theory, so that

$$g_4^{(0)} = \frac{n+8}{16\pi}\lambda_4, \quad g_6^{(0)} = \frac{3n+22}{16\pi^2}\lambda_6. \tag{23}$$

Rewriting the second equation of (22) in the form

$$\gamma_4 = \lambda_4/(1-g_6^{(0)}\ln\varkappa)^{a(n)}. \tag{24}$$

It is easy to verify that the renormalizations of both charges in the tricritical region has a purely logarithmic character, as should be the case in the band where the six-point diagram is the leading one. We note that expressions (22) and (24) coincide with their analogs obtained in Ref. 3 by summation of a three-dimensional parquet diagram.

Crossover region. In this region $g_4 \sim g_6 \ll 1$; therefore we take the RG equations in the form

$$\partial g_4/\partial t = g_4, \quad \partial g_6/\partial t = -b(n) g_4 g_6 - g_6^2. \tag{25}$$

It is seen that we use here for g_4 a cruder approximation than in the tricritical region. This approximation, however, is perfectly correct under the assumed restrictions on the values of g_4 and g_6, since the possible $a(n)$ do not exceed $2/3$. On the other hand, inclusion of the small term $a(n)g_4g_6$ in the first equation of (21) is explained by the fact that it is precisely this term which is responsible for the logarithmic renormalizations of g_4, which are of principal interest in the tricritical region.

The solution of the system (25) entails no difficulty whatever. The result is of the form

$$g_4 = g_4^{(0)}/\varkappa, \qquad g_6 = \frac{\exp[-b(n)g_4]}{g_6^{(0)-1}\exp[-b(n)g_4^{(0)}] + \mathrm{Ei}[-b(n)g_4] - \mathrm{Ei}[-b(n)g_4^{(0)}]}, \tag{26}$$

where $\mathrm{Ei}(x)$ is the integral exponential function. The expression obtained for g_6 can be simplified by recognizing that in the crossover region we have $b(n)g_4 \ll 1$. In fact, the transition from one asymptotic form to another takes place, as is seen from (25), at $b(n)g_4 \approx g_6$, and we regard g_6 as a quantity small compared with unity. From this we easily obtain that

$$g_6 \approx g_6^{(0)}\left[1 - b(n)\frac{g_4^{(0)}}{\varkappa}\right] \Big/ [1 - b(n)g_4^{(0)} - g_6^{(0)}\ln\varkappa]. \tag{27}$$

If $g_4^{(0)}$ is not too small, then the change of the asymptotic forms takes place relatively rapidly (at not too small values of $\varkappa$), and $\ln\varkappa$ in (21) does not have time to become large. In this case $g_6 \approx g_6^{(0)}$, i.e., the renormalizations of g_6 in the crossover region are inessential. On the other hand if the situation is close to tricritical, namely $|\ln[b(n)g_4^{(0)}]| \gg 1$, then the logarithmic renormalizations become noticeable, and expression (27), for the three-particle charge reduces to the first equation of (22).

Having expressions for g_4 and g_6 in the crossover region, we can find the so-called critical exponent (crossover exponent) ψ. Let $\varkappa_c$ be the characteristic value of $\varkappa$ at which the system goes over from the tricritical to the critical regime. Then the exponent ψ is determined by the relation $\varkappa_c \sim \lambda_4^{\psi}$. Substituting (23) and (26) in the condition for the asymptotic crossover $b(n)g_4 = g_6$, we can easily establish that $\psi = 1$. Allowance for the logarithmic renormalizations of g_4 and g_6 does not change the obtained value of ψ, but the very relation that connects $\varkappa_c$ with λ_4 is somewhat modified: $\varkappa_c \sim \lambda_4|\ln\lambda_4|^{1-a(n)}$. We present also the value of the crossover exponent in the case when the width of the critical region is measured not in units of the reciprocal correlation radius $\varkappa$, but in units of τ. Since $\varkappa \sim \tau^{\nu}$, where ν is the exponent of the correlation radius, the crossover exponent is here equal to $1/\nu$.

Critical region. In this region the principal role is played by pair interaction, i.e., $g_6 \ll g_4$. To remain within the framework of perturbation theory it is necessary, in principle, to impose the condition that the charge g_4 itself be small, $g_4 \ll 1$. This limitation, however, immediately obstructs the investigation of the vicinity of the second-order phase-transition line, where, as is well known, $g_4 \sim 1$. Yet it is precisely this region of the phase diagram which is of fundamental interest. It is therefore most desirable to relax somewhat the restriction on g_4. Fortunately, there are at present grounds for such a step. In fact, we know that in the three-dimensional theory of the $\lambda_4\varphi^4$ type the lowest approximation of the renormalized perturbation theory yields perfectly satisfactory results, both qualitative[8,15] and quantitative[6,7] (we have in mind the values of the critical exponents). A recent summation of the asymptotic series for the GML functions of three-dimensional models,[16,17] in which the highest terms were calculated with the aid of the Lipatov method,[18] has shown that the success of perturbation theory at $d = 3$ is apparently no accident. The point is that the first two terms of the expansion powers of g_4 approximate sufficiently well the exact GML function not only at $g_4 \ll 1$, but in the entire interval $0 < g_4 \lesssim 1$; in particular, the coordinate of the stable fixed point obtained in the lowest approximation in g_4 differs from its exact value by less than 30%. This fact enables us to relax the restriction imposed on the value of g_4, and assume henceforth $g_4 \lesssim 1$. One can then expect the results obtained on the basis of Eqs. (18) to be at least qualitatively correct.

Taking the foregoing into account, we can express the system of RG equations in the critical region in the form

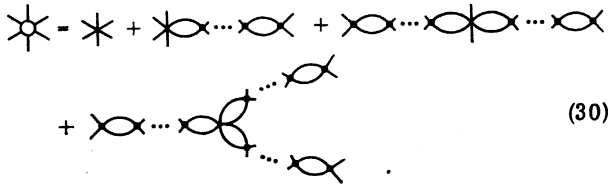
$$\partial g_4/\partial t = g_4 - g_4^2, \quad \partial g_6/\partial t = -b(n) g_4 g_6. \tag{28}$$

Its solution is

$$g_4 = \frac{g_4^{(0)}}{g_4^{(0)} + \varkappa(1-g_4^{(0)})}, \quad g_6 = g_6^{(0)}\left[\frac{\varkappa}{g_4^{(0)} + \varkappa(1-g_4^{(0)})}\right]^{b(n)}. \tag{29}$$

As $\varkappa \to 0$ the charge g_4, as can be easily seen from (29), takes on its asymptotic value unity, and the auxiliary six-point diagram vanishes like $\varkappa^{b(n)}$. Since the vertex $\Gamma_6^{(4)}(0,\ldots,0)$ at $d=3$ is dimensionless (neglecting the exponent η), it follows therefore that the dimensionality of the corrections to scaling, necessitated by the three-particle interaction, is equal to $\nu b(n)$. If we change over now to the more customary variable $\tau \sim \varkappa^{1/\nu}$, then the exponent of these corrections ω becomes equal to $\nu b(n)$. In the lowest order in g_4 we have $\nu = (n+8)/(n+14)$ (Ref. 6); consequently, in this approximation the exponent is $\omega = 3$. Thus, the contribution of the three-particle forces to the thermodynamics of our system decreases quite rapidly as the second-order phase-transition line is approached, and these forces are insignificant in the critical region.

The last result was obtained within the framework of a method which we known not to be rigorous. It is therefore desirable to verify it with some exactly solvable model as an example. For this purpose we calculate the dimensionality ω of the auxiliary six-point diagram in the limit as $n \to \infty$. At $g_6^{(0)} \ll 1$, in the zeroth approximation in $1/n$, the vertex g_6 is equal to a sum of graphs of the following form:



(30)

Summation of this triple ladder leads to the expression

$$g_6 = g_6^{(0)} \varkappa^3 / (\varkappa + g_4^{(0)})^3 \underset{\varkappa \to 0}{\to} g_6^{(0)} \varkappa^3. \tag{31}$$

Since $\nu = 1$ for the spherical model, the critical exponent ω turns out here to be equal to three. Thus, the quantity ω, which is exact at $n \to \infty$, coincides with the value obtained by the lowest approximation of the RG method at $d=3$. This fact confirms the reliability of the results given above.

To conclude this section, we present the phase diagram of the system of RG equations (18), which illustrates clearly the behavior of the effective coupling constants g_4 and g_6 in the fluctuation region. On this diagram (see the figure) there are two fixed points, saddle and stable node, which describe respectively

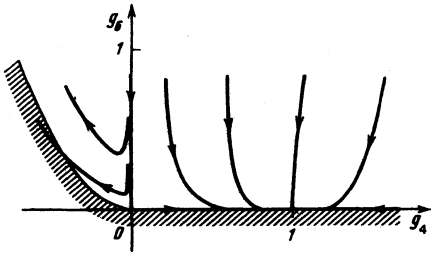


FIG. 1. Picture of the phase trajectories of the RG system of equations (18). The shaded area is the region of values of the charge g_4 and g_6 at which the disordered phase is thermodynamically unstable.

the tricritical and critical regimes. The shaded line bounds the region of values of g_4 and g_6 at which the phase without a condensate is thermodynamically stable within the framework of the employed approximation. At $g_4 < 0$ this curve is given by the equation

$$g_6 = 6 g_4^2 (3n+22)/(n+8)^2; \tag{32}$$

it can be easily obtained by analyzing the expression for the free energy of the considered system. As seen from the figure, the tricritical regime is realized only at $g_4^{(0)} = 0$. A second-order phase transition takes place in the system at $g_4^{(0)} > 0$, a first-order transition at $g_4^{(0)} < 0$, and the condensate drops out at a certain finite value of $\varkappa$.

We note that the system (18) has one more fixed point in addition to those shown in the figure. This point is a stable focus and its coordinates are

$$g_4 = -\frac{(n+8)(3n+22)}{3n^2+62n+160}, \quad g_6 = \frac{3(n+14)(3n+22)}{3n^2+62n+160}. \tag{33}$$

Since, however, this point lies far beyond the limits of the region where perturbation theory is applicable to any degree, it hardly pays to discuss seriously the corresponding critical-behavior regime.[7)]

4. SUSCEPTIBILITY AND HEAT CAPACITY

In this section we study the behavior the susceptibility and of the singular part of the heat capacity of our system in the disordered phase. These quantities will first be obtained as functions of the renormalization-group variables, after which we shall change over in succession from $\varkappa$ to τ and from $g_4^{(0)}$ to λ.

The susceptibility is expressed in the usual manner in terms of the propagator

$$\chi = G(0) \cong 1/\varkappa^2, \tag{34}$$

while $\varkappa^2$ is expressed in terms of the mass operator $\Sigma(\varkappa_0^2, q)$:

$$\varkappa^2 = \varkappa_0^2 - \Sigma(\varkappa_0^2, 0). \tag{35}$$

In the graphs for $\Sigma(\varkappa^2, 0)$ it is convenient to carry out the subtractions at the phase-transition point, i.e., to change over to the mass operator

$$\Sigma(\varkappa^2, q) = \Sigma(\varkappa^2, q) - \Sigma(0, q), \tag{36}$$

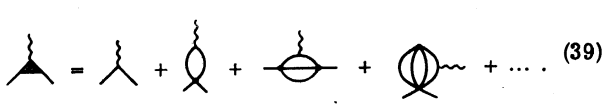
after which the formula for $\varkappa^2$ becomes

$$\varkappa^2 = \tau - \Sigma(\varkappa^2, 0). \tag{37}$$

Here $\tau = \varkappa_0^2 - \Sigma(0,0)$ is the linear measure of the distance to the second-order phase-transition line, and $\tau = 0$ at $T = T_c$. To find χ as a function of τ and $g_4^{(0)}$, we differentiate $\varkappa^2$ with respect to τ (under the assumption that $g_4^{(0)} \sim \lambda_4$ does not depend on τ):

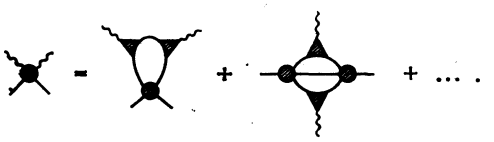
$$\partial \varkappa^2 / \partial \tau = 1 - \partial \Sigma / \partial \tau = T(\varkappa, 0). \tag{38}$$

The three-point diagram $T(\varkappa, q)$ is represented in the form of the following graphic series in the bare lines and coupling constants:



(39)

One more differentiation with respect to τ makes it possible to dress the propagators and vertices and obtain the renormalized graphic expansion for $\partial T(\varkappa, q)/\partial\tau$:



(40)

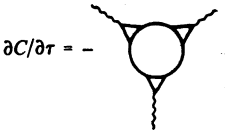
It is known that in the lowest order of perturbation theory the vertices at finite momenta coincide with the corresponding dressed charges. Consequently, the derivative of $\partial T(\varkappa, 0)/\partial\tau$ is given in this approximation by the first graph of (40), where the shaded vertex is replaced by the charge γ_4. In analytic form, the equation for $T(\varkappa) \equiv T(\varkappa, 0)$ is

$$\frac{\partial T(\varkappa)}{\partial\tau} = \frac{n+2}{32\pi\varkappa^3} T^2(\varkappa)\gamma_4. \tag{41}$$

Using (38) and (17), it can be rewritten in the form

$$\frac{\partial \ln T(\varkappa)}{\partial t} = -\frac{n+2}{n+8} g_4. \tag{42}$$

Solving (42) for $T(\varkappa)$, we can obtain with the aid of (38) the susceptibility above the phase-transition point. As to the heat capacity, in the assumed approximation it is determined by the single graph:



(43)

Therefore, knowing the forms of the functions $T(\varkappa)$ and $\varkappa(\tau)$, we can easily obtain also the functions $C(\tau)$.

We thus calculate the susceptibility in the tricritical region. Substituting the second equation of (22) in (42) and recognizing that $g \ll g_6^{(0)} \ll 1$, we obtain

$$T(\varkappa) \cong 1 - \frac{n+2}{n+8} \frac{g_4^{(0)}}{\varkappa(1-g_6^{(0)} \ln \varkappa)^{a(n)}}. \tag{44}$$

The integration constant is chosen here such that the Curie-Weiss constant is equal to unity in the region where the Landau theory is valid. Combining next (44), (38) and (34), we obtain

$$\chi = \frac{1}{\tau}\left[1 + 2\frac{n+2}{n+8} \frac{g_4^{(0)}}{\tau^{1/2}(1-g_6^{(0)} \ln \tau^{1/2})^{a(n)}}\right]. \tag{45}$$

For the heat capacity, the graph (43) yields

$$\partial C/\partial\tau = -nT^3(\varkappa)/32\pi\varkappa^3. \tag{46}$$

Substituting here the relations obtained for $T(\varkappa)$ and $\varkappa(\tau)$ we get

$$C = \frac{n}{16\pi\sqrt{\tau}} + O\left(\frac{g_4^{(0)2}}{\tau(1-g_6^{(0)} \ln \tau^{1/2})^{2a(n)}}\right). \tag{47}$$

Thus, when account is taken of the fluctuation of the order parameter in the tricritical region the expressions for $\chi(\tau)$ and $C(\tau)$ acquire additive corrections that contain logarithmic factors. These corrections are similar in form to their analogs obtained in Ref. 3, but it is easily verified that the structure and origin of the two types of correction terms are in some cases substantially different. In fact, the first correction to the susceptibility, obtained in Ref. 3, is proportional to $(g_4^{(0)}/\tau^{1/2})^2$, and the logarithmic factor contained in it is due in part to the logarithmic character of the corresponding bare diagram for the mass operator. In our case, on the other hand, the correction to $\chi(\tau)$ appears already in the first order in $g_4^{(0)}/\tau^{1/2}$, and its logarithmic character is exclusively the consequence of the logarithmic behavior of the charge g_4. Obviously at $g_4^{(0)}/\tau^{1/2} \ll 1$, i.e., in the region where perturbation theory is valid, the principal correction should be taken to be the one obtained in the present paper. At the same time, the approach proposed here makes it possible to reproduce also the result of Ref. 3. To this end it suffices to examine the graphs of second order in g_4 in the expansion of type (40).

In the case of the heat capacity, the correction terms obtained above and those obtained in Ref. 3 are of the same structure. The authors of Ref. 3, however, did not notice that the coefficient of the first-approximation correction for $C(\tau)$ is identically equal to zero, and therefore the principal correction factor is proportional to $G_4^{(0)2}$, and not to $g_4^{(0)}$ as stated in their paper.

We discuss next the form of the functions $\chi(\tau)$ and $C(\tau)$ in the crossover region. If the bare vertex $g_4^{(0)}$ is not too small and the crossover between the asymptotic forms occurs at small values of $|\ln\varkappa|$, then the fluctuation renormalizations of the charges γ_4 and γ_6 in the considered band are small. Therefore the expressions for $\chi(\tau)$ and $C(\tau)$ differ in this case from those obtained in the Landau theory only by trivial correlation corrections proportional to $g_4^{(0)}/\tau^{1/2}$. On the other hand if the asymptotic crossover takes place at large logarithms, then the renormalizations of the charges become substantial, and the expressions for $\chi(\tau)$ and $C(\tau)$ must be sought in a manner similar to that used in the transcritical region. The resultant formulas then coincide with expressions (45) and (47).

The calculation of $\chi(\tau)$ and $C(\tau)$ in the third—critical—region entails no difficulty in principle, but we must bear in mind here one circumstance. As we have seen, the three-point diagram $T(\varkappa)$, in the assumed approximation, does not depend explicitly on the charge g_6, and therefore, if we wish to ascertain how the three-particle forces influence the character of the thermodynamic anomalies, we must first find the correction that must be made to the charge g_4 because of the unrenormalized coupling constant $g_6^{(0)}$. For this purpose it suffices to add to the right-hand side of the first equation of (28) the term $-a(n)g_4g_6$, and to solve this equation under the condition $|1-g_4| \sim g_6 \ll 1$. The result is

$$g_4 \cong 1 + \frac{a(n)}{b(n)-1} g_6^{(0)} \left(\frac{\varkappa}{g_4^{(0)}}\right)^{b(n)}. \tag{48}$$

Substitution of (48) in (42) yields

$$T(\varkappa) = A\varkappa^{(n+2)/(n+8)} \exp[c(n) g_6^{(0)} (\varkappa/g_4^{(0)})^{b(n)}], \tag{49}$$

where

$$c(n) = \frac{(n+2)(n+4)(n+8)}{3(n+14)(n+17)(3n+22)}, \tag{50}$$

and A is an integration constant. Since the argument over the exponential in (49) is small in the investigated region,[8)] it can be expanded in a series:

$$T(\varkappa)\cong A\varkappa^{(n+2)/(n+8)}[1+c(n)g_6^{(0)}(\varkappa/g_4^{(0)})^{b(n)}]. \tag{51}$$

Using this expression for $T(\varkappa)$, we easily obtain

$$\chi(\tau)\sim\tau^{-2\nu}(1-\mu\tau^3), \tag{52}$$

and

$$C(\tau)\sim\tau^{(n-4)/(n+14)}\left[1+\frac{3(n-4)(2n+25)}{2(n+8)(2n+19)}\mu\tau^3\right], \tag{53}$$

where

$$\mu=A^3c(n)b^2(n)g_6^{(0)}/144g_4^{(0)b(n)}, \quad \nu=3/b(n). \tag{54}$$

Thus, allowance for the three-particle forces has brought about the appearance of specific corrections to scaling, the exponent of which is universal, and the amplitudes depend on the nature of the investigated thermodynamic quantities and on the atomic constants of the material. As expected, the amplitudes of the corrections turned out to be proportional to $g_6^{(0)}\sim\lambda_6$, and their exponent coincided with the one obtained in the preceding section.

We now have thus equations for the susceptibility and heat capacity in all the regions of interest to us. These equations, however, still do not provide the final solution of our problem. The point is that all were obtained under the condition $d\lambda_4/d\varkappa=0$, and consequently they are valid only in those cases when we approach the phase-transition line on trajectories along which $\lambda_4(\varkappa,\tilde\lambda_4,\lambda_6)=\text{const}$. To find the final answer we must obviously establish the forms of these trajectories, i.e., to solve the equation $\lambda_4=\text{const}$, say with respect to $\tilde\lambda_4$. But to this end it is necessary to know λ_4 as a function of its arguments. We shall find λ_4 in the lowest order of perturbation theory. From the expansion (7) we easily obtain

$$\lambda_4=\tilde\lambda_4+\frac{n+4}{4\pi^2}\lambda_6\left(1-\varkappa\operatorname{arctg}\frac{1}{\varkappa}\right). \tag{55}$$

(The cutoff momentum, as before is equal to unity.) Inasmuch as we chose the point $\varkappa=1$ for the matching to the Landau theory, it is convenient to choose as the linear measure of the distance to the TCP not $\tilde\lambda_4$ but the parameter

$$\lambda=\tilde\lambda_4+\frac{n+4}{4\pi^2}\lambda_6\left(1-\frac{\pi}{4}\right), \tag{56}$$

which vanishes at $\lambda_4(\varkappa=1)=0$. The phase-diagram trajectories, the evolution of the charges along which is described by the RG equations (18), are then specified by the relation

$$\lambda=\lambda_4+\frac{n+4}{4\pi^2}\lambda_6\left(\varkappa\operatorname{arctg}\frac{1}{\varkappa}-\frac{\pi}{4}\right). \tag{57}$$

With the aid of (34) and equations such as (45) and (52) we can change over in (57) from $\varkappa$ to τ. As a result we obtain the equation of the sought trajectories for the case when the metric of the phase diagram is linear with respect to both coordinates. In the critical region, for example, this equation takes the form

$$\lambda=\lambda_4+\frac{n+4}{16\pi}\lambda_6(B\tau^\nu-1), \tag{58}$$

where B is a certain constant.

Relation (57) is valid, strictly speaking, not at all values of $\varkappa$. The point is in a very narrow vicinity of the phase-transition line, where $|\lambda_6\ln\varkappa|\gtrsim 1$, it is necessary to take into account in the expansion (7), even at small $\tilde\lambda_4$ and λ_6, diagrams of higher order containing large logarithms. The diagrams that are principal in each order can obviously be summed in this case, as a result of which a logarithmic factor appears in front of that term of (57) which is proportional to $\varkappa$. Allowance for this factor will lead to a certain deformation of our trajectories in an exponentially narrow band adjacent to the second-order transition line.

The determination of the form of the trajectories on which the RG equations are valid, completes the solution of our problem. Knowing their shape, we can obtain the susceptibility and heat capacity of the system at any point of the phase diagram.

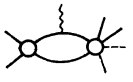
5. HIGHER VERTICES AND EQUATION OF STATE

In this section we discuss briefly how to describe the influence of the higher unrenormalized vertices $(\tilde\lambda_8,\tilde\lambda_{10},\dots)$ on the critical thermodynamics of our system within the framework of the proposed scheme, and also say a few words on the equation of state.

It is known that the fluctuation Hamiltonians of the type (2) contain as a rule, besides $\tilde\lambda_4$ and λ_6, unrenormalized vertices of higher orders. To take into account the contributions of the multiparticle coupling constants in the thermodynamic quantities, we can introduce additional many-point diagrams which are defined by the relation

$$\gamma_{2k}(q,\dots)=\Gamma_{2k}(q,\dots)-\Gamma_{2k}^{(2k-2)}(q,\dots), \tag{59}$$

where $\Gamma_{2k}(q,\dots)$ is the total vertex part with $2k$ external ends, and $\Gamma_{2k}^{(2k-2)}(q,\dots)$ is a $2k$-point diagram constructed of exact propagators and unrenormalized vertices of order not higher than $(2k-2)$. It is understood, of course, that the unrenormalized vertices λ_{2k} include all the ultraviolet diagrams with $2k$ ends. It is easily seen that $\gamma_{2k}(q,\dots)=0$ at $\lambda_{2k}=0$. For the charges $\gamma_{2k}\equiv\gamma_{2k}(0,0,\dots)$ we can obtain RG equations that describe the evolution in the fluctuation region when moving along trajectories on which $\lambda_{2k}=\text{const}$. Solving these equations, we easily find the exponents ω_{2k} of the scaling corrections necessitated by each of the vertices $\tilde\lambda_{2k}$. For the ordinary critical point, for example, the exponents ω_{2k} in the lowest order of perturbation theory are given by diagrams of the type

 (60)

and are equal to

$$\omega_{2k}=\frac{2k(3k+n+2)-3(n+8)}{n+14}. \tag{61}$$

We see that these exponents increase rapidly with increasing k. Similarly, by considering second-order

diagrams with triple sections at $\gamma_4 = 0$, we can calculate the corrections to the tricritical scaling, etc.

It is very easy to write down in terms of the supplementary vertices the equation of state near the TCP. In this equation, as is well known, the coefficients of the expansion of the field F in powers of the order parameter $\Phi = \langle \varphi \rangle$ are complete 1-irreducible vertices Γ_{2k} at zero momenta.[19] Since $\Gamma_6(0, \ldots, 0) = \Gamma_6^{(4)}(0, \ldots, 0) + \gamma_6$ and $\Gamma_6^{(4)}(0, \ldots, 0) \sim \gamma_4^3$, in the lowest approximation quadratic in the charges the total six-point diagram reduces to the additional vertex γ_6, and the equation of state takes the form

$$F = \varkappa\Phi + {}^1/_2\gamma_4\Phi^3 + {}^1/_8\gamma_6\Phi^5. \tag{62}$$

In the critical region γ_6 decreases rapidly, so that the last term can be neglected here; in the tricritical region, on the contrary, we can omit the second term. In the crossover region, obviously, all the terms of (62) are essential. A more detailed discussion of the equation of state, and also a study of the behavior of the system in the ordered phase, is planned for a separate article.

In conclusion I wish to express my sincere gratitude to A. A. Migdal, A. L. Korzhenevskiĭ, and B. N. Shalaev for very helpful discussions of questions touched upon in the present paper.

[1)]In some papers, the crossover of this system from the tricritical to the critical behavior was investigated within the framework of the ε expansion. It was treated there simply as a transition from a Gaussian fixed point of the RG equations to a nontrivial singular point. In our problem, however, this approach cannot be regarded as satisfactory, both from the point of view of the method and from the point of view of the result: in $(4-\varepsilon)$-dimensional models the operator φ^6 is really not taken into account as irrelevant in the renormalization-group sense, and it is impossible here in principle to study logarithmic renormalizations associated with it.

[2)]This graph was incorrectly calculated in Ref. 4: a logarithmic factor which is essential in the present case was left out.

[3)]In principle λ_4 depends also on the momenta $\mathbf{q}$, $\mathbf{q}'$ and $\mathbf{q}''$. This dependence, however, can be ignored, since it would lead to the appearance, in the effective Hamiltonian, of terms that are irrelevant from the point of view of the renormalization group. We therefore assume $\mathbf{q} = \mathbf{q}' = \mathbf{q}'' = 0$ in (7).

[4)]We note incidentally that the simpler linear transformation of the independent variables, used in Refs. 3 and 4, is in fact not reflected in the form of the diagram expansions for the GML functions. In particular, it does not make it possible to get rid of the skeleton diagrams that add logarithms to the RG equations.

[5)]We note that the usefulness of considering vertices made up of unrenormalized vertices of a definite type (λ_4 or λ_6) was indicated earlier in a paper by the author.[12] The so-called proper vertices introduced there turned out, however, to be inconvenient for the solution of the present problem.

[6)]We have in mind here, of course, total vertices corresponding to operators $\varphi(\mathbf{x}_1)\varphi(\mathbf{x}_2)\cdots\varphi(\mathbf{x}_n)$ with unequal arguments. Besides these vertices, the theory involves also vertices of a special type: with one corner, two corners, etc. They correspond to operators of type φ^n with partially coinciding arguments. The dimensionalities of these operators, as is well known, differ from the dimensionality of the complete n-point diagram.

[7)]It is appropriate to note here that not all the singular points of the RG equations contained in Ref. 4 were obtained there. The coordinates of the very point which in the authors' opinion corresponds to the critical asymptotic value were incorrectly calculated, as can be easily verified directly.

[8)]We note that besides the parametric smallness there is also here a numerical smallness connected with the factor $c(n)$.

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