[3]A. D. Buckingham and M. Pariseau, Trans. Faraday Soc. 62, 1 (1966).
[4]L. L. Boyle, A. D. Buckingham, R. L. Disch, and D. A. Dunmur, J. Chem. Phys. 45, 1318 (1966).
[5]A. D. Buckingham and B. J. Orr, Trans. Faraday Soc. 65, 673 (1969).
[6]A. M. Kung, J. F. Young, and S. F. Harris, Appl. Phys. Lett. 22, 301 (1973).
[7]A. Owyoung and R. W. Hellwarth, and M. George, Phys. Rev. A4, 2243 (1971).
[8]J. J. Song and M. D. Levenson, Appl. Phys. 48, 3496 (1977).
[9]A. Owyoung, IEEE, QE-9, 1064 (1973).
[10]A. Gerard and J. M. Birch, Introduction to Matrix Optics (Russ. transl.), Mir, 1978.
[11]D. V. Vlasov, V. V. Korobkin, R. V. Serov, Kvantovaya elektron. (Moscow) 5, 2457 (1978) [Sov. J. Quant. Electron. 8, 1380 (1978).
[12]J. F. Ward and G. H. C. New, Phys. Rev. 185, 57 (1959).
[13]D. V. Vlasov and V. V. Korobkin, and R. V. Serov, Kvantovaya elektron. (Moscow) 6, in press (1979) [Sov. J. Quantum Electron. 9, in press (1979).

Translated by J. G. Adashko

High pressure plasma pinch under conditions of ion "free flight" conditions

Yu. R. Alanakyan

All-Union Research Institute of Physicotechnical and Radiotechnical Measurements
(Submitted 1 November 1978)
Zh. Eksp. Teor. Fiz. 76, 2046–2051 (June 1978)

A plasma pinch in dynamic equilibrium with a surrounding dense gas is considered under conditions when the ion mean free path in the hot region of the pinch is much longer than the characteristic dimensions of the pinch. An ion flux that hinders the penetration of the neutral particles into this region is produced on the boundary of the hot region of the pinch so that in the ion "free flight" regime the energy losses of the electrons turn out to be lower than under the conditions of the diffusion regime. The structure of the plasma pinch is investigated.

PACS numbers: 52.55.Ez

1. High-power high-frequency electromagnetic radiation can produce in a dense gas a plasma pinch in which the electron temperature is several orders of magnitude higher than the temperature of the gas surrounding the pinch. This interesting phenomenon was observed by Kapitza.[1] He obtained and investigated a pinch in gas at atmospheric pressure at a power input ≈20 kW. The pinch thickness was several millimeters, the length several centimeters, and the electron temperature $T_e \approx 3 \cdot 10^5 - 10^6$ K.

The electrons are heated in the pinch under conditions of the anomalous skin effect, and the containment of the high-temperature electrons is due to the presence in the boundary region of the pinch of a constant electric field that is the result of the separation of the charges. In this boundary region, called in Ref. 1 a double layer, the densities of both the hot electrons and of the neutral particles are high. The structure of the double layer influences strongly the energy lost by the hot electrons. If it is assumed[1] that the thickness of the double layer is of the order of the pinch diameter, then the power of the RF field is by far not enough to compensate for the hot-electron energy loss due to elastic collisions with the neutral particles and to the collisions with the cold electrons, which result from impact ionization. The structure of the pinch was investigated in Refs. 2 and 3 on the basis of the balance equations for the number of particles under conditions of the diffusion regime, when the mean free path of the ions between the collisions with the neutral particles is small compared with the characteristic dimensions of the pinch. In this case the thickness of the double layer turns out to be smaller by two or three orders of magnitude than the dimension of the pinch. The inelastic energy lost by the hot electron makes in this case a negligible contribution to the energy balance of the system. However, the energy lost by the hot electrons in collisions with the cold ones is nevertheless large: the power necessary for the pinch to exist in the diffusion regime is 10–30 times larger than the experimental value of the RF field power fed to the pinch.

The present paper deals with a plasma pinch under conditions when the ion mean free path in the hot region of the pinch exceeds the characteristic dimensions of the pinch. It appears that this is precisely the case realized in Kapitza's experiment,[1] since the regime of the ion "free flight" is energywise favored over the diffusion regime. The point is that under ion free flight conditions the neutral particle density in the entire pinch region where hot electrons are present is much less than the density of the gas far from the pinch. The reason is that the ions accelerated by the constant electric field produce on the boundary of the pinch a convection pressure that balances the action of the neutral gas surrounding the pinch. Beyond the limits of the hot region of the pinch the ions lose velocity as they collide with the neutral particles, the convection pressure of the ions decreases, the partial pressure of the neutral particles increases. Thus, the partial pressure of the hot electrons on the pinch boundary is transformed into convection pressure of the ions, which prevents the neutral particles from penetrating

 0038-5646/79/061036-04$02.40

into the region of the pinch where the hot electrons are present. The number of neutral particles entering the pinch is just sufficient to compensate for the drift of the ions and electrons from the pinch.

We note that electrons of energy exceeding the potential barrier produced by the constant electric field are not retained in the pinch. Therefore under conditions when the mean free path of the hot electron between the collisions with the particles exceeds the dimensions of the pinch we have for the hot electron a truncated energy distribution function. The electrons that leave the hot region of the pinch are those which have acquired an excess energy by interacting with the RF field or with other electrons and could overcome the potential barrier. Since these interactions are practically continuous, we have cooled electrons behind the potential barrier. In the present paper, just as in Ref. 2, the hot-electron velocity distribution function is assumed to be Maxwellian with a truncated tail. This is valid if the partition of the energy among the electron is more intense than the heating of the electrons in the RF field. In the opposite case, when this conditions is not satisfied, the kinetic equation with account taken of the interaction of the electrons with the RF field and with the constant electric field and the electron collisions with the heavy particles must be taken into account in order to determine the electron distribution function. This problem for a diffusion-regime pinch was investigated in Ref. 3.

2. Considering the one-dimensional problem, we write down the kinetic equations for the ions, for the hot electrons, and for the neutral particles in the form

$$v_z\frac{\partial f_i}{\partial z}-e\frac{\nabla_z\varphi}{M}\frac{\partial f_i}{\partial v_z}=I_{in}+\beta n_e F, \quad (1)$$

$$v_z\frac{\partial f_e}{\partial z}+e\frac{\nabla_z\varphi}{m}\frac{\partial f_e}{\partial v_z}=0, \quad (2)$$

$$v_z\frac{\partial F}{\partial z}=I_{ni}-\beta n_e F, \quad (3)$$

where I_{in} and I_{ni} are the collision electrons of the ions with the neutral particles and of the neutral particles with the ions, β is a coefficient that characterizes the impact-ionization frequency, e is the absolute value of the electron charge. The potential of the constant self-consistent electric field is determined by the Poisson equation:

$$d^2\varphi/dz^2=-4\pi e(n_i-n_e). \quad (4)$$

We note that in this analysis we neglect the recombination of the hot electrons with the ions, since this process makes a negligible contribution to the particle-number balance. Intense recombination of the plasma takes place outside the boundaries of the hot region of the pinch.

If we add the first-moment equations corresponding to the kinetic equations (1)–(3) and take the Poisson equation (4) into account, we get

$$P_e+P_i+P+P_E=R=\text{const}, \quad (5)$$

where P_e is the partial pressure of the hot electron, $P_i=\int d\mathbf{v}Mv_z^2f_i$ is the convective pressure of the ions, P is the partial pressure of the neutral particles, $P_E=(\nabla\varphi)^2/8\pi$ is the pressure of the constant electric field, and R is the pressure of the neutral gas far from the pinch.

The plasma pinch consists of several regions with different structures. In the central region of the pinch we have a hot quasineutral plasma. In our analysis this is the region between the places $z=\pm a$. The partial pressure of the electrons makes here the main contribution to the total pressure of the system, and near the boundary of this region the convective pressure of the ions also becomes significant. Located beyond this is the region $a<|z|<b$, whose thickness is of the order of the Debye radius and in which the charge separation is large. The most important in this region is the convective pressure of the ions, and in contrast to the other region, a sizable contribution can be made here by the pressure P_E of the constant electric field. Beyond the limits of the hot region ($|z|>b$) the convective pressure of the ions decreases with increasing $|z|$, and the pressure of the neutral particles increases and tends to R. This takes place at a distance equal to several mean free paths of the ion in the gas of neutral particles with pressure R. The thickness of the layer of the low-temperature weakly ionized plasma, which is formed beyond the limits of the hot pinch, depends on the diffusion coefficient and on the recombination rate of the cold plasma.

3. We consider now the hot plasma region $|z|<b$. It is now necessary to neglect in Eqs. (1) and (3) the collisions between the heavy particles; this corresponds to the condition of "free flight" of the ions. With the aid of Eq. (3) we obtain the neutral-particle distribution function:

$$F^-=F^-(z=b)\exp\left(\frac{\beta}{v_z}\int_z^b n_e\,dz\right),\quad F^+=F^+(z=-b)\exp\left(-\frac{\beta}{v_z}\int_{-b}^z n_e dz\right). \quad (6)$$

Here and below functions of velocities directed in the positive and negative z directions are marked by the superscripts + and −, respectively.

Assuming that the probability of ion production outside the considered pinch region is negligibly small, we have for the ion distribution function the following boundary condition:

$$f^-_i(z=b)=f^+_i(z=-b)=0. \quad (7)$$

The kinetic equation (1) in the region $0<z<b$ admits of the following solution, which satisfies the boundary condition (7):

$$f_i^+=\beta\left[\int_{z_0}^{z}\frac{n_e(z')F^+(u,z')}{v_z(u,z')}dz'+\int_{z_0}^{b}\frac{n_e(z')F^-(u,z')}{v_z(u,z')}dz'\right],$$

$$f_i^-=\beta\int_z^b\frac{n_e(z')F^-(u,z')}{v_z(u,z')}dz'. \quad (8)$$

We have replaced here v_z by the variable $u=(Mv_z^2/2)+e\varphi(z)$. The quantity z_0 is the root of the equation $v_z(u,z)\equiv[2(u-e\varphi)(z)/M]^{1/2}=0$ at $u<0$ and $z_0=0$ at $u>0$. Symmetry considerations allow us to write down the relation

$$f^+_i(v_z,z)=f^-_i(-v_z,-z),$$

with the aid of which, using (8), we can obtain the solution in the region $-b<z<0$.

We integrate the ion distribution function over velocity space. After some transformations and calculations (change of the order of the integration, use of the saddle-point method, neglect of terms $\approx T/T_e$) we get

$$n_i=\frac{1}{2\sqrt{6}}n_b\left(\frac{M}{e}\right)^{1/2}\beta\int_0^z n_e(z')\exp\left[-\frac{3}{2}\left(\frac{M\beta^2}{T}\right)^{1/3}\times\left(\int_{z'}^{b}n_e(z'')dz''\right)^{2/3}\right][\varphi(z')-\varphi(z)]^{-1/2}dz', \tag{9}$$

where n_b is the neutral-particle density at the point $z=b$.

Under the considered conditions, when the distribution function of the hot electron velocity is assumed to be Maxwellian with a truncated tail, the spatial distribution of the density of the hot electrons is described by the formula

$$n_e=n_{e0}\left[\exp\left(\frac{e\varphi}{T_e}\right)-\exp\left(\frac{e\varphi_b}{T_e}\right)\right]\Big/\left[1-\exp\left(\frac{e\varphi_b}{T_e}\right)\right], \tag{10}$$

where $n_{e0}=n_e(z=0)$, $\varphi_b=\varphi(z=b)$, $\varphi(z=0)=0$.

In the region $|z|<a$ in the Poisson equation (4) we can neglect the left-hand side—the plasma is quasineutral in this region. The electron distribution function does not differ significantly here from a Boltzmann distribution, inasmuch as the electric-field potential at $|z|<a$ has an absolute value much lower than $|\varphi_b|$. Taking this into account, we obtain with the aid of (9) and (10) the following relation that determines the dependence of the electric-field potential on the spatial coordinate:

$$\exp(-\eta)=\int_0^\zeta\exp\left\{-\eta(\zeta')-\left[\alpha\int_{\zeta'}^{\zeta(b)}\exp(-\eta(\zeta''))d\zeta''\right]^{2/3}\right\}\times[\eta(\zeta)-\eta(\zeta')]^{-1/2}d\zeta'. \tag{11}$$

We have introduced here the dimensionless quantities

$$\eta=-\frac{e\varphi}{T_e},\quad \zeta=\frac{1}{2\sqrt{6}}\beta n_b\left(\frac{M}{T_e}\right)^{1/2}|z|,\quad \alpha=9\frac{n_{e0}}{n_b}\left(\frac{T_e}{T}\right)^{1/2}.$$

We note that Eq. (11) coincides at $\alpha=0$ with the equation obtained by Tonks and Langmuir[4] for a weakly ionized low-pressure gas-discharge plasma, when the ion "free flight" conditions are satisfied. In this case the density of the neutral particles is homogeneous in space.

Equation (11) reduces to an Abel integral equation, from which we get

$$\frac{d\eta}{d\zeta}=-\pi\left[\eta^{-1/2}e^{\eta}-2\int_0^{\eta^{1/2}}e^{y^2}dy\right]^{-1}\exp\left\{-\left[\alpha\int_\zeta^{\zeta(b)}e^{-\eta(\zeta')}d\zeta'\right]^{2/3}\right\}. \tag{12}$$

Putting $d\eta/d\zeta\to\infty$, we obtain the value of the potential at the point $|z|=a$. This quantity is independent of α; we have $\eta\approx0.85$. We shall not investigate in detail the function $\eta(\zeta)$. We note only that in contrast to the case $\alpha=0$, where the dependence of the potential on the coordinate has a smooth character (a plot of the solution Eq. (12) with $\alpha=0$ is given in Ref. 5), under conditions of the plasma pinch, when $\alpha\zeta(a)\gg1$, the spatial distribution of the potential of the constant electric field is close in form to a rectangular potential well.

The ion flux density

$$j_i=\int dv\int_0^z dz\,\beta n_eF=n_{e0}\left(\frac{2T_e}{M}\right)^{1/2}\int_0^\zeta\exp\left\{-\eta(\zeta')-\left[\alpha\int_{\zeta'}^{\zeta(b)}e^{-\eta(\zeta'')}d\zeta''\right]^{2/3}\right\}d\zeta' \tag{13}$$

at the point $z=a$ can be calculated in the following manner: We multiply both halves of (11) by $[\eta_a-\eta(\zeta)]^{-1/2}d\eta/d\zeta$ and integrate with respect to ζ from ζ' to $\zeta(a)$. Comparing the obtained relation with Eq. (13), we get

$$j_{ia}=(1/\pi\eta^{1/2})n_{e0}(2T_e/M)^{1/2}. \tag{14}$$

A flux of the same magnitude exists at the point $z=b$ if we can neglect in the region $A<z<b$ the change of the number of ion as a result of ionization processes. To this end it is necessary that the Debye radius of the hot plasma be much less than the characteristic length of the inhomogeneity of the neutral-particle density, a condition satisfied in our problem. The potential of the constant electric field at the point $z=b$ is determined by equating the obtained value of the ion flux at the point $z=b$ to the flux of the electrons that is produced because the electrons surmount the potential barrier. We get

$$\eta_b=\ln[4a\sigma_{ee}n_{e0}(M/m)^{1/2}\eta_a^{-1/2}],$$

where σ_{ee} is the effective cross section of electron-electron collisions. For the usual parameters of the plasma pinch, η_b is equal to several units (≈4).

Since the plasma pinch is in dynamic equilibrium with the surrounding neutral gas, the flux density of the neutral particles into the pinch at the point $z=b$ is equal to the flux density of the ions (electrons), as given by Eq. (14). From this we get the density of the neutral particles at the point $z=b$, i.e., on the boundary of the hot region of the pinch:

$$n_b=(1/\pi\eta_a^{1/2})(T/T_e)^{1/2}n_\infty,$$

where $n_\infty=R/T$ is the neutral-particle density far from the pinch.

We turn now to consideration of the region $|z|>b$, where there are no hot electrons and we can neglect the influence of the electric field on the ion motion. Under these conditions Eqs. (1) and (5) reduce to

$$dP_i/d|z|=-PP_i/\lambda R,\quad P+P_i=R, \tag{15}$$

where λ is the ion mean free path in a gas of density n_∞. The system (15) has the following solution:

$$P_i=\frac{P_{ib}R\exp\{-(|z|-b)/\lambda\}}{R-P_{ib}[1-\exp\{-(|z|-b)/\lambda\}]}. \tag{16}$$

From this we get the characteristic distance over which the convective pressure of the ions falls off with increasing distance from the hot region of the pinch:

$$l\approx\lambda\ln(n_\infty/n_b).$$

A detailed analysis of the region of the cold plasma must be based on the condition of the continuity equations for the particle number, with account taken of recombination of the diffusion of the cold plasma in the dense gas, and with account taken of the energy balance equation with allowance for the influence of the RF field on the weakly ionized gas. Similar problems were investigated by a number of workers (see, e.g., Ref. 6), and we shall therefore not dwell on this process.

The energy flux carried away by the electrons from the pinch can be estimated from the equation $Q = j_{ib}ST_e$, where S is the surface area of the pinch. We note that these losses are equal to the energy lost by the hot electrons to heating the cold electrons produced in the pinch as a result of ionization. We obtain $Q \approx 3 \cdot 10^{-13} SR \times (T_e/M)^{1/2}$ kW. At $R \approx 1$ atm, $S \approx 1$ cm^2, and $T_e \approx 3 \times 10^5$ K we have $Q \approx 10^2/\sqrt{\mu}$ kW, where μ is the molecular weight of the particle.

In conclusion, we note the following. It was observed in Kapitza's experiments[1] that the light hydrogen and deuterium impurities contribute to the appearance of a hot plasma pinch, and spectroscopic measurements have shown that there are no multiply charged ions in the pinch. The cause of these phenomena can be understood with the aid of the proposed theory. The point is that the only particles that can penetrate into the interior of the hot region of the plasma pinch are those of light impurities, which have a high thermal velocity and a sufficiently high ionization potential. This follows, for example, from Eq. (6) [or (9)]: the larger the parameter $M\beta^2$, the more difficult it is for the particle to land in the interior of the pinch.

[1]P. L. Kapitza, Zh. Eksp. Teor. Fiz. **57**, 1801 (1969) [Sov. Phys. JETP **30**, 973 (1970)].

[2]Yu. R. Alanakyan, Pis'ma v Zh. Eksp. Teor. Fiz. **26**, 133 (1977) [JETP Lett. **26**, 125 (1977)].

[3]Yu. Ar. Alanakyan, Fizika plazmy **5**, No. 4 (1979) [Sov. J. Plasma Phys. **5**, No. 4 (1979)].

[4]L. Tonks and G. Langmuir, Phys. Rev. **34**, 876 (1929).

[5]E. R. Harrison and W. B. Thompson, Proc. Phys. Soc. **74**, 476 (1959).

[6]B. É. Meĭerovich, Zh. Eksp. Teor. Fiz. **63**, 549 (1972) [Sov. Phys. JETP **36**, 291 (1973)].

Translated by J. G. Adashko

Singularities of the transition to a turbulent motion

G. M. Zaslavskiĭ and Kh.-R. Ya. Rachko

Institute of Physics, Siberian Division of the Academy of Sciences of the USSR

(Submitted 30 November 1978)
Zh. Eksp. Teor. Fiz. **76**, 2052–2064 (June 1979)

An elementary model of the onset of turbulence, corresponding to Landau's idea of the collapse of the stable limit cycle, is considered. We give qualitative estimates of the conditions for the appearance of the turbulent (stochastic) motion which has the structure of a strange attractor. We show that a strange attractor occurs when the effective coefficient for the stretching of the trajectories in phase space in the dissipationless case becomes larger than the effective dissipation coefficient. We performed a detailed numerical experiment on the model to elucidate the structure of the strange attractor, the transition regime, the local instability, the correlation function, and the stationary distribution function in phase space.

PACS numbers: 47.25. – c

1. INTRODUCTION

The analysis for the conditions for the transition of a dynamical system from the regime of a regular, conditionally peridoic motion to a regime of an irregular, stochastic motion has become more and more often the topic of study in the physical and mathematical literature. The generation of turbulence from a laminar motion is perhaps the most characteristic field of physics whose content is directly connected with the determination of a criterion for the appearance of stochasticity.

In the case of weak turbulence the dissipative processes in the system are weak and one can describe the dynamics of the system by Hamiltonian equations. An analysis of the conditions for the appearance of turbulence in this kind of system was given by Sagdeev and one of us[1] (see also Ref. 2). It is shown there that the basis for the mechanism for the generation of a stochastic (turbulent) component of the motion is the well known n-wave ($n \geq 3$) cluster wave interaction. The wave concept itself is quite well defined, since the dissipation is weak. The situation, however, changes abruptly under conditions of strong dissipation, when the system is no longer Hamiltonian. The analysis of the conditions for the appearance of stochasticity in this kind of system started with Lorenz's well known paper,[3] in which a highly simplified model of thermal convection was studied. A large number of papers, stimulated by Lorenz's paper (see, e.g., Refs. 4 to 9), led to the appearance in the physical literature of a new term—"the strange attractor"—introduced by Ruelle and Takens[8] to denote a particular form of stochasticity occurring in strongly dissipative systems (for details see the reviews by Monin[10] and Rabinovich[11]).

Great hopes were pinned on the study of strange attractors in attempts to construct a theory of the onset of turbulence. One should note that a strange attractor also occurs in problems in other fields of physics: laser systems, plasmas, and so on (see Ref. 11). The study of the stochasticity effect in strongly dissipative systems entails at present considerable difficulties. Real physical systems are as a rule studied numerically. It is also unclear how to apply to these systems the existing rigorous mathematical methods.

The aim of the present paper consists in a detailed numerical study of a greatly simplified model for the