Differential conductivity of semiconductors with inelastic scattering of electrons

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We calculate the longitudinal differential conductivity of a semiconductor in a strong constant magnetic field, when the dominant electron-scattering mechanism is spontaneous emission of optical phonons of frequency ω_0 by the electrons. It is assumed that the scattering in the passive region, where the electron energy is $\epsilon < \hbar\omega_0$, is weak, and that the penetration of the electron into the active region is also small ($\epsilon > \hbar\omega_0$). In this approximation, the differential conductivity differs from zero only in the resonant regions, at frequencies that are multiples of the "fly through" frequency $\omega_E = 2\pi/\tau_E$ (τ_E is the time of flight of the electron to the boundary of the passive region). The shape of the resonance line is obtained. It is determined by a competition between the weak resonance in the passive region and the finite penetration of the electrons into the active region. The conditions under which negative differential conductivity can occur are determined.

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1. INTRODUCTION

In pure semiconductors at low temperatures ($kT \ll \hbar\omega_0$, ω_0 is a limiting frequency of the optical phonon), the dominant electron-scattering mechanism may turn out to be spontaneous emission of optical phonons.[1] Since this scattering occurs in the active region of momentum space, where the electron energy is $\varepsilon > \hbar\omega_0$ and only weak scattering mechanisms are possible in the passive region ($\varepsilon < \hbar\omega_0$), there exists an electric-field interval

$$E_0^- \ll E \ll E_0^+, \quad E_0^\pm = p_0/e\tau^\pm, \tag{1.1}$$

in which the electron distribution function in momentum space deviates considerably from equilibrium. The notation used in (1.1) is the following: $p_0 = (2m\hbar\omega_0)^{1/2}$ is the electron momentum at the boundary of the passive region; τ^+ and τ^- are the characteristic electron scattering times in the active and passive regions, respectively, with $\tau^+ \ll \tau^-$. When the conditions (1.1) are satisfied, the electron moving under the action of the external field does not have time, on the one hand, to be scattered in the passive region, and on the other hand it does not manage to penetrate in it to a sufficient depth. Therefore all the kinetic properties of the semiconductor are determined mainly by the dynamics of the electron motion in the passive region. The electron system is not only strongly heated (the average electron energy is of the order of $\hbar\omega_0$) but is essentially anisotropic, since the electrons can be located only on some of the so-called invariant trajectories.

That this situation is realistic is attested by the observed increased anisotropy of the distribution of the carriers in p-Ge at nitrogen temperatures.[2] Particularly encouraging, however, were recent experiments performed with AgBr at helium temperatures.[3] The saturation of the drift velocity, the characteristic dependence of the Hall angle of the current on the field, and the linear field dependence of the cyclotron-resonance line width all indicate that scattering by optical phonons predominates, and the kinetic properties of the semiconductor are determined by the dynamics of the motion of the electrons in the passive region.

The presence, in momentum space, of a region with a clearly pronounced dynamics of the electrons gives rise to a number of interesting phenomena. First, threshold effects, which were considered in detail in Ref. 1, appear on the current-voltage characteristics. Second, resonance effects due to the presence of a characteristic time of flight τ_E of the electron from the point $p = 0$ to the boundary of the passive region, (in the case of a constant electric field we have $\tau_E = p/eE$). Thus, it is indicated in Ref. 4 that the noise spectra should have peaks at frequencies multiples of the "fly-through" frequency $\omega_E = 2\pi/\tau_E$. The longitudinal differential conductivity $\sigma_{zz}(\omega)$, in contrast to the current-fluctuation spectral density $(\delta j_z \delta j_z)$ calculated in the lowest approximation in the scattering in the passive region, is equal to zero. Thus, relations of the Callen-Welton type, which are satisfied under thermodynamic equilibrium conditions

$$(\delta j_i \delta j_k)_\omega = \frac{kT}{\pi} \operatorname{Re} \sigma_{ik}(\omega), \tag{1.2}$$

are strongly violated in the present situation.

More detailed calculations[5-7] have shown, however, that the differential conductivity differs from zero. Moreover, in the resonant regions the conditions are favorable for the appearance of negative differential conductivity (NDC). Thus, in Refs. 5 and 6 there were solved model problems in which account was taken of only the finite penetration of the electrons into the active region. The problem in a quantizing magnetic field was solved numerically in Ref. 7, with account taken of both the penetration of the electrons into the active region, and the scattering in the passive region. These studies have revealed regions in which NDC conductivity accurs. The calculations show that the NDC is very sensitive to the intensity of the weak scattering of the electrons in the passive region, as well as to the magnitude of the penetration of the electrons into the active region. But since the numerical calculations are always restricted to some single concrete set of material parameters, we deem it advisable to carry out an analytic treatment of the longitudinal differential conduc-

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tivity in the presence of a constant electric field that satisfies the condition (1.1). In this paper we present the results of the calculation of the differential conductivity in resonant regions, determine the shape of the resonance line, and obtain the conditions under which one can expect NDC to appear.

2. FORMULATION OF PROBLEM

To calculate the differential conductivity we shall solve the kinetic equation, assuming that besides the strong constant electric field $\mathbf{E}_0$ directed along the z axis there is also a weak alternating field $\mathbf{E}_1 e^{i\omega t} \| \mathbf{E}_0$. We assume that the only electron-scattering mechanism in the active region is spontaneous emission of optical phonons, characterized by the probability

$$w^{(0)}(\mathbf{p}, \mathbf{p}') = \frac{1}{\tau^+ p_0} \delta(p^2 - p'^2 - p_0^2), \tag{2.1}$$

and in view of the small penetration of the electrons into the active region τ^+ can be regarded as a constant. In the passive region we postulate the presence of weak scattering characterized by a probability $w(\mathbf{p}, \mathbf{p}')$. The electron concentration n is assumed to be so small that the interaction between the electrons can be disregarded. To simplify the derivations, the electron dispersion law is assumed to be parabolic and isotropic. To take into account the real dispersion law it is essentially necessary only to replace in (4.12) and (4.13) the integration region by the region determined from the condition $\varepsilon(\mathbf{p}) \leqslant \hbar\omega_0$.

We introduce the dimensionless variables

$$\mathbf{r} = \mathbf{p}/p_0, \quad \Omega = \omega\tau_E, \quad W(\mathbf{r}, \mathbf{r}') = p_0^3 \tau^- w(\mathbf{p}, \mathbf{p}'),$$
$$\gamma_0 = \tau_E/\tau^+, \quad \eta = \tau_E/\tau^-, \quad \alpha = E_1/E_0, \quad 1/\tau^- = p_0^{-3} \int d^3p \int d^3p' w(\mathbf{p}, \mathbf{p}'). \tag{2.2}$$

In the determination of τ^- the integration is carried out over the passive region of momentum space. Then, representing the distribution function normalized to unity in the form

$$f(\mathbf{r}, t) = F(\mathbf{r}) + \alpha e^{i\omega t} \varphi(\mathbf{r}) \tag{2.3}$$

and linearizing the kinetic equation with respect to the alternating field, we obtain a system of equations for the constant part of the distribution function

$$\frac{d}{dz} F = \hat{K} F^+ + \eta \hat{S} F, \tag{2.4}$$
$$(d/dz + \gamma) F^+ = 0, \tag{2.5}$$

and for the alternating part of the distribution function

$$(d/dz + i\Omega)\varphi = \hat{K}\varphi^+ + \eta\hat{S}\varphi - \hat{K}F^+ - \eta\hat{S}F, \tag{2.6}$$
$$(d/dz + i\Omega + \gamma)\varphi^+ = \gamma F^+. \tag{2.7}$$

A plus sign indicates that the particular function pertains to the active region. In (2.4)–(2.7) we have used the following operator notation:

$$\hat{K}F^+ = \gamma_0 \int_{r'>1} d^3r' \delta(r'^2 - r^2 - 1) F^+(\mathbf{r}') = {}^1/_2 \gamma_0 \int d^2\rho' \cdot F^+(\rho', (1 + r^2 - \rho'^2)^{1/2}) / (1 + r^2 - \rho'^2)^{1/2} \tag{2.8}$$

[in the last equality a change was made to the cylindrical coordinate system $F(\mathbf{r}) = F(\boldsymbol{\rho}, z)$],

$$\hat{S}F = \int d^3r' \{W(\mathbf{r}', \mathbf{r}) F(\mathbf{r}') - W(\mathbf{r}, \mathbf{r}') F(\mathbf{r})\}. \tag{2.9}$$

We have also used the notation

$$\gamma = 2\pi\gamma_0 (r^2 - 1)^{1/2}. \tag{2.10}$$

The differential conductivity can be represented in the form

$$\sigma(\omega) = \sigma_0 \int d^3r \varphi(\mathbf{r}), \quad \sigma_0 = nep_0/mE_0, \tag{2.11}$$

where n is the electron concentration.

We solve the system (2.4)–(2.7) in the following sequence. We first integrate the equations (2.5) and (2.7) for the active regions along the electron trajectories in momentum space. We represent the solution in terms of certain functions [$\Phi(\rho)$ and $\Psi(\rho)$ respectively for the constant and alternating parts of the distribution function) that characterize the distribution of the electrons across the trajectories of the motion on the boundary of the passive region. Substituting the indicated solutions into the equations for the passive region (2.4) and (2.6), integrating these equations, and matching the solutions on the boundary of passive region, we obtain integral equations for the function $\Phi(\rho)$ and $\Psi(\rho)$. These integral equations are solved by methods of perturbation theory, using two small parameters η and γ_0^{-1}. To simplify the calculations we used inverse operators defined as follows:

$$G^{\pm}(\Omega)F = e^{-i\Omega z} \int_{\pm(1-\rho^2)^{1/2}}^{z} dz' e^{i\Omega z'} F(\rho, z'),$$
$$G(\Omega)F = \exp\{-i\Omega(1-\rho^2)^{1/2}\} \int_{-(1-\rho^2)^{1/2}}^{(1-\rho^2)^{1/2}} dz e^{i\Omega z} F(\rho, z), \tag{2.12}$$
$$G^{\pm} \equiv G^{\pm}(0), \quad G \equiv G(0).$$

3. CONSTANT PART OF THE DISTRIBUTION FUNCTION

The solution of (2.5) can be represented in the form

$$F^+(\rho, z) = \Phi(\rho)\Gamma(\rho, z), \tag{3.1}$$

where

$$\Gamma(\rho, z) = \exp\{-G^+\gamma\}. \tag{3.2}$$

Substituting (3.1) in (2.4) and integrating the latter, we obtain the solution in the passive region:

$$F(\rho, z) = G^- \hat{K}\Gamma\Phi. \tag{3.3}$$

This solution was obtained using the boundary condition $F[\rho, -(1-\rho^2)^{1/2}] = 0$ brought about by the absence of a scattering arrival term in the active region. We have also discarded the term $\eta\hat{S}F$ in (2.4), since iteration with respect to this term leads to higher degrees of smallness than those taken into account hereafter in the equation for the alternating part of the distribution function.

Matching the solutions (3.1) and (3.3) on the boundary leads to the following homogeneous integral equation:

$$\Phi = \hat{M}\Phi \tag{3.4}$$

with the operator

$$\hat{M} = G\hat{K}\Gamma, \tag{3.5}$$

the kernal of which can be represented in the following form:

$$M(\rho, \rho') = {}^1/_2 \gamma_0 \int_{-(1-\rho^2)^{1/2}}^{(1-\rho^2)^{1/2}} dz (1 + \rho^2 + z^2 - \rho'^2)^{-1/2} \cdot \exp\left\{-2\pi\gamma_0 \int_{(1-\rho'^2)^{1/2}}^{(1+\rho^2+z^2-\rho'^2)^{1/2}} dz' (\rho'^2 + z'^2 - 1)^{1/2}\right\}. \tag{3.6}$$

The properties of the kernel (3.6) can be easily investigated, for owing to the presence of the large parameter $\gamma_0 \gg 1$ it is a sharp function of the argument ρ, whereas the dependence on the argument ρ' is a smooth one. Expanding the kernel in powers of ρ' and constructing a perturbation theory in the parameter $\gamma_0^{-2/3}$, we can verify that the spectrum of the operator $\hat{M}$ contains the level $\lambda_0 = 1$ and the levels $\lambda_n \sim (\gamma_0^{-2/3})^n, n \geq 1$. The function $\Phi(\rho)$ is thus an eigenfunction of this operator, corresponding to the eigenvalue $\lambda_0 = 1$. It can be approximately represented in the form

$$\Phi(\rho) \approx M(\rho, 0) \approx {}^1/_2\gamma_0 \int_{-\infty}^{\infty} dz \exp\{-{}^3/_8\pi\gamma_0(\rho^2+z^2)^{1/2}\}. \tag{3.7}$$

In the approximation $\gamma_0 \to \infty$ it goes over naturally into $\delta(\boldsymbol{\rho})$, which corresponds to a needle-shaped distribution of the electrons on the principal trajectory. The eigenfunction of the adjoint operator, corresponding to the eigenvalue $\lambda_0 = 1$, is approximately equal to $\chi_0(\boldsymbol{\rho}) \approx 1$.

4. ALTERNATING PART OF THE DISTRIBUTION FUNCTION

We represent the solution for the active region (2.7) in the form

$$\varphi^+(\rho, z) = \exp\{-i\Omega(z-\sqrt{1-\rho^2})\}\Gamma(\rho, z)\Psi(\rho) + \Gamma(\rho, z)G^+(\Omega)\gamma\Phi. \tag{4.1}$$

We have used here the identity

$$(d/dz + i\Omega + \gamma)^{-1}\gamma\Gamma\Phi = \Gamma G^+(\Omega)\gamma\Phi, \tag{4.2}$$

the validity of which can be easily verified by applying to both sides of this identity the operator $(d/dz + i\Omega + \gamma)$.

Substituting (4.1) in (2.6) and integrating, we get the following expression for the distribution function in the passive region:

$$\begin{aligned}\varphi(\rho, z) = {} & G^-(\Omega)\{\hat{K}\exp[-i\Omega(z-\sqrt{1-\rho^2})]\Gamma\Psi \\ & + \hat{K}\Gamma[G^+(\Omega)\gamma - 1]\Phi - \eta\hat{S}F + \eta\hat{S}\varphi\}.\end{aligned} \tag{4.3}$$

We now use the smallness of the parameter η and carry out one iteration of the term $\eta\hat{S}\varphi$ in (4.3). Since the third term of the expression already contains the small parameter η, while the second term, as will be shown below, contains the small parameter $\Omega\gamma_0^{-2/3}$, we substitute in $\hat{S}\varphi$ only the first term of the right-hand side of (4.3). Then the matching of the solutions on the boundary leads to the following integral equation for the function

$$\begin{aligned}&\{1 - G(\Omega)\hat{K}\exp[-i\Omega(z-\sqrt{1-\rho^2})]\Gamma - \eta G(\Omega)\hat{S}G^-(\Omega)\hat{K}\exp[-i\Omega(z \\ &\quad -\sqrt{1-\rho^2})]\}\Psi = \{G(\Omega)\hat{K}\Gamma[G^+(\Omega)\gamma - 1] - \eta G(\Omega)\hat{S}G^-\hat{K}\Gamma\}\Phi.\end{aligned} \tag{4.4}$$

In the zeroth approximation in η and at $\Omega = 0$ this equation takes the form

$$\{1 - \hat{M}\}\Psi = G\hat{K}\Gamma[G^+\gamma - 1]\Phi, \tag{4.5}$$

whose operator coincides with the operator of (3.5). Multiplying this equation by the eigenfunction of the adjoint operator $\chi_n(\boldsymbol{\rho})$ and integrating with respect to $\boldsymbol{\rho}$, we obtain the following identities:

$$(\chi_0, G\hat{K}\Gamma[G^+\gamma - 1]\Phi) = 0, \tag{4.6}$$

$$(\chi_n, G\hat{K}\Gamma[G^+\gamma - 1]\Phi) = -\lambda_n = O(\gamma_0^{-1/3}), \quad n \geq 1. \tag{4.7}$$

We have introduced here the notation

$$(\chi, \varphi) = \int d^2\rho\, \chi(\rho)\varphi(\rho). \tag{4.8}$$

It follows from (4.6) and (4.7) that the right-hand side of (4.4) contains the small parameters η, $\gamma_0^{-2/3}$, or $\Omega^2\gamma_0^{-2/3}$. If we assume that the solution Ψ is sought in the form of expansion in the eigenfunctions of the operator $\hat{M}$, then it is easy to verify that the action of the operators of the left-hand side of (4.4) on Ψ also leads to the appearance of small parameters, with the exception of the action of the first operator on the zeroth eigenfunction, which makes a contribution of the order of unity. Therefore in the lowest nonvanishing approximation in the parameters η and $\gamma_0^{-2/3}$ the function $\Psi(\boldsymbol{\rho})$ can be represented in the form

$$\Psi(\rho) = A_0\Phi(\rho), \tag{4.9}$$

and the coefficient A_0 can be obtained by averaging Eq. (4.4) with the function χ_0, and this leads to the expression

$$\begin{aligned}A_0 = {} & \{(\chi_0, G(\Omega)\hat{K}\Gamma[G^+(\Omega)\gamma - 1]\Phi) + \eta(\chi_0, G(\Omega)\hat{S}G^-\hat{K}\Gamma\Phi)\} \cdot \\ & \cdot\{1 - (\chi_0, G(\Omega)\hat{K}\exp[-i\Omega(z-\sqrt{1-\rho^2})]\Gamma\Phi) - \eta(\chi_0, \\ & G(\Omega)\hat{S}G^-(\Omega)\hat{K}\exp[-i\Omega(z-\sqrt{1-\rho^2})]\Gamma\Phi)\}^{-1}.\end{aligned} \tag{4.10}$$

In this expression, it suffices to confine oneself in the terms containing η to the approximation $\chi_0(\boldsymbol{\rho}) = 1, \Phi(\boldsymbol{\rho}) = \delta(\boldsymbol{\rho})$. In the remaining terms it is necessary to use expression (3.7). Because of the identity (4.6) it is necessary to take into account in the numerator of (4.10) only the terms that do not vanish at $\Omega = 0$, and this simplifies greatly the calculation of the integrals. Confining ourselves to the indicated approximations and substituting the obtained $\Psi(\boldsymbol{\rho})$ in (4.3) and next in the definition (2.11) of $\sigma(\omega)$ we obtain the final expression for the differential conductivity

$$\sigma(\omega) = -\frac{\sigma_0}{i\Omega}A_0 = \frac{\sigma_0}{i\Omega}\frac{{}^1/_3 a_0\gamma_0^{-2/3}[\Omega^2 + i\Omega] + \eta B_0}{e^{i\Omega} - 1 + {}^1/_2 a_0\gamma_0^{-1/3}[\Omega^2 + i\Omega] + \eta B_1}, \tag{4.11}$$

where

$$a_0 = \left(\frac{3}{2\pi}\right)^{1/3}\frac{1}{3}\Gamma\left(\frac{5}{3}\right) \approx 0.18,$$

and

$$\begin{aligned}B_0 = {} & -\int_{r<1} d^3r \int_0^1 dz' W(0, z', r)\{\exp[i\Omega(z'-1)] \\ & - \exp[i\Omega(z-\sqrt{1-\rho^2})]\},\end{aligned} \tag{4.12}$$

$$\begin{aligned}B_1 = {} & \int_{r<1} d^3r \int_0^1 dz' W(0, z', r)\{\exp[i\Omega(z'-1)] \\ & - \exp[i\Omega(z-\sqrt{1-\rho^2})]\}\exp[-i\Omega(z'-1)].\end{aligned} \tag{4.13}$$

Expression (4.11) was obtained accurate to terms $\Omega^2\gamma_0^{-2/3}$ and η. Therefore it is valid only in the resonance region, where $\Omega \approx \Omega_n \equiv 2\pi n$, when $e^{i\Omega} \approx 1$, and the entire fraction assumes a value of the order of unity.

5. DISCUSSION OF RESULTS

We investigate the behavior of $\sigma(\omega)$ determined by expression (4.11) in the resonant region. We put for this purpose $\Omega = \Omega_n + \Delta\Omega$ and expand the exponential in the denominator of the expression. We then get

$$\sigma(\Omega) = \frac{\sigma_0}{\Omega_n}\frac{a+ib}{i\Delta\Omega + c + id}, \tag{5.1}$$

where the expressions for the coefficients a, b, c, and d can be easily obtained in terms of the parameters B_0,

B_1, η, and γ_0.

The dependence of the differential conductivity on the frequency is easiest to trace by mapping (5.1) in the complex plane $\sigma=\sigma_1+i\sigma_2$, as shown in Fig. 1. Then $\Delta\Omega$ ranges from $-\infty$ to $+\infty$ and $\sigma(\Omega)$ traces in a clockwise direction a circle that passes through the origin 0 and whose center is located at the point

$$\sigma_1^c=\frac{\sigma_0}{\Omega_n}\frac{a}{2c},\quad \sigma_2^c=\frac{\sigma_0}{\Omega_n}\frac{b}{2c}, \tag{5.2}$$

while the point diametrically opposite to 0 corresponds to the shifted resonant frequency $\Delta\Omega=-d$. The detuning $\Delta\Omega$, of course, can not be too large ($\Delta\Omega\ll 1$), inasmuch as at $\Delta\Omega\sim 1$, on the one hand, the exponential cannot be expanded in a series, and on the other hand it is incorrect to retain only the resonant term in (4.11).

It is seen from Fig. 1 that the NDC region ($\sigma<0$) is always present when $\sigma_1^c<0$. To be able to discern the DNC against the background of the nonresonant terms at $\sigma_1^c<0$ it is necessary to satisfy the condition

$$|\sigma_2^c|^2\geqslant|\sigma_1^c|\max\,(\Omega^2\gamma_0^{-2/3},\eta). \tag{5.3}$$

It follows from (4.11) that two weak phenomena compete in the formation of the frequency dependence of $\sigma(\Omega)$, scattering in the passive region and penetration of the electron into the active region. If the penetration into the active region is decisive, then, putting $\eta=0$ in (4.11), we get

$$\sigma^c=\frac{\sigma_0}{\Omega_n}\left\{\frac{1}{3\Omega_n^2}-i\frac{1}{3\Omega}\right\} \tag{5.4}$$

The center of the circle lies in this case in the fourth quadrant and NDC should be observed at frequencies exceeding the resonant frequency, as was in fact determined in Ref. 5. According to (5.3) the conditions for the appearance of the NDC become worse with increasing number of the resonance.

If scattering in the passive region is decisive, then the frequency dependence of $\sigma(\Omega)$ is determined by the coefficients B_0 and B_1. We have estimated these coefficients numerically, assuming that the scattering in the passive region is elastic. For the scattering probability we have chosen the standard expression[8]

$$w_I(\mathbf{p},\mathbf{p}')=w_0|\mathbf{p}-\mathbf{p}'|^n\delta(p^2-p'^2). \tag{5.5}$$

We recall that an exponent $n=1$ corresponds to deformation scattering and $n=-1$ corresponds to polarization scattering by acoustic phonons. Scattering by neutral impurities corresponds to $n=-1$. To estimate the effect of scattering by ionized impurities we chose the expression[9]

$$w_{II}(\mathbf{p},\mathbf{p}')=\frac{w_0}{(|\mathbf{p}-\mathbf{p}'|^2+q_D^2)^2}\delta(p^2-p'^2). \tag{5.6}$$

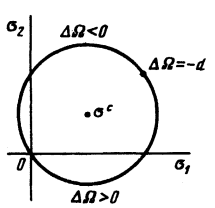

FIG. 1. Frequency dependence of Eq. (5.1) in the complex plane $\sigma=\sigma_1+i\sigma_2$.

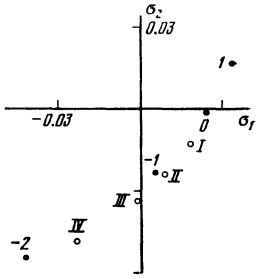

FIG. 2. Position of the center of the circle σ^c (in units of $\sigma_0/2\pi$): ○—scattering of type (5.5), the numbers indicate the values of n; ●—scattering of type (5.6), I—$q_D=p_0$, II—$q_D=0.5p_0$, III—$q_D=0.25p_0$, IV—$q_D=0.06p_0$.

The results of the calculation for the first resonance $\Omega_1=2\pi$ are shown in Fig. 2 in the form of the position of the circle σ^c as a function of the exponent n for $w_I(\mathbf{p},\mathbf{p}')$ and of the screening momentum q_D for $w_{II}(\mathbf{p},\mathbf{p}')$. The plot is in units of σ_0/Ω_1. The results indicate that the most favorable conditions for the determination of the NDC are negative values of n or small q_D, i. e., when the scattering is most effective at small momenta.

We present for comparison also an expression for $\sigma(\Omega)$ in the case of quantizing magnetic field, for which numerical calculations were made in Ref. 7. In the single-band approximation, the calculation can be performed in accordance with the proposed scheme with the only simplification that, because of the dimensionality of the problem, Eq. (4.4) becomes algebraic. The final expression for $\sigma(\Omega)$ agrees in form with (4.11), the only difference being that the expression $1/3a_0\gamma_0^{-2/3}$ in the numerator must be replaced by $2\gamma_0^{-2}$, and $1/2a_0\gamma_0^{-2/3}$ in the denominator must be replaced by γ_0^{-2}. If we confine ourselves again to elastic scattering in the passive region, assuming that

$$w(p_z,p_z')=\frac{1}{p_0\tau_E}\nu(z)\delta(z+z'),$$

then the coefficients B_0 and B_1 take the form

$$B_0=-\int_0^1 dz\nu(z)\sin\Omega z,\quad B_1=2i\int_0^1 dz\nu(z)\sin\Omega z e^{-i\Omega z}. \tag{5.7}$$

If the contribution of the scattering in the passive region prevails over the contribution of the penetration of the electrons into the active region then, recognizing that (5.7) means $\mathrm{Re}B_1>0$ and $\mathrm{Im}B_0=0$, we can conclude that the necessary condition for the onset of NDC is $B_0<0$. Thus, just as in the three-dimensional case, the favorable factor for the onset of NDC is scattering that prevails at small momenta.

As already indicated in the introduction, the most favorable experimental conditions for the realization of the model investigated above occur in silver halides at helium temperatures. In Ag, Br, for example,[10] the mobility changes from 50 to 20 000 cm^2/V·sec when the temperature changes from 300 to 6 K. This points to a rather intense scattering of the electrons by optical phonons in the active region. The ration E_0^+/E_0^- estimated from the mobility is of the order of 400, and condition (1.1) is easily satisfied. Estimates[3] yield $\tau^-\sim 5$

$\times 10^{-11}$ sec, which corresponds to $E_0^- \sim 50$ V/cm. At helium temperatures, scattering by ionized impurities predominates in the passive region, and at $n \sim 2.5 \times 10^{14}$ cm^{-3} the value of q_D/p_0 is of the order of 0.25, so that good conditions are obtained for the observation of NDC in a frequency region slightly exceeding the fly-through frequency. The fly-through frequency is $\nu_E \sim 30$ GHz at $E_0 \sim 74$ V/cm.

We note in conclusion that despite the strong disequilibrium in the electron system in this case, the spectral density of the current fluctuations, defined in Ref. 4, is closely connected with $\sigma(\omega)$. Both quantities differ substantially from zero only in resonant regions, where both have Lorentzian shapes. Therefore in the vicinity of each resonance the Callen-Welton relation (1.2) can be suitably defining the proportionality coefficient T, which is interpreted as the noise temperature. In the present situation T is large and is proportional to either $(\tau_E/\tau^+)^{2/3}$ or τ^-/τ_E; this indicates that the electron system is strongly heated. In addition, T must be chosen complex and written under the Re sign. The phase of T defined in this manner is equal to the phase of σ^e and is directly connected with the onset of the NDC or, in other words, with the possibility of development of unstable fluctuations. The electron system for which a complex T follows from (1.22) is a unique intermediate case between a stable system with real positive T and unstable fully inverted system with negative T for which NDC exists in the entire resonance region.

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[1]I. I. Vosilyus and I. B. Levinson, Zh. Eksp. Teor. Fiz. **50**, 1660 (1966); **52**, 1013 (1967) [Sov. Phys. JETP **23**, 1104 (1966); **25**, 672 (1967)].
[2]M. A. C. Brown and E. C. S. Paige, Phys. Rev. Lett. **7**, 84 (1961). R. Bray, and W. E. Pinson, Phys. Rev. Lett. **11**, 268 (1963); Phys. Rev. **A136**, 449 (1964).
[3]S. Komiyama, T. Masumi, and T. Kurosava, Proc. 13-th Int. Conf. Phys. Semicond. Rome, 1976, pp. 1222–1225; 14-th Int. Conf. Phys. Semicond., Edinburgh, 1978.
[4]I. B. Levinson and A. Yu. Matulis, Zh. Eksp. Teor. Fiz. **54**, 1466 (1968) [Sov. Phys. JETP **27**, 786 (1968)].
[5]A. A. Andronov and V. A. Kozlov, Pis'ma Zh. Eksp. Teor. Fiz. **17**, 124 (1973) [JETP Lett. **17**, 87 (1973)].
[6]Yu. V. Gulyaev and I. I. Chusov, Fiz. Tverd. Tela (Leningrad) **20**, 2637 (1978) [Sov. Phys. Solid State **20**, 1524 (1978)].
[7]Ya. I. Al'ber, A. A. Andronov, V. A. Valov, V. A. Kozlov, A. M. Kozlov, A. M. Lerner, and I. P. Ryazantseva, Zh. Eksp. Teor. Fiz. **72**, 1030 (1977) [Sov. Phys. JETP **45**, 539 (1977)].
[8]F. G. Bass and Yu. G. Gurevich, Goryachie élektrony i sil'nye élektromagnitnye volny v plazme poluprovodnikov i gazovogo razryada (Hot Electrons and Strong Electromagnetic Waves in Semiconductor and Gas-Discharge Plasma), Nauka, 1975, p. 20.
[9]V. Denis and Yu. Pozhela, Goryachie élektrony (Hot Electrons Electrons), Mintis, Vil'nyus, 1971, p. 25.
[10]D. S. Burnham, F. C. Brown, and R. S. Knox, Phys. Rev. **119**, 1560 (1960).

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Current fluctuations in a strong electric field under conditions of frequent interelectron collisions that ensure a Maxwellian distribution with drift

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A quantitative description is presented of noise in strong electric fields at high carrier densities when the form of the electron distribution both in energy and in momentum is governed by the collisions between the atoms. It is shown that under these conditions, in the general case, neither the longitudinal nor the transverse noise temperatures are equal to the electron temperature which fluctuates in a Maxwellian distribution with drift. The difference is due to an additional correlation of the occupation numbers of the electronic states which results from (and only from) the collisions between the electrons. The corresponding expressions are obtained for the spectral density of the current fluctuations in a wide temperature interval.

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1. INTRODUCTION

In nonequilibrium systems the current fluctuations and the carrier diffusion coefficients are no longer connected with the conductivity by the Nyquist and Einstein relations. These quantities contain new information, and their investigations provides a method for the diagnostics of the nonequilibrium electron gas in a semiconductor.

Price[1] was apparently the first to call attention to the important circumstance that although neither the Nyquist theorem nor the Einstein relation holds in the nonequilibrium state, there nevertheless remains in force a simple relation between the diffusion coefficient and the spectral density of the current fluctuations. Price's "nonequilibrium fluctuation-diffusion relation" is widely used, since it makes it possible to extract, from measurements of noise in a spatially homogeneous nonequilibrium system, the information on the response of a nonequilibrium system to a spatial gradient produced in it, and vice versa.[2-4]

The foregoing, however, is incontrovertibly true only

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