Magnetic oscillations of the momentum distribution of hot photoexcited electrons in semiconductors

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The effect of a magnetic field on the polarization spectrum of hot photoluminescence in *p*-type semiconductors is considered (in the Stokes and anti-Stokes regions of the spectrum). It is shown that in quasielastic energy relaxation the distribution function of the photoexcited electrons depends periodically on the magnetic field at a fixed energy and on the energy at a fixed field. This leads to the same dependences of the position of the luminescence-polarization plane in a longitudinal magnetic field (Faraday geometry) and the degree of linear polarization in a transverse magnetic field (Voigt geometry). It is shown that if the Stokes parameters of the photoluminescence in the magnetic field are known it is possible to determine the energy and momentum relaxation times of hot electrons.

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1. INTRODUCTION

The polarization characteristics of hot photoluminescence in *p*-type semiconductors were investigated experimentally and theoretically in Refs. 1–6. It was established[1,2] that in interband absorption of light the photoexcited electrons are anisotropically distributed in momentum near the excitation line, i. e., optical alignment of the momenta of these electrons takes place. The last circumstance leads to linear polarization and to an anomalously high degree of circular polarization of the recombination radiation in the short-wave region.

In the present paper we investigate the influence of a stationary homogeneous magnetic field on the alignment of the momenta of photoexcited electrons in *p*-type semiconductors under stationary illumination by linearly polarized light. The case is considered when the magnetic field H is not quantizing for the relaxing hot electrons ($\omega_c = |e|H/mc \ll \delta\varepsilon/\hbar$, $\delta\varepsilon$ is the average energy given up or acquired by the electron in one collision act, and m is the electron mass). It is shown in this article that when the electrons are excited by the light high into the conduction band ($T \ll \varepsilon_0$, where ε_0 is the electron energy at the instant of excitation and T is the temperature in energy units), when quasielastic energy relaxation becomes possible, the anisotropic part of the electron distribution function at fixed energy depends periodically on the magnetic field[1]. The reason for this is the following. Each electron, produced with an energy ε_0 and momentum $\mathbf{p}_0$, is cooled to a fixed energy ε after a certain random time τ, and its momentum rotates around the direction of the magnetic field through an angle $\omega_c\tau$. However, the scatter of the random quantity τ about the mean time $\bar{\tau}(\varepsilon)$ of the descent of the electron is

$$\Delta\tau=(\overline{(\tau-\bar{\tau}(\varepsilon))^2})^{1/2}\sim\bar{\tau}(\varepsilon)/n^{1/2},$$

where n is the number of scattering acts when the "distance" is traversed. If the field is such that

$$\omega_c\Delta\tau=\omega_c\bar{\tau}(\varepsilon)\left(\frac{\delta\varepsilon}{\varepsilon_0-\varepsilon}\right)^{1/2}\ll 1,$$

then it can be assumed that all the photoexcited electrons, which are anisotropically distributed in momentum at the instant of their production, are cooled to the energy ε within the same time $\bar{\tau}(\varepsilon)$ and their momenta rotate through the same angle $\omega_c\bar{\tau}(\varepsilon)$ around the direction of the magnetic field. The anisotropic part of the distribution function "rotates" around the field direction and becomes a periodic function of the parameter $\omega_c\bar{\tau}(\varepsilon)$.

In the present paper, the periodic character of the distribution function is established by solving the kinetic equation. In Sec. 2 of the article is derived a system of kinetic equations for the coefficients $f_{j_1j_2j_3}(\varepsilon, H)$ for spherical angular invariants that characterize the distribution of the photoexcited electrons in the magnetic field. In Sec. 3 this system of equations is solved near the excitation line (in both the Stokes and anti-Stokes regions), and then (in Sec. 4) the obtained solutions are continued into the energy region where the "diffusion" approximation is valid. In magnetic fields satisfying the relation

$$\omega_c\bar{\tau}(\varepsilon)(\delta\varepsilon/|\varepsilon_0-\varepsilon|)^{1/2}\ll 1,$$

the distribution of the electrons turns out to be periodically dependent on the parameter $\omega_c\bar{\tau}(\varepsilon)$; the time $\bar{\tau}(\varepsilon)$ is determined. Also determined is the magnetic momentum-distribution anisotropy damping due to the dephasing of the oscillations. In Sec. 5 are investigated the polarization characteristics of hot photoluminescence in a magnetic field. Two cases are considered: the Faraday geometry (radiation in a longitudinal magnetic field) and the Voigt geometry (magnetic field perpendicular to the outgoing light beam). It is established that in the case of the Voigt geometry the degree of linear polarization of the recombination radiation depends periodically on the field at a fixed radiation frequency or on the frequency at a fixed field. In the case of the Faraday geometry, the plane of the radiation polarization rotates when the magnetic field is increased or when the distance from the excitation line is increased.

The case of low temperatures, when the corrugation of the equal-energy surfaces in the valence band becomes important, is considered separately. Approximate expressions are obtained for the Stokes parameters at arbitrary directions of the magnetic field and of the excitation polarization plane relative to the crystal axes.

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It is shown in the Conclusion that the polarization characteristics of the luminescence in a magnetic field make it possible to determine the times of the energy and momentum relaxation of the hot photoexcited electrons.

2. KINETIC EQUATION

The kinetic equation for the distribution function $f(\mathbf{p}, \mathbf{H})$ of photoexcited electrons in a constant homogeneous nonquantizing magnetic field $\mathbf{H}$ is of the form

$$i\omega_c(\mathbf{e}_H\mathbf{L})f(\mathbf{p},\mathbf{H})+f(\mathbf{p},\mathbf{H})\int W(\mathbf{p},\mathbf{p}')d^3p'/(2\pi\hbar)^3 -\int f(\mathbf{p}',\mathbf{H})W(\mathbf{p}',\mathbf{p})d^3p'/(2\pi\hbar)^3=F(\mathbf{p}), \tag{1}$$

where $\mathbf{e}_H$ is a unit vector along the field $\mathbf{H}$, and $L=-i[\mathbf{p}\times\partial/\partial\mathbf{p}]$. The function $F(\mathbf{p})$ is the rate of electron photoexcitation. The quantity $W(\mathbf{p},\mathbf{p}')$ is the probability, per unit time, of the transition of the electron from a state with momentum $\mathbf{p}$ into a state with momentum $\mathbf{p}'$ under the influence of the collisions. The collisional relaxation is assumed to be isotropic, i.e., $W(\mathbf{p},\mathbf{p}')$ depends only on the initial and final energies ε and ε' and on the angle between $\mathbf{p}$ and $\mathbf{p}'$. Introducing the unit vectors $\boldsymbol{\nu}$ and $\mathbf{e}$ respectively along the momentum band along the exciting-light polarization vector, it is convenient to seek the function $f(\mathbf{p},\mathbf{H})$ in the form

$$f(\mathbf{p},\mathbf{H})=\sum_{j_1j_2j_3} f_{j_1j_2j_3}(\varepsilon,H)\,|j_1j_2(j_3)j_300\rangle. \tag{2}$$

Here the functions $|j_1j_2(j_3)j_300\rangle$ are all the possible variants made up of the vectors $\boldsymbol{\nu}$, $\mathbf{e}$, and $\mathbf{e}_H$ (Ref. 8) and are listed in the Appendix. Separating in Eq. (1) the angular part (see the Appendix) we obtain system of equations for the functions f_{j1j2j3}:

$$f_{000}(\varepsilon)\int W_0(\varepsilon,\varepsilon')\rho(\varepsilon')d\varepsilon'-\int f_{000}(\varepsilon')W_0(\varepsilon',\varepsilon)\rho(\varepsilon')d\varepsilon'=F_{000}(\varepsilon), \tag{3}$$

$$-2^{1/2}i\omega_c f_{221}+If_{220}=F_{220}, \tag{4}$$

$$-i\omega_c[2^{1/2}f_{220}+(7/5)^{1/2}f_{222}]+If_{221}=0, \tag{5}$$

$$-i\omega_c[(7/5)^{1/2}f_{221}+(6/35^{1/2})f_{223}]+If_{222}=0, \tag{6}$$

$$-i\omega_c[(6/35^{1/2})f_{222}+(2/7^{1/2})f_{224}]+If_{223}=0, \tag{7}$$

$$-2/7^{1/2}i\omega_c f_{223}+If_{224}=0. \tag{8}$$

Here $F_{000}(\varepsilon)$ and $F_{220}(\varepsilon)$ are the coefficients of the expansion of the pump[2] $F(\mathbf{p})$ in a series such as (2),

$$If=f(\varepsilon,H)\int W_0(\varepsilon,\varepsilon')\rho(\varepsilon')d\varepsilon'-\int f(\varepsilon',H)W_2(\varepsilon',\varepsilon)\rho(\varepsilon')d\varepsilon', \tag{9}$$

$$W_l(\varepsilon,\varepsilon')=\frac{1}{2}\int_0^\pi W(\mathbf{p},\mathbf{p}')P_l(\cos\theta)\sin\theta\,d\theta, \tag{10}$$

$\rho(\varepsilon)$ is the state density, and θ is the angle between $\mathbf{p}$ and $\mathbf{p}'$. It is more convenient to represent the system (4)–(8) in a different form, expressing all the functions f_{j1j2j3} in terms of the single function f_{220}:

$$IIII(If_{220}-F_{220})+5\omega_c^2II(If_{220}-F_{220}) +4\omega_c^4(If_{220}-F_{220})=-2\omega_c^2IIF_{220}-{}^{16}/_5\,\omega_c^4F_{320}, \tag{11}$$

$$f_{221}=-\frac{i}{2^{1/2}}\frac{1}{\omega_c}(If_{220}-F_{220}), \tag{12}$$

$$f_{222}=-\sqrt{\frac{5}{14}}\left[2f_{220}+\frac{1}{\omega_c^2}I(If_{220}-F_{220})\right], \tag{13}$$

$$f_{223}=\frac{i}{6\sqrt{2}}\left[10\frac{1}{\omega_c}F_{220}+17\frac{1}{\omega_c}(If_{220}-F_{220})+\frac{5}{\omega_c^3}II(If_{220}-F_{220})\right], \tag{14}$$

$$f_{224}=\frac{1}{12\sqrt{14}}\left[72f_{220}+\frac{70}{\omega_c^2}IF_{220}+\frac{155}{\omega_c^2}I(If_{220}-F_{220}) +\frac{35}{\omega_c^4}III(If_{220}-F_{220})\right]. \tag{15}$$

3. DISTRIBUTION OF PHOTOEXCITED ELECTRONS NEAR THE EXCITATION LINE ($|\varepsilon-\varepsilon_0|\ll\varepsilon_0$)

We assume that the electrons are produced with an energy $\varepsilon_0(T\ll\varepsilon_0, T$ is the temperature in energy units), i.e.,

$$F_{000}(\varepsilon)=F_{000}\delta(\varepsilon-\varepsilon_0),\quad F_{220}(\varepsilon)=F_{220}\delta(\varepsilon-\varepsilon_0).$$

To solve Eqs. (3) and (11)–(15) we shall follow the procedure used in Ref. 8. We assume the energy relaxation to be quasielastic. Then the quantities $|\varepsilon_0-\varepsilon|\ll\varepsilon_0$, near the excitation line depend only on the difference $\varepsilon'-\varepsilon$ and on the energy ε_0, i.e.,

$$W_l(\varepsilon',\varepsilon)=u_l(\varepsilon_0,\varepsilon'-\varepsilon).$$

The detailed balancing principle then takes the form

$$u_l(\varepsilon_0,x)=e^{x/T}u_l(\varepsilon_0-x). \tag{16}$$

In addition, the state density $\rho(\varepsilon')$ can be replaced by $\rho(\varepsilon_0)$ and the integration limits in the collision integral can be taken to be $-\infty$ and $+\infty$. These approximations are valid in the region $|\varepsilon_0-\varepsilon|\ll\varepsilon_0$, if the energy $\delta\varepsilon$ given up or acquired by the electron in one collision act is much less than ε_0. Using these assumptions, we can solve the system (3) and (11)–(15) by the Fourier method. Equation (3) was treated by the described scheme in Ref. 6. The solution of Eq. (11) will be sought in the form

$$f_{220}(\varepsilon,H)=f_{220}^{(0)}(\varepsilon)+f_{220}^{(1)}(\varepsilon,H),$$

with the function $f_{220}^{(0)}(\varepsilon)$ subject to the condition

$$If_{220}^{(0)}=F_{220}. \tag{17}$$

The solution of Eq. (17) was obtained also in Ref. 6.

Solving the equation for $f_{220}^{(0)}(\varepsilon,H)$ by the Fourier method, we obtain

$$f_{220}^{(1)}(\varepsilon,H)=\frac{1}{2\pi}\left(-\frac{2}{5}F_{220}\right) \times\int_{-\infty}^{\infty}dt\,e^{i(\varepsilon-\varepsilon_0)t}\frac{5\omega_c^2\chi^2(t)+8\omega_c^4}{\chi(t)[\chi(t)+i\omega_c][\chi(t)-i\omega_c][\chi(t)+2i\omega_c][\chi(t)-2i\omega_c]}, \tag{18}$$

where

$$\chi(t)=\rho(\varepsilon_0)\int_{-\infty}^{\infty}[u_0(\varepsilon_0,x)-u_2(\varepsilon_0,x)e^{ixt}]dx.$$

The integrand in (18) has no singularities in the band $0<\operatorname{Im} t<1/T$, since $\operatorname{Re}\chi(t)$ does not vanish in this region. On the other hand it follows from the detailed balancing principle (16) that

$$\tilde{u}_l(\varepsilon_0,-t)=\tilde{u}_l\left(t+\frac{i}{T}\right),\quad \tilde{u}_l(\varepsilon_0,t)=\int_{-\infty}^{\infty}u_l(\varepsilon_0,x)e^{ixt}dx.$$

Using these circumstances, we can show that near the excitation line ($|\varepsilon_0-\varepsilon|\ll\varepsilon_0$)

$$f_{220}^{(1)}(\varepsilon_0+\Delta\varepsilon,H)=e^{-\Delta\varepsilon/T}f_{220}^{(1)}(\varepsilon_0-\Delta\varepsilon,H). \tag{19}$$

In similar fashion, using (12)–(15), we can establish relations of the type (19) also for the functions f_{221}, f_{222}, f_{223}, f_{224}. Similar relations for the functions f_{000} and $f_{220}^{(0)}$ were obtained in Ref. 6. Thus, all the quantities

$f_{j1j2j3}(\varepsilon,H)$ that characterize the distribution function (2) near the excitation line in the anti-Stokes region ($\varepsilon>\varepsilon_0$) are expressed in terms of the values of the same functions in the Stokes region ($\varepsilon<\varepsilon_0$):

$$f_{j_1j_2j_3}(\varepsilon_0+\Delta\varepsilon,\ H)=e^{-\Delta\varepsilon/T}f_{j_1j_2j_3}(\varepsilon_0-\Delta\varepsilon,\ H). \tag{20}$$

It should be noted that relations (20) hold at $|\Delta\varepsilon|\ll\varepsilon_0$ independently of the relaxation mechanism and of the temperature.

We now obtain the asymptotic expression for $f_{220}(\varepsilon_0-\Delta\varepsilon,H)$ at $\Delta\varepsilon>0$ and $\Delta\varepsilon\gg T$, $\delta\varepsilon$[2)] (but, as before, $\Delta\varepsilon\ll\varepsilon_0$). The asymptotic function $f_{220}^{(0)}(\varepsilon_0-\Delta\varepsilon)$ is given in Ref. 6, and the asymptotic expression for $f_{220}^{(1)}(\varepsilon_0-\Delta\varepsilon,\ H)$ will be determined by those poles of the integrand of (18) which have the smallest absolute values. To find these poles we can use the small parameter $\delta\varepsilon/\varepsilon_0$ and expand the function $\chi(t)$ in (18) in a series in the "diffusion" approximation [i.e., neglecting quantities of order $(\delta\varepsilon/\varepsilon_0)^3$]:

$$\chi(t)=\frac{1}{\tau_{p2}(\varepsilon_0)}-i\frac{\varepsilon_0}{\tau_\varepsilon'(\varepsilon_0)}t+D_2(\varepsilon_0)t^2. \tag{21}$$

The relaxation times $\tau_\varepsilon'(\varepsilon_0)$ and $\tau_{p2}(\varepsilon_0)$ and the "diffusion" coefficient $D_2(\varepsilon_0)$ are determined here by the relations

$$\frac{\varepsilon_0}{\rho(\varepsilon_0)\tau_\varepsilon'(\varepsilon_0)}=\int_{-\infty}^{\infty}xu_2(\varepsilon_0,x)\,dx, \tag{22}$$

$$\frac{1}{\rho(\varepsilon_0)\tau_{p2}(\varepsilon_0)}=\int_{-\infty}^{\infty}[u_0(\varepsilon_0,x)-u_2(\varepsilon_0,x)]\,dx, \tag{23}$$

$$\frac{D_2(\varepsilon_0)}{\rho(\varepsilon_0)}=\frac{1}{2}\int_{-\infty}^{\infty}x^2u_2(\varepsilon_0,x)\,dx. \tag{24}$$

The sought poles of the integrand of (18), which contribute to the asymptotic form of $f_{220}^{(1)}$, will then be the following values of t:

$$\begin{gathered}t=-it_0,\quad t=-i(t_0+\Delta t)\pm\omega_c\tau_\varepsilon'(\varepsilon_0)/\varepsilon_0,\\ t=-i(t_0+4\Delta t)\pm2\omega_c\tau_\varepsilon'(\varepsilon_0)/\varepsilon_0,\\ t_0=\tau_\varepsilon'(\varepsilon_0)/\varepsilon_0\tau_{p2}(\varepsilon_0),\ \Delta t=D_2(\varepsilon_0)(\tau_\varepsilon'(\varepsilon_0)/\varepsilon_0)^3\omega_c^2.\end{gathered} \tag{25}$$

The imaginary parts of the poles (25) describe the damping of the anisotropic part of the electron distribution function with increasing energy distance from the excitation line. This damping is due both to momentum relaxation and to the presence of the magnetic field. In the expression for Δt we have discarded the diffusion correction terms that do not depend on the magnetic field. The poles (25) yield the correct asymtotic form of $f_{220}^{(1)}$ if $|t|\ll(\delta\varepsilon)^{-1}$.

Closing the integration contour in (18) in the lower half-plane and using the values of the poles (25) and the asymptotic representation of the function $f_{220}^{(0)}(\varepsilon_0-\Delta\varepsilon)$ (Ref. 6) we obtain the asymptotic form of $f_{220}(\varepsilon_0-\Delta\varepsilon,H)$:

$$f_{220}(\varepsilon_0-\Delta\varepsilon,H)={}^1/_5f_{220}^{(0)}(\varepsilon_0-\Delta\varepsilon)[1+2\Phi^4\cos2\varphi+2\Phi\cos\varphi], \tag{26}$$

$$\Phi=\exp(-\Delta t\Delta\varepsilon),\ \varphi=\omega_c\tau_\varepsilon'(\varepsilon_0)\Delta\varepsilon/\varepsilon_0. \tag{27}$$

Similar calculations yield readily the asymptotic representations of the other functions f_{j1j2j3} near the excitation line.

4. DISTRIBUTION OF PHOTOEXCITED ELECTRONS

We obtain now the distribution of the electrons in the energy region $|\varepsilon-\varepsilon_0|\gg\delta\varepsilon$, where the diffusion approximation is valid. In this energy region (the diffusion region), which is far from the excitation line, the anisotropic part of the distribution function satisfies the system of equations (11)–(15), in which we can neglect that contain the source function F_{220}. Taking into account the quasicontinuous character of the energy relaxation of the photoexcited electrons, we write down the collision term in Eqs. (11)–(15) in the diffusion approximation ($\varepsilon<\varepsilon_0$):

$$If=\frac{1}{\tau_{p2}}f-\frac{1}{\rho}\frac{\partial}{\partial\varepsilon}\left(\frac{\varepsilon}{\tau_\varepsilon'}\rho f\right)-\frac{1}{\rho}\frac{\partial^2}{\partial\varepsilon^2}(D_2\rho f). \tag{28}$$

The relaxation times $\tau_\varepsilon'(\varepsilon)$ and $\tau_{p2}(\varepsilon)$ and the diffusion coefficient $D_2(\varepsilon)$ are defined here by the relations

$$-\frac{\varepsilon}{\tau_\varepsilon'(\varepsilon)}=\int(\varepsilon'-\varepsilon)W_2(\varepsilon,\varepsilon')\rho(\varepsilon')\,d\varepsilon', \tag{29}$$

$$\frac{1}{\tau_{p2}(\varepsilon)}=\int[W_0(\varepsilon,\varepsilon')-W_2(\varepsilon,\varepsilon')]\rho(\varepsilon')\,d\varepsilon', \tag{30}$$

$$D_2(\varepsilon)=\frac{1}{2}\int(\varepsilon'-\varepsilon)^2W_2(\varepsilon,\varepsilon')\rho(\varepsilon')\,d\varepsilon'. \tag{31}$$

Equations (29)–(31) with $\varepsilon=\varepsilon_0$ go over, in the approximation assumed in Sec. 3, into Eqs. (22)–(24).

We now obtain the solution of Eq. (11) in the diffusion region. Since the function $f_{220}(\varepsilon,H)$ takes near the excitation line the form (26), it is natural to seek this function in the energy region $\varepsilon<\varepsilon_0$ in the form

$$f_{220}(\varepsilon,H)=Cf_{220}^{(0)}(\varepsilon)\exp\left[-\gamma^2\omega_c^2\int_\varepsilon^{\varepsilon_0}D_2\left(\frac{\tau_\varepsilon'}{\varepsilon}\right)^3d\varepsilon+i\gamma\omega_c\int_\varepsilon^{\varepsilon_0}\frac{\tau_\varepsilon'}{\varepsilon}d\varepsilon\right], \tag{32}$$

where C and γ are constants. Substituting (32) in (28) and recognizing that $If_{220}^{(0)}=0$ in the "diffusion" region, we obtain, neglecting quantities of order $\delta\varepsilon/\varepsilon_0$

$$If_{220}=i\gamma\omega_cf_{220},\ \varepsilon<\varepsilon_0. \tag{33}$$

Substituting now in Eq. (11) the collision term in the form (33), we obtain for the parameter γ the equation $\gamma^5-5\gamma^3+4\gamma=0$, whose solutions are the numbers 0, ±1, ±2. Returning to Eq. (32) and considering its real part with the obtained values of γ in the region of energies close to ε_0, we match the solution to Eq. (26). As a result we find that $f_{220}(\varepsilon,H)$ is determined as before by Eq. (26), in which we replace $f_{220}^{(0)}(\varepsilon_0-\Delta\varepsilon)$ by $f_{220}^{(0)}(\varepsilon)$, and the functions Φ and φ are defined not by (27) but as

$$\Phi=\exp\left[-\omega_c^2\int_\varepsilon^{\varepsilon_0}D_2\left(\frac{\tau_\varepsilon'}{\varepsilon}\right)^3d\varepsilon\right],\quad\varphi=\omega_c\int_\varepsilon^{\varepsilon_0}\left(\frac{\tau_\varepsilon'}{\varepsilon}\right)d\varepsilon. \tag{34}$$

We now solve Eq. (11) in the anti-Stokes region ($\varepsilon>\varepsilon_0$) far from the excitation line. Using the detailed balancing principle, it is easy to show that in this region the collision term (9) for a function in the form $f=e^{-\varepsilon/T}\Psi$ in the "diffusion" approximation is written in the form

$$If=e^{-\varepsilon/T}\left(\frac{1}{\tau_{p2}}+\frac{\varepsilon}{\tau_\varepsilon'}\frac{\partial}{\partial\varepsilon}-D_2\frac{\partial^2}{\partial\varepsilon^2}\right)\Psi. \tag{35}$$

Using expressions (20) and (26) for the asymptotic representation of the function $f_{220}(\varepsilon,H)$ near the excitation line at $\varepsilon>\varepsilon_0$, we seek the solution of (11) in the form

$$f_{200}(\varepsilon,H)=Cf_{200}^{(0)}(\varepsilon)\exp\left[-\gamma^2\omega_c^2\int_{\varepsilon_0}^{\varepsilon}D_2\left(\frac{\tau_\varepsilon'}{\varepsilon}\right)^3d\varepsilon+i\gamma\omega_c\int_{\varepsilon_0}^{\varepsilon}\left(\frac{\tau_\varepsilon'}{\varepsilon}\right)l\varepsilon\right], \tag{36}$$

where C and γ are constants.

Substituting (36) in (35), we easily verify that in this case, just as in the Stokes region, the following relation hold if quantities of the order $\delta\varepsilon/\varepsilon_0$ are neglected:

$$If_{220}=i\gamma\omega_c f_{220}, \quad \varepsilon>\varepsilon_0. \tag{37}$$

Substitution of the collision term in the form (37) in (11) and the subsequent matching of the real part of (36) to the function f_{220} near the excitation line make it possible to obtain the solution of (11) in the anti-Stokes region. Determining the explicit form of $f_{220}(\varepsilon, H)$ in the energy regions to the left and to the right of the excitation line, we can obtain with the aid of (12)–(15) the anisotropic part of the distribution function in the entire "diffusion" region. Thus, in the Stokes and anti-Stokes regions of the energy spectrum, where the "diffusion" approximation ($|\varepsilon-\varepsilon_0|\gg T$, $\delta\varepsilon$) is valid, the functions f_{j1j2j3}, which characterizes the distribution (2) of the photoexcited electrons, are of the form

$$f_{220}(\varepsilon, H)=\frac{1}{5}f_{220}^{(0)}(\varepsilon)\left[1+2\Phi^4\cos 2\varphi+2\Phi\cos\varphi\right], \tag{38}$$

$$f_{221}(\varepsilon, H)=\frac{2^{1/2}}{5}if_{220}^{(0)}(\varepsilon)\left[2\Phi^4\sin 2\varphi+\Phi\sin\varphi\right], \tag{39}$$

$$f_{222}(\varepsilon, H)=-\left(\frac{2}{35}\right)^{1/2}f_{220}^{(0)}(\varepsilon)\left[1-2\Phi^4\cos 2\varphi+\Phi\cos\varphi\right], \tag{40}$$

$$f_{223}(\varepsilon, H)=\frac{2^{1/2}}{5}if_{220}^{(0)}(\varepsilon)\left[\Phi^4\sin 2\varphi-2\Phi\sin\varphi\right], \tag{41}$$

$$f_{224}(\varepsilon, H)=\frac{1}{5}\left(\frac{2}{7}\right)^{1/2}f_{220}^{(0)}(\varepsilon)\left[3+\Phi^4\cos 2\varphi-4\Phi\cos\varphi\right], \tag{42}$$

where

$$\Phi=\exp\left[-\omega_c^2\left|\int_{\varepsilon_0}^{\varepsilon}D_2\left(\frac{\tau_\varepsilon'}{\varepsilon}\right)^3 d\varepsilon\right|\right], \quad \varphi=\omega_c\left|\int_{\varepsilon_0}^{\varepsilon}\left(\frac{\tau_\varepsilon'}{\varepsilon}\right)d\varepsilon\right|. \tag{43}$$

The expressions for the functions $f_{220}^{(0)}(\varepsilon)$ and for the isotropic part of the electron distribution $f_{000}(\varepsilon)$ in the diffusion region are given in Ref. 6.

In magnetic fields, for which the relation[3)] is satisfied

$$\omega_c^2\left|\int_{\varepsilon_0}^{\varepsilon}D_2\left(\frac{\tau_\varepsilon'}{\varepsilon}\right)^3 d\varepsilon\right|\ll 1, \tag{44}$$

the magnetic damping is small ($\Phi\approx 1$) and the electron distribution depends periodically on the parameter $\omega_c\bar{\tau}(\varepsilon)$, where

$$\bar{\tau}(\varepsilon)=\left|\int_{\varepsilon_0}^{\varepsilon}\left(\frac{\tau_\varepsilon'}{\varepsilon}\right)d\varepsilon\right|.$$

It can be directly verified that in this case the distribution function can be represented in the form

$$f(\mathbf{p}, \mathbf{H})=\frac{1}{8\pi^{3/2}}f_{000}(\varepsilon)\left[1+\alpha(\varepsilon)\frac{3(\mathbf{ve}')^2-1}{2}\right], \quad \alpha(\varepsilon)=\frac{5^{1/2}\overset{0}{f}_{220}(\varepsilon)}{f_{000}(\varepsilon)}, \tag{45}$$

where $\mathbf{e}'$ is a unit vector obtained from the excitation-polarization vector $\mathbf{e}$ by rotation through an angle $\varphi=\omega_c\bar{\tau}(\varepsilon)$ in the positive direction around the vector $\mathbf{e}_H$. In other words, the anisotropic part of the distribution function at fixed energy is described as before (just as at the instant of excitation) by the second Legendre polynomial, but the direction of the anisotropy axis (of the vector $\mathbf{e}'$) depends on the magnetic field and on the energy. The reason why the electron distribution function at fixed energy in a magnetic field is obtained by the action of a rotation operator in momentum space on the initial distribution function (in the absence of the field) is that at the instant of excitation the anisotropic part of the electron distribution is a definite spherical harmonic (the second Legendre polynomial). At an arbitrary initial distribution, the distribution function in the magnetic field can not be obtained by rotation, inasmuch in the latter case the different spherical harmonics at fixed energy will be rotated through different angles φ_l. The rotation angle of the l-th harmonic is $\varphi_l=\omega_c\bar{\tau}_l$, where

$$\bar{\tau}_l=\left|\int_{\varepsilon_0}^{\varepsilon}(\tau_{\varepsilon l}'/\varepsilon)d\varepsilon\right|$$

and the $\tau_{\varepsilon l}'$ are determined by a formula obtained from (29) by replacing W with W_l. One can speak of rotation of a distribution function of arbitrary form in a magnetic field only in the case of small-angle scattering, when the values of $W_l(\varepsilon, \varepsilon')$ (10) for different l are close and the differences between the rotation times $\bar{\tau}_l(\varepsilon)$ for the different spherical harmonics can be neglected.

5. POLARIZATION CHARACTERISTICS OF HOT PHOTOLUMINESCENCE IN A MAGNETIC FIELD

We consider the recombination of hot electrons excited by linearly polarized light in p-type semiconductors with the band structure of GaAs in a magnetic field. We are interested in the recombination of electrons with equilibrium heavy and light holes in the valence band or with holes that are frozen out to the ground level of a shallow acceptor. The valence band is assumed for the time being to be spherically symmetrical. The magnetic field is assumed to be non-quantizing for the recombining electrons and holes. It is convenient to characterize the partially polarized recombination radiation by the Stokes parameters[9] ξ_1, ξ_2, and ξ_3, which are connected with the mutually perpendicular polarization unit vectors $\mathbf{e}_1$ and $\mathbf{e}_2$ that form a right-hand triad with the unit vector $\mathbf{n}$ directed along the emerging light beam ($\mathbf{e}_1\times\mathbf{e}_2=\mathbf{n}$). The parameters ξ_1 and ξ_2 determine the degree of maximal linear polarization l and the angle ψ between the direction of the maximum polarization and the vector $\mathbf{e}_1$ (Ref. 9):

$$\xi_1=l\sin 2\psi, \quad \xi_3=l\cos 2\psi.$$

The third parameter ξ_2 determines the degree of circular polarization, and in our case is equal to zero, since we assume the magnetic field to be non-quantizing and by the same token we disregard its action on the distribution of the electrons and holes over the spin sublevels.

We assume that the observation direction coincides with the excitation direction or with the opposite direction, and choose as the polarization vectors $\mathbf{e}_1=\mathbf{e}$, $\mathbf{e}_2=\mathbf{n}\times\mathbf{e}$ ($\mathbf{e}$ is the unit vector of the excitation polarization). We consider the case when the radiation emerges parallel to the magnetic field ($\mathbf{n}\parallel\mathbf{e}_H$, Faraday geometry). Using the explicit form of the anisotropic part of the distribution function (38)–(42) and the expressions given in Ref. 3 for the Stokes parameter, we can show that in this case

$$\xi_1(\omega)=(\mathbf{n}\mathbf{e}_H)P_l\Phi^4\sin 2\varphi, \quad \xi_3(\omega)=P_l\Phi^4\cos 2\varphi, \tag{46}$$

where $P_l(\omega)$ is the degree of linear polarization of the luminescence in the absence of a field. Here and below the argument $\varepsilon(\omega)$ in the quantities Φ and φ [Eq. (43)], which enter in the polarization characteristics, is taken to mean the energy of the electron that takes part in the production of a luminescence quantum of frequency ω. From (46) it is seen that in the Faraday geometry the degree of maximal linear polarization l is $|P_l|\Phi^4$ and the vector of the maximum polarization is rotated through an angle

$$\varphi=\omega_c\left|\int_{\varepsilon_0}^{\varepsilon(\omega)}\left(\frac{\tau_\varepsilon'}{\varepsilon}\right)d\varepsilon\right|$$

relative to the vector **e** in a plane perpendicular to the observation direction (to the vector **n**), at a fixed frequency, with increasing field or in a fixed field with increasing distance from the excitation line. The spectral dependence of the degree of linear polarization $P_l(\omega)$ at $\mathrm{H}=0$ was investigated earlier in Refs. 5 and 6.

We consider now the Voigt geometry, when $\mathbf{e}_H\perp\mathbf{n}$. We assume also that the excitation-polarization plane is so oriented that $\mathbf{e}\perp\mathbf{e}_H$. In this case, as above, using the expressions for the Stokes parameters,[3] we can show that $\xi_1=0$ and

$$\xi_3(\omega)=P_l(1+\Phi^4\cos 2\varphi)/[2-P_l(1-\Phi^4\cos 2\varphi)], \qquad (47)$$

i. e., the axis of the maximum polarization is oriented in the same manner as in the absence of a field, and the degree of maximal polarization l changes in accord with formula (47) ($l=|\xi_3|$). In magnetic fields that satisfy the condition (44), the degree of maximum polarization is a periodic function of the parameter $\omega_c\bar{\tau}(\varepsilon(\omega))$.

Interest attaches to the case of low temperature, when an important role is played by the corrugation of the equal-energy surfaces in the valence band. We assume that the particles taking part in the recombination are only the heavy "diagonal" holes, whose quasimomenta are directed along [111], [$\bar{1}$11], [$\bar{1}\bar{1}\bar{1}$], etc. This situation obtains, for example, in GaAs.[4)] To gain an idea of the dependence of the radiation polarization on the magnetic field, we shall assume that the condition (440 is satisfied and that the entire distribution function of the electrons in the magnetic field is rotated at fixed energy through an angle $\varphi=\omega_c\bar{\tau}(\varepsilon)$ (this is strictly speaking not the case, as indicated above, since the distribution of the electrons photoexcited from the corrugated valence band are characterized by cubic rather than spherical harmonics[3]). Then, using the expressions given in Ref. 3 for the Stokes parameters, we can obtain the following expressions for the Stokes parameters of the luminescence due to heavy diagonal holes:

$$\xi_1=\frac{2P_l[(\mathbf{e}'\mathbf{e}_1)(\mathbf{e}'\mathbf{e}_2)-R_{12}']}{1+P_l[(\mathbf{e}'\mathbf{e}_1)^2+(\mathbf{e}'\mathbf{e}_2)^2-R_{11}'-R_{22}']}, \qquad (48)$$

$$\xi_3=\frac{P_l[(\mathbf{e}'\mathbf{e}_1)^2-(\mathbf{e}'\mathbf{e}_2)^2-R_{11}'+R_{22}']}{1+P_l[(\mathbf{e}'\mathbf{e}_1)^2+(\mathbf{e}'\mathbf{e}_2)^2-R_{11}'-R_{22}']}, \qquad (49)$$

where

$$R_{ij}'=e_x'^2e_{ix}e_{jx}+e_y'^2e_{iy}e_{jy}+e_z'^2e_{iz}e_{jz},$$

the coordinate axes X, Y, and Z are directed respectively along [100], [010], [001]. The quantity $P_l(\omega)$ in (48) and (49) is the maximum degree of linear polarization of luminescence of frequency at $\mathrm{H}=0$, when the radiation emerges along the [001] direction perpendicular to the crystal surface, and the excitation polarization vector **e** lies in the plane of the surface of the crystal and makes an angle $\pi/4$ with the [010] axis. (The theoretical value near the excitation line is[3] $P_l=0.25$.) The vector $\mathbf{e}'(\omega)$ in (48) and (49) is the vector $S_\varphi\mathbf{e}$, where S_φ is the operator of rotation through an angle $\varphi=\omega_c\bar{\tau}(\varepsilon(\omega))$ about the magnetic-field direction.

We consider now the case when the radiation emerges along the [001] axis perpendicular to the surface of the crystal and parallel to the direction of the excitation. It follows from symmetry considerations that the vector of the maximal linear polarization (if it is not equal to zero) lies in the plane of the surface of the crystal and makes an angle $\pi/4$ with the [100] axis independently of the excitation polarization and of the magnetic-field direction. This can be verified by writing down the Stokes parameters in a reference frame in which the vectors $\mathbf{e}_1$ and $\mathbf{e}_2$ are directed along [100] and [010], respectively. In this system $\xi_3=0$ and $\xi_1=2P_l(\mathbf{e}'\mathbf{e}_1)(\mathbf{e}'\mathbf{e}_2)$. In the case of Faraday geometry $\xi_1=P_l\sin 2\beta$, where β is the angle between the vector $\mathbf{e}'$ and the [100] axis. It is seen that the maximum degree of polarization at a fixed frequency can either increase when the magnetic field grows from zero (if the vector **e** is directed along [100] or [010], or else decrease (if **e** is directed along [110] or [$\bar{1}$10]). In the Voigt geometry, if the exciting light is polarized in the (100) plane, the expression for in the reference frame connected with the axes [100] and [010] takes form

$$\xi_1=2P_l[\cos\varphi+(\mathbf{e}\mathbf{e}_H)^2(1-\cos\varphi)](1-\cos\varphi)(\mathbf{e}\mathbf{e}_H)([n\times e]\mathbf{e}_H).$$

It is seen that the luminescence will be completely unpolarized in the absence of a field, and also in the presence of a field H if $\mathbf{e}\perp\mathbf{e}_H$ or $\mathbf{e}\parallel\mathbf{e}_H$.

We note in conclusion that knowing the Stokes parameters of the recombination radiation in a magnetic field, we can determine the relaxation time $\bar{\tau}_\varepsilon$. From the shape of the line $P_l(\omega)$ at $\mathrm{H}=0$ we can assess the time ratio $\tau_\varepsilon'/\tau_{p_2}$ (Ref. 6). Consequently, by studying the polarization of the photoluminescence in a magnetic field we can determine the times τ_ε' and τ_{p_2} of the energy and momentum relaxations of the hot electrons.

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APPENDIX

The spherical invariants made up of the vectors ν, **e**, and $\mathbf{e}_H$ are of the form

$$|j_1j_2(j_3)j_300\rangle=(-1)^{j_1+j_2+j_3}\sum_{m_1m_2m_3}\begin{pmatrix}j_1 & j_2 & j_3\\ m_1 & m_2 & m_3\end{pmatrix}Y_{j_1m_1}(\mathbf{v})Y_{j_2m_2}(\mathbf{e})Y_{j_3m_3}(\mathbf{e}_H),$$

where Y_{jm} are the spherical functions. With the aid of the relations

$$\langle j_1j_2(j_3)j_300|\mathbf{e}_H\mathbf{L}|j_1'j_2'(j_3')j_3'00\rangle=$$
$$=\delta_{j_1j_1'}\delta_{j_2j_2'}(-1)^{j_1+j_2+j_3+j_3'}\begin{pmatrix}1 & j_3 & j_3'\\ 0 & 0 & 0\end{pmatrix}\begin{Bmatrix}j_3 & 1 & j_3'\\ j_1 & j_2 & j_1\end{Bmatrix}.$$
$$[j_1(j_1+1)(2j_1+1)(2j_3+1)(2j_3'+1)]^{1/2},$$
$$\langle j_1j_2(j_3)j_300|j_1'j_2'(j_3')j_300\rangle=\delta_{j_1j_1'}\delta_{j_2j_2'}\delta_{j_3j_3'}$$

we can separate in the kinetic equation (1) the angular part and obtain the system of equations (3)–(8) for the coefficients f_{j1j2j3} for the following invariants:

$$|00(0)000\rangle=\frac{1}{8\pi^{3/2}},\qquad |22(0)000\rangle=\frac{5^{1/2}}{16\pi^{3/2}}(3(\mathbf{ev})^2-1),$$

$$|22(1)100\rangle=-\frac{3\cdot 10^{1/2}}{16\pi^{3/2}}i(\mathbf{ev})([\mathbf{ev}]\mathbf{e}_H),$$

$$|22(2)200\rangle=\frac{5\sqrt{14}}{112\pi^{3/2}}\{1-3(\mathbf{ev})(\mathbf{ve}_H)(\mathbf{ee}_H)-3([\mathbf{e}\times\mathbf{v}]\mathbf{e}_H)^2\},$$

$$|22(3)300\rangle=-i\frac{3\sqrt{10}}{32\pi^{3/2}}\{(\mathbf{ev})([\mathbf{e}\times\mathbf{v}]\mathbf{e}_H)-5(\mathbf{ee}_H)(\mathbf{ve}_H)([\mathbf{e}\times\mathbf{v}]\mathbf{e}_H)\},$$

$$|22(4)400\rangle=\frac{15}{32\cdot70^{1/2}\pi^{3/2}}\{-4+7(\mathbf{ev})^2-30(\mathbf{ve})(\mathbf{ve}_H)(\mathbf{ee}_H)+35(\mathbf{ve}_H)^2(\mathbf{ee}_H)^2+5([\mathbf{e}\times\mathbf{v}]\mathbf{e}_H)^2\}.$$

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1) The situation is similar to the one discussed in Refs. 7 in connection with spin relaxation of photoexcited electrons and excitons in a magnetic field.

2) Here and below we are interested in the asymptotic representation of the function f_{220} and of the other quantities $f_{j_1j_2j_3}$ in that energy region where the asymptotic representation of the function f_{000} is valid. The latter takes place when[6] $\Delta\varepsilon \gg T$ and $\Delta\varepsilon \gg \delta\varepsilon$.

3) It should be noted that since $D_2(\varepsilon)\sim\varepsilon\delta\varepsilon/\tau_\varepsilon'$, the condition (44) written for the Stokes region coincides with the condition for the existence of the periodic distribution obtained from intuitive considerations in Sec. 1.

4) It was shown in Refs. 3–5 that in GaAs the heavy diagonal holes have the lowest energy at a given quasimomentum p, and that the wave function of the ground state of the shallow acceptor in the momentum representation is made up mainly of wave functions of the heavy diagonal holes.

[1] V. I. Zemskiĭ, B. P. Zakharchenya, and D. N. Mirlin, Pis'ma Eksp. Teor. Fiz. **24**, 96 (1976) [JETP Lett. **24**, 82 (1976)] B. P. Zakharchenya, V. I. Zemskiĭ, and D. N. Mirlin, Fiz. Tverd. Tela (Leningrad) **19**, 1725 (1977) [Sov. Phys. Solid State **19**, 1006 (1977)].

[2] V. D. Dymnikov, M. I. D'yakonov, and V. I. Perel', Zh. Eksp. Teor. Fiz. **71**, 2373 (1976) [Sov. Phys. JETP **44**, 1252 (1976)].

[3] V. D. Dymnikov, Fiz. Tekh. Poluprovodn. **11**, 1478 (1977) [Sov. Phys. Semicond. **11**, 868 (1977)].

[4] D. N. Mirlin and I. I. Reshina, Zh. Eksp. Teor. Fiz. **73**, 859 (1977) [Sov. Phys. JETP **46**, 451 (1977)].

[5] V. D. Dymnikov, D. N. Mirlin, V. I. Perel', and I. I. Reshina, Fiz. Tverd. Tela (Leningrad) **20**, 2165 (1978) [Sov. Phys. Solid State **20**, 1250 (1978)].

[6] V. D. Dymnikov and V. I. Perel', Fiz. Tekh. Poluprovodn. **13**, 707 (1979) [Sov. Phys. Semicond. **13**, 415 (1979)].

[7] G. Fishman and C. Hermann, Phys. Status Solidi B **63**, 307 (1974). E. M. Gamarts, E. L. Ivchenko, M. I. Karaman, V. P. Mushinskiĭ, G. E. Pikus, E. S. Razbirin, and A. N. Starukhin, Zh. Eksp. Teor. Fiz. **73**, 1113 (1977) [Sov. Phys. JETP **46**, 590 (1977)].

[8] D. A. Varshalovich, A. N. Moskalev, and V. K. Khersonskiĭ, Kvantovaya teoriya uglovogo momenta (Quantum Theory of the Angular Momentum), Nauka, 1975.

[9] L. D. Landau and E. M. Lifshitz, Teoriya polya (Field Theory), Nauka, 1973 [Pergamon].

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Change of phonon energy in germanium at pressures up to 3 GPa

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Tunnel spectroscopy is used to measure the phonon energies in germanium at pressures up to ~3 GPa. It is shown that the restructuring of the germanium bands at ~1.8 GPa, wherein the minimum Δ_1 of the conduction band drops below the L_1 minimum, is accompanied by a change in the character of the tunneling with phonon participation. The values of the Grüneisen constants are obtained for the acoustic modes of the phonons with wave vector in the [100] direction; they are found to be −0.8 and 1.2 for the *TA* and *LA* modes, respectively.

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The participation of phonons in electron tunneling through a *p-n* junction which manifests itself by singularities on the current-voltage characteristics, yields information on the phonon spectra and the band structure of solids.[1]

In germanium at normal pressure, the tunneling is accompanied by a transition of the electrons to the valence band ($\Gamma_{25'}$) in the minimum of the conduction band L_1 (Ref. 2). Therefore, in accordance with the momentum conservation law, the positions of the singularities on the tunnel characteristics correspond to the phonon energies on the boundary of the Brillouin zone in the [111] direction (see Fig. 1).

The change of the phonon frequencies under hydrostatic compression to 1.8 GPa was investigated in Ref. 3 on a Ge tunnel diode (of *n*-type). In Ref. 4, using *n*-Ge samples with a Schottky barrier they investigated, in approximately the same pressure range, the shift of the singularities connected only with the optical branches of the spectrum. Finally, Payne[5] investigated the change of the phonon frequencies in a germanium *p-n* diode under uniaxial compression. These data were used to calculate the Grüneisen constants in Ge in the linear approximation.

Besides the change in the phonon frequencies, the band structure of Ge also changes under pressure. Various experiments have established that the minima of the conduction band L_1, $\Gamma_{2'}$ and Δ_1 under pressure are shifted relative to the valence band at different rates dE/dP, equal respectively to $5\cdot10^{-11}$, $14\cdot10^{-11}$ and $-(0-2)\cdot10^{-11}$ eV/Pa.[6] At a pressure ~1.5–1 GPa, as a result, the minimum Δ_1 of the conduction band with