In the general case it is not clear whether the symmetry relative to the transformation (6.4) can serve as a source of exact data on the positions of the singularities of the statistical sum in the space of the parameters γ_1, γ_2, and γ_3. We note only the special case of the cube, in which these parameters are connected by the relations

$$\gamma_2=\gamma_1^2, \quad \gamma_3=\gamma_1^2. \tag{6.6}$$

It corresponds to one of the discrete approximations for the classical Heisenberg model; the sum of states can be written in the form

$$Z(\beta)=\sum_{\mathbf{n}_x}\exp\left\{\beta\sum_{x,\alpha}[\mathbf{n}_x\mathbf{n}_{x+\hat{\alpha}}-1]\right\}, \tag{6.7}$$

where the three-dimensional vectors $\mathbf{n}_x$ run through the vertices of a cube inscribed in the unit sphere. The connection (6.6) between the parameters is preserved after the transformation (6.4), and the transition temperature for the model (6.7) can be calculated exactly. This case is trivial, however, since the statistical sum (6.7) reduces to three noninteracting Ising models.

[1)]We omit the factor $\beta=(kT)^{-1}$ in the exponent in Eq. (2.6), including it in the definition of H.

[2)]If we regard $g_{x,\alpha}$ as connectivities in a G-stratification over the lattice, the constraints $Q_{\Sigma}=I$ are the condition for zero curvature.

[3)]It can easily be seen that the relation between the groups G and $\hat{G}$ is of the nature of a duality; i. e., if $\hat{G}$ is the group of characters of G, then $\hat{\hat{G}}=G$ is the group of characters of the group $\hat{G}$.

[4)]In the model P_N the existence of two different phases is possible. The KW symmetry enables us to determine the critical value β_c, which satisfies the equation $\beta_c=\tilde{\beta}(\beta_c)$. This gives $\beta_c=\ln(1+N^{1/2})$. Some rigorous results of the existence of two phase in P_N have been obtained recently.[18]

[5)]It is easily seen that the spin system on the tetrahedron is equivalent to the model P_4 (see Sec. 4).

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Anomaly of the magnetic susceptibility near the ferroelectric phase transition point in narrow-gap A_4B_6 semiconductors

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The magnetic susceptibility in a system that is unstable against singlet electron-hole pairing or a structural phase transition is computed. The magnetic susceptibility has a fluctuation-induced singularity near the transition temperature: it undergoes a finite jump in the region of applicability of the Landau-Ginzburg theory and obeys a power law with an exponent equal to $d\nu-1$, where ν is the exponent of the correlation length, in the scaling region. The diamagnetism is found in the mean-field approximation to decrease smoothly below the transition temperature. The results qualitatively explain the experiments that have been performed on SnTe samples in the vicinity of the ferroelectric transition point.

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1. INTRODUCTION

The occurrence of a ferroelectric phase transition in A_4B_6 crystals and solid solutions based on them has been well established. In particular, the dependence of the transition temperature, T_c, for SnTe on the carrier concentration is well known.[1,2] A structural-phase-transition-induced anomaly has been observed[3] in the magnetic susceptibility (MS) of SnTe at $T=T_c$.

The existence of such an anomaly is not *a priori* apparent within the framework of the conventional notions about a structural transition, which does not affect the magnetic properties of the system. Since it

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has been observed, the question arises whether it has received a more or less adequate theoretical interpretation.

The problem consists in the computation of the MS in narrow-gap crystals that undergo a structured transition in a weak magnetic field and the determination of the conditions under which the indicated anomaly manifests itself sufficiently distinctly for it to be experimentally observable. This is the object of the present paper.

It is *a priori* clear that a structural transition ocurring in highly degenerate narrow-gap crystals, and leading to the distortion of the electronic spectrum, should alter the characteristics of the electrons at the Fermi level (mass, density of states), and this will have an effect on the MS of the electron gas. In this paper we compute the magnitude of such a correction in the mean-field approximation; it depends on the parameters of the electron spectrum, and cannot explain the characteristic shape of the observed jump. We also compute the fluctuation correction to the MS in the vicinity of T_c. It is precisely the fluctuation correction to the MS in the region of applicability of the Ginzburg-Landau theory and in the scaling region that can account for the magnitude and shape of the anomaly observed by Baginskii *et al.*[3]

Let us note that the order parameter associated with the structural phase transition is not the conjugate of the susceptibility computed in the presented paper. The determination of the effect of the fluctuations on such a susceptibility is, apparently, of general interest.

2. THE SPECTRUM OF THE SYSTEM AND THE THERMODYNAMIC POTENTIAL

Let us consider the Hamiltonian of a two-band narrow-gap semiconductor (semimetal) with an interband electron-phonon interaction in a magnetic field:

$$H=\sum_{\alpha i\sigma}E_{\alpha i\sigma}c^{+}_{\alpha i\sigma}c_{\alpha i\sigma}+\sum_{\mathbf{q}t}\omega_{0}{}^{t}(\mathbf{q})\left(b^{+}_{\mathbf{q}t}b_{\mathbf{q}t}+\frac{1}{2}\right)$$
$$+\sum_{\alpha\alpha'}\sum_{\mathbf{q}t}\sum_{i\neq j}B^{ij}_{\alpha\alpha'}(\mathbf{q})c^{+}_{\alpha i\sigma}c_{j\alpha'\sigma}(b^{+}_{\mathbf{q}t}+b_{-\mathbf{q}t}). \quad (1)$$

Here α numbers the electron states in the magnetic field, σ is the spin, t is the phonon polarization, and i, $j=1,2$ are the band numbers.

The electron-phonon coupling constant for $q=0$ possesses the property $B_{\alpha\alpha'}(0)\sim\delta_{\alpha\alpha'}$. Below we assume that $B(0)$ does not depend on α.[4]

The spectrum of a narrow-gap semiconductor in zero magnetic field is described by the Cohen-Blount model:

$$E_{1,2}=\pm(s^2k^2+\Delta^2/4)^{1/2}, \quad (2)$$

where Δ is the gap in the electronic spectrum, k is the three-dimensional momentum, and s is the interband hybridization parameter.

Since, below, we consider a degenerate semiconductor (or semi-metal) located in a weak magnetic field, the spectrum in the magnetic field can be constructed in the quasiclassical approximation. The applicability of this approximation is guaranteed by the smallness of the Landau-level spacing in comparison with the magnitude of the chemical potential μ, as measured from the allowed-band edge (for definiteness, from the lower edge of the c band).

In the quasiclassical approximation the electronic spectrum is determined by the equation

$$S=\frac{2\pi eH}{c}\left(m+\frac{1}{2}\pm\frac{g}{4}\right), \quad (3)$$

where H is the field, g is the gyromagnetic ratio, m is the Landau band number, and S is the cross section of the region enclosed by the constant-energy surface at th the place where it is cut by a plane perpendicular to the field.

The spectrum $E_{\alpha i\sigma}$ in (1) is determined by the cross section

$$S^{t}=\pi\left[\frac{E^2-p^2s^2}{s^2}-\frac{\Delta^2}{4s^2}\right], \quad (4)$$

where p is the component of the momentum in the direction of the field.

It is known that the active TO mode with $q=0$ softens in A_4B_6 at $T=T_c$, so that it is sufficient to take the condensation of only the phonon mode with $q=0$ into consideration at the first stage of the calculations in the self-consistent field approximation.[5] In this case the distortion of the electronic spectrum at $T<T_c$ leads to the replacement $E_{1,2}\to\varepsilon_{1,2}=\pm(s^2k^2+1/4\Delta^2+\varkappa^2)^{1/2}$, and the spectrum in the magnetic field is determined by Eq. (3) with the cross section

$$S=\pi\left[\frac{\varepsilon^2-s^2p^2}{s^2}-\frac{\Delta^2}{4s^2}-\frac{\varkappa^2}{s^2}\right], \quad (5)$$

where $\varkappa=2|B|\langle b_0\rangle$ is the order parameter, which is proportional to the sublattice displacement $B=E^{12}(0)$.

Knowing the spectrum, we can easily write down an expression for the thermodynamic potential of the crystal in the approximation in question:

$$\Omega=-\frac{eHTV}{4\pi^2c}\sum_{m\sigma}\int_{-\infty}^{\infty}dp\left\{\ln\left[1+\exp\left(\frac{\mu-\varepsilon_{m}{}^{\sigma}}{T}\right)\right]\right\}+\frac{\omega_0\varkappa^2}{4B^2}. \quad (6)$$

The second term in (6) corresponds to the elastic energy due to the lattice distortion and V is the volume of the base region of the crystal.

In (6) we perform the summation over the magnetic quantum number with the aid of the well-known formula

$$\sum_{m=0}^{\infty}\varphi\left(m+\frac{1}{2}\right)=\int_{0}^{\infty}\varphi(l)\,dl+\frac{1}{24}\varphi'(0), \quad (7)$$

and, further, integrating in (7) by parts, we obtain

$$\frac{\Omega}{V}=-\frac{1}{(2\pi)^3}\sum_{\sigma}\int dp\left\{\mp\frac{\pi eHgT}{2c}\ln\left[1+\exp\left(\frac{\mu-\varepsilon_0{}^{\sigma}}{T}\right)\right]\right.$$
$$\left.+\int_{\varepsilon_0{}^{\sigma}}^{\infty}S(\varepsilon,p,\varkappa)f(\varepsilon)\,d\varepsilon-\left(\frac{eH}{c}\right)^2\frac{\pi f(\varepsilon_0{}^{\sigma})}{12m_0{}^{\sigma}}\right\}+\frac{\omega_0\varkappa^2}{4B^2V}. \quad (8)$$

Here $f(x)$ is the Fermi function; the ε_0^{σ} are the solutions of the equation $S=\pm\pi egH/2c$, the mass being connected with S by the well-known relation:

$$m=\frac{1}{2\pi}\frac{\partial S}{\partial\varepsilon},\qquad m_0{}^{\sigma}\equiv m(\varepsilon_0{}^{\sigma}). \quad (9)$$

3. THE MAGNETIC SUSCEPTIBILITY

Using (8), we can easily calculate the MS in a weak field. Differentiating (8) twice with respect to the field, we obtain for $H=0$ the equation

$$\frac{1}{V}\frac{d^2\Omega}{dH^2}=-\frac{1}{(2\pi)^3}\sum_i\int dp\left\{\frac{\pi e^2g^2f(\varepsilon_0^i)}{4m_0^ic^2}-\left(\frac{e}{c}\right)^2\frac{\pi f(\varepsilon_0^i)}{3m_0^i}\right.$$
$$+2\frac{d^2\varkappa^2}{dH^2}\int_{\varepsilon_0^i}^{\varepsilon_\infty^i}\frac{\partial S}{\partial\varkappa^2}f(\varepsilon)\,d\varepsilon+\frac{f(\varepsilon_0^i)}{\pi m_0^i}\left(\frac{\partial S}{\partial\varkappa^2}\right)^2\left(\frac{d\varkappa^2}{dH}\right)^2$$
$$\left.+2\left(\frac{d\varkappa^2}{dH}\right)^2\int_{\varepsilon_0^i}^{\varepsilon_\infty^i}\frac{\partial^2S}{\partial^2\varkappa^2}f(\varepsilon)\,d\varepsilon\right\}+\frac{\omega_0}{4B^2V}\frac{d^2\varkappa^2}{dH^2}. \tag{10}$$

The summation over i in (10) includes only the band numbers.

Let us note at once that, when $\varkappa=0$, the formula (10) coincides with the result obtained by Beneslavskiĭ and Fal'kovskiĭ.[6]

Entering into (9) are the derivatives of the equilibrium order parameter $\varkappa(H)$, which satisfies the equation $\partial\Omega/\partial\varkappa=0$:

$$\frac{2}{(2\pi)^3}\sum_\sigma\int dp\left\{\int_{\varepsilon_0^\sigma}^{\varepsilon_\infty^\sigma}\frac{\partial S}{\partial\varkappa^2}f(\varepsilon)\,d\varepsilon-\left(\frac{eH}{c}\right)^2\frac{\pi}{12}\frac{d}{d\varkappa^2}\left(\frac{f(\varepsilon_0^\sigma)}{m_0^\sigma}\right)\right\}=\frac{\omega_0}{2B^2V}. \tag{11}$$

Regarding (11) as a given implicit function of $\varkappa^2(H)$, we can easily compute the derivative and verify that $d\varkappa^2/dH=0$ for $H=0$. As to the coefficient attached to $d^2\varkappa^2/dH^2$ in (10), it vanishes on account of Eq. (11).

Thus,

$$\frac{\chi}{V}=\frac{1}{8}\left(\frac{e}{\pi c}\right)^2\left(\frac{g^2}{4}-\frac{1}{3}\right)\sum_i\int dp\frac{f(\varepsilon_0^i)}{m_0^i}, \tag{12}$$

where the ε_0^i are the solutions of the equation $S=0$. It follows from (12) that the SM will feel the phase transition through a change in the electronic spectrum.

Taking (5) and (9) into account, we obtain

$$m_0^i=\pm\frac{1}{s^2}\left(s^2p^2+\frac{\Delta^2}{4}+\varkappa^2\right)^{1/2},\qquad \varepsilon_0^i=m_0^is^2, \tag{13}$$

and, with the aid of (13), we can rewrite (12) in the following form:

$$\frac{\chi}{V}=\frac{1}{8}\left(\frac{e}{\pi c}\right)^2\left(\frac{g^2}{4}-\frac{1}{3}\right)\int dp\frac{f(\varepsilon_0^+)-f(\varepsilon_0^-)}{(s^2p^2+{}^1/_4\Delta^2+\varkappa^2)^{1/2}}, \tag{14}$$

where the plus and minus signs correspond to the c and v bands respectively.

If $g\approx 2$, then in (14) $\chi<0$ (diamagnetism). This result agrees with the well-known fact that SnTe is diamagnetic. In order to obtain a quantitative agreement between χ, as given by (14), and experiment, we should, apparently, use the Dimmock, instead of the Cohen-Blount, spectrum, but the computation of the MS from the formula (14) is not the object of the present paper, since we are interested in only the correction to the MS due to the lattice distortion.

We should take into account in (14) the fact that, when $T\ll T_c$, we have $\mu=\mu_0+\Delta\mu$, where μ_0 is the chemical potential in the undistorted phase. The correction $\Delta\mu$ can be found from (8) by constructing the neutrality equation $-\partial\Omega/\partial\mu=N_e$:

$$L\equiv\frac{1}{(2\pi)^3}\sum_\sigma\int dp\left\{\mp\frac{\pi egH}{2c}f(\varepsilon_0^\sigma)+\int_{\varepsilon_0^\sigma}^{\varepsilon_\infty^\sigma}S(\varepsilon)\frac{\partial f(\varepsilon)}{\partial\mu}d\varepsilon\right.$$
$$\left.-\left(\frac{eH}{c}\right)^2\frac{\pi}{12m_0^\sigma}\frac{\partial f(\varepsilon_0^\sigma)}{\partial\mu}\right\}-n=0, \tag{15}$$

$N_e/V=n$ is the electron concentration.

Using (15), we can compute $d\mu/d\varkappa^2$:

$$\frac{d\mu}{d\varkappa^2}=-\frac{L'_{\varkappa^2}}{L'_\mu}=\sum_\sigma\int dp\left\{\int_{\varepsilon_0^\sigma}^{\varepsilon_\infty^\sigma}\frac{\partial S}{\partial\varkappa^2}f_\varepsilon'(\varepsilon)\,d\varepsilon\right\}$$
$$\times\left(\sum_\sigma\int dp\left\{\int_{\varepsilon_0^\sigma}^{\varepsilon_\infty^\sigma}S(\varepsilon)f_\varepsilon''(\varepsilon)d\varepsilon\right\}\right)^{-1}. \tag{16}$$

In the degenerate (i.e., $\mu_0\gg T$) case being considered by us, (16) is easy to compute:

$$d\mu/d\varkappa^2=1/2\mu\quad\text{or}\quad\mu^2=\mu_0^2+\varkappa^2. \tag{17}$$

The behavior of χ at $T\leqslant T_c$ follows from the expansion of (14) in powers of $\varkappa^2$:

$$\chi/\chi_0=1-A\varkappa^2, \tag{18}$$

where

$$A^{-1}=2\mu_0^2[1+(1-y^2)^{1/2}]\ln[\Lambda/\mu_0(1+(1-y^2)^{1/2}]. \tag{19}$$

In (18) and (19), χ_0 is the MS in the undistorted phase and Λ, the cutoff energy for the integral in (14), is of the order of the width, $y=\Delta/2\mu_0$, of the allowed band.

If the phase transition that occurs is of second order, then

$$\varkappa^2=\alpha_0(T_c-T),\quad \alpha_0>0,\quad T\leqslant T_c. \tag{20}$$

The MS is continuous at $T=T_c$, and the diamagnetism decreases linearly below T_c:

$$-|\chi|=-|\chi_0|+A\alpha_0|\chi_0|(T_c-T),\quad T\leqslant T_c. \tag{21}$$

The linear decrease of the diamagnetism below T_c agrees with a result obtained earlier by us.[7] In Ref. 7 it is shown that this variation can be critical when the parameters of the system are realistic, but the shape of the singularity is, as noted above, not reproduced. The experimental data indicates an MS jump at T_c; it may be due to either of two causes: either the fluctuations, or the $\varkappa$ jump that occurs in a first-order transition.

4. EFFECT OF THE FLUCTUATIONS ON THE MS NEAR T_c

Near T_c the correction, due to the lattice distortion, to the thermodynamic potential has the form

$$\Omega-\Omega_0=-T\ln\int\exp\left(-\frac{H_{\text{eff}}}{T}\right)d\eta_k, \tag{22}$$

where η_k is the Fourier transform of the nonhomogeneous order parameter; H_{eff} is the Ginzburg-Landau Hamiltonian:

$$H_{\text{eff}}=\int dV\left[\alpha\eta^2t+\frac{b}{2}\eta^4+\bar g(\nabla\eta)^2\right],\quad t>0,$$
$$H_{\text{eff}}=V\left[\alpha|t|\varkappa_0^2+\frac{b}{2}\varkappa_0^4\right]$$
$$+\int dV\left[2\alpha|t|\eta^2+\frac{b}{2}\eta^4+\bar g(\nabla\eta)^2\right],\quad t<0, \tag{23}$$

where $t=T-T_c$, $\varkappa=\varkappa_0+\eta(\mathbf{r})$, $\eta(\mathbf{r})$ is the fluctuation at the point r, and $\varkappa_0$ is the homogeneous order parameter.

We obtain from (22) and (23) in the Gaussian approximation the relations

$$\Omega-\Omega_0=\begin{cases} T\sum_k \ln\frac{V(\alpha t+\bar g k^2)}{\pi T}, & t>0 \\ -\frac{(\alpha t)^2}{2b}+T\sum_k \ln\frac{V(2\alpha|t|+\bar g k^2)}{\pi T}, & t<0 \end{cases} \tag{24}$$

To compute χ, let us differentiate (24) with respect to the field, bearing in mind that we should make the substitution $\alpha t \to \alpha_1 t + \alpha_2 H^2$ in going over to a nonzero field. We obtain the MS:

$$\chi=\begin{cases} -\frac{V\alpha_2}{2\pi^2}\int_0^{k_0}\frac{k^2\,dk}{\alpha_1 t+\bar g k^2}, & t>0 \\ -\frac{V\alpha_2}{\pi^2}\int_0^{k_0}\frac{k^2\,dk}{2\alpha_1|t|+\bar g k^2}+\frac{2\alpha_1\alpha_2|t|}{b}, & t<0. \end{cases} \tag{25}$$

Here we have introduced the cutoff momentum $k_0 = (\alpha_1 T_c/\bar g)^{1/2}$ (Refs. 8 and 9), the use of which makes sense in the region of applicability of the Ginzburg-Landau theory (the Gaussian approximation), i.e., at temperatures not too close to T_c. Evaluating the integrals in (25), we obtain (below we have set $r=(\alpha_1|t|/\bar g)^{1/2}$

$$\frac{\chi}{V}=\begin{cases} -\frac{\alpha_2(\alpha_1 T_c)^{1/2}}{2\pi^2\bar g^{1/2}}\left[k_0-r\,\mathrm{arctg}\frac{k_0}{r}\right], & t>0 \\ -\frac{\alpha_2(\alpha_1 T_c)^{1/2}}{\pi^2\bar g^{3/2}}\left[k_0-r\,\mathrm{arctg}\frac{k_0}{r}\right]+\frac{2\alpha_1\alpha_2|t|}{b}, & t<0 \end{cases} \tag{26}$$

The temperature behavior of the MS, as given by (26), is depicted in Figs. 1a and 1b by the solid curves. We obtain the jump $\Delta\chi$ at $t=0$ in the Gaussian approximation from (26):

$$\Delta\chi=\chi|_{T_c+0}-\chi|_{T_c-0}=\frac{V\alpha_2(\alpha_1 T_c)^{1/2}}{2\pi^2\bar g^{1/2}}. \tag{27}$$

Without allowance for the fluctuations, the jump is determined by the second term in the lower line of formula (26), and is equal to zero at T_c, which agrees with the result of the preceding section.

The result (26), (27) makes sense in the region of Gaussian fluctuations, i.e., at temperatures not too close to T_c. The behavior of χ in the vicinity of T_c can be estimated qualitatively from the following arguments. In scaling theory the free energy is proportional to the number of "clusters" formed by the volume ξ^d: $F\sim\xi^{-d}$, where ξ is the correlation length and d is the dimensionality of the space. The classical value of the correlation length, $\xi=(\alpha|t|/\bar g)^{-1/2}$, near T_c should be replaced by $(\alpha|t|/\bar g)^{-\nu}$, where ν is the critical exponent. Thus, $F\sim(\alpha|t|/\bar g)^{\nu d}$, and MS has the form

$$\chi=-\frac{d^2F}{dH^2}\sim-\frac{2\alpha_2\nu d}{\bar g^{d\nu}}(\alpha_1|t|)^{d\nu-1}, \tag{28}$$

i.e., a power-law dependence of χ on t obtains in the vicinity of T_c (in the three-dimensional case $d\nu>1$).

The power-law behavior (28) is depicted in Figs. 1a and 1b by the dashed curves within the scaling region, which is marked off by the vertical broken lines. This solution should join smoothly onto the $|t|^{1/2}$ dependences

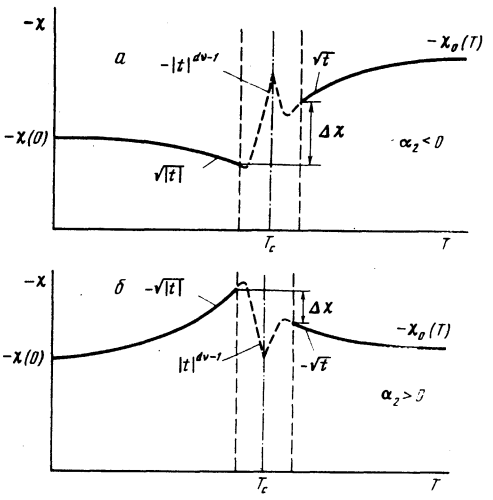

FIG. 1.

in the region of applicability of the Ginzburg-Landau theory.

5. DISCUSSION OF THE RESULTS

The MS behavior obtained for the temperature region $T<T_c$ in §3 without allowance for the fluctuations amounts to the smooth variation of $-\chi$ from $-\chi_0$ at $T>T_c$ to $-\chi(0)$ at $T=0$. This change is of the order of $\sim\varkappa^2/\mu_0^2$, and the maximum value is attained at $T=0$: $T_c^2/\mu_0^2\ll1$. For this reason, the magnitude of the χ jump can hardly be accounted for by the second cause noted after formula (21), i.e., by the $\varkappa$ jump that occurs at T_c in a first-order phase transition.

Allowance for the fluctuations in the Gaussian approximation allows us to find the fluctuation-induced jump, $\Delta\chi$, shown in Figs. 1a and 1b. In the scaling region χ varies according to a power law $\sim|t|^{d\nu-1}$. The magnitude of $\Delta\chi$, (27), and the dimension of the fluctuation region depend on the parameters of the Hamiltonian (23). It is well-known that, for phase transitions connected with the electron subsystem, these parameters depend on the level of doping. This dependence is such that $\bar g$ decreases with increasing level of doping, and the situation in which $\bar g\leq0$ for $\mu\geq\mu_{cr}$ is possible. The vanishing of $\bar g$ corresponds to the advantageousness of the nonhomogeneous ordered state. The nonhomogeneous state arises, as the level of doping is raised, in transitions of the Peierls[10,11,12] and exciton types, and this is generally a common situation for systems with any type of pairing.[12]

The considered system with the singlet order parameter is analogous to the exciton dielectric, and the possible smallness to $\bar g$ as a result of doping can ensure the necessary fluctuation-region size and $\Delta\chi$ value.

As Figs. 1a and 1b show, the behavior of χ depends on the sign of α_2. To the real situation corresponds, apparently, $\alpha_2<0$, the curve shown in Fig. 1a being then close in shape to the experimental curve.[3]

On the other hand, $\alpha_2<0$ implies the possibility of T_c being raised by a magnetic field. This conclusion is, on the whole, confirmed by the calculations for SnTe in a weak field,[7] as well as by experiments performed on

SnGeTe samples in a strong magnetic field.[2] In fact, the increase of T_c with increasing field intensity does occur in the case of the excition transition.[13]

In conclusion, let us note that the behavior of a system's responses not associated with the order parameter is not universal for all phase transitions: the responses have to be calculated in each specific case. For example, in Ref. 14 the magnetic-phase-transition-induced electrical-resistance anomalies are studied. In the present paper we have considered another example of the computation of a system's response to an order-parameter-unrelated perturbation; to wit, we have elucidated the magnetic properties of a crystal in the vicinity of a structural phase transition.

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Magnetic-ordering temperatures and direct-exchange integral in the rare-earth terbium–yttrium and terbium–gadolinium alloys

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Measurements were made of the temperatures of the magnetic phase transitions and of the paramagnetic Curie point in single-crystal rare-earth terbium–yttrium and terbium–gadolinium alloys. Concentration intervals with different coefficients of proportionality of the paramagnetic Curie point to the mean square of the spin projection on the total mechanical angular momentum $\bar{G}$ were observed in these alloys. It is established that the difference between the indirect-exchange integrals in the ferromagnetic and antiferromagnetic states in the rare-earth alloys is a universal function of the spin parameter $\bar{G}$. The contributions of the magnetocrystalline interaction and of the indirect exchange interaction to the magnetic ordering temperatures and to the paramagnetic Curie points in terbium–yttrium and terbium–gadolinium alloys are determined.

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INTRODUCTION

An investigation of the dependences of the magnetic-ordering temperatures of rare-earth metal (REM) alloys on the atomic number and on the concentration of the alloyed components is needed to elaborate on the theory of the nature of exchange interactions in REM. According to the indirect-exchange theory,[1,2,3] the magnetic ordering temperature and the paramagnetic Curie point are relatively simply connected with the indirect-exchange integral. This permits a comparison of the conclusions of the theory with experiment, which was indeed made in a number of studies.[4-7] The comparison, however, was made by analyzing the experimental data for polycrystalline samples, and the accuracy of the determination of the magnetic phase-transition temperatures was not high enough in a number of cases.

The aim of the present study was to obtain more exact values for the magnetic phase transition temperatures and the paramagnetic Curie point from measurements on single crystals of REM alloys, to determine the

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