Evolution of electron distribution function in high-frequency breakdown of a gas

P. E. Moroz

P. N. Lebedev Physics Institute, USSR Academy of Sciences
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The nonstationary kinetic equation is solved under conditions of pulsed high-frequency gas breakdown with account taken of the real cross sections of the elementary processes in helium. The distribution function of the electrons in energy at various instants of time is obtained. The time dependences of the electron density and of the avalanche development constant are calculated, and the time required for the distribution function to assume a stationary value is determined.

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1. In the course of gas breakdown, time variation occurs not only in the density of the electron particles, but also in the very distribution function of the particles in energy. Usually, however, one solves the stationary kinetic equation (see, e.g., Refs. 1–4), and the initial transient stage is neglected. It is of interest (particularly for short pulses, when the electron density has not yet increased by many orders) to examine in detail the evolution of the distribution function. To this end we solve in the present paper the nonstationary Boltzmann equation with the real cross sections[4,5] of the elementary processes in helium.

2. Turning to the literature, we note that the nonstationary kinetic equation was solved earlier by a number of workers.[6–8]

Naĭdis[6] used the infinitely strong sink approximation at an energy equal to the excitation potential I_{ex}. This is permissible only in relatively weak fields, when the following inequality is rigorously satisfied:

$$\varepsilon_0\nu/I_{ex}\nu_{ex}\ll 1. \tag{1}$$

Here ν is the transport collision frequency of the electrons with the gas atoms, ν_{ex} is the frequency of the collisions that excite the atom, and ε_0 is the electron oscillation energy:

$$\varepsilon_0=e^2E^2/m(\omega^2+\nu^2), \tag{2}$$

where e and m are the charge and mass of the molecule, and ω and E are the cyclic frequency and the effective value of the electric field intensity. In addition, Naĭdis[6] neglected the energy loss in elastic collisions and assumed that ν is independent of energy. Under these conditions the problem could be solved analytically to conclusion. The result was a calculation of the time required to reach the stationary regime, $\tau_s=I_{ex}/\varepsilon_0\nu$.

Polman[7] solved numerically the nonstationary kinetic equation with model-dependent cross sections of the elementary processes, without allowance for the ionization collision term. He pointed out that in the nonstationary heating regime the distribution function is enriched with fast electrons compared with the stationary distribution at the same average energy.

Reshetnyak and Shelepin[8] developed a quasistationary distribution function (QDF) method for use with equations of the Fokker-Planck type and solved the kinetic equation under conditions when the principle role in the elastic collisions is played by excitation of the rotational and vibrational levels of the molecules. The collision term was written in differential form, and the total number of electrons remains constant in time.

3. However, in the course of the breakdown it is necessary to take into account the ionization collision term and the possibility of particle multiplication. The latter can become decisive in the case of relaxation in strong fields, when the avalanche development constant γ is of the order of or larger than $\delta\nu$ (Ref. 4):

$$\delta\nu=\delta_{el}\langle\nu_{el}\rangle+I_{ex}\langle\nu_{ex}/\varepsilon\rangle+I_i\langle\nu_i/\varepsilon\rangle. \tag{3}$$

Here $\delta_{el}=2m/M$ (M is the mass of the gas atom), ν_{el} and ν_i are the frequencies of the elastic and ionizing collisions of the electrons with the gas atoms, and I_i is the ionization potential. The angle brackets denote averaging over the distribution function.

We consider now the homogeneous problem, when the distribution function does not depend on the spatial coordinates. This is admissible under conditions when the drift of the electrons from the discharge region can be neglected (large diffusion times τ_d):

$$\gamma\tau_d\gg 1. \tag{4}$$

In addition, the electron distribution function will be assumed to be spherically symmetrical; this is possible in moderate fields, when the following inequalities are satisfied

$$\varepsilon_0\ll\langle\varepsilon\rangle,\quad \gamma\ll(\omega^2+\langle\nu\rangle^2)^{1/2}. \tag{5}$$

Under these conditions, the electron distribution function in energy $F(\varepsilon,t)$ satisfies the kinetic equation

$$\frac{\partial F}{\partial t}=\frac{1}{\varepsilon^{1/2}}\frac{\partial}{\partial\varepsilon}\left\{\varepsilon^{3/2}\left[\frac{2}{3}\varepsilon_0\nu\frac{\partial F}{\partial\varepsilon}+\delta_{el}\nu_{el}F\right]\right\}+S_{in}. \tag{6}$$

We use here the normalization

$$\int_0^\infty F(\varepsilon,t)\varepsilon^{1/2}\,d\varepsilon=n(t)/n_0, \tag{7}$$

where $n(t)$ is the electron density at the instant of time t, $n_0=n(0)$. When writing down the collision term S_{in} for the inelastic collisions, we make the simplifying assumption that the energy losses are I_{ex} in each exciting collision and I_i in each ionizing collision.

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We introduce the distribution function $f(\varepsilon, \tau)$ for an electron produced at the instant of time $\tau = 0$. The total distribution function $F(\varepsilon, t)$ can be connected with $f(\varepsilon, \tau)$:

$$F(\varepsilon, t) = \int_0^t f(t-\tau) R(\tau) d\tau + f(\varepsilon, t), \tag{8}$$

where $R(\tau)$ is the frequency of electron production at the instant:

$$R(\tau) = \int_0^\infty F(\varepsilon, \tau) \nu_i(\varepsilon) \varepsilon^{1/2} d\varepsilon. \tag{9}$$

The function $f(\varepsilon, t)$ satisfies the equation

$$\varepsilon^{1/2} \frac{\partial f}{\partial t} = -\frac{\partial J}{\partial \varepsilon} + \delta(t)\delta(\varepsilon), \tag{10}$$

where $J(\varepsilon, t)$ is the flux in energy space:

$$J = -\frac{2}{3} \varepsilon^{3/2} \varepsilon_0 \nu_* \frac{\partial f}{\partial \varepsilon} - \varepsilon^{3/2} \delta_{el} \nu_{el} f - \int_\varepsilon^{\varepsilon + I_{ex}} f \nu_{ex} \varepsilon^{1/2} d\varepsilon - \int_\varepsilon^{\varepsilon + I_i} f \nu_i \varepsilon^{1/2} d\varepsilon. \tag{11}$$

Here $\delta(t)$ and $\delta(\varepsilon)$ are the Dirac delta functions which determine the initial condition. The function $f(\varepsilon, t)$ is convenient because it has a constant normalization

$$\int_0^\infty f(\varepsilon, t) \varepsilon^{1/2} d\varepsilon = 1. \tag{12}$$

4. To solve Eqs. (10) and (6) we take the Laplace transforms:

$$\varphi(\varepsilon, s) = \int_0^\infty e^{-st} f(\varepsilon, t)\, dt, \quad \Phi(\varepsilon, s) = \int_0^\infty e^{-st} F(\varepsilon, t)\, dt. \tag{13}$$

For $\varphi(\varepsilon, s)$ we have the equation

$$s\varphi\varepsilon^{1/2} = -dJ\{\varphi\}/d\varepsilon + \delta(\varepsilon), \tag{14}$$

where $J\{\varphi\}$ denotes expression (11) with f replaced by φ. Equation (14) is an ordinary differential equation and can be solved by an iteration method.[4] Knowing $\varphi(\varepsilon, s)$ we can easily obtain also $\Phi(\varepsilon, s)$:

$$\Phi(\varepsilon, s) = \varphi(\varepsilon, s) \Big/ \left(s \int_0^\infty \varphi \varepsilon^{1/2} d\varepsilon - \int_0^\infty \varphi \nu_i \varepsilon^{1/2} d\varepsilon \right). \tag{15}$$

To obtain the original $F(\varepsilon, t)$ we must take the inverse Laplace transform of $\Phi(\varepsilon, s)$. We use here the Papoulis numerical method.[9]

5. We demonstrate the results of the calculations in three figures. Figure 1 shows the establishment of the distribution function in time at $\varepsilon_0 = 2$ eV. With respect to the parameter ε_0 it must be stated immediately that

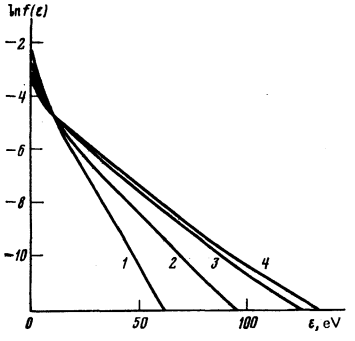

FIG. 1. Electron energy distribution function at $\varepsilon_0 = 2$ eV for different pt (in units of 10^{-8} Torr-sec): curves: 1—0.25, 2—0.6, 3—5, 4—stationary value.

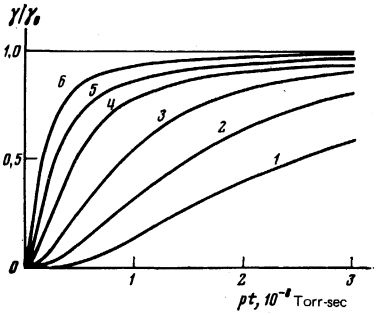

FIG. 2. Change of avalanche development constant with time at different ε_0(eV): 1—0.5, 2—1, 3—2, 4—5, 5—10, 6—20.

it has the same meaning as in Ref. 4, i.e., to calculate it it is necessary to substitute in (2) the value $\nu = 2.4 \times 10^9 p$ (sec^{-1}). Then, as shown in Refs. 4 and 5, the parameter ε_0 for helium characterizes the action of the electric field both at high ($\omega > \nu$) and low ($\omega < \nu$) frequencies.

Figure 2 shows the change of the avalanche development constant with time, due to relaxation of the distribution function, while γ_0 denotes the stationary value.[4]

Figure 3 shows a plot of the growth of the electron density with time:

$$n(t) = n_0 \exp\left(\int_0^t \gamma(t)\, dt \right); \tag{16}$$

with the straight lines showing the corresponding stationary values.

6. An analysis of the calculations shows that the time τ_s required for the distribution function to settle at its stationary value is close to

$$1/\tau_s \sim \max\{(\delta\nu)_0, \gamma_0\} \tag{17}$$

and for helium in a wide range of the parameter ε_0, 10^{-3} eV $\leq \varepsilon_0 \leq 20$ eV, it is expressed by the simple analytic relation

$$1/p\tau_s = 5 \cdot 10^7 \varepsilon_0^{0.7} \quad (\text{Torr}^{-1}\,\text{sec}^{-1}). \tag{18}$$

τ_s was determined here in accord with the level γ:

$$\gamma_0 - \gamma(\tau_s) = \gamma_0/e. \tag{19}$$

In the narrow region $\varepsilon_0 \sim 0.01 - 0.1$ eV, the values of τ_s calculated from (18) are close to the result of Naĭdis.[6]

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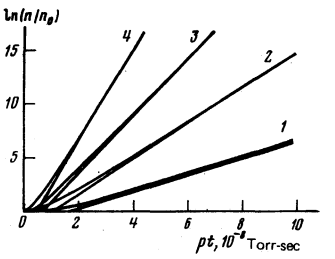

FIG. 3. Growth of electron density with time at different ε_0 (eV): 1—2, 2—5, 3—10, 4—20.

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Influence of oscillations of plasma particles in a radial potential well on the beam-plasma interaction

L. P. Katsubo, V. V. Lisitchenko, I. A. Soloshenko, and Ya. M. Shuba

Physics Institute, Ukrainian Academy of Sciences
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We investigate the influence of the limited radial dimensions of a plasma on beam-plasma interaction in the absence of a magnetic field. It is shown theoretically and experimentally that in the presence of a static transverse electric field, which causes transverse oscillations of the plasma, the limited radial dimensions of the system greatly influence the dispersion of the excited electronic oscillations if two conditions are satisfied: $k_z r_0 < 1$ and $\omega < v_T/r_0$, where k_z is the longitudinal wave number and r_0 is the inhomogeneity dimension (the radius of the plasma), v_T is the thermal velocity of the plasma electrons, and ω is the oscillation frequency. The observed effect, that the frequency is lower than the electron Langmuir frequency, is due to the decrease of the transverse conductivity of the plasma as a result of the indicated oscillations..

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One of the fundamental problems in beam-plasma interaction is the influence of the limited radial dimensions of the system on the dispersion of the excited oscillations. In the one-dimensional case, which is realized in a strong magnetic field ($\Omega_e \ll \Omega_c$, where Ω_e and Ω_c are the plasma and cyclotron frequencies of the electrons), it was shown theoretically and experimentally that the radial restriction leads to a substantial decrease of the frequencies and increments of the oscillations, and critical values of the parameters it stops the instability.[1-4] As to systems without a magnetic field, the situation remained unclear until the very latest time. On the one hand, a rigorous theoretical analysis with account of both the fact that the oscillations are not potential[5] and of the inhomogeneity of the system[3] has led inevitably to the conclusion that at all $k_z r_0$ (k_z is the longitudinal wave number and r_0 is the characteristic dimension of the inhomogeneity or the radius of the plasma) the increments and frequencies of the volume oscillations are equal to the corresponding values in an unbounded homogeneous system with particle concentrations close to the corresponding concentrations on the axis of the considered system.

On the other hand there exist experimental indications that in some conditions and in the absence of an external magnetic field the decrease of r_0 can lead to a decrease of the frequencies and increments of the excited volume oscillations.[6,7] This contradiction is resolved in the present paper, in which we construct a theory of beam-plasma interaction with account taken of the transverse static electric fields which confine the plasma particles within the beam, and show that under certain conditions these fields, in analogy with longitudinal magnetic field, lead to a substantial dependence of the dispersion of the excited oscillations on the characteristic dimension of the inhomogeneity (radius) of the system. The experimental data obtained in the present paper in an investigation of the collective interaction of a beam of positive ions with plasma electrons have confirmed the main conclusions of theory.

THEORY OF BEAM-PLASMA INTERACTION WITH ALLOWANCE FOR TRANSVERSE OSCILLATIONS OF THE PLASMA ELECTRONS IN AN ELECTROSTATIC WELL

We consider an axially symmetrical system in which a cold beam of charged particles passes with velocity V_0 along the Z axis through a plasma with electron temperature T_e. The potential of the plasma varies along the radius like

$$\varphi_0(r) = -T_e r^2/e r_0^2$$

and is the potential well for the electrons.

An investigation of the wave processes in the plasma will be carried out in a quasistatic approximation ($\mathbf{E} = -\nabla\varphi$) on the basis of linearized equations–the Vlasov equation for the electrons captured in the radial potential well, the hydrodynamic equations for the beam particles, and the Poisson equation. For simplicity we consider only axially symmetrical perturbations,

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