ities of the magnetocaloric effect (see Figs. 1 and 3). It is obvious that the above-described behavior of the magnetic system in the vicinity of the Curie point should manifest itself also in others of its physical properties, such as, for example, the specific heat, the magnetostriction, and the scattering of light.

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Translated by W. F. Brown, Jr.

Effect of fluctuations on the properties of the phase transition from a nematic liquid crystal to an isotropic liquid

A. L. Korzhenevskiĭ and B. N. Shalaev

Leningrad Electrotechnical Institute

(Submitted 28 December 1978)
Zh. Eksp. Teor. Fiz. **76**, 2166–2177 (June 1979)

The critical properties are investigated of a model in which the order parameter is a symmetrical zero-trace $n \times n$ tensor. The particular case $n = 3$ corresponds to the model of the nematic liquid crystal–isotropic liquid phase transition. It is shown that the critical anomalies near this transition can be due to the specific properties of the interaction of the fluctuations of the order-parameter tensor field. An experimental method is proposed with which to establish the cause of the pre-transition anomalies.

PACS numbers: 64.70.Ew

INTRODUCTION

We employ in this paper the method of field renormalization groups (RG) for the investigation of the critical behavior of the Q-model, in which the order parameter is by definition a zero-trace $n \times n$ tensor. Particular interest attaches to the case $n = 3$, which corresponds to the model proposed by de Gennes for the description of phase transitions of the type nematic liquid crystal-isotropic liquid (NLC–IL).[1] Most of them are accompanied by strong pre-critical phenomena, and the Landau expansion for the free energy contains a third-order invariant. Therefore, in the spirit of the predictions of the phenomenological theory, the anomalous behavior of the thermodynamic quantities near the NLC–IL transition was previously attributed to the presence, on the curve of the first-order phase transitions (or near this curve), of an isolated singular point at which the coefficient of this invariant vanishes. The critical properties of the NLC–IL transition, assuming that such a point exists, were investigated by Vigman, Larkin, and Filev, as well as by Lubensky and Priest.[3,4]

Recently Gorodetskiĭ and Zaprudskiĭ,[5] and independently of them one of us[6] have proposed another explanation for the appearance of critical anomalies near this phase transition, without resorting to the assumption that is close to an isolated singular point. It was shown that in contrast to the conclusions of the phenomenological theory the NLC–IL transition can be continous if the system of the RG equations has a scale-invariant solution satisfying definite conditions, and the assumption was advanced that the experimental values of the critical exponents at $\gamma \approx 1.0$ for the susceptibility and $\alpha = 0.3-0.5$ for the heat capacity can pertain to the fluctuation region and not to the region of the Landau theory. In this case the proximity of the exponent γ to unity can be attributed to mutual cancellation of the contribution made to it by the triple or quadruple vertices. For the value $n = 3$ the scale-invariant solution of the system of the RG equations in the single-loop approximation was obtained by Gorodetskiĭ and Zaprudskiĭ[5] without taking into account the renormalization of the Green's function, a procedure which in general is inconsistent for the given problem. Nor did they ascertain whether this solution satisfies the condition that the phase transition be continuous.

 0038-5646/79/061094-07$02.40

In the present paper we show that for an analysis of the critical behavior of the Q model at $n>2$ it is necessary to consider the evolution, with temperature of the vertices of third and fourth order simultaneously, and we present a solution of the corresponding solution of Kallen–Symanzik equations (without neglecting the renormalization of the Green's function), and use them to demonstrate the possibility that the NLC–IL transition can be of second order.

We pay particular attention to those experimentally observed qualitative differences in the critical behavior, with the aid of which it is possible to establish whether the anomalies in the behavior of the thermodynamic quantities are connected with the proximity of the transition to an isolated singular point or whether they are the consequence of the possibility of a continuous NLC–IL phase transition.

1. HAMILTONIAN AND SCALE-INVARIANT PROPERTIES OF THE *Q* MODEL

The effective Hamiltonian of the Q model is

$$H=\int d\mathbf{x}\left\{\frac{1}{2}\left[r_0\,\mathrm{Sp}\,Q^2+\sum_{\alpha,\beta}(\partial_\alpha Q_{\alpha\beta})^2+h\sum_{\alpha,\beta,\gamma}(\partial_\alpha Q_{\alpha\gamma})\times(\partial_\beta Q_{\beta\gamma})\right]-\frac{g_0}{3!}\mathrm{Sp}\,Q^3+\frac{u_0}{4!}(\mathrm{Sp}\,Q^2)^2+\frac{v_0}{4!}\mathrm{Sp}\,Q^4\right\}. \quad (1)$$

Here Q is a symmetrical zero-trace $n\times n$ tensor and r_0 is a linear function of the temperature. The third term in (1) describes the contribution made to the free energy by the anisotropic fluctuations. In the subsequent calculations we shall neglect them, since usually the correlation function in the liquid phase of the NLC is close to isotropic, and the coefficient h is small; we shall only qualitatively discuss the influence of this term on the precritical properties of the system. Owing to the difference in the expansion (1) of the third-degree invariant (the cause of its appearance is the physical nonequivalence of the states $-Q_{\alpha\beta}$ and $Q_{\alpha\beta}$) the phase transition in the Q model, in accordance with the Landau theory, should be of first order, with the exception of an isolated singular point at which $g_0=0$.

However, the interaction of the fluctuations in the Q model has a unique property: at $n\leqslant 4$ it leads to an effective decrease of the triple vertex $\Gamma_3(q_1=q_2=q_3=0,\varkappa)\equiv\Gamma_3(0,\varkappa)$ in the critical region, where $\varkappa$ (the reciprocal correlation radius) decreases rapidly. In fact, the sign of the fluctuation correction to the dimensionless invariant charge

$$g_R(\varkappa)\sim Z^{3/2}\varkappa^{(d-6)/2}\Gamma_3(0,\varkappa)$$

(Z is the renormalization factor of the Green's function) at these values of n is negative, i.e., it is opposite in sign to the analogous correction in the scalar of vector models. Therefore the Gell-Mann and Low equation $g_R(\varkappa)$ without allowance for the fourth-order terms in (1) has in first-order approximation[4,6]

$$-\frac{\partial g_R}{\partial\ln\varkappa}=\frac{6-d}{2}g_R+\frac{3(n^2+6n-40)}{16n}g_R^3 \quad (2)$$

and has at $n\leqslant 4$ a nontrivial fixed point (FP)

$$g_R^{*2}=\frac{8n(d-6)}{3(n^2+6n-40)}. \quad (3)$$

This circumstance suggests that the total system of the RG equations for the Q model also admits a scale-invariant solution, in which the value of g_R differs from zero as $\varkappa\to 0$, and consequently the critical behavior of the model is not necessarily connected with the proximity to an isolated singular point.

Equation (2) and its singular solution (3) describe asymptotically exactly the scale-invariant properties of the Q model, strictly speaking, only in a $(6-\varepsilon)$-dimensional space ($\varepsilon\ll 1$) in which the fourth-order vertices are not critical and the dimensionless charge g_R^* is small, $g_R^{*2}\sim\varepsilon$. On going over to a real three-dimensional space, the solution (3) turns out to be unstable to terms of fourth order in (1). To verify this, it suffices to calculate the anomalous dimensionalities of all the operators of the type Q^4 which are needed for a complete renormalization of the model, on a non-Gaussian basis (3). Using the general calculation scheme proposed in Ref. 9, we have calculated these dimensionalities in first order in ε and obtained

$$\lambda_1=\varepsilon,\quad \lambda_2=\frac{(n-2)(n+4)\varepsilon}{3(n^2+6n-40)}+\varepsilon,$$
$$\lambda_\pm=\frac{\varepsilon}{12n}\{(4n^2+19n-132)\pm(n^4-18n^3+745n^2-696n+4624)^{1/2}\}. \quad (4)$$

The additions to the Hamiltonian of the operators of type Q^4 do not influence the critical behavior of this system, which is determined by the solution (3) in the case when the contribution of the diagrams containing the inserts of all the operators tends to zero as $\varkappa\to 0$ more rapidly than the contribution of the diagrams made up of only triple vertices. To this end it is necessary to satisfy the inequality[1)]

$$\lambda_i>\varepsilon-2. \quad (5)$$

From expressions (4) we see that at $n=2$ the function $\lambda_-(n)$ vanishes, and at $n>2$ it becomes negative,[2)] so that to study the critical behavior of the three-dimensional Q model with $n>2$ it is necessary to consider the evolution, with temperature, of the vertex parts of the operators $g_0\mathrm{Sp}Q^3$, $u_0(\mathrm{Sp}Q^2)^2$, $v_0\mathrm{Sp}Q^4$ simultaneously.

2. KALLEN-SYMANZIK EQUATION FOR THE *Q* MODEL IN THREE-DIMENSIONAL SPACE

In the theories where the effective Hamiltonian contains a triple vertex in g (for example, in the microscopic theory for the percolation problem[10]), the lowest-approximation corrections for the Green's function and for this vertex itself are proportional to g^2, so that renormalization of the Green's function must be taken into account from the very outset (in contrast to the case of the parquet approximation for theories of the $u\varphi^4$ type). It is therefore convenient to use the RG method in the Kallen-Symanzik scheme.[11]

Within the framework of this scheme, the entire information of interest to us is extracted from the renormalization constants Z, Z_g, Z_u, and Z_v, which are defined below. The constant Z connects the 1-irreducible vertex functions (without the lower ends) $\Gamma_R^{(N)}$ with the nonrenormalized functions $\Gamma^{(N)}$:

$$\Gamma_R^{(N)}(\mathbf{p}_i;\varkappa^2,g_R,u_R,v_R)=Z^{N/2}\Gamma^{(N)}(\mathbf{p}_i;r_0,g_0,u_0,v_0), \quad (6)$$

where g_R, u_R, and v_R are dimensionless invariant charges, with respect to which the expansion is carried out in the renormalized perturbation theory; they are expressed in terms of the bare charges with the aid of the constants Z_g, Z_u, Z_v $(d=3)$:

$$g_0=\varkappa^{-3/2}g_RZ_RZ^{-3/2},\quad u_0=\varkappa^{-1}u_RZ_uZ^{-2},\quad v_0=\varkappa^{-1}v_RZ_vZ^{-2}. \tag{7}$$

The constants Z_g, Z_u, Z_v, and Z as well as the renormalized mass $\varkappa$ can be determined by imposing the normalization conditions on the function $\Gamma_R^{(N)}$ at zero external momenta:

$$\Gamma_R^{(2)}(\mathbf{p}=0)=\varkappa^2,\quad \frac{\partial\Gamma_R^{(2)}(\mathbf{p}^2=0)}{\partial p^2}=1,\quad \Gamma_R^{(3)}(\mathbf{p}_i^2=0)=\varkappa^{3/2}g_R,$$
$$\Gamma_R^{(4)u}(\mathbf{p}_i^2=0)=\varkappa u_R,\quad \Gamma_R^{(4)v}(\mathbf{p}_i^2=0)=\varkappa v_R. \tag{8}$$

It is important that within the framework of perturbation theory the renormalization constants depend only on the invariant charges (there is no explicit dependence on $\varkappa$), this being a direct consequence of the renormalizability of the theory. In the single-loop approximation we obtain for these constants the expressions[3)]

$$Z=1-\frac{n^2+2n-8}{24n}g_R^2, \tag{9}$$

$$Z_g=1+\frac{n^2+3n-12}{n}v_R+4u_R-\frac{n^2+4n-24}{8n}g_R^2, \tag{10}$$

$$u_RZ_u=u_R+\frac{n^2+n+14}{3}u_R^2+\frac{2(2n^2+3n-6)}{3n}u_Rv_R$$
$$+\frac{n^2+6}{n^2}v_R^2+\frac{3(3n^2+32)}{32n^2}g_R^4-\frac{n^2+6}{n^2}g_R^2v_R-\frac{n^2+2n-16}{2n}g_R^2u_R, \tag{11}$$

$$v_RZ_v=v_R+\frac{2n^2+9n-36}{3n}v_R^2+8u_Rv_R+\frac{3(n^2+8n-64)}{32n}g_R^4$$
$$-\frac{n^2+6n-36}{2n}g_R^2v_R-4g_R^2u_R. \tag{12}$$

Substituting these values in (7) and differentiating both halves of these equations with repsect to $t=\ln\varkappa$, we obtain a system of three linear equations for the determination of the functions of Gell-Mann and Low.

Thus, we arrive at a system of renormalization-group equations describing the evolution of the invariant charges in the critical region:

$$\frac{dg_R}{dt}=\beta_g(g_R,u_R,v_R)$$
$$=-\frac{3}{2}g_R+\frac{n^2+3n-12}{n}g_Rv_R+4g_Ru_R-\frac{3(n^2+6n-40)}{16n}g_R^3, \tag{13}$$

$$\frac{du_R}{dt}=\beta_u(g_R,u_R,v_R)=-u_R+\frac{n^2+n+14}{3}u_R^2+\frac{n^2+6}{n^2}v_R^2$$
$$+\frac{2(2n^2+3n-6)}{3n}u_Rv_R+\frac{15(3n^2+32)}{32n^2}g_R^4$$
$$-\frac{3(n^2+6)}{n^2}g_R^2v_R-\frac{5n^2+10n-88}{4n}g_R^2u_R, \tag{14}$$

$$\frac{dv_R}{dt}=\beta_v(g_R,u_R,v_R)=-v_R+\frac{2n^2+9n-36}{3n}v_R^2+8u_Rv_R$$
$$-12g_R^2u_R-\frac{5n^2+34n-208}{4n}g_R^2v_R+\frac{15(n^2+8n-64)}{32n}g_R^4. \tag{15}$$

Before we proceed to discuss the properties of the solution of the system (13)–(15), let us explain briefly how to calculate the critical exponents in the Kallen–Symanzik scheme.[11,12] The constant Z is proportional to $\varkappa^\eta$ as $t\to-\infty$, we therefore have for the Fisher exponent η

$$\eta=\frac{d\ln Z}{dt}\bigg|_{t=-\infty}=\left\{\frac{\partial\ln Z}{\partial g_R}\beta_g+\frac{\partial\ln Z}{\partial u_R}\beta_u+\frac{\partial\ln Z}{\partial v_R}\beta_v\right\}, \tag{16}$$

and the expression in the curly brackets is taken at $g_R=g_R^*$, $u_R=u_R^*$, $v_R=v_R^*$, where g_R^*, u_R^*, v_R^* are the coordinates of the fixed points of the system of equations (13)–(15). From (9)–(16) it follows that in first-order approximation in the invariant charges we get

$$\eta=\frac{n^2+2n-8}{8n}g_R^{*2}. \tag{17}$$

To calculate the critical exponent of the correlation length ν we use the Ward identity[13]

$$\frac{\partial\Gamma^{(2)}(\mathbf{p}=0)}{\partial\tau}=\Gamma^{(1,2)}(\mathbf{p}_1=\mathbf{p}_2=0),\quad \tau=\frac{T-T_c}{T_c}, \tag{18}$$

which connects the derivative of the reciprocal susceptibility $\Gamma^{(2)}$ with respect to τ with the Green's function (without external ends) $\Gamma^{(1,2)}$ that contains the composite operator $\mathrm{Sp}Q^2$. The transition from $\Gamma^{(1,2)}$ to $\Gamma_R^{(1,2)}$ is effected with the aid of a new renormalization constant Z_1, which does not reduce to a combination of constants[9,12]:

$$\Gamma^{(1,2)}(\mathbf{p}_1,\mathbf{p}_2;r_0,g_0,u_0,v_0)=Z_1\Gamma_R^{(1,2)}(\mathbf{p}_1,\mathbf{p}_2;\varkappa^2,g_R,u_R,v_R). \tag{19}$$

To calculate the constant Z_1, which also depends only on the invariant charges, it is necessary to superimpose an additional condition on $\Gamma_R^{(1,2)}$, for example

$$\Gamma_R^{(1,2)}(\mathbf{p}_1=\mathbf{p}_2=0)=1, \tag{20}$$

and then we obtain in the approximation linear in g_R^2, u_R, and v_R

$$Z_1=1-\frac{n^2+n+2}{3}u_R-\frac{2n^2+3n-6}{3n}v_R+\frac{n^2+2n-8}{4n}g_R^2. \tag{21}$$

In the critical region $Z_1\sim\varkappa^{2-\eta-1/\nu}$, whence

$$2-\eta-\nu^{-1}=\frac{\partial\ln Z_1}{\partial t}\bigg|_{t=-\infty}=\left\{\frac{\partial\ln Z_1}{\partial g_R}\beta_g+\frac{\partial\ln Z_1}{\partial u_R}\beta_u+\frac{\partial\ln Z_1}{\partial v_R}\beta_v\right\}$$
$$=\frac{n^2+n+2}{3}u_R^*+\frac{2n^2+3n-6}{3n}v_R^*-\frac{3(n^2+2n-8)}{4n}g_R^{*2}. \tag{22}$$

The expression in the curly bracket is calculated at $g_R=g_R^*$, $u_R=u_R^*$, $v_R=v_R^*$.

An investigation of the system of the renormalization group equations (13)–(15) becomes much simpler at $n=2$ or 3, for in this case there is only one fourth-order invariant [inasmuch as $2\mathrm{Sp}S^4=(\mathrm{Sp}Q^2)^2$]. The case $n=2$ is trivial: the Q model degenerates into the XY model. At $n=3$ the RG equation for the single isotropic vertex of the fourth order is obtained by multiplying (15) by 1/2, adding it to (14), and replacing $(u_R+v_R/2)$ by u_R. By a similar substitution in (13), we obtain a system of two coupled equations:

$$dg_R/dt=-{}^3/_2g_R+4g_Ru_R+{}^{13}/_{16}g_R^3,$$
$$du_R/dt=-u_R+{}^{26}/_3u_R^2$$
$$-{}^{59}/_{12}g_R^2u_R+{}^{125}/_{192}g_R^4. \tag{23}$$

The system (23) has, besides the unstable Gaussian fixed point, also three nontrivial fixed points:

$$O(5):\ g_R^{*2}=0,\quad u_R^*={}^3/_{26};$$
$$A_+:\ g_R^{*2}=1.09;\quad u_R^*=0.15;$$
$$A_-:\ g_R^{*2}=0.39,\quad u_R^*=0.3. \tag{24}$$

The first of them is a saddle and corresponds to the fixed point of a five-component Heisenberg model, which

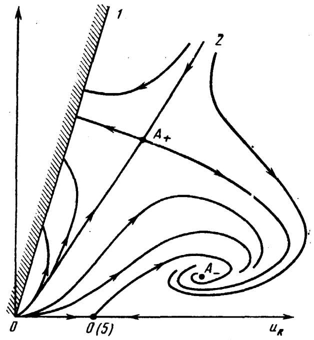

FIG. 1. Phase diagram of system of renormalization group equations (23). The shaded region is that of the instability of the isotropic phase.

is isomorphic to the Q model at $n=3$ and $g_0=0$. This fixed point describes the critical behavior of a nematic liquid crystal near an isolated singular point. Linearizing Eqs. (23) with respect to g_R we obtain for $\Gamma_R^{(3)}(0, \varkappa)$ in the vicinity of the isolated singular point the following solution[2,3]:

$$\Gamma_R^{(3)}(0,\varkappa)\sim(\varkappa)^{\Delta_3},\quad \Gamma_R^{(4)}(0,\varkappa)\sim\varkappa,\quad \Delta_3=6/13. \tag{25}$$

In a number of papers[14-16] the Padé-Borel method (matching of the first seven terms of the expansions in powers of the charge u_R with the asymptotic estimates of the higher orders of the perturbation-theory series) was used to obtain the critical exponents for an n-component three-dimensional Heisenberg model at $n=0$, 1, 2, 3, which agreed with the most reliable values obtained by numerical methods for lattice systems. Although the case $n=5$ was not considered, but since the susceptibility exponent $\gamma(n)$ increases monotonically with increasing n, one should expect

$$\gamma(n=5)>\gamma(n=3)\approx1.39, \tag{26}$$

i.e., near the isolated singular point the susceptibility should have a rather complicated nonlinearity as a function of the temperature or pressure.

Expressions (25) are solutions of the system (23) only at small g_R, namely $g_R^2 \ll u_R$ or $[\Gamma_R^{(3)}(0,\varkappa)]^2 \ll \Gamma_R^{(4)}(0,\varkappa)\varkappa^2$. The qualitative behavior of the phase trajectories at finite values of g_0 is shown in the figure. The fixed points A_+ and A_- are respectively a saddle and a stable focus. They are inside a region that is stable to precipitation of a condensate. In the first approximation in the renormalization group, the boundary of this region, the straight line 1–0 is given by the equation[6,17] $g_R^2 < 9u_R$. The phase trajectory 2–0, which passes through the fixed point A_+, divides the phase plane into two parts: the trajectories from the region 1–0–2 go over to the line 1–0, on which a first-order transition takes place (in our approximation), and in the region 2–0–u_R they tend to the fixed point A_-, and the phase transition is of second order. Substituting the values of coordinates (g_R^*, u_R^*) of the fixed points in (17) and (22) we obtain the values of the critical exponents

$$\eta=0.11,\quad \nu=0.85,\quad \gamma=(2-\eta)\nu=1.6. \tag{27}$$

The remaining exponents can be calculated from the ordinary scaling relations. The values for ν and γ are much higher than obtained from measurements near the NLC–IL transition; $\nu=0.57$ and $\gamma\approx1.0$.[8,18] We note, however, that the contributions from the triple and quadruple vertices to the exponent γ are of opposite sign and are quite large in absolute magnitude, 0.4 and 0.8, respectively.[4)] It can therefore be assumed that in the higher orders of renormalized perturbation theory almost complete cancellation of the contributions from both vertices can occur, as the result of which we get $\nu\approx1/2$ and $\gamma\approx1.0$.

Equations similar to our equations at $n=3$ (23) were derived by another method without allowance for the renormalization of the Green's function (i.e., under the assumption $\eta=0$) by Gorodetskiĭ and Zaprudskiĭ.[5] These equations also had fixed points of the type A_+ and A_-. However, the qualitative phase-trajectory picture shown in[5] is incorrect: the regions of values of u_R for which the phase transition is of first order at arbitrarily small value of g_R does not exist in reality. The phase transition is of first order for coupling constants in the region 1–0–2 (see the figure), and the transition itself into the ordered phase (in first-order transition in the renormalization group) takes place on the 1–0 line and not on the $u_R=0$ line as indicated in Ref. 5. It can be shown that allowance for the anisotropy of the correlation function [see (1)] leads to a certain increase of the region 1–0–2.

To determine the critical properties of the Q model at $n>3$ it is necessary to find the phase transitions of the system (13)–(15). We have verified that $n=4$ and 5, and also as $n\to\infty$, this system does not have real stable fixed points with $g_R^*\neq0$. It is natural to assume that there are no such stable fixed points also at arbitrary values $n<5$. Since it is known that the fixed points with $g_R^*=0$ are unstable for all $n>3$, it follows that the Q model phase transitions of only first order are possible at $n>3$, and the continuous phase transition is possible only at $n=3$, corresponding to the model of the NLC–IL transition. The presence of a stable fixed point A_- for this case is apparently due to the special symmetry properties of the Q model at $n=3$, which lead to degeneracy of the system (13)–(15) into the system (23) and transform the space of the renormalization group variables into a two-dimensional space.

In the next section we consider the experimental results that point to the need for taking into account the fluctuations near the NLC–IL transition, and also discuss the experiments with which it is possible to determine the type of the scale-invariant solution realized by this transition in the region of strongly developed fluctuations.

3. CHARACTER OF THE PRE-TRANSITION ANOMALIES IN NLC

In recent years the singularities of phase transitions into the liquid-crystal state, including the properties of the NLC–IL transition, have been intensively investigated experimentally. It is well known that in the immediate vicinity of this transition there are strong temperature anomalies: a strong increase of the birefringence in electric and magnetic fields, a strong increase of the intensity of the Rayleigh scattering of light, etc.[20] From experiments on light scattering in the pre-transition region of the liquid phase it was found that the correlation length is $\xi \sim \tau^{-\nu}$, where $\nu = 0.5$ and $\Delta T = T_c - T_p$ ~ 1 K (T_p is the transition temperature).[7] More accurate measurements performed in Ref. 8 have revealed a weak anisotropy of the correlation function (characterized by an approximate difference of 15% between the two correlation lengths ξ_n and ξ_t in the case of MBAA), and yielded a critical-exponent value $\nu = 0.57$. The coefficient of the magnetic birefringence behaves like $\tau^{-\gamma}$, where the values of the exponent γ, which plays the role of the susceptibility exponent, turned out to be close to 1.0.[18] It was established that the coefficient of thermal expansion

$$\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p \sim |\tau|^{-0,6},$$

also has singularities near the transition (on both sides), and the value obtained for ΔT was smaller by one or two orders of magnitude (several hundredths of a degree) than obtained from optical experiments.[21] The results of calorimetric measurements of the temperature dependence of the heat capacity, obtained in various studies, differ strongly (apparently because of the influence of the impurities), it can nevertheless be concluded from them that near the transition there appears a single increment to the specific heat $\Delta C_{\text{sing}} \sim |\tau|^{-\alpha}$ and the exponent α is large enough, $\alpha \gtrsim 0.3$.[22]

The thermodynamic properties of the NLC–IL phase transition are customarily described with the aid of an expansion of the free energy in powers of the order parameter $Q_{\alpha\beta}$,[20] which is a particular case of the phenomenological Landau theory

$$\Phi - \Phi_0 = {}^1/_2 A(T)\,\mathrm{Sp}\,Q^2 - {}^1/_3 B\,\mathrm{Sp}\,Q^3 + {}^1/_4 C(\mathrm{Sp}\,Q^2)^2 + \ldots \qquad (28)$$

Within the framework of this approach, the anomalous growth of the birefringence, the strong scattering of light in the isotropic phase near T_p, other similar effects can be connected with the strong temperature dependence of the coefficient $A(T) \sim (T - T_c)^\gamma$, and the small values of the jump of the heat capacity and of the heat of transition can be connected with the smallness of the coefficient B. The results that follow from the phenomenological theory are in fair agreement with most experimental data. However, more accurate measurements of the temperature dependence of the heat capacity,[22] of the correlation length,[8] and of the coefficient of thermal expansion[21] show that the fluctuations of the order parameter near the transition are not small and the Landau expansion is insufficient for a quantitative description of the pre-transition phenomena.

From the solution of the renormalization group system of equations (23) it follows that the critical anomalies can be connected either with the presence of the fixed point $O(5)$, which described the phase transition near the isolated singular point, or with the stable fixed point A_-, the existence of which is due to the specific character of the interaction of the fluctuations in the nematic liquid crystal. In either case the theory predicts a rather strong nonlinear susceptibility ($\gamma > 1.39$ or $\gamma = 1.6$), and as a consequence, the absence of a divergence of the heat capacity ($\alpha < 0$), although its anomalous growth is observed experimentally.[22] We note, however, that for the solution connected with the phase transition A_-, when account is taken of the next higher orders of the renormalized perturbation theory, a cancellation can take place of the contributions of the vertices g_R^* and u_R^* in the exponent γ, in which case α becomes larger than zero and there will be no disparity with experiment, whereas for the Heisenberg fixed point $O(5)$ the estimate (26) is valid and the value of α remains less than zero. The presence of a scale-invariant solution, in which, on account of the interaction of the fluctuations, the triple vertex decreases rapidly as the transition point is approached $\Gamma_R^{(3)}(0, \varkappa) \sim \varkappa^{3/2}$), has made it possible to explain in a unified manner the nearly-continuous character of the NLC–IL transitions in various substances. In fact, the fluctuation region in this case is wider ($\tau_B \lesssim B^{4/3}$) than the critical region near the isolated singular point, determined in accordance with the usual Ginzburg–Levanyuk criterion ($\tau_c \lesssim C^2$), and τ_B can be several times larger than τ_c at $B \sim C < 1$. Therefore, although the NLC–IL transition is in fact of first order, when an attempt is made to describe it with the aid of the expansion (28) it is necessary to assign to the coefficient B the value of the rapidly decreasing function $\Gamma_R^{(3)}(0, \varkappa)$, averaged over the measured temperature interval which is much less than the value of the function $B(T)$ outside the fluctuation region.[5)] At the same time, if it is assumed that the NLC–IL transition is close to an isolated singular point, then it becomes necessary to regard the systematic smallness of B for all NLC to be accidental and it is difficult to indicate for it any common cause.[6)] We therefore assume that the pre-transition critical anomalies are more readily described by a solution of the renormalization equations that is connected with the stable fixed point A_- rather than the Heisenberg fixed point.

How can experiment show which of the two scale invariant solutions is responsible for these anomalies? Measurements of any critical exponent by traditional procedures are hardly suitable for this purpose, for in either case the theory yields for it close values (not too reliable, although the inequality (26) should be satisfied), and furthermore the accuracy of the measurements in modern experiments is not high enough.[7)] It is therefore natural to attempt to use those experimentally observed consequences which depend on the type of the scaling solution not quantitatively but qualitatively. It seems to us that it is possible to establish the character of the critical behavior in the NLC–IL transition from experiments on the scattering of electromagnetic waves in the nematic phase near the transition point.

Pokrovskiĭ and Katz[24] and Stratonovich[25] have shown that the expression for the differential scattering by fluctuations in a frequency interval $d\omega$ and in a solid-angle interval $d\Omega$ can be represented as a sum of three parts: uniaxial transverse, longitudinal, and biaxial transverse:

$$\frac{d\sigma}{d\omega\,d\Omega} = \frac{\omega^4}{32\pi^3}\langle\delta\varepsilon_{\alpha\beta}\delta\varepsilon_{\gamma\sigma}\rangle e_\alpha e_\beta e_\gamma' e_\sigma' = \frac{\omega^4}{32\pi^3}[a_1 B^{(1)}_{\alpha\beta\gamma\sigma} + a_2 B^{(2)}_{\alpha\beta\gamma\sigma} + a_3 B^{(3)}_{\alpha\beta\gamma\sigma}] e_\alpha e_\beta e_\gamma' e_\sigma', \qquad (29)$$

where **e** and **e′** are the polarization vectors of the incident and scattered waves, and the quantities a_1, a_2, and

a_3 characterize the contribution made to the cross section by the scattering from biaxial, longitudinal, and transverse uniaxial fluctuations, respectively. The matricess $B^{(i)}_{\alpha\beta\gamma\delta}$ are projectors on the mutually orthogonal subspaces

$$B^{(i)}_{\alpha\beta\gamma\sigma}B^{(k)}_{\gamma\sigma\mu\nu}=\delta_{ik}B^{(i)}_{\alpha\beta\mu\nu}$$

(the explicit form of $B^{(i)}_{\alpha\beta\gamma\delta}$ is given in Ref. 25). Their convolutions with the polarization vectors are given by[25]

$$\begin{aligned} B^{(1)}_{\alpha\beta\gamma\sigma}e_\alpha e_\beta e_\gamma' e_\sigma' &= {}^1/_2+{}^1/_2(\mathbf{en}^0)^2(\mathbf{e'n}^0)^2-{}^1/_2(\mathbf{en}^0)^2-{}^1/_2(\mathbf{e'n}^0)^2, \\ B^{(2)}_{\alpha\beta\gamma\sigma}e_\alpha e_\beta e_\gamma' e_\sigma' &= {}^3/_2(\mathbf{en}^0)^2(\mathbf{e'n}^0)^2-(\mathbf{en}^0)(\mathbf{e'n}^0)(\mathbf{ee'})+{}^1/_6(\mathbf{ee'}), \\ B^{(3)}_{\alpha\beta\gamma\sigma}e_\alpha e_\beta e_\gamma' e_\sigma' &= (\mathbf{en}^0)(\mathbf{e'n}^0)(\mathbf{ee'})+{}^1/_2(\mathbf{en}^0)^2+{}^1/_2(\mathbf{e'n}^0)^2-2(\mathbf{en}^0)^2(\mathbf{e'n}^0)^2, \end{aligned} \quad (30)$$

where $\mathbf{n}^0$ is the director vector.

The anomalously large intensity of the scattered light in the NLC is due primarily to scattering by transverse uniaxial fluctuations (rotations of the director), which have no gaps because of the degeneracy of the state of the system with respect to the direction of the director $\mathbf{n}^0$. The contribution of the longitudinal fluctuations is also anomalously large for small wave vectors $\mathbf{q}$, as follows from the modulus-conservation principle, but its singularity is much weaker. Scattering by biaxial fluctuations become noticable only in the region of sufficiently short waves, since the barrier $\Delta \sim B^2/C$ must be surmounted in order to excite them. If, as above, we disregard the anistropy of the Green's function,[8] then the dependences of the contributions of a_i on $\mathbf{q}$ take the form[24]

$$a_1\sim 1/(Kq^2+\Delta), \quad a_2\sim 1/q, \quad a_3\sim 1/Kq^2, \quad (31)$$

where K is a constant connected with the Frank modulus.

From the microscopic point of view in the critical region the functions $B(T)$ and $C(T)$ are 1-irreducible vertex parts $\Gamma_3(0)$ and $\Gamma_4(0)$ at zero momenta for the effective Hamiltonian (1). If it is assumed that the NLC–IL transition takes place near an isolated singular point, then the gap Δ in the spectrum of the biaxial fluctuations should not change in practice with temperature [see (25)]:

$$\Delta \sim \frac{[\Gamma^{(3)}(0,\varkappa)]^2}{\Gamma^{(4)}(0,\varkappa)} \sim \frac{(\varkappa)^{2\Delta_3}}{\varkappa} \sim \varkappa^{-1/13} \sim \tau^{-1/26}. \quad (32)$$

On the other hand, if the near continuous character the NLC–IL transition is due to the specific character of the interaction of the fluctuations, then a scale-invariant solution is realized, connected with the fixed point A_-, we obtain and for the gap Δ

$$\Delta \sim \frac{[\Gamma^{(3)}(0,\varkappa)]^2}{\Gamma^{(4)}(0,\varkappa)} \sim \frac{\varkappa^{3-2\eta}}{\varkappa^{1-2\eta}} \sim \varkappa^{2-\eta} \sim \tau^{\gamma}. \quad (33)$$

We see that in this case Δ decreases rapidly (at a rate equal to the reciprocal susceptibility) when the transition point is approached. This sharp difference in the qualitative behavior of the gap Δ can be used to ascertain which of the two possible scale-invariant solutions are realized in the NLC–IL transition.

As shown in Refs. 24 and 25, when certain conditions are satisfied for the polarization, the transverse uniaxial fluctuations do not lead to scattering of light. In particular, there is no scattering if the vectors $\mathbf{e}$ and $\mathbf{e}'$ are perpendicular to the direction of the director $\mathbf{n}^0$ [see (30)]. A contribution to the scattering will then be made only by the longitudinal and biaxial fluctuations and, as seen from (30), these contributions depend differently on the angle between $\mathbf{e}$ and $\mathbf{e}'$. If this angle is chosen close to 90°, then the longitudinal scattering is suppressed and only biaxial scattering remains. By measuring the corresponding scattering cross sections as a function of the temperature, we can establish the $\Delta(T)$ dependence. Generally speaking, the condition $\mathbf{e} \perp \mathbf{e}'$ is not obligatory for this purpose. Measurements can be performed at different values of $\mathbf{q}$ (i.e., by varying the scattering angle), and the contribution to the cross section from the biaxial fluctuations can be separated by using the difference between the angular dependences of the longitudinal and biaxial scattering.

We emphasize that the main purpose of such an experiment is not so much a quantitative measurements as an observation of a qualitative effect: in one case (isolated singular point) Δ is practically independent of temperature, and in the other (critical behavior) this dependence should be very strong (33). It is important that the last conclusion follows already from the very existence of a fixed point of the type A_-, and is connected with the possible numerical inaccuracies of the RG method in the single-loop approximation when it comes to determining the exact position of this fixed point and of the critical exponents corresponding to it.

In conclusion, we discuss briefly the influence exerted on the properties of the NLC–IL transition by the impurities contained in the sample. Assuming that the correlation radius of the impurities is small near the transition compared with the correlation length ξ, we have derived the renormalization-group equations with account taken of the indirect interaction of the fluctuations via the randomly disposed nonequilibrium impurities. It turned out that the fixed point A_- remains stable as before and consequently the critical behavior is isomorphic to the behavior of the pure system. The situation can change, however, if the impurity produces near itself an effective ordering of the molecules.

The authors are grateful to A. G. Aronov, S. L. Ginzburg, and A. I. Sokolov for useful discussions, and to S. A. Brazovskiĭ, P. B. Vigman, E. I. Kats, A. I. Larkin, S. V. Maleev, S. A. Pikin, and V. M. Filev for a discussion of the results.

[1)] Our definition of the anomalous dimensionalities λ_i differs in sign from that used in Ref. 9.

[2)] The conclusion that the scale-invariant properties of the Q model at $n>2$ and of its analytic continuation to $n<2$ are substantially different is obviously not connected with perturbation theory in $\varepsilon = 6-d$, since the equation $\lambda_-(n=2)=0$ is the consequence of the degeneracy of the Q model into the XY model at $n=2$.

[3)] In Eqs. (13)–(15) we have changed over for the sake of convenience from the renormalized charges (8) to the variables $g_R \to g_R/\Delta$, $u_R \to u_R/\Delta$, v_R/Δ, where $\Delta = 1/32\pi$.

[4)] In the Ising field model the analogous contribution is equal to 0.2.[19]

[5)] We recall that at the transition point we have $\tau^* = (T_c - T_p)/T_p \sim 10^{-3} \div 10^{-4}$.

[6)] It was suggested in Ref. 22 that NCL–IL transition is close to the tricritical point. In this case, however, it must be assumed that both B and C are "accidentally" systematically small compared with the coefficient of Q^6.

7) We note that Filev[23] has proposed a method of measuring the exponent γ by an optical procedure, with which it is possible to distinguish between the behavior near the isolated point and the tricritical point.

8) This can be done by using the formulas obtained by Pokrovskiĭ and Kats,[24] but we are trying to avoid additional complications which are of no fundamental significance for us.

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Translated by J. G. Adashko

μ^+-meson spin relaxation in rare earth metals at various temperatures

V. G. Grebinnik, I. I. Gurevich, V. A. Zhukov, V. A. Nikol'skiĭ, V. I. Selivanov, and V. A. Suetin

I. V. Kurchatov Institute of Atomic Energy

(Submitted 29 December 1978)
Zh. Eksp. Teor. Fiz. **76**, 2178–2184 (June 1979)

We measured the temperature dependences of the rates of relaxation of the μ^+-meson spin in Pr, Nd, Sm, Eu, Tb, Dy, Ho, and Er, at $T = 5$–300 K. We demonstrate the possibility of identifying antiferromagnetic phase transitions and of measuring the Néel temperature T_N by the μ^+-meson method. The value of T_N of praseodymium measured by the μ^+ method was found to be 6 K. A method is proposed for measuring the frequency of the oscillations of the electron spin of the atoms of a metal in the paramagnetic state.

PACS numbers: 75.30.Kz, 75.50.Cc, 75.50.Ee, 76.90.+d

The spin of a μ^+ meson in a metal relaxes because of the interaction of the magnetic moment of the μ^+ meson with the magnetic moments of the surrounding electrons and nuclei. A study of these interactions is of interest both for the investigation of the properties of a singly charged impurity particle in a metal and for the investigation of the properties of the metal itself. An example of a study of ferromagnets (iron, nickel, cobalt and gadolinium) with the aid of μ^+ mesons is Ref. 1.

In this paper we study the relaxation of the μ^+-meson spin in rare-earth metals. The metals of this group, depending on the temperature, can be in a paramagnetic, antiferromagnetic, and ferromagnetic state. We used polycrystalline samples of metals with less than 0.2% impurities. The relaxation rate Λ of the spin of the μ^+

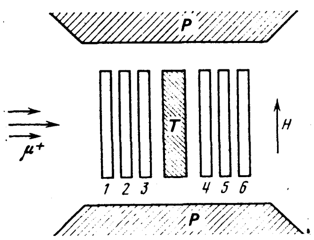

FIG. 1. Experimental setup: T—target, P—poles of electromagnet, 1-6—scintillation counters.