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Thermomagnetic forces in a rarefied gas

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A kinetic theory of the thermomagnetic force (TMF) effect has been constructed and is capable of accounting for the known experimental data. The TMF effect consists of the influence of an external magnetic field on the thermal force acting on a body immersed in a nonuniformly heated gas. To calculate the TMF we use an approach based on the solution of the integral kinetic equation that was previously proposed by the authors for investigating thermomagnetic phenomena in rarefied polyatomic gases. It is shown that in addition to the TMF mechanism associated with the lack of spherical symmetry in collisions between the gas molecules, there is a second TMF mechanism, which is associated with the lack of spherical symmetry in the reflection of polarized molecules from a surface. By taking the second mechanism into account one can explain the observed dependence of the strength of the TMF on the material of the body.

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1. INTRODUCTION

It was recently found that a body immersed in a nonuniformly heated rarefied polyatomic gas experiences a force when a magnetic field is applied.[1-4] The investigators measured the normal and tangential components of the force that, when the field H is applied, acts on a thin disk (of radius r) of nonmagnetic material suspended in a gas between two surfaces a distance $L \sim 2r$ apart that are maintained at different temperatures. The thermomagnetic force (TMF) is found only in a region of intermediate gas pressures (when $\bar{l} \sim L$, where $\bar{l}$ is the mean free path) and vanishes in the high-pressure limit (as $\bar{l}/L \to 0$). The strength of the TMF depends on the ratio H/p of the field strength to the gas pressure. In some gases (N_2 and CO) the strength of the TMF depends substantially on the material of the disk,[3] but in other gases (O_2 and NO) there is no such dependence.[4]

The earlier theoretical treatment of the TMF[5,6] was limited to the idealized case in which $r \ll \bar{l} \ll L$. The origin of the force was sought in the nonspherical character of the collisions between the gas molecules and in the precession of the molecules in the field. The interaction of the molecules with the walls was assumed to be entirely diffuse. The strength of the force was related phenomenologically to the translational part of the field-dependent[7] heat flux (the Senftleben-Beenakker effect). That theory gave only a qualitative explanation of the effect; it could not account for the observed field dependence (it predicted the onset of the TMF at higher H/p values than were observed), and it could not explain the dependence of the TMF on the material of the disk.

As will be shown below, there is still another contribution to the TMF effect—a contribution due to the nonspherical character of the interaction of the molecules with the surface—that was not taken into account in the explanation proposed earlier. In fact, as a result of the lack of spherical symmetry in the collisions between the molecules the distribution function for the molecules striking the surface becomes anisotropic in M (M is the angular momentum of the molecules) and H dependent. If the interaction of the molecules with the surface also lacks spherical symmetry, the distribution of the molecules reflected from the surface may become isotropic in M while retaining its H dependence. As a result of such scattering processes, the thermal force will change when the field is applied. It turns out that the observed dependence of the TMF on the material of the disk[3] is indeed associated with the lack of spherical symmetry in the collisions of the molecules with the surface.

To describe the effect one should solve Boltzmann's integrodifferential equation with integral boundary conditions at the walls. As the authors showed in a recent paper,[8] the mathematical difficulties that arise when the problem is approached in that way can be avoided by employing the integral kinetic equation. In Section II we discuss the principal points in solving that equation by iteration for gas pressures such that $\bar{l} \lesssim L/10$. The resulting distribution function for a gas between two infinite surfaces makes it possible to calculate the thermomagnetic forces acting on the walls and to analyze the contributions from various collision processes (Section III). Features of the TMF in a finite geometry are discussed in Section IV, where a comparison is also made with experimental results.

2. CALCULATION OF THE DISTRIBUTION FUNCTION FOR A POLYATOMIC GAS IN A MAGNETIC FIELD

Let us consider a polyatomic gas in the gap between two infinite surfaces located a distance L apart and having different temperatures T_0 and T_L such that $|T_0 - T_L| = \Delta T \ll T_0$. We shall consider only pressures such that $\bar{l} \lesssim L/10$. In this case we can neglect the effect of any molecules that fly from wall to wall without collision. Then if we are interested in the state of the gas near one of the surfaces (say the lower one, at $z = 0$) we may assume for simplicity that the gas occupies the entire half space $z > 0$ and that the gas far from the surface $z = 0$ is held at a different temperature than the gas near $z = 0$. Such a system is described by an integral kinetic equation for the distribution function f, which can be obtained by integrating Boltzmann's equation along characteristics (using the boundary conditions at the surface) and then linearizing the equation thus obtained. The resulting equation has the form[8]

$$f=f \qquad), \quad \chi=\hat{G}\chi+\hat{S}\chi, \quad |\chi|\ll 1, \tag{2.1}$$

where

$$\hat{G}\chi=\int\limits_{\substack{0\,(v_z>0)\\ \infty\,(v_z<0)}}^{z}\frac{dz_1}{v_z}\exp\left[-\int\limits_{z_1}^{z}\frac{dz_2}{v_z}\hat{P}(z\leftarrow z_2)\Omega(\Gamma)\right]\hat{P}(z\leftarrow z_1)$$

$$\times\iiint f_M(\Gamma_1)\left[\chi(\Gamma_1',z_1)+\chi(\Gamma',z_1)-\chi(\Gamma_1,z_1)\right]W'\,d\Gamma_1'\,d\Gamma'\,d\Gamma_1,$$

$$\hat{S}\chi=\Theta(v_z)\exp\left[-\int\limits_{0}^{z}\frac{dz_2}{v_z}\hat{P}(z\leftarrow z_2)\Omega(\Gamma)\right]\hat{P}(z\leftarrow 0)$$

$$\times\int\limits_{v_z'<0}\frac{|v_z'|f_M(\Gamma')}{v_zf_M(\Gamma)}V(\Gamma'\to\Gamma)\chi(\Gamma',0)\,d\Gamma',$$

$$\Omega(\Gamma)=\iiint f_M(\Gamma_1)W'\,d\Gamma_1'\,d\Gamma'\,d\Gamma_1, \quad \Theta(a)=\begin{cases}1, & a>0\\ 0, & a\leqslant 0.\end{cases}$$

Here $\Gamma=(\mathbf{v},\mathbf{M})$, $d\Gamma=d\mathbf{v}\cdot d\mathbf{M}$, $\mathbf{v}$ is the molecular velocity, Ω is the equilibrium frequency of collisions between gas molecules, $W'=W'(\Gamma_1', \Gamma'\to\Gamma, \Gamma_1)$ and $V = V(\Gamma'\to\Gamma)$ are the probabilities for scattering of a molecule from a molecule and from the wall, respectively, and f_M is the Maxwell distribution for the temperature T_0 of the wall. The operator $\hat{P}$ in (2.1) describes the periodic change in the orientation of the angular momentum of the molecule as it processes in the field; it is defined as follows:

$$\hat{P}(z\leftarrow z_0)A(\Gamma)=\sum_{lmk\sigma}d_{m\sigma}^{(l)}(\theta_H)d_{mk}^{(l)}(\theta_H)Y_{lk}\left(\frac{\mathbf{M}}{M}\right)\exp\left[i(\sigma-k)\varphi_H\right.$$

$$\left.+im\frac{\omega(z-z_0)}{v_z}\right]\int Y_{l\sigma}^{*}\left(\frac{\mathbf{M}}{M}\right)A(\Gamma)\,do_{\mathbf{M}}, \tag{2.2}$$

where $\omega=\gamma H$ is the precession frequency (γ is the gyromagnetic ratio of the molecule), and θ_H and φ_H are the spherical angles specifying the orientation of the magnetic field.

Equation (2.1) can be solved by iteration. It is convenient to choose the expression for the heat flux in the system in the absence of a field as the initial function χ^0:

$$\chi^{(0)}=\varphi\frac{\delta T}{T_0}+\frac{(\nabla T)_\Gamma}{T_0}\psi z-\frac{(\nabla T)_\Gamma}{T_0}\frac{\psi v_z}{\Omega_0}, \tag{2.3}$$

where

$$\varphi=E/T_0-c_v, \quad \psi=E/T_0-c_p, \quad \Omega_0=\iiint f_M(\Gamma_1)W_0'\,d\Gamma_1'\,d\Gamma'\,d\Gamma_1.$$

Here $E=mv^2/2+M^2/2I$ is the energy of a molecule, c_v and c_p are the heat capacities of the gas, $(\nabla T)_\Gamma$ is the temperature gradient in the gas far from the wall (i. e. for $z \gg \bar{l}$), W_0' is the spherically symmetric part of the intermolecular-collision probability, and δT is the temperature discontinuity at the surface, which enters here as a parameter.

The successive iterations can be found from the scheme

$$\chi^{(n)}=\hat{G}\chi^{(n-1)}+\hat{S}\hat{G}\chi^{(n-1)}, \quad n=1, 2, 3,\ldots \tag{2.5}$$

(n numbers the iterations). Then the requirement that there be no accumulation of gas at the wall is satisfied for each n (since the probability V is normalized).

The solution to Eq. (2.1) constructed in accordance with (2.3) and (2.5) contains the unknown quantities δT and $(\nabla T)_\Gamma$ (it is assumed that the difference ΔT between the temperature of the walls is given). The parameter δT can be evaluated in the n-th approximation by making use of the relation

$$Q_z^{(n)}|_{z=0}=Q_z^{(n)}|_{z\gg\bar{l}},$$

in which $Q_z^{(n)}$ is the heat flux calculated with the function $\chi^{(n)}$. If $|\Delta T| \ll T_0$ and the surfaces confining the gas are identical so that the temperature discontinuities at the walls can be regarded as equal, we have

$$(\nabla T)_r=(\Delta T-2\delta T)/L.$$

It is shown below that in calculating the TMF effect, as in calculating the thermomagnetic pressure effect[9,8] and the Senftleben-Beenakker effect[7,8], it is sufficient to consider only the first iterated solution.

According to Refs. 10 and 11, the lack of spherical symmetry in the scattering of molecules from molecules and at the surface is small and the probabilities W and V can be written in the form

$$W=W_0+\nu W_1,\quad V=V_0+\varepsilon V_1, \tag{2.6}$$

where W_0 and V_0 are independent of the molecules' angular momenta, and ν and ε are small parameters—the asphericity parameters—that specify the magnitudes of the deviations from spherical symmetry (ν^2, $\varepsilon^2\sim 10^{-3}$–$10^{-2}$). With the aid of (2.6), we can express the function $\chi^{(1)}$, obtained as described above, as well as the temperature discontinutiy, as series in powers of the asphericity parameters:

$$\chi^{(1)}=\chi_0^{(1)}+\chi_1^{(1)}+\chi_2^{(1)}+\dots,\quad \delta T=(\delta T)_0+(\delta T)_1+(\delta T)_2+\dots, \tag{2.7}$$

where the subscripts 0, 1, 2, . . . indicate the terms of the zeroth, first, second, . . . degree in the asphericity parameters, the first degree terms being proportional to ν or ε, the second degree terms to ν^2 or $\nu\varepsilon$, etc. The field dependence of the large-scale fluxes in the gas (not associated with angular-momentum transport) appears only in the second approximation in the asphericity parameters and is determined by the function $\chi_2^{(1)}$ (the explicit form of this function is given in Ref. 8). The field dependence of δT also appears in the second approximation.

3. FORCES ACTING ON INFINITE SURFACES

We have the following expression for the forces per unit area acting, for example, on the lower surface (at $z=0$):

$$F_i=-k_jP_{ij}|_{z=0},\quad P_{ij}=m\int v_iv_jf_M\chi^{(1)}d\Gamma+\delta_{ij}p \tag{3.1}$$

(the k_j are the components of the unit vector normal to the surface). In the case of spherically symmetric interactions ($\nu=\varepsilon=0$) only the component of the force normal to the surface differs from zero, and for it we obtain

$$\begin{gathered}F_z|_{\nu=\varepsilon=0}=-p+F_1{}^T+F_2{}^T,\\ F_1{}^T=-(\nabla T)_0\frac{m}{T_0}\Big[\int\limits_{v_z<0}v_z^2f_Ma_1(\Gamma)\,d\Gamma\\ +\int\limits_{v_z>0}v_z\,d\Gamma\int\limits_{v_z'<0}|v_z'|f_M'V_0(\Gamma'\to\Gamma)a_1(\Gamma')\,d\Gamma'\Big],\\ F_2{}^T=(\delta T)_0\frac{m}{T_0}\Big[\int\limits_{v_z<0}v_z^2f_M\varphi\,d\Gamma\\ +\int\limits_{v_z>0}v_z\,d\Gamma\int\limits_{v_z'<0}|v_z'|f_M'V_0(\Gamma'\to\Gamma)\varphi\,d\Gamma'\Big].\end{gathered} \tag{3.2}$$

Here $\bar\Gamma=(\mathbf{v}, M^2)$, $d\Gamma=4\pi d\mathbf{v}MdM$, and

$$\begin{gathered}(\nabla T)_0=\frac{\Delta T-2(\delta T)_0}{L},\quad a_1(\Gamma)=\frac{\psi v_z}{\Omega_0}+\left(\frac{2T_0}{m}\right)^{1/2}\frac{\tilde\Omega_0}{\Omega_0^2},\\ \tilde\Omega_0=\left(\frac{m}{2T_0}\right)^{1/2}\iiint f_M(\Gamma_1)[\psi_1'v_{z1}'+\psi'v_z'-\psi_1v_{z1}]W_0'd\Gamma_1'd\Gamma'd\Gamma_1.\end{gathered} \tag{3.3}$$

The quantity $(F_1^T+F_2^T)$ in (3.2) is due to the fact that the walls are held at different temperatures and represents the thermal force (TF) acting on the surface. The contributions F_1^T and F_2^T to the TF are associated with the mean temperature gradient in the gas and with the temperature discontinuity at the surface, respectively.

The field dependence of the force appears only in the second approximation in the asphericity parameters. Following Ref. 8, we retain in the expression for $\chi_2^{(1)}$ only terms corresponding to polarization of the molecules far from the wall (where $z\gg\bar l$) such that the distribution function depends on the directions of $\mathbf{v}$ and $\mathbf{M}$ only through the combination[1] $[\mathbf{v}][\mathbf{M}]^2$ (here $[\mathbf{a}]^m$ represents the irreducible tensor of rank m constructed from the components of the vector $\mathbf{a}$). Then in accordance with (3.1) and (2.7) we obtain the follwoing expression for the field dependent part of the force:

$$\begin{gathered}\mathbf{F}(\mathbf{H})=\sum_{s=1}^{4}\mathbf{F}_s{}^{\mathrm{TM}},\\ \mathbf{F}_s{}^{\mathrm{TM}}=\frac{(\nabla T)_0m}{T_0}\sum_{lmk\sigma}R_{mk-\sigma}^{(l)}\left(\frac{\mathbf{H}}{H}\right)\mathbf{F}_s^{(lmk\sigma)}\quad(s=1,2,3),\\ \mathbf{F}_1^{(lmk\sigma)}=-\nu^2\int\limits_{v_z<0}\frac{\mathbf{v}v_zf_Ma_2}{\Omega_0}\frac{\Omega_0-im\omega}{\Omega_0^2+m^2\omega^2}\Omega_{1lk}\tilde\Omega_{1l\sigma}d\Gamma,\\ \mathbf{F}_2^{(lmk\sigma)}=-\nu^2\int\limits_{v_z>0}\mathbf{v}d\Gamma\int\limits_{v_z'<0}\frac{|v_z'|f_M'a_2'}{\Omega_0'}\frac{\Omega_0'-im\omega}{\Omega_0'^2+m^2\omega^2}V_0(\Gamma'\to\Gamma)\Omega_{1lk}'\tilde\Omega_{1l\sigma}d\Gamma',\\ \mathbf{F}_3^{(lmk\sigma)}=\nu\varepsilon(4\pi)^{1/2}\int\limits_{v_z>0}\mathbf{v}d\Gamma\int\limits_{v_z'<0}|v_z'|f_M'a_2'\\ \times\frac{\Omega_0'+im\omega}{\Omega_0'^2+m^2\omega^2}V_{lk00}(\Gamma'\to\Gamma)\tilde\Omega_{1l\sigma}'d\Gamma',\\ (F_4{}^{\mathrm{TM}})_i=-\delta_{zi}\frac{(\delta T)_2m}{T_0}\Big[\int\limits_{v_z>0}v_iv_z\varphi f_Md\Gamma\\ +\int\limits_{v_z>0}v_id\Gamma\int\limits_{v_z'<0}|v_z'|f_M'V_0(\Gamma'\to\Gamma)\varphi'd\Gamma'\Big],\quad i=x,y,z.\end{gathered} \tag{3.4}$$

We have used the following notation in (3.4):

$$\begin{gathered}R_{mk\sigma}^{(l)}\left(\frac{\mathbf{H}}{H}\right)=\frac{(-1)^{l+\sigma}}{4\pi}d_{mk}^{(l)}(\theta_{\mathbf{H}})d_{m\sigma}^{(l)}(\theta_{\mathbf{H}})\exp[i(k-\sigma)\varphi_{\mathbf{H}}],\\ a_2=\left(\frac{2T_0}{m}\right)^{1/2}\frac{1}{\Omega_0},\\ \Omega_{1lk}(\bar\Gamma)=\int Y_{lk}^*\left(\frac{\mathbf{M}}{M}\right)do_{\mathbf{M}}\iiint f_M(\Gamma_1)W_1'd\Gamma_1'd\Gamma'd\Gamma_1,\\ \tilde\Omega_{1lk}(\Gamma)=\int Y_{lk}^*\left(\frac{\mathbf{M}}{M}\right)do_{\mathbf{M}}\left(\frac{m}{2T_0}\right)^{1/2}\\ \times\iiint f_M(\Gamma_1)(\psi_1'v_{z1}'+\psi'v_z'-\psi_1v_{z1})W_1'd\Gamma_1'd\Gamma'd\Gamma_1,\\ V_{lk00}(\bar\Gamma'\to\Gamma)=(4\pi)^{-1/2}\iint Y_{lk}^*\left(\frac{\mathbf{M}'}{M'}\right)V_1(\Gamma'\to\Gamma)do_{\mathbf{M}'}do_{\mathbf{M}}.\end{gathered} \tag{3.5}$$

The primed quantities in (3.4) represent the corresponding functions of the argument $\bar\Gamma'$.

The terms $\mathbf{F}_s^{TM}$ $(s=1,2,3)$ in expression (3.4) for the TMF are due to the presence of a temperature gradient in the gas near the surface and are related, respectively, to the following collision processes: 1) incidence on the wall of molecules whose distribution is isotropic in $\mathbf{M}$ but depends on H (as a result of the lack of spherical symmetry in the intermolecular collisions); 2) spherically symmetric reflection of such molecules from the surface: and 3) aspherical reflection from the surface of molecules polarized as a result of intermolecular collisions in the gas. The quantity F_4^{TM} in (3.4) represents an additional contribution to the component of the TMF normal to the surface, associated with the field dependence of the temperature discontinuity.

It can be shown that a calculation of the TF and TMF acting on a small flat specimen immersed in the gas and whose size r is small as compared with the mean free path $\bar{l}$ (so that the molecules reflected from its surface will not affect the distribution of the incident molecules) leads to expressions (3.2) with $F_2^T=0$ and (3.4) with $F_4^{TM}=0$. Thus, the terms F_2^T and F_4^{TM} associated with the temperature discontinuity are due to the size of the wall and take into account (through δT) the effect of the interaction of the molecules with the surface on the distribution of the incident molecules.

It follows from (3.2) and (3.4) that the TF and TFM are independent of the pressure p at high enough pressures ($F\sim(l/L)n_0\Delta T$, where n_0 is the equilibrium particle density). As the pressure falls the TF and TMF decrease, the extent of the decrease being described by the terms of order $(l/L)^2 n_0\Delta T$ in (3.2) and (3.4).

4. COMPARISON WITH EXPERIMENT

In the experiments reported in Refs. 1–4, the investigators measured the force acting on a thin disk suspended in a gas in the gap between two surfaces that were held at different temperatures (the plane of the disk was perpendicular to the temperature gradient ∇T). The strength of the TMF was determined by measuring the deviation of the disk from its equilibrium position in zero field observed when the magnetic field is applied. Disks of aluminum foil and paper of radius $r\approx 1$ cm were used, and the disks were at a distance of $\tilde{L}\approx 1$ cm from the surfaces. The measurements were made at gas pressures such that $\bar{l}/L\sim 10^{-2}$–1.

In examining the experimental results one must take account of a number of phenomena associated with the finite size of the disk. First, one must take account of the fact that forces act on both sides of the disk. It can be shown that the total force on the disk is twice the force acting on one side of it (the thickness of the disk is neglected) provided the scattering probabilities W and V are invariant under space inversion.

Second, since there is gas on both sides of the disk, the experimental p dependences of the TF and TMF should differ at high pressures from the corresponding dependences given by (3.2) and (3.4). Indeed, in the limit $\bar{l}/L\to 0$ the disk is in a "continuous medium"; then the pressure will be isotropic and the TF and TMF will vanish. For large disks ($r\gg\bar{l}$), as Einstein noted long ago,[12] thermal forces can act only in edge regions of size $\sim\bar{l}$. Then the observed TF and TMF should be proportional to r/p. Such behavior of the TF was observed experimentally for $\bar{l}/r\lesssim 0.01$.[13] It is clear that as the pressure decreases ($\bar{l}/r$ increases) the actions of the molecules incident onto the disk from opposite sides become independent. Experiment shows[3,4,13] that the thermal force is proportional to r^2, beginning at $\bar{l}/r\sim 0.05$ and is only weakly pressure dependent in the range $0.05\lesssim\bar{l}/r\lesssim 1$. Hence we may suppose that under these conditions one can correctly describe the TF and TMF by calculating the force acting on the other side. Thus, it is clear that although to calculate the TF and TMF correctly at higher pressures one must solve a complicated problem that would take into account the coupling of the gas volumes on the two sides of the disk, one can use the formulas (3.2) and (3.4) obtained above in a narrow pressure interval in which $\bar{l}/L\sim 0.1$.

Still another feature of the experimental situation consists in the fact that when $r\sim\tilde{L}$ the transport of heat in the gas around the edge of the disk affects the temperature profile near the disk's surface. As a result, this case differs from the case $r\gg\tilde{L}$ in that the temperature discontinuity at the surface of the disk may be small. This conclusion is confirmed by experiment. As was shown in Ref. 13, the dependence of the TF on the nature of the gas is best described by the product $F\sim\lambda_{\text{trans}}\sqrt{m}$ (λ_{trans} is the translational part of the heat conductivity of the gas and m is the mass of a molecule). In calculating the contribution F_1^T (see Eq. (3.2)) to the TF (assuming that $\Omega_0=\text{const}$ and, for simplicity, that V_0 represents specular-diffuse reflection) we may associate F_1^T with the translational part of the heat flux and obtain $F_1^T\sim\lambda_{\text{trans}}\sqrt{m}$. At the same time, since the discontinuity $(\delta T)_0$ in (3.2) is associated with the balance of the total heat flux, we have $F_2^T\sim\lambda\sqrt{m}$. Thus, the dependence of the TF on the nature of the gas can be accounted for if the temperature discontinuity at the disk be neglected.

In accordance with what was said above we shall first assume for simplicity that the temperature discontinuities at the surfaces of the disk are small and shall set $F_4^{TM}=0$ in (3.4). As is shown below, Eq. (3.4) can then be used to calculate the field dependence of the TMF within the experimental errors. It turns out that δT at the surface of the disk need be taken into account only at lower values of $\tilde{L}/r$.

Let us calculate the TMF ($F_1^{\text{TM}}+F_2^{\text{TM}}$) associated with the lack of spherical symmetry in the collisions between the molecules. We shall use the model expression introduced in Ref. 10 for the probability for intermolecular collisions, in which the dependence of W_1 on the mutual orientation of the relative velocity of the molecules (g) and their angular momenta is represented by the Legendre polynomial P_2. We shall assume the spherically symmetric interaction with the wall to be specular-diffuse with the diffuseness factor α. The integration over Γ and Γ' in (3.4) can be performed approximately under the assumption that the collision frequency Ω_0 [Eq. (2.4)] varies only weakly as v and M vary but remain close to their thermal values.[8] In the case of nonparamagnetic linear molecules, we obtain, in accordance with (3.4), the following expressions for the changes in the normal force acting on the disk when the field $\mathbf{H}=(H_x, 0, 0)$ is applied:

$$\mathbf{F}=\mathbf{F}_1^{\text{TM}}+\mathbf{F}_2^{\text{TM}},$$

$$\Delta F_z^{\parallel}=c_1(2-\alpha)\frac{\xi_1^2}{1+\xi_1^2},\quad \Delta F_z^{\perp}=\frac{c_1}{2}(2-\alpha)\left[\frac{\xi_1^2}{1+\xi_1^2}+\frac{10\xi_1^2}{1+4\xi_1^2}\right],$$

$$F_y^{tr}=\frac{c_1}{4}(2-\alpha)\left[\frac{\xi_1}{1+\xi_1^2}+\frac{5\xi_1}{1+4\xi_1^2}\right]. \tag{4.1}$$

Here $\Delta F_z^{\parallel}$ and $\Delta F_z^{\perp}$ are the changes in the components parallel and perpendicular, respectively, to ∇T of the normal force on the disk, F_y^{tr} is the component of the tangential force on the disk transverse to ∇T ($F_y^{tr}=0$ when $H=0$), $\xi_1 = c_1' H/p$, ΔT and L are the temperature difference and distance, respectively, between the surfaces limiting the gas, $(\delta T)_0$ is the temperature discontinuity at these surfaces ($\delta T = 0$ at the disk), and the quantities c_1 and c_1' ($|c_1| \sim \nu^2$) can be calculated if the dependence of W on g^2 and M^2 is known. Similar calculations for NO (it must be understood that in the case of a paramagnetic gas the integration over Γ in (3.4) includes a summation over a variable associated with the projection of the spin onto the direction of the total angular momentum of the molecule) also leads to Eqs. (4.1). In view of the dependence of ω on M^2, however, the parameter c_1' will have a different value for NO than for a nonparamagnetic gas.

To calculate the TMF associated with the aspherical reflection of polarized molecules from the surface ($\mathbf{F}_3^{\mathrm{TM}}$), we must have an expression for the probability V_1 for scattering of the gas at the wall. We shall use the expansion of V_1 in spherical functions of the vectors $\mathbf{v}'$, $\mathbf{M}'$, $\mathbf{v}$, $\mathbf{M}$, and $\mathbf{k}$:[11]

$$V_1(\mathbf{v}', \mathbf{M}' \to \mathbf{v}, \mathbf{M}; \mathbf{k}) = |\mathbf{v}\mathbf{k}| \exp\left(-\frac{E}{T}\right) \sum_s \beta_S(v'^2, M'^2, v^2, M^2) A_S,$$
$$S = \{(l_1' l_1) j_1,\ (l_2' l_2) j_2,\ l_3\}, \tag{4.2}$$
$$A_S = [Y^{l_1'}(\mathbf{v}') \times Y^{l_1}(\mathbf{v})]^{j_1} [Y^{l_2'}(\mathbf{M}') \times Y^{l_2}(\mathbf{M})]^{j_2} Y^{l_3}(\mathbf{k}).$$

Within the framework of a semiphenomenological approach to the description of the TMF we can obtain a model expression for V_1 by retaining the necessary number of terms in (4.2) and evaluating the expansion parameters by comparison with experiment. We note that in accordance with (3.4) and (3.5) only those terms of expansion (4.2) for which $l_2 = 0$ contribute to F_3^{TM}. When speaking of the use of some term $\{(l_1' l_1) j_1, (l_2' 0) l_2', l_3\}$ in (4.2) we mean automatically to imply that the expression for V_1 also contains the term $\{(l_1 l_1') j_1, (0 l_2') l_2', l_3\}$ symmetric to it (as required by the reciprocity theorem). Below we shall use model expressions for the probability for scattering at the surface obtained from study of the effect of the field on the heat flux in a Knudsen gas.[11,14]

Let us first consider the TMF acting on aluminum and paper disks in O_2 and NO. Since the paper surface has a porous filamentary structure, the reflection from the paper surface should evidently be fully diffuse as a result of the complete averaging over the escape directions and orientations of the molecules. We shall therefore assume that the asphericity parameter ε vanishes for the paper surface, so that we have $F_3^{\mathrm{TM}} = 0$ in (3.4). Assuming that the Al surface is coated with a layer of chemisorbed oxygen and noting that the scattering probability is determined primarily by the composition of the adsorbed layer,[11] we shall assume that the principal terms in the expansion (4.2) of the probability for scattering NO and O_2 molecules from the Al surface are those with $l_2' = 1$.[14] In this case, as is not difficult to see, we again have $F_3^{\mathrm{TM}} = 0$.

Thus, in NO and O_2, the TMF should not change when a paper disk is substituted for the aluminum one, and Eqs. (4.1) should correctly give the field dependence of the TMF in NO. Indeed, in these gases (but not, for example, in N_2) the observed TMF remains practically unchanged on substituting the paper disk for the aluminum one.[4] The experimental points in Fig. 1 (taken from Ref. 4) show the H/p dependences of $\Delta F_z^{\parallel}$, $\Delta F_z^{\perp}$, and F_y^{tr} for NO at $\bar{l}/r \sim 0.1$. The figure also shows the theoretical dependences according to (4.1) (for suitable choice of the values of the parameters c_1 and c_1'). The observed agreement between the theoretical and experimental results indicates that the TMF in NO and O_2 is due to the deviations from spherical symmetry in the collisions between gas molecules.

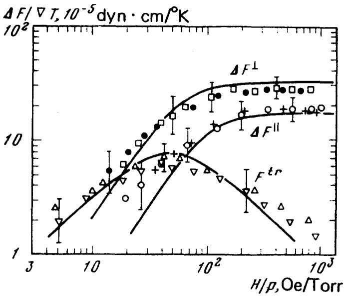

FIG. 1. The TMF in NO vs H/p. The experimental points were taken from Ref. 4 and were measured at various gas pressures for which $\bar{l}/r \sim 0.1$: The points □, ●, ○, and + were measured with an aluminum disk, and the points ▽ and △, with a paper disk.

We note that the observed agreement also confirms the assumption that the temperature discontinuity at the surface of the disk is small. The term in (3.4) associated with δT (i.e., $\mathbf{F}_4^{\mathrm{TM}}$) contributes only to the TMF component normal to the surface. Taking this term into account, therefore, changes the value of the ratio $F^{tr}/\Delta F^{\parallel}$ and only worsens the agreement with experiment.

Let us consider the TMF in N_2 and CO. For these gases only the normal component of the TMF in a field $\mathbf{H}$ parallel to ∇T has been measured.[3] In N_2 and CO (though not in NO and O_2) $\Delta F^{\parallel}$ increases when the paper disk is replaced by an aluminum one (in N_2 it increases by a factor of 1.7—see Fig. 2). We note that the TF and TMF have similar dependences on the spherically symmetric interaction with the wall (in the case of specular-diffuse interaction the strengths of the TF [Eq. (3.2)] and the TMF [Eq. (3.4)] are proportional to $2-\alpha$). No change in the TF on altering the material of the disk

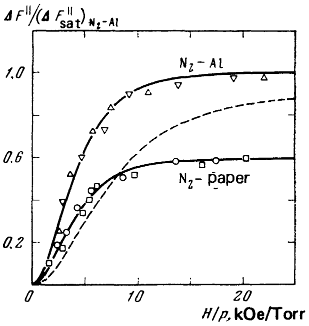

FIG. 2. The TMF on aluminum and paper surfaces in N_2 vs H/p. The full curves represent Eq. (4.4) with $c_3/c_1 = 0.7$ for aluminum and $c_3 = 0$ for paper. The experimental points were taken from Ref. 3 and were measured at various gas pressures for which $\bar{l}/r \sim 0.1$. The dashed curve represents the theory of Ref. 6.

has been observed.[13] We may therefore assert that the dependence of the TMF on the material of the disk is associated with nonspherical scattering from the surface.

We shall assume that the principal term in the probability for scattering of N_2 and CO molecules from the chemisorbed oxygen coating of the Al surface is the $\{(11)1, (20)2, 2\}$ term.[11] Then for the change in a field **H** parallel to ∇T of the force normal to the surface, corresponding to the contribution $\mathbf{F}_3^{TM}$ in (3.4), we obtain

$$[\Delta F_3]_z^{\parallel} = c_3 \frac{\xi_3^2}{1+\xi_3^2}, \qquad \xi_3 = c_3' \frac{H}{p}, \tag{4.3}$$

where the parameters c_3 and c_3' ($|c_3| \sim \nu\varepsilon$) can be calculated if the dependences of the probabilities W and V_1 on v^2 and M^2 are known. We note that taking account of the additional terms $\{(00)0, (20)2, 2\}$, $\{(00)0, (10)1, 1\}$, $\{(11)0, (20)2, 2\}$, and others that occur in the expressions for the probabilities for scattering of N_2 and CO molecules from oxygen-coated surfaces[11] does not result in any change in expression (4.3). For the scattering of molecules from a paper surface we assume, as above, that $\varepsilon = 0$ and correspondingly that $\mathbf{F}_3^{TM} = 0$ in (3.4). Assuming for simplicity that $c_1' = c_3'$, we obtain the following expression for the TMF in N_2 and CO in accordance with (4.1) and (4.3):

$$\Delta F_z^{\parallel} = c_1 \left(1 + \frac{c_3}{c_1}\right) \frac{\xi_1^2}{1+\xi_1^2}, \qquad \xi_1 = c_1' \frac{H}{p}, \tag{4.4}$$

moreover, $c_3 = 0$ for a paper disk and $c_3 \neq 0$ for an aluminum disk (here the factor $2 - \alpha$ in (4.1) is absorbed in the parameter c_1). The ratio c_3/c_1 can be determined from the experimental value of the ratio of the effects for aluminum and paper at saturation:

$$(\Delta F_{sat}^{\parallel})_{N_2-Al} / (\Delta F_{sat}^{\parallel})_{N_2-paper} \approx 1.7.$$

As a result, we have $c_3/c_1 = 0.7$. We evaluate the parameters c_1 and c_1' by equating the values of $(\Delta F^{\parallel}_{sat})_{N_2-Al}$ and $(H/p)_{1/2}$ (the value of H/p for which $\xi_1^2/(1+\xi_1^2) = 1/2$) corresponding to Eq. (4.4) to the experimental values. The resulting field dependence of $\Delta F^{\parallel}$ is shown in Fig. 2 by the full curves and agrees with experiment within the experimental errors. Similar agreement is also found for the TMF in CO.

It follows from what has been said that in the case of a paper disk the TMF in N_2 is due to the lack of spherical symmetry in the intermolecular collisions, while in the case of an aluminum disk, for which $c_3/c_1 = 0.7$, about 60% of it is due to molecular collisions and 40% is related to the lack of spherical symmetry in the reflection of molecules from the surface.

Comparison of the TMF in N_2 with experiment also confirms the assumption adopted above that the temperature discontinuity at the surface of the disk is small. Since the temperature discontinuity is determined from the condition for heat flux balance near the wall, the field dependence of δT should be close to the field dependence of the heat flux.[7,10] Hence we can write the following approximate expression for the contribution F_4^{TM} to the effect:

$$[\Delta F_4]_z^{\parallel} \approx c_4 \xi_4^2 / (1+\xi_4^2), \tag{4.5}$$

where $\xi_4 = c_4' H/p$, and c_4' is known from studies of the Senftleben-Beenakker effect. To be able to compare the positions of the curves on the H/p axis we have normalized the theoretical and experimental dependences plotted in Fig. 3 to the corresponding saturation values. The dashed curve on Fig. 3 represents Eq. (4.5); it obviously lies to the right of the experimental curve.

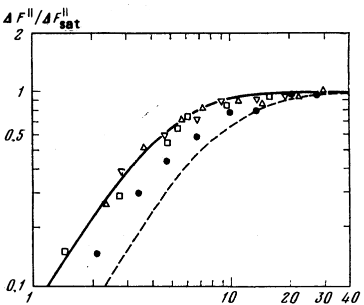

FIG. 3. The component of the TMF in N_2 normal to the surface vs H/p. The full curve and the open experimental points are the same as in Fig. 2 ($r/\tilde{L} \approx 1$). The dashed curve represents Eq. (4.5) and the theory of Ref. 6. The black circles are experimental points measured with the disk shifted to such a position that $r/\tilde{L}_{min} \approx 3.5$ (Ref. 3).

A more realistic analysis using the experimentally known[15] Knudsen corrections to the Senftleben-Beenakker effect shows that the H/p dependence of $\Delta F_4^{\parallel}$ is shifted somewhat farther to the right than the dashed curve in Fig. 3. This, however, only strengthens the assertion made above.

The temperature discontinuity at the disk surfaces becomes significant only at larger values of $r/\tilde{L}$. Thus, when the disk was displaced so far toward one of the surfaces that the ratio of the radius of the disk to the gap between the disk and the nearer surface became $r/\tilde{L} \approx 3.5$ (instead of $r/\tilde{L} \approx 1$ as in the experiments cited above) the $\Delta F^{\parallel}$ vs H/p curve was observed[3] to shift to the right (see Fig. 3). Under the assumption that in this situation δT at the surface of the disk facing the shorter gap differs from zero, the theoretical dependence will be a linear combination of expressions (4.4) and (4.5). In this case the resulting curve will lie between the full and dashed curves in Fig. 3 and will obviously agree with experiment if the ratio c_4/c_1 is suitably chosen.

The calculations presented above show that if the interaction between the molecules or between a molecule and the surface is not invariant under space inversion [for example, for O_2–Au (Ref. 14)], thermomagnetic forces having opposite directions on opposite sides of the disk may arise. These forces are also described by the general expression (3.4). Such forces could not be detected in the experiments reported in Refs. 3 and 4 because of the symmetry of the transducer employed. To measure such forces one might, for example, use a sensitive disk whose surfaces differ from one another.

[1] As is well known, such polarization is basic for a nonuniformly heated diatomic gas.[7,10]

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Translated by E. Brunner

Large-scale fluctuation potential and state density in doped and strongly compensated semiconductors

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A method is proposed for a self-consistent calculation of the large-scale fluctuation potential in doped and strongly compensated semiconductors with shallow impurities at $T=0$. It is shown that the fluctuation potential can range from zero to the energy difference Δ between the donor and acceptor levels. The position of the Fermi level and the mean value of the potential $\bar{V}$ are determined. The potential probability density function $\mathscr{P}(V)$ is calculated in the energy interval $0 \leq V \leq \Delta$ for a degree of compensation $0.95 \leq K \leq 1$. The large-scale potential causes all the neutral donors to be on the Fermi level at the energy $E=0$, and causes the empty donor states to spread into a band with energies from zero to Δ. The electronic states below the Fermi level arise in the case $a_B^3\bar{N}_d \ll 1$ only as a result of the Coulomb potential of the nearest empty donor, and in the case $a_B^3\bar{N}_d \gg 1$ they are due to quantization of the electron in small-radius fluctuations. The tails of the state densities of the valence and empty bands differ in character. The valence band has a deep state-density tail on account of the large-scale potential (when $\bar{N}_d > \bar{N}_a$), while the conduction-band tail is due only to the small-radius fluctuations, in which the quantization is significant.

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INTRODUCTION

An inhomogeneous random distribution of the impurities in doped semiconductors violates the local electroneutrality and accordingly leads to the appearance of an inhomogeneous large-scale potential. It is known from experiment that when the degree of compensation is increased, the activation energy for the conductivity increases.[1-5] The influence of a fluctuation potential on the electron spectrum was considered in Refs. 6–13. Shklovskii and Éfros[10-12] calculated the fluctuation potential in the following manner. They considered fluctuations of arbitrary radius R under the assumption that the impurities are uniformly distributed inside the fluctuation. Since fluctuations of sufficiently small radius cannot be screened by electrons, a "bare" charge appears in the region of the fluctuations, and is equal to $q\Delta N\Omega$, where $\Delta N = N_d - N_a - (\bar{N}_d - \bar{N}_a)$, where N_d and N_a are the concentrations of the donors and acceptors in the fluctuation, $\bar{N}_d$ and $\bar{N}_a$ are the average concentrations of the impurities in the crystal, and Ω is the volume of the fluctuation. The potential in the region of the fluctuation is in this case approximately equal to $(q^2/\varepsilon)\Delta N R^2$.

Assuming a Gaussian distribution for the fluctuations, the mean squared potential is calculated to be

$$\gamma = (q^2/\varepsilon)(\bar{N}_d R)^{1/2},$$

from which it follows that without allowance for the screening γ diverges when R is increased.

The characteristic radius at which screening comes into play is estimated in Refs. 10 and 11 from the following considerations: since the mean squared deviation of the concentration decreases when the radius is increased, $\overline{\Delta N^2} = \bar{N}_d/R^3$, it follows that at $n > (\overline{\Delta N^2})^{1/2}$ ($n = \bar{N}_d - \bar{N}_a$ is the average concentration of the electrons in the free or impurity donor band) the electrons easily screen such fluctuations, and the latter make no contribution to the potential. Thus, the characteristic radius R_c is determined from the relation