- ²A. P. Kazantsev, Zh. Eksp. Teor. Fiz. **66**, 1599 (1974) [Sov. Phys. JETP **39**, 784 (1974)].
- ³T. W. Hansh and A. L. Shawlow, Opt. Commun. **13**, 68 (1975).
- ⁴I. V. Krasnov and N. Ya. Shaparev, Pis'ma Zh. Tekh. Fiz.
 2, 301 (1976) [Soviet Tech. Phys. Letters 2, 116 (1976)]; Kvantovaya Elektron. (Moscow) 4, 176 (1977) [Sov. J. Quantum Electron. 7, 100 (1977)].
- ⁵V. S. Letokhov, V. G. Minogin, and B. D. Pavlik, Zh. Eksp. Teor. Fiz. **72**, 1328 (1977) [Sov. Phys. JETP **45**, 698 (1977)].
- ⁶V. S. Letokhov and V. G. Minogin, Zh. Eksp. Teor. Fiz. 74, 1318 (1978) [Sov. Phys. JETP 47, 690 (1978)].
- ⁷S. Stenholm and J. Javanieneam, Appl. Phys. 16, 159 (1978).
- ⁸V. E. Shapiro, Zh. Eksp. Teor. Fiz. **72**, 1328 (1977) [Sov. Phys. JETP **45**, 698 (1977)].
- ⁹Yu. L. Klimontovich and S. N. Luzgin, Zh. Tekh. Fiz. 48,

2217 (1978) [Sov. Phys. Tech. Phys. 23, 1269 (1978)].

- ¹⁰I. V. Krasnov and N. Ya. Shaperev, Opt. Commun. 27, 239 (1978).
- ¹¹A. Yu. Pusep, A. B. Doktorov, and A. I. Burshtein, Zh. Eksp. Teor. Fiz. **72**, 98 (1977) [Sov. Phys. JETP **45**, 52 (1977)].
- ¹²E. V. Baklanov and B. Ya. Dubetskil, Opt. Spektrosk. 41, 3 (1976).
- ¹³J. D. Cole, Perturbation Methods in Applied Mathematics, Xerox College, 1968.
- ¹⁴A. P. Kol'chenko, S. G. Rautian, and G. I. Smirnov, Preprint No. 42, Inst. of Automation and Electrometry, 1977.
- ¹⁵M. I. D'yakonov, Zh. Eksp. Teor. Fiz. **51**, 612 (1966) [Sov. Phys. JETP **24**, 408 (1967)].

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Excitation of ion oscillations by a freely propagating electron beam

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We calculate the mechanism and singularities of the excitation of ion oscillations in a plasma produced by a steady electron beam that propagates in a gas in the absence of an external magnetic field. It is shown that they are governed by the effect of spatial enhancement of system-charge-density perturbations that are almost transverse to the beam velocity. The established experimental facts are explained satisfactorily, in the main, on the basis of a dispersion relation that takes into account, on the one hand, the buildup of the ion oscillations by the cold beam, and on the other, the collisionless damping of these oscillations by the ions and electrons of the plasma.

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Ion oscillations excited in a neutralized electron beam were investigated in a number of studies both in the presence of an external longitudinal magnetic field and without such a field. Whereas for a beam in a magnetic field there exists a fairly complete picture of the excitation of the ion oscillations of different types,⁴ the data obtained in the absence of a magnetic field are insufficient to establish on the basis of the results of the theory⁶⁻⁸ the character of the oscillations and the processes that determine their onset in this case. Yet freely propagating beams are quite frequently encountered physical systems (electron-beam welding, active experiments in the upper atmosphere, their transport is apparently quite strongly subjected to action of oscillations produced in the beams.

We have experimentally investigated ion oscillations in a plasma that is produced by an electron beam passing in the absence of a magnetic field through a gas. The measurements were performed with a setup for electron-beam welding, comprising a metallic cubic chamber measuring $50 \times 50 \times 50$ cm, in which it was possible to vary the air pressure in the range from 8×10^{-5} to 1×10^{-3} Torr. In one of the steps of the measuring chambers we placed an electron gun. In almost all the measurements we used a diode focusing system that shaped a practically parallel beam of 2-3 mm diameter with current up to 250 mA and energy 10-20 keV. The only exceptions were experiments with variation of the beam diameter. The latter can be easily effected only by using guns with magnetic focusing lens; to be sure, this resulted in a beam with a clearly pronounced crossover and one could speak of a certain average beam diameter.

A system of three diaphragms with holes of 10 mm diameter, approximately coaxial with the beam was located 18 cm away from the gun anode. The outer diaphragms were grounded, and when necessary it was possible to apply to the inner diaphragm a high-frequency voltage to modulate the beam. The modulator length was 20 mm. The oscillations excited in the beam were received with a high-frequency probe or with a diaphragm-loaded collector. The signal was next applied to the input of an oscilloscope or an S-4-8 spectrum analyzer. Since the investigated beam carried a rather large specific power (up to 10^5 W/cm^2 in a welding beam), the probe was moved in the course of the measurements in a direction transverse to the beam, with linear velocity $1.6 \times 10^3 \text{ cm/sec}$, to prevent



FIG. 1. Oscillograms of signals from a probe moving across the beam (a, b) and spectrum of the current-density oscillations (c). Oscillograms 1, 2, 3, and 4 correspond to different diameters of the beam and are numbered in increasing order of the beam diameter. a) beam current I=250 mA, accelerating voltage $U_0=18$ kV; b) I=30 mA, $U_0=6.5$ kV; c) I=180 mA, $U_0=20$ kV, $p=5\times10^{-4}$ Torr.

melting the probe. On the other hand, when the measurement was with a collector, the diaphragm employed was an intensely cooled tungsten disk with a hole $\sim 0.1-0.2$ mm in diameter.

Figure 1a shows a typical oscillogram of a signal received by a probe that moves rapidly across the beam. These oscillograms were used in the present study to measure the beam diameter. It can be seen that the probe registers also high-frequency noise oscillations localized in this case in the axial region of the beam. The oscillograms of these oscillations, with a larger time resolution, are shown in Fig. 1b for somewhat different beam parameters. The oscillations are in fact most easily produced near the axis, but when the current is increased they can occupy the total volumé of the beam. Figure 1c shows the characteristic spectrum of the excited oscillations. The frequency of the maximum of the spectral distribution varies as a function of the pressure in the range 0.5-2 MHz and corresponds approximately to the plasma frequency of the ion background that neutralizes the beam.

For further evaluation of the experimental results, we analyze the oscillatory properties of the considered plasma system at frequencies ω close to the plasma ion frequency ω_i . We choose the z axis along the direction of beam motion. Since the time of flight of the electrons of the beam along this axis over the entire drift space is much shorter than the oscillation period, it is natural to expect the oscillations to result from small transverse displacements of the electrons along the axes x and y, and accordingly neglect the electric field component E_z compared with E_x and E_y . In the general case, the systematic consideration consists of three components: a fast cold electron beam and the ions and electrons of the quiescent plasma. We assume that the plasma contains waves of the form

$$E_{x,y} = E_{1,2} \exp \left[-i(\omega t - k_x x - k_y y - k_z z)\right],$$

with

$$k_z \ll (k_x^2 + k_y^2)^{\frac{1}{2}} \approx (k_x^2 + k_y^2 + k_z^2)^{\frac{1}{2}} \equiv k_z$$

and we use the expression for the complex longitudinal dielectric constant of the quiescent components of the plasma at $\omega \ll k v_{Te}$,⁹ adding to this expression the hydrodynamic beam term. We then obtain the following dispersion relation

$$\frac{1+2\gamma^{2}(1+\mu)-4\gamma^{3}\alpha\exp\left(-\alpha^{2}\gamma^{2}\right)\psi(\alpha\gamma)}{i}+i\left[\frac{2\pi^{\prime\prime_{h}}\mu\gamma^{3}\upsilon_{\tau,\alpha}}{\upsilon_{\tau*}}+2\pi^{\prime\prime_{h}}\gamma^{3}\alpha\exp\left(-\alpha^{2}\gamma^{2}\right)\right]-\frac{\omega_{b}^{2}}{(\omega-k_{z}\upsilon_{0})^{2}}=0,$$
(1)

where v_{Ti} and v_{Te} are the average thermal velocities of the ions and electrons of the plasma;

$$\gamma = \omega_i / |k| v_{Ti}, \quad \mu = (d_i / d_e)^2,$$

 d_i and d_e are the Debye radii of the ions and electrons of the plasma;

$$\alpha = \frac{\omega}{\omega_i}, \quad \psi(\alpha\gamma) = \int_{\alpha}^{\alpha\gamma} e^{x^2} dx,$$

 ω_b is the plasma frequency of the beam; ν_0 is the beam velocity.

For the steady state following stationary injection of the beam in the system, it is necessary to solve the problem of the spatial amplification of the oscillations; this can be done by determining from Eq. (1) the value of k_x as a function of the real values ω and k, which correspond to a certain component of fluctuation perturbations at the start of the beam. We obtain

$$k_{z} = \frac{\omega}{v_{0}} \pm \frac{\omega_{b}}{v_{0}} q(\omega, k) \mp i \frac{\omega_{b}}{v_{0}} s(\omega, k).$$
⁽²⁾

The calculated quantities q and s, which determine respectively the axial phase velocity of the waves and their spatial growth rate, are shown in Figs. 2 and 3. Figure 2 illustrates the character of the frequency dependences of q and s for different γ , while Fig. 3 shows the dependences of the frequency of the most rapidly increasing wave and of its growth rate on the parameters γ and μ .

Thus, our analysis shows that waves with longitudinal phase velocity $v_{ph} \approx \omega v_0 / \omega_b q \ll v_0$ should increase exponentially along the beam. In contrast to the spatial amplification of the electron waves in the plasmabeam system, there is no synchronism whatever between the wave and the beam in our problem $(v_0 \gg v_{ph})$.



FIG. 2. Calculated dependences of the coefficients q (dashed) and s (solid curves) in formula (2) on the dimensionless frequency $\alpha = \omega/\omega_i$.

As seen from Fig. 3, the frequencies of the waves that increase with maximum growth rate lie near ω_i . In particular, they are very close to ω_i in the case $\mu = 0$, which corresponds to a two-component system the (electron beam and the ions that compensate it). We call attention to the substantial difference between this result and the conclusions that follow from an analysis of the instability of such a system (Buneman instability), when one considers not the spatial but the temporal evolution of the oscillations. In the latter case we have for the frequency ω_B of the resonant excitations, as is well known, the expression⁹

$\omega_{B}=2^{-4/3}\omega_{pi}(m_{i}/m_{e})^{1/4}\approx 2.5\omega_{pi}$

(for nitrogen). At the same time, within the framework of the spatial problem, the Landau damping, which is included in Eq. (1), leads to a strong change of the wave properties of the system compared with the case of a cold plasma. For example, in the two-component system $\operatorname{Re}k_{z}$ is no longer equal to ω/v_{0} , but has a much higher value determined from expression (2).

We note that the results obtained here can be used also in an investigation of ion oscillations in compressed beams of negative ions. Such oscillations were also observed by Gabovich *et al.*¹⁰

We proceed now to compare the experimental facts with the theoretical results.

1. According to the theory, the particles oscillate in a direction perpendicular to the beam velocity. This should lead to oscillations of the density distribution of the beam current over its cross section. The total beam current, on the other hand, should not oscillate. Such a structure of the oscillations was in fact observed in experiment. The current of a beam received by a collector without a diaphragm contains practically no high-frequency component. If, however, the collector is covered with a diaphragm (in which case one



FIG. 3. Dependence of the frequency α_{M} of the maximally amplified wave (dash) and of its dimensionless spatial growth rate s_{M} (Fig. 2) on the parameter γ at different μ and v_{Te}/v_{Ti} : curves $1-\mu=0$; $2-\mu=10^{-2}$, $v_{Te}/v_{Ti}=1.65 \cdot 10^{3}$, $3-\mu=2 \cdot 10^{-2}$, $v_{Te}/v_{Ti}=1.65 \cdot 10^{3}$; $3-\mu=2 \cdot 10^{-2}$, $v_{Te}/v_{Ti}=4.3 \cdot 10^{2}$.



FIG. 4. Oscillograms of current density at two points of the cross section of the beam. On the right are shown arbitrarily the coordinates of the points. $I_0 = 160$ mA, $U_0 = 14$ kV, diameter 2.5 mm.

measures the beam current density uniquely), the current oscillations on the collector reach 20-30%.

To determine the phase relations between the oscillations of the current density at different points of the beam cross section, we obtained simultaneous oscillograms of signals received from two diaphragmed collectors, which could be moved together with the diaphragms across the beam. The diaphragm openings were located in a single cross-section plane and separated by 1.2 mm. If some definite transverse oscillation mode was excited spontaneously in the system (for example compression and expansion of the beam, bending in one plane, etc.), then this mode could be determined with the aid of these measurements. It turned out that regardless of the location of the diaphragms the correlation of the oscillations received through two channels is noticeably disturbed, and the difference between the phases of these oscillations is generally speaking arbitrary. The oscillograms of Fig. 4 can serve as an example. It appears that a random set of oscillations of different spatial modes is present in the cross-section plane of the beam.

To establish, whether in particular, conditions exist in the system for the growth of axially symmetrical modes, corresponding perturbations were produced artificially with the aid of the modulator described above. The spectra shown in Fig. 5 give grounds for an affirmative answer to this question. In fact, it can be seen that the amplitude of the coupled oscillations in the beam depends on the modulation frequency, and the amplitude is maximal when the modulation frequency coincides with the frequency of the spontaneously excited oscillations. We note that inasmuch as the modulator causes transverse displacements of the electron beam, the experiments with modulation confirm additionally the established character of the oscillations.

2. Figure 6 shows plots of the amplitude of the oscillations against the distance to the electron beam, obtained both with an unmodulated and a modulated



FIG. 5. Spectra of the oscillation of beam current density oscillations under external modulations. Broad maximum—selfexcited oscillations, narrow peaked—entrained oscillations. $I = 120 \text{ mA}, E_0 = 13 \text{ keV}, p = 1.5 \times 10^4 \text{ Torr}$. Coordinate of observation point z = 35 cm. Amplitude of modulator voltage 1 V.

beam. The oscillations were recorded with a diaphragmed collector, and in each point along z the diaphragm was placed on the beam axis at the location where the amplitude was maximal.

In contrast to the experiments in the presence of an external longitudinal magnetic field, when the axial distribution of the amplitude corresponds to a standing wave,⁴ in our case the amplitude grows exponentially.



FIG. 6. Dependence of the intensity of the oscillations on the distance to the electron gun at modulation amplitudes equal to zero (curve 1) and 5 V (curve 2). The arrow marks the co-ordinate of the modulator. I=120 mA, $U_0=13$ kV, $p=2\times10^{-4}$ Torr, f=18 MHz, diameter 2.5 mm.

f, MHz; A, rel. un



The spatial increments determined from the plots of Fig. 6 are equal to 0.8 cm⁻¹ for the spontaneous oscillations and to 1.25 cm⁻¹ for the induced oscillations. They coincide with the theoretical calculations if we assume the parameter s in (2) to be respectively equal to 1.7 and 2.7.

For a more detailed comparison of the experimental data with the theoretical ones it is necessary to estimate the parameters of the produced beam plasma. We present such an estimate starting with the beam-ion balance equation, which can be written for a positive beam potential approximately in the form

$$\pi r^2 n_{\nu} v_0 \varepsilon(v_0) p = 2\pi r n_i v_i, \tag{3}$$

where r is the radius of the beam n_b and n_i are the densities of the beam electrons and of the plasma ions, $\varepsilon(v_0)$ is the specific ionization, p is the gas pressure, and v_i is the average velocity at which the ions leave the beam. The value of v_i can be determined from the experimental data on the static radial electric fields,¹¹ which are equal to ~1 V/cm in the pressure range 10^{-4} - 10^{-3} Torr. It follows therefore that for a beam of 0.25 cm diameter the potential difference between its axis and the edges of the order of 0.1 V, while the value of v_i for air ions turns out to be ~8×10⁴ cm/sec. Substituting this value of v_i in Eq. (3), we can determine the approximate dependence of the ion density on the pressure and the corresponding dependence of the parameter μ under typical conditions of our experiments (Fig. 7a). In the estimate of μ we assumed v_{T_i} $\sim v_i$, $T_e \sim 5 \text{ eV}$, and $n_e = n_i - n_b$ (n_e and T_e are the concentration and temperature of the slow electrons). On the basis of this estimate, we can see that for a quantitative agreement between the measured and theoretically derived growth rates it is necessary to assume that the parameter γ amounts under the experimental conditions (Fig. 6) to 2 for the spontaneously excited waves and to 2.3 for waves excited by a modulator. Satisfactory agreement is then obtained also between the measured (1.8 MHz) and calculated (~2 MHz) frequencies of the maximally amplified waves.

From the obtained values of γ we find that the transverse wavelengths of the growing perturbations of the order of 1 mm, somewhat less than the beam radius. This agrees with the significant breakdown, over such a distance of the correlation of the oscillations in the unmodulated beam.

3. The theory agrees also with the experimental de-

pendence of the frequency of the spontaneously excited oscillations on the air pressure in the system, see Fig. 7a. The dashed line in this figure shows how the ion Langmuir frequency, calculated from the data of Fig. 7b varies with pressure. We note also that when the air is replaced with the helium and the value of ϵp remains the same, the frequency of the excited oscillations increases, as is predicted also by the theory.

Unlike the Buneman oscillations in a magnetic field, the frequency of the oscillations under the conditions of our experiments was independent of the length of the beam. At the same time, it decreases with increasing beam diameter, (see Fig. 1). These facts also agree with the present analysis of the spatial amplification of the ion Langmuir oscillations.

As to the experimental dependence of the amplitude of the oscillations on the pressure (Fig. 7a), we can note that it agrees qualitatively with the theoretical relations of Fig. 3, inasmuch as the values of γ and μ increase with increasing p. This notwithstanding, the increase of γ in these experimental conditions can be attributed apparently only to the ascending section of the A(p) curve. Estimates show that in the pressure range 10^{-12} -10⁻³ Torr the imaginary part of the dielectric constant of the electron-beam plasma begins to receive a substantially increasing contribution from the collisions of the ions with the neutral atoms. This should lead to a faster decrease, compared with the relations in Fig. 3, of the spatial growth rate with increasing pressure, as is in fact observed in experiment.

In conclusion, a remark must be made concerning the possible influence of the secondary electrons from the collector on the oscillations in question. It seems that by producing an opposing beam these electrons can play an important role if they are confined by a longitudinal magnetic field, or else in short beams of length of the order of the beam width.^{2,12} On the other hand, in the experiments described here, with a long and thin beam, we did not notice any process connected with secondary electrons, a fact that can apparently be attributed to the strong decrease of the density of the

secondary electrons with increasing distance from the collector.

Thus, we have shown in this paper that free propagation of an electron beam is accompanied by a spatial amplification of ion oscillations that are almost transverse to the beam velocity, of the produced electronbeam plasma, and of the corresponding oscillations of the beam-current density. The experimental data are in satisfactory agreement with the presented analysis of the spatial beam amplification of such a wave, with account taken of their collisionless damping by the ions and electrons.

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- ¹V. D. Fedorchenko, B. N. Rutkevich, V. I. Muratov, and B. M. Chernyĭ, Zh. Tekh. Fiz. **32**, 958 (1962) [Sov. Phys. Tech. Phys. **7**, 696 (1963)].
- ²K. G. Hernqvist, J. Appl. Phys. 26, 544 (1955).
- ³A. Vermeer, T. Matitti, H. J. Opman, and J. Kistemaker, Plasma Phys. 9, 241 (1967).
- ⁴M. V. Nezlin, M. I. Taktakishvili, and A. S. Trubnikov, Zh. Eksp. Teor. Fiz. 55, 397 (1968) [Sov. Phys. JETP 28, 208 (1968)].
- ⁵L. I. Bolotin, E. A. Kornilov, O. K. Nazarenko, S. K. Pats'ora, and Ya. B. Fainberg, Zh. Tekh. Fiz. **42**, 1620 (1972) [Sov. Phys. Tech. Phys. **17**, 1294 (1973)].
- ⁶G. I. Budker, Atomnaya energiya 5, 9 (1956).
- ⁷O. Buneman, Phys. Rev. **115**, 503 (1959).
- ⁸A. A. Vedenov, E. P. Velikhov, and R. Z. Sagdeev, Usp. Fiz. Nauk **73**. 701 (1961) [Sov. Phys. Usp. **4**, 332 (1961)].
- ⁹A. B. Mikhailovskii, Teoriya plazmennykh neustoichivostei (Theory of Plasma Instabilities), Vol. 1, Atomizdat, 1975, p. 38.
- ¹⁰M. D. Gabovich, L. S. Simonenko, I. A. Soloshenko, and N. V. Shkorina, Zh. Eksp. Teor. Fiz. 67, 1710 (1974) [Sov. Phys. JETP 40, 851 (1975)].
- ¹¹M. D. Gabovich, V. P. Kovalenko, O. A. Metallov, O. K. Nazarenko and S. K. Pats'ora, Zh. Tekh. Fiz. 47, 1569 (1977) [Sov. Phys. Tech. Phys. 22, 901 (1977)].
- ¹²E. G. Linder and K. G. Hernqvist, J. Appl. Phys. **21**, 1088 (1950).

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