Cyclotron resonance in metals in weak magnetic fields

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An asymptotically exact solution is obtained for the problem of the surface impedance of a metal with an arbitrary electron-reflection coefficient ρ from the boundary, in weak magnetic fields and under the conditions of the anomalous skin effect, when the characteristic arc of the electron trajectory in the skin layer is much longer than the effective mean free path length. The dependence of the impedance on the magnetic field and the singularities of cyclotron resonance (CR) are analyzed in detail for different ratios of the collision frequency ν and the frequency ω of the electromagnetic wave. The influence of the delay effect on the CR line shape is investigated in the entire interval of magnetic fields. It is shown that the amplitude of the cyclotron oscillations increases sharply near $\omega=\nu$ on going into the low-frequency region $\omega<\nu$. This increase is due to the fact that a at $\nu>\omega$ the delay effect does not manifest itself in the dependence of the impedance on the magnetic field. An asymptotic form, uniform in the delay parameter, is obtained for the current density for diffuse reflection of the electrons from the metal surface.

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1. INTRODUCTION

Cyclotron resonance (CR), discovered more than twenty years ago,[1] turned out to be quite useful and informative for the clarification of the physical picture of the interaction between conduction electrons and the electric field in a metal. Owing to the anomalous skin effect, the character of the interaction of the electrons with the skin-layer field, and therefore also the singularities of the CR, are determined by the relation between the free-path time $1/\nu$, the time $\tau_0=(8R\delta)^{1/2}/v$ of flight of the electrons through the skin layer δ, and the half-life π/ω of the high-frequency field. Here R is the radius of the electron orbit in the magnetic field H, and v is the electron Fermi velocity.

Cyclotron resonance is usually observed in the region of microwave frequencies and in relatively strong magnetic field, when the time of flight through the skin layer is much less than the half-period of the electromagnetic wave, which in turn is small compared with the free path time, i. e.,

$$(8R\delta)^{1/2}/v\ll\pi/\omega\ll1/\nu. \tag{1.1}$$

These inequalities mean that the resonant electrons move inside the skin layer without collisions in the static field of the wave. The conditions (1.1) are the most favorable for the observation of the CR. The situation of the "usual" cyclotron resonance has been investigated in sufficient detail in many theoretical papers and is described, in particular, in Ref. 2, where the pertinent references are given (see also Ref. 3).

With increasing frequency or with decreasing magnetic field, the relation between the time of flight and the half-period of the wave is violated. The left-hand inequality of (1.1) is then reversed,

$$\pi/\omega\ll(8R\delta)^{1/2}/v, \tag{1.2}$$

i. e., during the time τ_0 the electrons are acted upon by the rapidly varying field of the electromagnetic wave, and they absorb the wave energy effectively only during a small fraction of the entire time of stay in the skin layer. In other words, the electrons are late in leaving the skin layer by the time a substantial change occurs in the field. One speaks then of the delay effect.

The important role of the relation between τ_0 and $|\omega+i\nu|^{-1}$ was first indicated in Ref. 4, where the surface-impedance component that depends smoothly on H was obtained in the form of an expansion of the reciprocal powers of the parameter $|\omega+i\nu|\tau_0$. The change of the character of the interaction of the electrons with the high-frequency field at $\omega\tau_0\sim1$ was used by Koch and Kip,[5] and later by Smith,[6] to explain the observed broad maximum on the plot of the derivative of the surface impedance with respect to the magnetic field, a maximum that precedes the CR oscillations. Later Kamgar, Heningsen, and Koch[7] observed and investigated CR in gallium at submillimeter wavelengths under conditions of the strong delay effect (1.2). In this case the CR amplitude decreases because of the decrease of the effectiveness of the interaction of the electrons with the high-frequency field. Somewhat earlier, Drew 8 published a qualitative theory of the delay effect.

A consistent asymptotic theory of the delay effect was constructed in Ref. 2, with account taken of the real distribution of the electromagnetic field in the metal, and in particular its dependence on the character of the scattering of the electrons by the sample surface. In Ref. 2, however, the resonance was assumed to be quite sharp, $\nu\ll\Omega$ (Ω is the cyclotron frequency), and only the immediate vicinity of the resonances ($|\omega-n\Omega|\lesssim\nu$, n is the number of the resonance) was investigated. This approximation corresponds to representing the current density in the form of a sum of two terms, one corresponding to the current in the absence of a magnetic field, and the other to the resonant current. Actually, however, the asymptotic expansion of the current density in terms of the para-

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meter $|\omega+i\nu|\tau_0$ contains also other less strongly varying terms, which were not taken into account before.[2] They exert a strong influence on the impedance and on the shape of the resonance line.

Summarizing the foregoing, it must be stated that at present the delay effect has been theoretically investigated only near a sharp resonance. It is therefore of interest to study the dependence of the impedance on H and the CR in weak magnetic fields and at other relations between the parameters τ_0, $1/\omega$, and $1/\nu$.

We obtain in this paper an asymptotically exact solution of the problem of the surface impedance of a metal at an arbitrary coefficient of electrons from the sample boundary in weak magnetic fields and under the conditions of the anomalous skin effect. By weak magnetic fields we mean those in which the parameter

$$|\Lambda|=\left|\frac{\omega+i\nu}{\Omega}\right|^2\frac{\delta}{2R}=\left(\frac{|\omega+i\nu|\tau_0}{4}\right)^2\gg 1 \tag{1.3}$$

independently of the ratio of the wave frequency ω to the collision frequency ν. The condition for the anomaly of the skin effect is as usual

$$v/|\nu-i\omega|\gg\delta, \tag{1.4}$$

i.e., the effective mean free path $v/|\nu-i\omega|$ is large compared with the skin-layer thickness δ. It follows from inequalities (1.3) and (1.4) that $\Omega\ll|\nu-i\omega|\ll v/\delta$, and they can be written in the form of the following chain:

$$|\gamma|^2\gg R/\delta\gg|\gamma|\gg 1,\quad \gamma=(\nu-i\omega)/\Omega. \tag{1.5}$$

In the high-frequency region, where $\omega\gg\nu$, the condition (1.3) coincides with (1.2) and means that a strong delay effect takes place. At low frequencies $\omega<\nu$, the interaction of the electrons with the wave in the skin layer takes place on a small segment, compared with the arc length $v\tau_0$, of the order of the mean free path $l=v/\nu$. During the time of this interaction the change of the phase of the electromagnetic field is negligible. This means that the delay effect plays no role at low frequencies. Therefore that part of the impedance which depends on the magnetic field turns out to be proportional to the probability of a single return of the electrons to the skin layer $e^{-2\pi\gamma}$ without collision in the interior of the metal. In addition, the amplitude of such damped harmonic oscillations is independent in this case of the parameter Λ. It follows directly from this physical picture that there should exist near $\omega=\nu$ a transition region in which the interaction with the wave during the free-path time gives way to interaction under conditions of the delay effect. The analysis presented below shows that the characteristic width of such a transition region is of the order of $1/\tau_0$, i.e.,

$$|\omega-\nu|\sim 1/\tau_0, \tag{1.6}$$

and is small compared with ω and ν by virtue of (1.3). The presence of the region (1.6) manifests itself in an abrupt change of the amplitude of the damped harmonic oscillations, of the type $e^{-2\pi\gamma}$.

In the next section we obtain the asymptotic Fourier component of the current density for different ratios of ω and ν within the framework of the inequalities (1.5). In the case of diffuse reflection of the electrons from the metal surface, we obtain, for the first time ever, an asymptotic current density that is uniform relative to the parameter Λ. In Sec. 3 is calculated the surface impedance and an analytic description is obtained for the entire dependence of the surface impedance of the metal on the magnetic field at any sharpness of the resonance. In the concluding fourth section we analyze the results.

2. CURRENT DENSITY

To solve Maxwell's equations and to calculate the surface impedance it is necessary to know the connection between the Fourier components of the current density $j_\alpha(k)$ and the electric field $\mathscr{E}_\alpha(k)$. In the case of a spherical Fermi surface this connection takes the form

$$j_\alpha(k)=K_\alpha(k)\mathscr{E}_\alpha(k)-\frac{1}{\pi}\int_0^\infty dk'\,Q_\alpha(k,k')\mathscr{E}_\alpha(k'). \tag{2.1}$$

Here $\alpha=y$ or z, the x axis is directed towards the interior of the metal, and the z axis is parallel to the magnetic field $\mathbf{H}$; $K_\alpha(k)$ are the tangential components of the conductivity of the unbounded metal:

$$K_\alpha(k)=\frac{3\omega_0^2}{4\pi^2\Omega}\int_0^{\pi/2}d\vartheta\sin\vartheta\int_0^\infty dx\,e^{-\gamma x}\int_0^\pi d\lambda\,n_\alpha(\vartheta,\lambda)$$
$$\times n_\alpha(\vartheta,\lambda-x)\cos[kR_\perp(\cos\lambda-\cos(\lambda-x))], \tag{2.2}$$

where $\omega_0=(4\pi Ne^2/m)^{1/2}$ is the plasma frequency, N, m, and e are respectively the concentration, effective mass and the absolute value of the charge of the electron, $R_\perp=R\sin\vartheta$ is the radius of the electron orbit in the magnetic field ($R=v/\Omega$), v is the Fermi velocity, $\Omega=eH/mc$, ϑ is the polar angle with the plaro axis z, and $n_\alpha(\vartheta,\lambda)$ are the components of the velocity unit vector ($n_y=\sin\vartheta\cos\lambda$, $n_z=\cos\vartheta$).

The integral part of the conduction operator takes into account the influence of the surface and is given by[9,10]

$$Q_\alpha(k,k')=\frac{3\omega_0^2R}{2\pi^2\Omega}\int_0^{\pi/2}d\vartheta\sin^2\vartheta$$
$$\times[I_\alpha^{(1)}(kR_\perp,k'R_\perp)-I_\alpha^{(2)}(kR_\perp,k'R_\perp)-2\rho I_\alpha^{(3)}(kR_\perp,k'R_\perp)], \tag{2.3}$$

where

$$I_\alpha^{(1)}(z,z')=\frac{1}{\operatorname{sh}\pi\gamma}\int_0^\pi d\varphi\sin\varphi\int_0^\pi d\lambda\,n_\alpha(\vartheta,\pi-\lambda)\operatorname{ch}\gamma\lambda\cos[z(\cos\varphi+\cos\lambda)]$$
$$\times\int_0^\varphi d\lambda'\,n_\alpha(\vartheta,\lambda')\operatorname{ch}\gamma\lambda'\cos[z'(\cos\varphi-\cos\lambda')], \tag{2.4}$$

$$I_\alpha^{(2)}(z,z')=\int_0^\pi d\varphi\sin\varphi\int_0^\varphi d\lambda\,n_\alpha(\vartheta,\lambda)e^{-\gamma\lambda}\cos[z(\cos\varphi-\cos\lambda)]$$
$$\times\int_0^\varphi d\lambda'\,n_\alpha(\vartheta,\lambda')\operatorname{ch}\gamma\lambda'\cos[z'(\cos\varphi-\cos\lambda')],$$

$$I_\alpha^{(3)}(z,z')=\int_0^\pi\frac{d\varphi\sin\varphi}{e^{2\gamma\varphi}-\rho}\int_0^\varphi d\lambda\,n_\alpha(\vartheta,\lambda)\operatorname{ch}\gamma\lambda\cos[z(\cos\varphi-\cos\lambda)]$$
$$\times\int_0^\varphi d\lambda'\,n_\alpha(\vartheta,\lambda')\operatorname{ch}\gamma\lambda'\cos[z'(\cos\varphi-\cos\lambda')], \tag{2.6}$$

and the parameter ρ is the probability of specular reflection of the electrons from the metal boundary ($0 \leq \rho \leq 1$).

We must first find the asymptotic current density (2.1) under the conditions of the normal skin effect

$$1+|\gamma| \ll kR \tag{2.7}$$

and in the region of weak magnetic fields (1.5). The asymptotic form of $K_\alpha(k)$ was calculated before a number of times (see, e.g., Ref. 9) and is given by the known formula

$$K_\alpha(k) \approx \frac{3\omega_0^2}{16kv} \operatorname{cth} \pi\gamma. \tag{2.8}$$

We emphasize that this result does not depend on the parameter (1.3) and is valid of only the inequality (2.7) holds.

The calculation of the asymptotic form of $Q_\alpha(k, k')$ is much more complicated because its form depends explicitly on the parameter (1.3). We use first only the condition (2.7) that the skin effect be anomalous. We start with the calculation of $I_\alpha^{(1)}(z, z')$. If we first integrate with respect to φ by parts, then it is easy to verify that the matter reduces to finding the asymptotic forms of Bessel-type integrals

$$\int_0^\pi dx \operatorname{ch} \gamma x \exp(iz \cos x).$$

We use for the calculation the stationary phase method and expand the cosine in the argument of the exponential in the vicinity of the stationary points $x=0$ and $x=\pi$ up to quadratic terms inclusive. The integral is then expressed in terms of the error function

$$\Phi(x) = 2\pi^{-1/2} \int_0^x dt \exp(-t^2)$$

by the following formula:

$$\int_0^\pi dx \operatorname{ch} \gamma x \exp(iz \cos x) \approx \left(\frac{\pi}{2|z|}\right)^{1/2} \left\{ \exp\left(iz - \frac{i\gamma^2}{2z} \mp \frac{i\pi}{4}\right) + \exp\left(-iz + \frac{i\gamma^2}{2z} \pm \frac{i\pi}{4}\right)\left[\operatorname{ch} \pi\gamma - \operatorname{sh} \pi\gamma \Phi\left(\frac{\gamma}{(2|z|)^{1/2}} e^{\pm i\pi/4}\right)\right]\right\}, \tag{2.9}$$

where the signs + and − correspond to positive (upper) and negative (lower) values of z.

The result of these basically straightforward but somewhat cumbersome calculations takes the form

$$\begin{aligned} \frac{I_\alpha^{(1)}(z,z')}{n_\alpha^2(\vartheta,0)} \approx & -\frac{\pi}{z^2 - z'^2} \operatorname{Im} \Phi\left(\frac{\gamma}{(2z)^{1/2}} e^{i\pi/4}\right) + \frac{\pi^2}{4z} \delta(z-z') \operatorname{Re}\left[\operatorname{cth} \pi\gamma - \Phi\left(\frac{\gamma}{(2z)^{1/2}} e^{i\pi/4}\right)\right] \\ & - \frac{\pi}{4} \frac{(zz')^{-1/2}}{z-z'} \operatorname{Im}\left\{\exp\left[-\frac{i\gamma^2}{2}\left(\frac{1}{z} - \frac{1}{z'}\right)\right]\left[\operatorname{cth} \pi\gamma - \Phi\left(\frac{\gamma}{(2z')^{1/2}} e^{i\pi/4}\right)\right]\right\} \\ & + \frac{\pi}{4} \frac{(zz')^{-1/2}}{z+z'} \operatorname{Re}\left\{\exp\left[\frac{i\gamma^2}{2}\left(\frac{1}{z} + \frac{1}{z'}\right)\right]\left[\operatorname{cth} \pi\gamma - \Phi\left(\frac{\gamma}{(2z')^{1/2}} e^{i\pi/4}\right)\right]\right\}. \end{aligned} \tag{2.10}$$

When performing the operations Re and Im the parameter γ must be regarded as real. We neglect in (2.10) the terms containing rapidly oscillating exponentials such as $\exp[\pm i(z \pm z')]$, $\exp(\pm 2iz)$, and $\exp(\pm 2iz')$. An exception is the term with $\exp[2i(z-z')]/(z-z')$, which in fact lead to the term with the delta function $\delta(z-z')$.

The determination of the asymptotic form of $I_\alpha^{(2)}(z, z')$ is somewhat more complicated because integration by parts with respect to φ yields, besides the already considered integrals, also double integrals in the form

$$\int_0^{\pi/2} d\varphi \int_0^\varphi dx \operatorname{ch}[\gamma(\varphi - x)]\{z \sin[z(\cos x - \cos\varphi)] - z' \sin[z'(\cos x - \cos \varphi)]\}. \tag{2.11}$$

The distinguishing feature of this integral is that the main contribution is made to it by small φ and by the entire interval of integration with respect to x. Its asymptotic form contains also logarithmic terms and is given by

$$\frac{1}{2} \ln \frac{z}{z'} + \pi^{1/2} \operatorname{Re} \int_{\gamma/(2z')^{1/2}}^{\gamma/(2z)^{1/2}} dt \exp\left(it^2 + \frac{i\pi}{4}\right) \Phi(te^{i\pi/4}). \tag{2.12}$$

After calculating the asymptotic form of $I_\alpha^{(2)}(z, z')$ we can obtain the following expression for the combination $I_\alpha^{(1)}(z, z') - I_\alpha^{(2)}(z, z')$:

$$\begin{aligned} \frac{I_\alpha^{(1)}(z,z') - I_\alpha^{(2)}(z,z')}{n_\alpha^2(\vartheta,0)} \approx & \frac{\pi^2}{2z} \frac{e^{-2\pi\gamma}}{1-e^{-2\pi\gamma}} \delta(z-z') + \frac{3+e^{-2\pi\gamma}}{4(z^2-z'^2)} \ln \frac{z}{z'} \\ & + \frac{\sqrt{\pi}}{2} \frac{1-e^{-2\pi\gamma}}{z^2-z'^2} \operatorname{Re} \int_{\gamma/(2z')^{1/2}}^{\gamma/(2z)^{1/2}} dt \exp\left(it^2 + \frac{i\pi}{4}\right) \Phi(te^{i\pi/4}) + \frac{\pi}{8} \frac{1-e^{-2\pi\gamma}}{(z+z')(zz')^{1/2}} \\ & \times \operatorname{Re} \exp\left[\frac{i\gamma^2}{2}\left(\frac{1}{z} + \frac{1}{z'}\right)\right]\left[\operatorname{cth} \pi\gamma - \Phi\left(\frac{\gamma}{(2z)^{1/2}} e^{i\pi/4}\right)\right]\left[\operatorname{cth} \pi\gamma - \Phi\left(\frac{\gamma}{(2z')^{1/2}} e^{i\pi/4}\right)\right] \\ & - \frac{\pi}{4} \frac{1+e^{-2\pi\gamma}}{z^2-z'^2} \operatorname{Im}\left[\Phi\left(\frac{\gamma}{(2z)^{1/2}} e^{i\pi/4}\right) - \Phi\left(\frac{\gamma}{(2z')^{1/2}} e^{i\pi/4}\right)\right] - \frac{\pi}{8} \frac{1-e^{-2\pi\gamma}}{(z-z')(zz')^{1/2}} \\ & \times \operatorname{Im} \exp\left[-\frac{i\gamma^2}{2}\left(\frac{1}{z} - \frac{1}{z'}\right)\right]\left[\operatorname{cth} \pi\gamma - \Phi\left(\frac{\gamma}{(2z)^{1/2}} e^{-i\pi/4}\right)\right]\left[\operatorname{cth} \pi\gamma - \Phi\left(\frac{\gamma}{(2z')^{1/2}} e^{i\pi/4}\right)\right]. \end{aligned} \tag{2.13}$$

It must be emphasized that in the derivation of (2.13) we used only the condition (2.7) for the anomaly of the skin effect. In other words, this expression is valid at all values of the parameter Λ (of the parameters $\gamma/(2z)^{1/2}$ and $\gamma/(2z')^{1/2}$). In the case of diffuse scattering ($\rho=0$). when the last term with $I_\alpha^{(3)}(z, z')$, is missing from Eq. (2.3), the expression (2.13) provides the uniform asymptotic form of the entire integral part of the conduction operator $Q_\alpha(k, k')$. At $|\gamma| \ll (2z)^{1/2}$ Eq. (2.13) goes over into a known expression.[1]

Equation (2.13) can be further simplified under the conditions (1.3) and (1.5). For this purpose it is necessary to replace the function $\Phi(x)$ in (2.13) by its asymptotic form at large values of the argument:

$$\Phi(x) \approx \operatorname{sign} \operatorname{Re} x - \frac{\exp(-x^2)}{x\pi^{1/2}}\left(1 - \frac{1}{2x^2} + \frac{3}{4x^4} - \dots\right). \tag{2.14}$$

In all the terms except the third we can then confine ourselves to the first two terms in the parentheses of (2.14), and in the third one we must retain all the asymptotic terms written out in (2.14). As a result we get

$$\frac{I_\alpha^{(1)}(z,z')-I_\alpha^{(2)}(z,z')}{n_\alpha^2(\vartheta,0)}\approx\frac{\ln(z/z')}{z^2-z'^2}+\frac{1-e^{-2\pi\gamma}}{8\gamma^4}+\frac{\pi}{2}\frac{(zz')^{-1/2}}{e^{2\pi\gamma}-1}$$
$$\times\left\{\pi\delta(z-z')+\frac{1}{z-z'}\sin\left[\frac{\gamma^2}{2}\left(\frac{1}{z}-\frac{1}{z'}\right)\right]\right.$$
$$\left.+\frac{1}{z+z'}\cos\left[\frac{\gamma^2}{2}\left(\frac{1}{z}+\frac{1}{z'}\right)+2\pi i\gamma\right]\right\}+\frac{\pi}{8}\frac{e^{-2\pi\gamma}}{(zz')^{1/2}}[I(z,z')+I(z',z)], \tag{2.15}$$

where

$$I(z,z')=\frac{1}{z+z'}\exp\left[\frac{i\gamma^2}{2}\left(\frac{1}{z}+\frac{1}{z'}\right)\right]\left[1-\Phi\left(\frac{\gamma}{(2z)^{1/2}}e^{i\pi/4}\right)\right]$$
$$-\frac{2i(zz')^{1/2}}{z^2-z'^2}\Phi\left(\frac{\gamma}{(2z)^{1/2}}e^{-i\pi/4}\right)$$
$$-\frac{1}{z-z'}i\exp\left[-\frac{i\gamma^2}{2}\left(\frac{1}{z}-\frac{1}{z'}\right)\right]\left[1+\Phi\left(\frac{\gamma}{(2z)^{1/2}}e^{-i\pi/4}\right)\right].$$

In the last expression we have neglected the nonresonant terms that oscillate rapidly relative to the parameter (1.3) (relative to $\gamma/(2z)^{1/2}$, $\gamma/(2z')^{1/2}$), which lead to terms of higher order of smallness in the calculation of the impedance. In the expression for (z,z') the error functions were not replaced by the asymptotic form (2.14) because the character of the asymptotic behavior of $I(z,z')$ is substantially different, depending on the ratio of ω and ν (i.e., the phase of the parameter γ). As will be shown below, this causes a strong change in the dependence of the impedance on ω and ν in the region where $\omega\sim\nu$.

We proceed now to find the asymptotic form of $I_\alpha^{(3)}(z,z')$. The main contribution to the integral (2.6) is made by the vicinity of the points $\lambda=\varphi$ and $\lambda'=\varphi$. We obtain accordingly for $I_\alpha^{(3)}(z,z')$ the expression

$$I_\alpha^{(3)}(z,z')\approx\gamma^2\int_0^\pi\frac{d\varphi\sin\varphi}{e^{2\gamma\varphi}-\rho}\frac{n_\alpha^2(\vartheta,\varphi)\,\mathrm{sh}^2\gamma\varphi}{(z^2\sin^2\varphi+\gamma^2)(z'^2\sin^2\varphi+\gamma^2)}. \tag{2.16}$$

We next expand the integrand in powers of ρ and calculate the integral with respect to φ. The latter is determined by small regions $\varphi\sim1/|\gamma|$ and $\pi-\varphi\sim1/|\gamma|$ near the lower and upper ends of the integration interval. An exception is that term of the series in powers of ρ, which does not contain exponentials of the type $e^{-2n\gamma\varphi}$ and is due to integration over small regions of scale $|\gamma|/z$ near the points $\varphi=0$ and $\varphi=\pi$. This term of the series yields a logarithmic asymptotic form. The final result for $I_\alpha^{(3)}(z,z')$ is

$$\frac{I_\alpha^{(3)}(z,z')}{n_\alpha^2(\vartheta,0)}\approx\frac{1}{2}\frac{\ln(z/z')}{z^2-z'^2}+\frac{\rho-2}{16\gamma^4}[1+e^{-2\pi\gamma}]$$
$$+\frac{(1-\rho)^2}{16\gamma^4}\sum_{n=2}^\infty\rho^{n-2}\frac{1+e^{-2\pi n\gamma}}{n^2}. \tag{2.17}$$

Thus, all the formulas needed to write down the asymptotic current density have been obtained. The asymptotic form of $j_\alpha(k)$ in the region of the anomalous skin effect and in weak magnetic fields (1.5) is represented by a sum of three terms:

$$j_\alpha(k)=j_\alpha^{(0)}(k)+j_\alpha^{(1)}(k)+j_\alpha^{(2)}(k). \tag{2.18}$$

Here

$$j_\alpha^{(0)}(k)=\frac{3\omega_0^2}{16v}\left[\frac{\mathscr{E}_\alpha(k)}{k}-\frac{2}{\pi^2}(1-\rho)\int_0^\infty dk'\mathscr{E}_\alpha(k')\frac{\ln(k/k')}{k^2-k'^2}\right] \tag{2.19}$$

is the current density in the absence of the magnetic field. It is due to the part of $K_\alpha(k)$ which is independent of H and is obtained by taking the limit $\gamma\to\infty$ in (2.8), as well as to the first (logarithmic) terms of the asymptotic forms (2.15) and (2.17).

The quantity

$$j_\alpha^{(1)}=-\frac{3\omega_0^2}{2^{10}\pi v}\mu_\alpha E_\alpha(0)\left(\frac{\gamma^2}{2R}\right)^{-2}\left[2-(1-\rho)^2\sum_{n=1}^\infty\rho^{n-1}\frac{1+e^{-2\pi n\gamma}}{n^2}\right],$$
$$\mu_y=3,\quad\mu_z=1,\quad E_\alpha(0)=\pi^{-1}\int_0^\infty dk\,\mathscr{E}_\alpha(k) \tag{2.20}$$

is the increment to the current $j_\alpha^{(0)}(k)$, and depends relatively smoothly on the magnetic field. It is made up of the second term of (2.15) and of all terms but the first of (2.17).

Finally, all the remaining terms of the asymptotic forms (2.8) and (2.15) make up the increment $j_\alpha^{(2)}(k)$. The latter differs in form and changes substantially, depending on the ratio of ω and ν. We present first an expression for $j_\alpha^{(2)}(k)$ in the high-frequency region $\omega\gg\nu$, when a strong delay effect takes place. In this case the term containing $I(z,z')+I(z',z)$ in (2.15) can be disregarded and

$$j_\alpha^{(2)}(k)=\frac{3\omega_0^2}{4\pi^2v}\frac{1}{e^{2\pi\gamma}-1}\int_0^{\pi/2}d\vartheta\,n_\alpha^2(\vartheta,0)\int_0^\infty\frac{dk'}{(kk')^{1/2}}\mathscr{E}_\alpha(k')$$
$$\times\left\{\pi\delta(k-k')-\frac{1}{k-k'}\sin\left[\frac{\gamma^2}{2R_\perp}\left(\frac{1}{k}-\frac{1}{k'}\right)\right]\right.$$
$$\left.-\frac{1}{k+k'}\cos\left[\frac{\gamma^2}{2R_\perp}\left(\frac{1}{k}+\frac{1}{k'}\right)+2\pi i\gamma\right]\right\} \tag{2.21}$$

at $\omega\gg\nu$.

The dependence on the magnetic field (both resonant and smooth) under the conditions of the delay effect is contained in the last two terms of (2.18), whereas the first term is independent of H. Therefore $j_\alpha^{(2)}$ and $j_\alpha^{(2)}(k)$ must be taken into account, even though they are small compared with $j_\alpha^{(0)}(k)$. The smallness of $j_\alpha^{(2)}(k)$ in the high-frequency region $\omega\gg\nu$ is ensured by the fact that the rapidly oscillating term (the term with the sine function) is subtracted from the delta function, but is the near-limiting expression for this function relative to the parameter (1.3).

It is of interest to compare the obtained formulas with the previously known results. Equation (2.21) generalizes, to the case of an arbitrary ratio of ν and Ω, the expression previously obtained[2] in the case of sharp resonance. The term (2.20) is not contained in Ref. 2. Although it is large compared with (2.21), in the case of sharp resonance this term is a much smoother function of the magnetic field than $j_\alpha^{(2)}(k)$ and describes the far wings of the resonance line. Equation (2.20) generalizes the result obtained in Ref. 4 to include the case of an arbitrary reflection coefficient ρ. In addition, in contrast to Ref. 4, where the oscillations of the current relative to the magnetic field were neglected, expression (2.20) does take these oscillations into account.

At low frequencies, when ω is smaller than or of the order of the collision frequency ν, the correction

$j_\alpha^{(2)}(k)$ is given by

$$j_\alpha^{(2)}(k)=\frac{3\omega_0^2}{16\pi^2 v}e^{-2\pi\gamma}\int_0^{\pi/2}d\vartheta\, n_\alpha^2(\vartheta,0)\int_0^\infty\frac{dk'}{(kk')^{1/2}}\mathscr{E}_\alpha(k')$$
$$\times\Big\{\pi\delta(k-k')\Big[1+\Phi\Big(\frac{\gamma}{(2kR_\perp)^{1/2}}e^{-i\pi/4}\Big)\Big]$$
$$+\Big[\pi\delta(k-k')-\frac{i}{k-k'}\exp\Big(-\frac{i\gamma^2}{2R_\perp}\Big(\frac{1}{k}-\frac{1}{k'}\Big)\Big)\Big]\Big[1-\Phi\Big(\frac{\gamma}{(2kR_\perp)^{1/2}}e^{-i\pi/4}\Big)\Big]$$
$$-\frac{1}{k+k'}\exp\Big(\frac{i\gamma^2}{2R_\perp}\Big(\frac{1}{k}+\frac{1}{k'}\Big)\Big)\Big[1-\Phi\Big(\frac{\gamma}{(2kR_\perp)^{1/2}}e^{i\pi/4}\Big)\Big]$$
$$+\frac{2i(kk')^{1/2}}{k^2-k'^2}\Phi\Big(\frac{\gamma}{(2kR_\perp)^{1/2}}e^{-i\pi/4}\Big)+(k\leftrightarrow k')\Big\} \tag{2.22}$$

at $\omega\lesssim\nu$. The symbol $(k\leftrightarrow k')$ means addition, in the curly brackets, of the terms obtained from those written out by interchaning k and k'. To obtain (2.22) it is necessary to expand the third term of (2.15) in terms of $e^{-2\pi\gamma}$ and neglect the cosine in the curly brackets. In the approximation linear in $e^{-2\pi\gamma}$ we arrive at (2.22). It must be noted that the asymptotic form of the current density in this nonresonant low-frequency region was not investigated previously with account taken of oscillations of the type $e^{-2\pi\gamma}$.

3. SURFACE IMPEDANCE

The dependence of the impedance $Z_\alpha(H)$ on the magnetic field can be obtained by perturbation theory, using as the zeroth approximation the impedance $Z_\alpha(\theta)$ determined by the current density $j_\alpha^{(0)}(k)$ in the absence of a magnetic field. In the approximation linear in $j_\alpha^{(1)}$ and $j_\alpha^{(2)}$ the increment to the impedance is

$$Z_\alpha(H)-Z(0)=\frac{8\pi\omega^2}{c^4E_\alpha'^2(0)}\int_0^\infty dk\mathscr{E}_\alpha(k)\,[j_\alpha^{(1)}+j_\alpha^{(2)}(k)], \tag{3.1}$$

where $\mathscr{E}_\alpha(k)$ is the Fourier component of the electric field $E(x)$ in the zeroth approximation (i.e., at $H=0$), and $E'_\alpha(0)$ is the derivative with respect to the argument on the surface of the metal.

The surface impedance $Z(0)$ at $H=0$ and at arbitrary reflection of the electrons from the metal boundary is given by[11]

$$Z(0)=\frac{4\pi\cdot 3^{1/2}\omega}{c^2k_0}\frac{\sin^3(\pi z_0/3)}{\sin^2(\pi z_0/2)}e^{-\pi i/3},\quad k_0=\Big(\frac{3\pi\omega_0^2\omega}{4vc^2}\Big)^{1/3}, \tag{3.2}$$

where $k_0=\delta^{-1}$ is the reciprocal thickness of the skin layer at $H=0$, and $0=\cos\pi z_0$.

To calculate the impedance due to the current $j_\alpha^{(1)}$ we do not need the explicit form of the distribution of the field $\mathscr{E}_\alpha(k)$, since the current $j_\alpha^{(1)}$ does not depend on k and is proportional to $E_\alpha(0)\propto Z(0)$. Accordingly,

$$\frac{\Delta Z_\alpha^{(1)}(H)}{Z(0)}=\frac{\mu_\alpha c^2k_0}{2^9\pi^2\omega}Z(0)\Big[2-(1-\rho)^2\sum_{n=1}^\infty\rho^{n-1}\frac{1+e^{-2\pi n\gamma}}{n^2}\Big]\Lambda^{-2}. \tag{3.3}$$

We have introduced here the complex parameter

$$\Lambda=-\frac{\gamma^2}{2k_0R}=\Big(\frac{\omega+i\nu}{\Omega}\Big)^2\Big/2k_0R, \tag{3.4}$$

which in the case of a sharp CR or near it ($\omega\approx n\Omega$, $\omega\gg\Omega\ll\nu$) coincides with the previously obtained delay parameter.[2]

The calculation of the impedance resonant component $\Delta Z_\alpha^{(2)}(H)$, connected with the current $j_\alpha^{(2)}(k)$, is perfectly analogous in the case $\omega\gg\nu$ with that given in Ref. 2. We therefore write down directly the final formula for the resonant part of the impedance at an arbitrary reflection of the electrons from the sample boundary:

$$\frac{\Delta Z_\alpha^{(2)}(H)}{Z(0)}=A_\alpha e^{-\pi i/3}\frac{2\pi\,\mathrm{ch}^2\pi\gamma}{e^{2\pi\gamma}-1}\sin^2\frac{\pi z_0}{2}\sin^2\frac{\pi z_0}{3}\Lambda^{-8}$$
$$\times\Big[1+a_\alpha e^{-\pi i/6}\frac{\cos^3(\pi z_0/2)}{\sin^2(\pi z_0/3)}\Lambda^{-1}+b_\alpha e^{-\pi i/3}\frac{\cos^4(\pi z_0/2)}{\sin^4(\pi z_0/3)}\Lambda^{-2}\Big] \tag{3.5}$$

at $\omega\gg\nu$. The values of the constants $A_\alpha a_\alpha$, and b_α, taken from Ref. 2, are the following:

$A_y=9A_z/\approx0.93$; $a_y=10a_z/9\approx4.414$; $b_y=11b_z/9=4.95$.

Equation (3.5) differs from the analogous equation of Ref. 2 in the form of the resonant factor and in the definition of Λ. Therefore Eq. (3.5) is valid not only near resonance, but also far from it. In the case of specular reflection, owing to the relatively simple form of the function $_\alpha(k)$, it is possible to find impedance corrections that have not only a power-law form but also an exponential form in terms of Λ. They can play an important role if the delay effect is not strong, when $|\Lambda|$ is not very large. The corresponding expressions are obtained by using the already indicated change of the form of the resonant factor and of the parameter Λ in Eqs. (3.9) and (3.10) of Ref. 2.

We proceed now to calculate the impedance in the region of low frequencies $\omega\lesssim\nu$. Since the asymptotic form of the current $j_\alpha^{(1)}$ does not change in this region, the impedance $\Delta Z_\alpha^{(1)}(H)$ has likewise the same form (3.3) as in the region of high frequencies $\omega\to\nu$. Owing to the change of the character of the asymptotic behavior of $j_\alpha^{(2)}(k)$, the corresponding impedance $\Delta Z_\alpha^{(2)}(H)$ changes, too. To calculate the latter we must substitute (2.22) in (3.1). We take into account the fact that the main contribution to $\Delta Z_\alpha^{(2)}(H)$ is made by the first term of (2.2) with the function

$$2\pi\delta(k-k')\Big[1+\Phi\Big(\frac{\gamma}{(2kR_\perp)^{1/2}}e^{-i\pi/4}\Big)\Big].$$

The other terms of the asymptotic form (2.22) make a contribution that is small in the parameter Λ. In fact the next term contains as a factor the difference $\delta(k-k')$ and expressions in the form of delta functions in $k-k'$. Therefore the calculation of the integrals with respect to k and k' produces in this term a corresponding smallness relative to Λ. The remaining terms of the asymptotic form (2.22) have no delta-function singularities at $k=k'$ and vary rapidly as functions of k and k' over intervals proportional to $|\Lambda|^{-1}$. This leads to the appearance of additional small factors in the parameter (3.4). Therefore the impedance $\Delta Z_\alpha^{(2)}(H)$ can be written in the form

$$\Delta Z_\alpha^{(2)}(H)=\frac{16\omega}{\pi c^2 k_0}e^{-2\pi\gamma}\int_0^{\pi/2}d\vartheta\, n_\alpha^2(\vartheta,0)$$

$$\times\int_0^\infty\frac{d\xi}{\xi}F^2(\xi)\left[1-\Phi\left(\left(\frac{|\Lambda|}{\xi\sin\vartheta}\right)^{1/2}e^{i\beta+i\pi/4}\right)\right] \tag{3.6}$$

at $\omega\lesssim\nu$. The function $F(\xi)$ is here the dimensionless Fourier component of the electric field,

$$\mathscr{E}_\alpha(k)=-2E_\alpha'(0)k_0^{-2}F(k/k_0),$$

and the phase shift β is determined by the ratio of ν and ω,

$$\beta=\mathrm{arctg}(\nu/\omega),\quad 0<\beta<\pi/2. \tag{3.7}$$

The explicit form of the function $F(\xi)$ is determined by the value of the reflection coefficient ρ of the electrons at the metal boundary, and is given, for example, in Ref. 1.

Equation (3.6) describes the effect mentioned in the preceding sections, that of the change of the surface impedance as a function of the sign of the frequency difference $\nu-\omega$. This effect is analogous to the known Stokes phenomenon and is due to the jumplike change of the asymptotic form of the error function $\Phi(x)$ at large values of $|x|$, when Re x goes through zero [see (2.14)]. The real part of the argument of the function Φ in (3.6) is equal to $(\omega-\nu)[|\Lambda|/2(\omega^2+\nu^2)\xi\sin\vartheta]^{1/2}$. Inasmuch as in the integration with respect to ϑ and ξ all the values of ϑ are significant and $\xi\sim 1$, the abrupt change of $\Delta Z_\alpha^{(2)}(H)$ should occur in the transition region

$$\frac{|\omega-\nu|}{(\omega^2+\nu^2)^{1/2}}\sim|\Lambda|^{-1/2}. \tag{3.8}$$

Far from this transition region we can confine ourselves to the first term of the asymptotic form of (2.14) for $\Phi(x)$ and replace the square bracket in (3.6) by double the unit step function $2\Theta(\nu-\omega)$. The integrals with respect to ϑ and ξ can then be calculated explicity-and the final expression for $\Delta Z_\alpha^{(2)}(H)$ becomes

$$\frac{\Delta Z^{(2)}(H)}{Z(0)}=-\frac{2-\cos(\pi z_0/3)}{3\cos^2(\pi z_0/2)}e^{-2\pi\gamma}\Theta(\nu-\omega). \tag{3.9}$$

Thus, (3.9) plays an essential role when the collision frequency ν exceeds the wave frequency ω, and the difference $\nu-\omega>[(\omega^2+\nu^2)/|\Lambda|]^{1/2}$. The impedance (3.9) does not depend on α (on the polarization of the external wave) because of the isotropy of the Fermi surface. It described damped harmonic surface-impedance oscillations due to CR at frequencies $\omega<\nu$. Their amplitude is determined by the value of the reflection coefficient ρ.

We note that in this region the analogous oscillations of $\Delta Z_\alpha^{(1)}(H)$ (the term proportional to $e^{-2\pi\gamma}$) are small compared with (3.9). The distinguishing feature of the impedance (3.9) is that it does not depend explicitly on the parameter Λ. At low frequencies $\omega<\nu$ the parameter $|\Lambda|=R\delta/2l^2$ is the square of the ratio of the characteristic length of the arc $(8R\delta)^{1/2}$ of the electron trajectory inside the skin layer to the mean-free-path length $l=v/\nu$. At $|\Lambda|\gg 1$ the arc length is large compared with l, i.e., the bending of the electron orbit by the magnetic field inside the skin layer is negligible. Therefore the current density is independent of H in the first-order approximation and coincides with the current in the absence of a magnetic field.

The impedance $\Delta Z^{(2)}(H)$ is due to electrons that execute a complete revolution in the magnetic field without colliding in the interior of the metal. This is precisely why there is no delay effect at $\omega<\nu$, and the dependence of $\Delta Z^{(2)}$ on H is given by the factor $e^{-2\pi\gamma}$, which is the probability of a single return of the electron to the skin layer (with account taken of the change of the phase of the electromagnetic wave). A dependence on the parameter Λ can appear only as a result of a change in the form of the distribution of the electromagnetic field inside the skin layer. At $|\Lambda|\lesssim 1$ this leads to a change of a numerical factor and to a dependence of $\Delta Z^{(2)}(H)$ on the reflection coefficient ρ. Thus, as expected from physical considerations, the delay effect exists only in a region of sufficiently high frequencies $\omega>\nu$.

4. DISCUSSION OF RESULTS

In this section we discuss the expected dependence of the surface impedance on the magnetic field in the region of weak magnetic fields (1.5), at different ratios between ω and ν, which in turn is determined by the temperature. In accord with the results of the preceding section, the impedance component that depends on the magnetic field, $\Delta Z_\alpha(H)=Z_\alpha(H)-Z(0)$, can be represented as a sum of a smooth part $\Delta Z_\alpha^{sm}(H)$ and a part $\Delta Z_\alpha^{osc}(H)$, that oscillates as a function of the field H and is due to the CR.

I. The expression for the field part is obtained directly from Eq. (3.3), if we let $e^{-2\pi\gamma}$ in it tend to zero:

$$\Delta Z_\alpha^{sm}(H)=\frac{\mu_\alpha c^2 k_0}{2^7\pi^2\omega}\left(\frac{k_0 v\Omega}{\omega^2+\nu^2}\right)^2|Z(0)|^2\left[2-(1-\rho)^2\sum_{n=1}^\infty\frac{\rho^{n-1}}{n^2}\right]e^{-i(2\pi/3+4\beta)} \tag{4.1}$$

Other smooth increments [e.g., from $\Delta Z_\alpha^{(2)}(H)$], are small in the parameter Λ. The expression for $\Delta Z_\alpha^{sm}(H)$ is valid for an arbitrary ratio of ω and ν and describes the quadratic change of the impedance with changing magnetic field; the sign of the change depends on β $=\arctan(\nu/\omega)$.

II. In contrast to the smooth part $\Delta Z_\alpha^{sm}(H)$, the impedance part $\Delta Z_\alpha^{osc}(H)$ that oscillates because of the CR is determined essentially by the ratio of ω and ν.

1. At sufficiently low temperatures in pure metals, when $\omega\gg\nu$, the form of the resonant oscillations is described by the sum of expression (3.5) and the terms oscillating in H from (3.3) (proportional to $e^{-2\pi n\gamma}$). These terms, while larger than $\Delta Z_\alpha^{(2)}(H)$, are smoother functions of the magnetic field H (or of the frequency ω). Therefore in the case of a sharp resonance $\Omega\gg\nu$ the line wings are described by $\Delta Z_\alpha^{(1)}(H)$, and (3.5) plays the essential role in the immediate vicinity of the resonant singularity ($|\omega-n\Omega|\lesssim\nu$). On the other hand if the resonance is not very sharp, it is necessary to take into account both terms $\Delta Z_\alpha^{(1)}$ and $\Delta Z_\alpha^{(2)}$. The smallness of the CR amplitude in this high-frequency case is due to the strong delay effect.

2. At a fixed frequency of the wave, the relaxation frequency ν increases with rising temperature and may turn out to be of the order of ω. Then the contribution (3.5) to the oscillating part of the impedance decreases much faster than (3.3). Consequently in the region

$$\omega-\nu\gg\left(\frac{\omega^2+\nu^2}{|\Lambda|}\right)^{1/2}=(2k_0\nu\Omega)^{1/2},\quad \omega\sim\nu \tag{4.2}$$

the CR manifest itself in the form of the damped harmonic oscillations from (3.3):

$$\Delta Z_\alpha^{osc}(H)=\frac{\mu_\alpha c^2k_0}{2^7\pi^2\omega}\left(\frac{k_0\nu\Omega}{\omega^2+\nu^2}\right)^2|Z(0)|^2(1-\rho)^2 \times\exp\left\{-\frac{2\pi\nu}{\Omega}+2\pi i\left(\frac{\omega}{\Omega}-\frac{2\beta}{\pi}+\frac{1}{6}\right)\right\}. \tag{4.3}$$

3. On going to the low-frequency region, where $\nu>\omega$, the delay effect ceases to play any role, as a result of which the amplitude of the CR oscillations increases sharply. In the limit

$$\nu-\omega\gg(2k_0\nu\Omega)^{1/2} \tag{4.4}$$

the oscillating part of the impedance does not depend on the parameter Λ and is determined, according to (3.9) by

$$\Delta Z^{osc}(H)=\frac{2-\cos(\pi z_0/3)}{3\cos^2(\pi z_0/2)}|Z(0)|\exp\left\{-\frac{2\pi\nu}{\Omega}+2\pi i\left(\frac{\omega}{\Omega}+\frac{1}{3}\right)\right\}. \tag{4.5}$$

The foregoing results were obtained for a quadratic and isotropic dispersion law. They can be easily generalized, however, to include the case of an arbitrary Fermi surface, in analogy with what was done earlier in Ref. 2.

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New approach to the theory of the dielectric constant of a system of interacting electrons

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An exact formula is obtained for the correction, which depends on the wave vector and on the frequency and must be introduced into the dielectric constant of a homogeneous system of interacting electrons to account for the local field $G(\mathbf{q},\omega)$. The simplest approximation for the exact result for $G(\mathbf{q},\omega)$, which takes into account both two-particle and three-particle exchange-correlation effects, is considered. A table is presented of the numerical values of the approximate correction for the local field in the static limit $\omega=0$.

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1. INTRODUCTION

A homogeneous system of N interacting electrons that move in a volume Ω against the background of a uniformly distributed neutralizing postive charge was considered in physics more than 40 years ago[1] as the simplest model of the metallic state of matter. Corresponding to such a state is a Hamiltonian

$$H=\sum_j\frac{\mathbf{p}_j^2}{2m}+\frac{1}{2}\sum_{\mathbf{q}}v(\mathbf{q})\sum_{j\neq j'}\exp[i\mathbf{q}(\mathbf{r}_j-\mathbf{r}_{j'})], \tag{1}$$

where $\mathbf{p}_j$ and $\mathbf{r}_j$ are respectively the momentum operator and the coordinate of the j-th electron, and

$$v(\mathbf{q})=\begin{cases}4\pi e^2/q^2\Omega, & q\neq0,\\ 0, & q=0\end{cases}$$

is the Fourier component of the electron-electron

