ever, the magnetic field can cancel $\alpha_0$ (in our case $\alpha_0 < 0$). It is seen from (38) that at $h \sim -\alpha_0$ the susceptibility has a $\delta$-like burst, and the heat capacity has two bursts at $|\varepsilon_1| \sim T$ and a dip at $|\varepsilon_1| \ll T$. Let now $p_0 > 1$ in our case. It is seen from (37) that altogether, including the pure ferromagnetic state, we have $p_0 + 1$ levels. It is easily seen that susceptibility bursts of the type (38) appear only upon intersection of only the two lower levels, and the two bursts and the dip of the heat capacity will occur when any two levels intersect. Therefore when the magnetic field is increased the susceptibility will have $p_0$ $\delta$-like bursts, and the heat capacity will have $p_0(p_0+1)/2$ minima each bracketed by two bursts, i.e., in all $p_0(p_0+1)/2$ minima and $p_0(p_0+1)$ maxima. Since the described picture is connected only with the presence of $p_0$ magnetic levels, it should be observed also for other magnetic systems. In particular, the same picture is observed in the antiferromagnetic-impurity problem[10] (not to be confused with the antiferromagnetic coupling).

We note that if the antiferromagnetic exchange integrals are random quantities, then we obtain near $|h + \alpha_0| \sim T$, averaging over the distribution of this random quantity:

$$C \sim N_0 T, \quad \chi \sim N_0. \tag{39}$$

We consider in conclusion the case $s \gg 1$. Here we have a region $\omega \ll T \ll T_c$ in which the described oscillations are smeared out. The sum in (37) must then be replaced by an integral. Calculating the heat capacity and the susceptibility by the saddle-point method, we get at $h \gg T$

$$C = N_0/2, \quad \chi = -N_0 \partial p_1/\partial h, \tag{40}$$

where $p_1$ is the minimum of $\varepsilon_p$ at $h \neq 0$. Since $\partial p_1/\partial h < 0$, it follows that $\chi$ in (40) is positive. Thus, $C$ and $\chi$ do not depend on temperature in this region.

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# Cross section for electron capture by a charged dislocation in a semiconductor


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The cross section for multiphoton capture of an electron by an edge dislocation in an $n$-type semiconductor is calculated in the quasiclassical approximation. Analytic dependence of the cross section on the electron-phonon interaction constant, on the temperature, and on the charge per dislocation unit are obtained.


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## 1. INTRODUCTION

According to the experimental data, an edge dislocation in a semiconductor leads to the onset of a one-dimensional band with a bottom $E_D$ located in the forbidden band. The investigations performed to date are still insufficient for final conclusions concerning the carrier dispersion in this band, but it can be regarded as established that the width of the dislocation band $E_0$ is much less than its depth $E_D$, which is comparable with the width of the forbidden band $E_g$. In accordance with the general concepts, such a "deep one-dimensional band" should contribute to the effective recombination of the excess carriers. The statistics of the electron and hole recombination in semiconductors were considered by Gulyaev,[2] who obtained the dependence of the lifetime of an electron-hole pair on the concentration of the excess carriers and capture cross sections. It was noted that the quantity corresponding to the capture cross section should be referred to the length of the dislocation, in view of the extended structure of the latter, and the concept of capture radius was introduced by the same token. The purpose of the present study was to find the radius for the capture of an electron by a

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dislocation.

It was shown by the author[3] in an earlier calculation of the hole capture radius that the edge dislocation in an $n$-type semiconductor, by acting as an acceptor and being negatively charged, leads to a bending of both the valence and the conduction band. This bending of the bands, by contributing to the hole capture, prevents the dislocation from capturing the like charged majority carriers—the electrons. Therefore the electron capture proceeds in two stages. The first is the penetration of the electron through the repulsion barrier of the charged dislocation, an elastic process, and the other is an energy transfer to a two-dimensional "chemical" well with a ground level corresponding to the depth $E_D$. A survey of the presently known energy outflow mechanism in carrier capture processes is contained in the paper of Bonch-Bruevich and Landsberg.[4] Particular interest attaches to the phonon mechanism, wherein the energy of the captured carrier is transferred to the acoustic lattice vibrations. It precisely such an interaction that is realized in the homopolar semiconductors Ge and Se, as well as in some other crystals of the CdS type. Inasmuch as the Lax capture mechanism[5] does not hold for deep traps, all that remains possible within the framework of the phonon model is a quantum multiphonon transition. The general methods of modern theory of multiphonon transitions are given in the review of Perlin.[6] In the present exposition we shall follow Rickayzen's original paper[7] where he considered carrier capture by deep point traps. To take into account the electron interaction with acoustic phonons we use the simplest approximation, assuming a Debye phonon spectrum and terminating it at the limiting wave number $q_m$ connected with the unit-cell volume $V_0$ by the equation $q_m^3V_0=6\pi^2$. We neglect the contribution of the local oscillations connected with the dislocations. As the potential of the interaction of the electrons with the phonons we use the usual isotropic deformation potential

$$H_{int}=\frac{iE_c}{\sqrt{N}}\sum_{\mathbf{q}} qQ_{\mathbf{q}}\exp(i\mathbf{qr}), \tag{1}$$

where $E_c$ is the constant of the deformation potential, $N$ is the number of unit cells in the fundamental volume, and $Q_q$ is the normal coordinate of a phonon with wave vector **q**.

## 2. WAVE FUNCTIONS

To find the wave functions that characterize the states of an electron in a one-dimensional dislocation band and in the continuum of the conduction band, we introduce cylindrical coordinates $(r, \theta, z)$ with the dislocation axis as the polar axis. Avoiding additional assumptions, we use the simplest model for the choice of the dislocation potential, and represent the dislocation as a two-dimensional square well of depth $U_0$ and of radius of the order of the lattice constant $a$, surrounded by an electrostatic barrier described in the region of sufficiently high temperatures $\alpha<1$, with high degree of accuracy, by a function of the form[3]

$$U(r)=2\alpha kT\ln(2r_D/\gamma r),\quad a<r<2r_D/\gamma. \tag{2}$$

Here $\alpha=e^2/\varepsilon akT$ is a parameter that characterizes the ratio of the Coulomb energy of the interaction of neighboring electrons that have settled on the dislocation to their thermal energy $kT$, $\varepsilon$ is the dielectric constant, $r_D$ is the Debye screening radius, and $\ln\gamma=0.577$ is the Euler constant. We note that the use of the concept of the dielectric constant is justified only at sufficiently large distances from the dislocation axis. In the investigation of the influence of the electrostatic barrier on the capture process, however, it is precisely such distances that are significant.

In the temperature region $\alpha<1$ of interest to us the height of the Coulomb barrier is much less than the well depth $U_0$. Therefore when finding the wave function $\psi_n(r,\theta,z)$, which characterizes the free motion of the electron along the dislocation with two-dimensional localization on the ground level $E_D$, we neglect the influence of the electrostatic field and assume that this function is well approximated by

$$\psi_n(r,\theta,z)=(\beta/\pi rL_z)^{1/2}\exp(ik_zz+il\theta-\beta r), \tag{3}$$

where $n\equiv\{E_D, l, k_z\}$, $l$ is the orbital number, $k_z$ is the wave number of the electron moving along the dislocation, $\beta\equiv(2mE_D/h^2)^{1/2}$, and the normalization is carried for a cylinder of radius $R$ and length $L$. This wave function is the simplest one that follows from the variational principle, and constitutes the asymptotic solution of the wave equation in the case of a cylindrically symmetrical potential.

To find the wave function of the electron $\psi_m(r,\theta,z)$ in the continuum of the conduction band we must solve the Schrödinger equation at short $(0<r<a)$ and long $(r>a)$ distances and then join together the obtained solutions. In view of the cylindrical symmetry of the potential, the variables separate, and the solution is sought in factorized form

$$\psi_m(r,\theta,z)=CR_{kl}(r)\exp(ik_zz+il\theta), \tag{4}$$

where $C$ is the normalization constant, $m\equiv\{k, l, k_z\}$, $k\ (2mW/\hbar^2)^{1/2}$ is the radial wave number, $W=E-\hbar^2k^2/2m$, and $R_{kl}(r)$ is the solution of the radial part of the wave equation. At short distances corresponding to the region of action of the "chemical forces," the solution satisfying the condition that the wave function be finite is

$$R_{kl}(r)=J_l(\varkappa r),\quad 0<r<a, \tag{5}$$

where $J(x)$ is a Bessel function of order $l$, and $\varkappa=[2m(W+U_0)/\hbar^2]^{1/2}$. In the case of narrow and deep one-dimensional band of interest to us $E_D\sim U_0\sim E_g\gg\max\{E_0, kT, \hbar\omega_D\}$ is the Debye frequency, neglecting the small corrections, we can assume that $\varkappa\approx\beta$.

At large distances the radial part of the wave function satisfies the equation

$$\frac{d^2R}{dr^2}+\frac{1}{r}\frac{dR}{dr}+\frac{2m}{\hbar^2}\left[W-U(r)-\frac{\hbar^2l^2}{2mr^2}\right]R=0. \tag{6}$$

We make the usual substitution $R_{kl}(r)=u_{kl}(r)/r^{1/2}$, and obtain then in place of (6)

$$\frac{d^2u}{dr^2}+\frac{2m}{\hbar^2}\left[W-U(r)-\frac{\hbar^2(l^2-1/4)}{2mr^2}\right]u=0. \tag{7}$$

The obtained equation readily reduces to the Riccati equation, which has no exact solution for the potential (2). We therefore use a quasiclassical approximation[8] that is valid when the condition $|d\lambda/dr| \ll 1$ is satisfied, where $\lambda$ is the de Broglie wavelength. It is impossible to apply the quasiclassical approximation directly to Eq. (7). The reason is that this equation has a singularity near the origin. At small $r$ only the centrifugal potential is significant, and consequently $d\lambda/dr \sim l^{-1}$ and the condition for the quasiclassical approach at small $l$ is violated. To obtain a solution that is valid at the origin at all $l$ and yields the correct value of the phase of the wave function at infinity, we make the change of variable $r=e^x$ and use the transformation $u_{kl}(r) = e^{x/2}\chi_{kl}(x)$ (the Langer method[9]). In terms of the new variables we have

$$\frac{d^2\chi}{dx^2}+\frac{2m}{\hbar^2}\left[W-U(e^x)-\frac{\hbar^2 l^2}{2m}e^{-2x}\right]e^{2x}\chi=0. \tag{8}$$

The use of the methods of quasiclassical approximation to Eq. (8) with subsequent transition to the variable $r$ leads to the following expressions for the radial part of the wave function:

$$R_{kl}(r)=r^{-1/2}|k(r)|^{-1/2}\exp\left(-\int_r^{r_l(k)}|k(r)|dr\right); \quad a<r<r_l(k),$$
$$R_{kl}(r)=r^{-1/2}2[k(r)]^{-1/2}\cos\left(\int_{r_l(k)}^{r}k(r)dr-\frac{\pi}{4}\right); \quad r>r_l(k), \tag{9}$$

which corresponds to motion of the particle in the classically accessible and inaccessible regions. Here

$$k(r)=\{2m[W-U(r)-\hbar^2l^2/2mr^2]\}^{1/2},$$

and $r_l(k)$ is the turning point determined from the condition $k(r)=0$. It is easily seen that the Langer correction reduce to the substitution $(l^2-1/4)\to l^2$ in the centrifugal potential. The obtained solution (9) must be joined to the solution ($f$) in the region $0<r<a$. The joining of the wave functions and of their derivatives at the point $r=a$ yields

$$R_{kl}(r)=\frac{1}{[|k(a)|a]^{1/2}}\frac{J_l(\beta r)}{J_l(\beta a)}\exp\left(-\int_a^{r_l(k)}|k(r)|dr\right); \quad 0<r<a,$$
$$R_{kl}(r)=\frac{1}{[|k(r)|r]^{1/2}}\exp\left(-\int_r^{r_l(k)}|k(r)|dr\right); \quad a<r<r_l(k), \tag{10}$$
$$R_{kl}(r)=\frac{2}{[k(r)r]^{1/2}}\cos\left(\int_{r_l(k)}^{r}k(r)dr-\frac{\pi}{4}\right); \quad r>r_l(k).$$

To determine the normalization constant we can replace $R_{kl}(r)$ by its asymptotic value at large $r$:

$$R_{kl}(r)\underset{r\to\infty}{=}2(kr)^{-1/2}\cos(kr-l\pi/2-\pi/2) \tag{11}$$

and to carry out the integration with any finite value of $r$ as the lower limit, e.g., zero. This leads to neglect of a finite quantity compared with an infinitely large one.[8] Normalization to a cylinder with account taken of the fact that $R_{kl}(R)=0$ yields

$$C=(k/4\pi RL_z)^{1/2}. \tag{12}$$

The wave numbers $(k, k_z)$ run through a series of discrete values chosen from the conditions that the wave function vanish on the boundary of the normalization volume. The final equation for the wave function of the continuous spectrum is given by expressions (4), (10), and (12). We note that the quasiclassical solution pertains only to radial motion, and the solution for the remaining dimensions is exact.

## 3. QUASICLASSICAL CAPTURE RADIUS

By definition, the capture cross section is given by the ratio of the flux of captured particles to their average velocity. As noted in the Introduction, the quantity corresponding to the capture cross section should be referred to a unit dislocation length and be of the dimension of length. We assume that the gas of the captured particles is nondegenerate and is at thermal equilibrium with the lattice. Confining ourselves to the capture of particles with angular momentum $l=0$ ($s$ capture) we get for the capture radius

$$\rho_t(T)=\int w_{mn}f(\mathbf{k})\frac{Rdk}{\pi}\frac{dk_z}{2\pi}\Big/\int\frac{\hbar|\mathbf{k}|}{m}f(\mathbf{k})\frac{d^3\mathbf{k}}{(2\pi)^3}, \tag{13}$$

where $f(\mathbf{k})$ is the Boltzmann function and $w_{mn}$ is the probability of the transition of the electron from the continuum to a one-dimensional dislocation band. Such a two-dimensional localization of the captured electron produces near the dislocation a lattice polarization that corresponds to a change of the normal coordinates of the phonons. An exact expression for the probability of a multiphoton transition is given in Rickayzen's paper.[7] In the quasiclassical limit, when the temperature is high enough and (or) the coupling of the captured carrier with the lattice is not small according to Ref. 7 we have for $w_{mn}$, neglecting the quantities of order of $E_0/E_D$ and $kT/E_D$, we have

$$w_{mn}=\omega_D\left(\frac{2\pi}{\sigma^2}\right)^{1/2}\sum_{\mathbf{q}}|P_{\mathbf{q}}|^2\Omega_{\mathbf{q}}^2(2n_{\mathbf{q}}+1)\exp\left[-\frac{(\varepsilon_D-\Delta)^2}{2\sigma^2}\right] \tag{14}$$

where

$$\sigma^2=\sum_{\mathbf{q}}|A_{\mathbf{q}}|^2\Omega_{\mathbf{q}}^2(2n_{\mathbf{q}}+1), \quad \Delta=2\sum_{\mathbf{q}}|A_{\mathbf{q}}|^2\Omega_{\mathbf{q}}, \quad A_{\mathbf{q}}=(\hbar/2M\omega_{\mathbf{q}})^{1/2}a_{\mathbf{q}},$$
$$P_{\mathbf{q}}=(\hbar/2M\omega_{\mathbf{q}})^{1/2}Q_{mn}(q), \quad Q_{mn}(q)=\frac{q}{N^{1/2}}\frac{iE_c}{E_D}\int\frac{dp_z'}{2\pi\hbar}\int d^3\mathbf{r}\psi_n^* e^{i\mathbf{qr}}\psi_m, \tag{15}$$
$$a_{\mathbf{q}}=\frac{1}{N^{1/2}}\frac{iE_c}{\hbar s}\int\frac{dp_z'}{2\pi\hbar}\int d^3\mathbf{r}|\psi_n|^2e^{i\mathbf{qr}}, \quad n_{\mathbf{q}}=[\exp(\hbar\omega_{\mathbf{q}}/kT)-1]^{-1}.$$

Here $M$ is the mass of the unit cell, $s$ is the speed of sound, $\Omega_q=\omega_q/\omega_D$ and $\varepsilon_D=E_D/\hbar\omega_D$ are the dimensionless frequency of the phonon and the binding energy of the electron with the dislocation, and $p_z'$ is the momentum of the electron in the dislocation band. $P_q$ is the matrix element of the electronic transition in the so-called Franck-Condon approximation,[6] and $a_q$ is the change of the normal coordinates of the phonons. Starting from symmetry considerations, it is convenient to introduce the phonon quasimomentum components $q_\parallel$ and $q_\perp$ along and across the dislocation axis, respectively. Direct calculation of $a_q$ with the aid of (3) yields

$$a_{\mathbf{q}}=\frac{1}{N^{1/2}}\frac{iE_c}{\hbar s}\left[1+\left(\frac{q_\perp}{2\beta}\right)^2\right]^{-1/2}. \tag{16}$$

We see therefore that an electron localized on a dislocation interacts strongly only with the transverse lattice vibrations, whose wavelength $q_\perp^{-1}$ is of the order of the localization radius. Substituting the wave functions in the expression for $Q_{mn}(q)$ and integrating with respect to $r$, we obtain after cumbersome but quite straightforward calculations for the matrix element $P_q$

$$P_q = \frac{1}{(4R\beta)^{1/2}} \frac{E_c}{E_D} \left( \frac{\hbar\omega_D}{Ms^2N} \right)^{1/2} \frac{\Omega_q^{1/2}}{(1+(q_\perp/\beta)^2)^{1/4}} \left( \frac{k}{|k(a)|} \right)^{1/2} \times \exp\left( - \int_a^{r_0(k)} |k(r)|dr \right). \quad (17)$$

Using the relations (15) and (16) and changing over, as usual, from summation to integration with respect to $q$, we get

$$\Delta = 3 \frac{E_c^2}{\hbar\omega_D Ms^2} \left( \frac{2\beta s}{\omega_D} \right)^2 \ln \frac{\omega_D}{\beta s}. \quad (18)$$

The dimensionless parameter $\hbar\omega_D\Delta/E_D$, which characterizes the force of the interaction of the electrons with the lattice vibrations, is equal to the ratio of the energy of the redistribution (i.e., the energy transferred to the lattice in the course of the capture) to the ionization energy $E_D$.

To obtain an analytic expression for the transition probability, we consider two limiting cases

1) $kT \gg \hbar\omega_D$—the classical limit. Expanding the exponential in the phonon distribution function in powers of $\hbar\omega_q/kT$, we get

$$\sum_q |P_q|^2 \Omega_q^2 (2n_q+1) = \frac{1}{2R\beta} \left( \frac{E_c}{E_D} \right)^2 \frac{kT}{Ms^2} \left( \frac{\beta s}{\omega_D} \right)^2 \ln\left( \frac{\omega_D}{\beta s} \right) \frac{k}{|k(a)|} \times \exp\left( -2 \int_a^{r_0(k)} |k(r)|dr \right), \quad (19)$$

$$\sigma^2 = 2\Delta kT/\hbar\omega_D. \quad (20)$$

2) $(8ms^2E_D)^{1/2} \ll kT \ll \hbar\omega_D$—the quasiclassical limit. The strong inequality condition $kT \ll \hbar\omega_D$ must be understood in the sense of smallness of the exponential $\exp(-\hbar\omega_D/kT)$. Neglecting corrections of the type $(\hbar\omega_D/kT)^2 \exp(-\hbar\omega_D/kT)$, we then have

$$\sum_q |P_q|^2 \Omega_q^2 (2n_q+1) = \frac{3}{16R\beta} \left( \frac{E_c}{E_D} \right)^2 \frac{\hbar\omega_D}{Ms^2} \left( \frac{\beta s}{\omega_D} \right)^2 \ln\left( \frac{\omega_D}{\beta s} \right) \times \left[ 1 + \frac{4\pi^4}{15} \left( \frac{kT}{\hbar\omega_D} \right)^4 \frac{\ln(kT/\hbar\beta s)}{\ln(\omega_D/\beta s)} \right] \frac{k}{|k(a)|} \exp\left( -2 \int_a^{r_0(k)} |k(r)|dr \right), \quad (21)$$

$$\sigma^2 = \frac{\Delta}{2} \left[ 1 + \frac{\pi^2}{3} \left( \frac{kT}{\hbar\omega_D} \right)^2 \frac{\ln(kT/2\hbar\beta s)}{\ln(\omega_D/\beta s)} \right]. \quad (22)$$

The exponentials in (19) and (21) characterize the tunneling of the electron through the repulsion barrier of the charged dislocation. Direct calculation of the argument of the tunnel exponential yields

$$\int_a^{r_0(k)} |k(r)|dr = kr_0(W)(2\alpha(W))^{1/2}\gamma(3/2, \ln(r_0/a)), \quad (23)$$

where $r_0(W) = (2r_D/\gamma)\exp[-1/2\alpha(W)]$, $\alpha(W) = \alpha kT/W$, and $\gamma(n, x)$ is the incomplete gamma function. To obtain the temperature dependence of the electron capture radius we must substitute the transition probability $w_{mn}$ given by (14) and (19)–(23) in (13) and average over the initial states of the continuum of the conduction band. The resultant integral, which characterizes the flux of the particles captured by the dislocation, can be easily calculated by the saddle-point method. The saddle point

$$\overline{W} = 2\alpha kT \ln(A/2\alpha), \quad A = \frac{2r_D}{\gamma}(4\pi\alpha mkT/\hbar^2)^{1/2} \quad (24)$$

corresponds to the energies of the most effectively captured particles.

We have followed in this exposition the Franck-Condon approximation, in which the electronic matrix element does not depend on the normal coordinates of the phonons. Kovarskii[10] has shown that by going outside the framework of the Franck-Condon approximation one obtains a much larger capture cross section, namely

$$\rho_t = (E_D/\hbar\beta s)^2 \rho_{t\Phi-K}.$$

The quantity $\hbar\beta s$ can be interpreted as the energy of the "resonant" phonon whose wavelength is of the order of the localization radius of the carrier on the trap. Taking the foregoing into account, the final expressions for the capture radius in the classical and quasiclassical limiting cases are

$$\rho_t(T) = \rho_{t0}(T)(kT/\hbar\omega_D\Delta)^{1/2} \exp(-E^*/kT), \quad kT \gg \hbar\omega_D,$$

$$\rho_t(T) = \rho_{t0}(T) \frac{3}{8} \left[ 1 + \frac{4\pi^4}{15} \left( \frac{kT}{\hbar\omega_D} \right)^4 \frac{\ln(kT/\hbar\beta s)}{\ln(\omega_D/\beta s)} \right] \times \left( \frac{2\pi}{\sigma^2} \right)^{1/2} \exp\left[ -\frac{(\varepsilon_D-\Delta)^2}{2\sigma^2} \right]; \quad (8ms^2E_D)^{1/2} \ll kT \ll \hbar\omega_D, \quad (25)$$

where

$$\rho_{t0}(T) = \frac{\pi}{2\sqrt{2}\beta} \frac{E_c^2}{\hbar\omega_D Ms^2} \frac{\hbar\omega_D}{kT} \ln\frac{\omega_D}{\beta s} \frac{\exp\{-2\alpha[1+\ln(A/2\alpha)]\}}{[\ln(2r_D/\gamma a) - \ln(A/2\alpha)]^{1/2}},$$

$$E^* = \frac{(\varepsilon_D-\Delta)^2}{4\Delta} \hbar\omega_D.$$

It is seen from (25) that at high temperatures the capture is via an activated state with activation energy $E^*$, and at low temperatures we have a pure quantum tunneling between the electron-vibrational states, as a result of which the temperature dependence of the inelastic process becomes weaker.

Under thermodynamic equilibrium conditions, the capture of the carrier by the dislocation is offset by the inverse process of ionization of a carrier from the dislocation. As shown by Rickayzen,[7] despite the strong interaction of the localized electron with the lattice, the Einstein formula remains in force. Consequently, the ratio of the corresponding radii is given by

$$\rho_i/\rho_t = \exp(-E_D/kT). \quad (26)$$

If the dislocation concentration in the semiconductor is $N_D$, then the lifetime of the nonequilibrium carrier is, by definition, connected with the radius $\rho_t$ in the following manner:

$$\tau_t^{-1} = N_D\rho_t\langle v\rangle, \quad (27)$$

where $\langle v\rangle = (8kT/m)^{1/2}$ is the average carrier velocity in the band. The region of applicability of (27) is bounded on the high-dislocation concentration side, where the contributions of the dislocations to the capture process become non-additive.

## 4. CONCLUDING REMARKS

In the treatment of electron tunneling through a repulsive electrostatic barrier of a charged dislocation we have confined ourselves to the case of $s$ capture, corresponding to isotropy of the process in a plane perpendicular to the dislocation axis. The results of the numerical calculations show that this limitation is justified, since the probabilities of the capture of the particle with $l \neq 0$, owing to the repulsive nature of the centrifugal potential, turns out to be much less than the $s$-capture probability.

According to the results of Ref. 3, the potential of the charged dislocation can be approximated by expression (2) only at sufficiently high temperatures $\alpha < 1$. At the same time, for the quasiclassical approximation to be valid it is necessary that many wavelengths of the captured carrier be spanned by the barrier thickness that the following inequality be satisfied

$$kr_0 = 2\alpha\pi^{-1/2} \ln^{1/2}(A/2\alpha) \gg 1,$$

which imposes an upper bound on the temperature interval. A joint analysis of the obtained criteria shows that for Ge with $E_c = 10$ eV, $\hbar\omega_D = 0.03$ eV, $m \sim 10^{-28}$ g, $M \sim 2.5 \cdot 10^{-22}$ g, $\varepsilon = 16$, $s \sim 5 \cdot 10^5$ cm/sec, with dnoro density $n_d \sim 10^{13}$ cm$^{03}$, and $0.3 < \alpha < 1$ the results of the present paper are qualitatively correct in the temperature interval 200 K $< T <$ 500 K. The absolute value of the capture radius turns out in this case to be of the order of $10^{-15}$ cm $< \rho_t(T) < 10^{-9}$ cm. If the dislocation density is $N_D = 10^6$ cm$^{-2}$, then according to (27) we have for the lifetime of the excess electrons $\tau_t \sim (10^{-6} - 1)$ sec.

The stronger temperature dependence of the capture radius than in the case of the point centers[10] can be explained by the fact that when the temperature changes a change takes place not only in the energy of the most effectively captured electrons, but also in the charged state of the dislocation, i.e., the shape of the electrostatic barrier that surrounds the edge dislocation.

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# Localized degrees of freedom in a ferromagnet with resonant impurities

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The Heisenberg model of a ferromagnet with an admixture of negative exchange integrals of a small, but definite magnitude is investigated. It is shown that this system exhibits low-temperature properties of the "glass" type along with the occurrence of a spontaneous moment close to the norminal moment. Long-wave spin waves are found to be abnormally strongly damped. The problem of elementary excitations in spin glasses is discussed.

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## 1. INTRODUCTION

There has been in recent years quite an intensive study of the properties of the models of disordered magnetic substances with random alternating-sign exchange interaction between the spins located on the regular lattice.[1,2] It is assumed that there is no spontaneous moment when the mean exchange value is close to zero, and the low-temperature phase in such models is similar to the spin-glass phase observed in disordered solutions of the Cu-Mn type.[2] The thermal capacity of spin glasses has a maximum at some temperature $T_f$, and is linear at $T \ll T_f$, while the magnetic susceptibility assumes a constant value at $T \ll T_f$.

In the present paper we show that the low-temperature, "glass" type properties are not necessarily due to the absence of a spontaneous moment, but can also be observed in a ferromagnet with a moment of magnitude close to the maximum value. Below we propose for the Heisenberg ferromagnet a model in which the magnitude and sign of a small part of the exchange integrals are


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