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Quantum theory of stimulated processes in a free-electron laser in a strong field

J. K. McIver and M. V. Fedorov

P. N. Lebedev Physics Institute, USSR Academy of Sciences
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General quantum-mechanical equations are derived for an electron in a spatially periodic magnetic field and in the field of an electromagnetic wave. The limits of applicability of the calculations in the lowest order of perturbation theory are obtained. It is shown that for gain calculations these limits are much wider than in the general case owing to the substantial cancellation of the higher-order corrections that contribute to the gain. The saturation parameter μ is determined. The asymptotic dependence of μ on the gain is obtained at $\mu > 1$. The spectral properties of the gain, namely the width and the shift of the resonant maximum, which depend on the field intensities, are investigated. An analytic expression is obtained, at $\mu > 1$, for the maximum (in the spectrum) gain, which decreases in proportion to $E_0^{-3/2}$ with increasing intensity E_0 of the amplified wave.

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1. INTRODUCTION

Amplification[1] and generation[2] of radiation produced when a beam of relativistic electrons is scattered by a spatially-periodic potential of a time-independent magnetic field were recently observed experimentally, for the first time ever, at Stanford University. The theory of the process was developed in a large number of papers[3-15] both on the basis of a classical description[5-11] and a quantum one.[3,4,12-15] At the same time, and independently, a theoretical and experimental investigation was made of spontaneous radiation in systems of this type–undulator (Wiggler) radiation.[16-18]

One of the principal results of Refs. 3–15 is the derivation of formulas for the gain of a weak trial wave (in the "weak signal" approximation). As shown by us earlier[15] these calculations are equivalent to a direct quantum-electrodynamic calculation in the lowest order of perturbation theory (in first order in the magnetic field and first order in the field of the electromagnetic wave). Saturation effects were considered in a number of papers by way of qualitative estimates[6,9,12] and also on the basis of a numerical solution of simplified equations in a classical model.[10,11] No analytic solutions have been apparently published to date. The present paper is devoted to a theoretical description of the amplification of an intense external wave when electrons are scattered by a spatially periodic strong magnetic field. We shall use one of the simplest variants of the quantum-mechanical description of the electron motion in the classical fields, proposed in our earlier papers[15] and based on the interpretation of these phenomena in terms of stimulated bremsstrahlung and absorption. On the basis of the analysis of the exact equations we find the conditions for applicability of the calculations in the lowest order of perturbation theory. In the general case these conditions are much more stringent than in the calculation of the gain. We shall obtain the solutions for the equations in different ranges of variation of the field intensities, including the asymptotic solution for a strong field and a description of the saturation effect.

To estimate various parameters we shall frequently use below the data of Ref. 2. We shall therefore assume that the following relations hold,

$$\varepsilon \gg m \gg \omega \gg q_0,$$

where ε and m are respectively the energy and mass of the electron, $q_0 = 2\pi/\lambda_0$, λ_0 is the period of the magnetic field, and ω is the frequency of the amplified wave; we use a system of units in which $\hbar = c = 1$. Just as in all the preceding papers[1-14] (with the exception of Ref. 12), we use the approximation of the given field of the electromagnetic wave, assuming its amplitude to be constant; this is justified by the *a posteriori* smallness of the gain per pass.

2. FORMULATION OF PROBLEM. PRINCIPAL EQUATIONS

We consider the initial problem, assuming that the interaction of the electron with the magnetic field is turned on at a certain instant of time (the instant when it enters the region where the magnetic field exists), and lasts for a limited time $t = L/v \approx L$, where v is the electron velocity and L is the length of the magnet. A direct check shows that the equations obtained in this

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manner are identical with those that can be derived within the framwork of a different formulation—by solving the boundary-value problem, if we consider the stationary incident and transmitted electron fluxes and we introduce in place of the time t the running coordinate $z=vt$.

We consider thus an electron in an external magnetic field and an external electromagnetic field, defined by the vector potentials

$$\mathbf{A}_H=\frac{B_0}{2^{1/2}q_0}\{\mathbf{a}e^{iq_0z}+\mathbf{a}^*e^{-iq_0z}\}, \tag{1}$$

$$\mathbf{A}_w=\frac{E_0}{2^{1/2}\omega}\{\mathbf{e}e^{i\omega(z-t)}+\mathbf{e}^*e^{-i\omega(z-t)}\}, \tag{2}$$

where a and e are unit vectors of the polarization, B_0 and E_0 are the amplitudes of the intensity of the constant magnetic field and of the electric field of the wave. In accordance with Ref. 2, we assume that the wave propagation direction, as well as the electron direction, coincides with the direction along which the magnetic field varies (the z axis).

The behavior of the electron is determined by the solution of the Dirac equation with account taken of the external fields (1) and (2). This equation can be simplified by neglecting the spin effects. It is known[19] that by squaring it is possible to transform the Dirac equation in an external field into a Klein-Gordon equation with spin-dependent correction terms. The relative values of these terms in our case are determined by very small parameters, $eE_0/m^2\sim10^{-12}$ and $eB_0/m^2\sim10^{-10}$, where $E_0=1.3\times10^5$ V/cm and $B_0=2.4\times10^3$ G are the values of the intensities in Ref. 2. Therefore the spin corrections can be neglected, so that we can assume as the initial equation the Klein-Gordon equation

$$\left\{\frac{\partial^2}{\partial z^2}-\frac{\partial^2}{\partial t^2}-e^2(\mathbf{A}_w+\mathbf{A}_H)^2-m^2\right\}\Psi=0 \tag{3}$$

for one-dimensional motion of the electron along the z axis in the fields (1) and (2). It is assumed here that the initial momentum of the electron p is directed along the z axis. The scatter over the propagation directions of the electrons in the beam is neglected.

We consider next the most interesting case, which corresponds to the experiment of Ref. 2, of circular polarization of the wave and of a helical magnetic field. Let

$$\mathbf{e}=(\mathbf{x}+i\mathbf{y})/\sqrt{2},\quad \mathbf{a}=(\mathbf{x}-i\mathbf{y})/\sqrt{2},$$

where x and y are unit vectors along the axes x and y. (It is precisely in the case $\mathbf{a}=\mathbf{e}^*$ that the gain obtained within the framework of perturbation theory and proportional to $|a\cdot e|^2$ is maximal.[15]) The quadratic terms $\propto A_H^2$ and A_w^2 in (3) are in this case constant and determine the mass shift

$$m^2\to m_*^2=m^2+e^2B_0^2/q_0^2+e^2E_0^2/\omega^2. \tag{4}$$

We assume that this mass shift is included in m and leave out the * index. Equation (3) consequently transforms into

$$\left\{\frac{\partial^2}{\partial z^2}-\frac{\partial^2}{\partial t^2}-2e^2\mathbf{A}_H\mathbf{A}_w-m^2\right\}\Psi=0. \tag{5}$$

We note that within the framework of the assumed model (neglecting the spin effects and when the electrons move in a direction perpendicular to $\mathbf{A}_H$ and $\mathbf{A}_w$) the mass shift is the only effect that arises when an electron is acted upon by an arbitrarily strong field, either $\mathbf{A}_H$ or $\mathbf{A}_w$. The wave functions in the field $\mathbf{A}_w$ (Ref. 19) and the exact solutions in the field $\mathbf{A}_H$, when account is taken of the shift of the mass, do not differ in this case from the wave functions of the free electron. However, the joint action on the electron by the two fields $\mathbf{A}_H$ and $\mathbf{A}_w$ leads to much more substantial changes of the character of its motion.

We expand the electron wave function Ψ in plane waves:

$$\Psi=\sum_p C_p(t)\exp[i(pz-\varepsilon_p t)], \tag{6}$$

where $\varepsilon_p=(p^2+m^2)^{1/2}$ is the energy of the free electron with momentum p.

Equation (5) is identical with the following system of equations for the coefficients $C_p(t)$:

$$\ddot{C}_p-2i\varepsilon_p\dot{C}_p=\frac{e^2E_0B_0}{q_0\omega}\exp(i\varepsilon_p t)\{C_{p-q_0-k}\exp[-i(\omega+\varepsilon_{p-q_0-k})t]+C_{p+q_0+k}\exp[i(\omega-\varepsilon_{p+q_0+k})t]\}. \tag{7}$$

Comparing the term $\propto\ddot{C}_p$ with the right-hand side of (7), we can estimate the characteristic time scale of the variation of $C_p(t)$:

$$t_0\sim q_0\omega\varepsilon_p/e^2E_0B_0.$$

It follows therefore that the relative value of the term $\ddot{C}_p$ in (7) is determined by the parameter

$$(\varepsilon_p t_0)^{-1}\sim e^2E_0B_0/q_0\omega\varepsilon_p^2,$$

which is equal to 10^{-9} under the conditions of the experiment of Ref. 2. This makes it possible to neglect, with good accuracy, the second derivatives of the coefficients $C_p(t)$ with respect to time. Taking this circumstance into account, putting

$$p+n(q_0+k)=p_n,\ \varepsilon_{p_n}=\varepsilon_n,\ n=0,\pm1,\pm2,\ldots,\qquad C_{p_n}=\exp\{i(\varepsilon_n-\varepsilon_0-n\omega)t\}a_n, \tag{8}$$

we rewrite the system (7) in the form

$$i\dot{a}_n-(\varepsilon_n-n\omega-\varepsilon_0)a_n=\frac{e^2E_0B_0}{2q_0\omega\varepsilon_n}(a_{n-1}+a_{n+1}). \tag{9}$$

Taking into account the definition of ε_n and the smallness of q_0 and ω compared with p, we use for ε_n the expansion

$$\varepsilon_n=\varepsilon_0+n(q_0+\omega)p/\varepsilon_0+n^2m^2(q_0+\omega)^2/2\varepsilon_0^3. \tag{10}$$

As a result, Eqs. (9) take the form

$$i\dot{a}_n+\left[n\left(\frac{m^2\omega}{2\varepsilon^2}-q_0\right)-n^2\frac{m^2\omega^2}{2\varepsilon^3}\right]a_n=\frac{e^2E_0B_0}{2q_0\omega\varepsilon}(a_{n-1}+a_{n+1}), \tag{11}$$

where we have put $\varepsilon_n\approx\varepsilon_0\equiv\varepsilon$ in the right-hand side of the equation. The possibility of such a substitution, as well as the possibility of confining ourselves to the lowest powers of n in the expansion (10), is connected with the relative smallness of the number of effective excited levels: $|n|\ll\varepsilon/\omega$. As will be shown below (see Sec. 5), under the experimental conditions of Ref. 2 we have $n\sim10^6$, whereas $\varepsilon/\omega\sim10^6$, so that the indicated condition is well satisfied.

Equations (11) are similar to the equations that describe the excitation of an anharmonic oscillator by a

resonant field. The role of the natural frequency of the system, with which the external field is at resonance, is played by the frequency of the wave that is amplified in a free-electron laser,[3-15] $\omega_{res}=2q_0\varepsilon^2/m^2$. The conditions for the resonances on the first excited levels ($n=\pm1$), $\varepsilon_{\pm1}\mp\omega-\varepsilon_0=0$, determine more accurately those frequencies at which absorption or emission of a photon is possible in first-order perturbation theory:

$$\omega_{a,e}=2q_0\frac{\varepsilon^2}{m^2}\left(1\pm\frac{\omega}{\varepsilon}\right)=\omega_{res}\left(1\pm\frac{\omega_{res}}{\varepsilon}\right) \tag{12}$$

(this case is discussed in greater detail in the next section).

We point out a number of differences between the system (11) and the ordinary equations for the anharmonic oscillator. First, the matrix elements of the transitions $n\to n\pm1$ do not contain the usual square root dependences $\sqrt{n+1}$ and $\sqrt{n}$, and do not depend at all on n within the framework of the assumed approximation $|n|\omega/\varepsilon\ll1$.

Another difference from the ordinary oscillator is that the quantum number n can assume both positive and negative integer values.

Finally, it is important to emphasize that the parameters of the equivalent anharmonic oscillator (ω_{res} and the anharmonicity constant) depend on the field frequency, on the electron energy, and on the period of the magnetic field. When these external factors change, the levels of the anharmonic oscillator are shifted in a wide range, since the real spectrum of the electron is continuous in the continuum.

The fact that the analysis of the transitions of the electrons in external periodic fields leads to a system of equations that is typical of a transition in a discrete spectrum, was noted also earlier.[20] This result is connected with the fact that in the periodic fields (1) and (2) the momentum conservation single out in the entire continuous spectrum only a definite sequence of levels which are connected with the initial state (with a given momentum p).

If it is assumed that the interaction is turned on instantaneously at the instant $t=0$ (the corresponding conditions will be estimated below, see Sec. 5), then the initial conditions for Eqs. (9) take the usual form $a_n(t=0)=\delta_{n,0}$. The total energy $\Delta\mathscr{E}$ radiated in this case by the electron after a time $t\approx L$, with allowance for the rule for the normalization of the wave functions that satisfy the Klein-Gordon equation,[21] can be expressed in terms of the coefficients $a_n(t)$ in the following manner:

$$\Delta\mathscr{E}=-\omega\sum_n n\frac{\varepsilon_n}{\varepsilon_0}|a_n|^2\approx-\omega\sum_n n|a_n|^2. \tag{13}$$

The gain per pass is

$$G=4\pi n_e E_0^{-2}\Delta\mathscr{E}, \tag{14}$$

where n_e is the electron density in the beam.

Let us determine the principal-parameters that characterize the system (11). Obviously, these parameters include the energy of interaction of the electron with the fields (1) and (2)

$$\mathscr{E}_{int}=e^2E_0B_0/2q_0\omega\varepsilon,$$

The anharmonicity energy

$$\mathscr{E}_{anh}=m^2\omega^2/2\varepsilon^3,$$

The detuning from resonance

$$q_0-m^2\omega/2\varepsilon^2\approx2q_0\Delta/\varepsilon\approx m^2\omega\Delta/\varepsilon^3;$$

$$\Delta=\varepsilon-m\sqrt{\frac{\omega}{2q_0}}=\frac{\varepsilon}{2\omega_{res}}(\omega_{res}-\omega)$$

and the interaction time t. We introduce also the characteristic dimensionless parameters

$$\rho=\frac{\mathscr{E}_{int}}{\mathscr{E}_{anh}}=\frac{e^2E_0B_0\varepsilon^2}{m^2\omega^3q_0},\quad \beta=t\mathscr{E}_{anh}=\frac{m^2\omega^2t}{2\varepsilon^3},$$

$$\mu=2t(\mathscr{E}_{int}\mathscr{E}_{anh})^{1/2}=\frac{m^2}{\varepsilon^2}\omega t\left(\frac{e^2E_0B_0}{q_0\omega m^2}\right)^{1/2}\approx\frac{et(2E_0B_0)^{1/2}}{\varepsilon}. \tag{15}$$

Under the conditions of the experiment[2] the values were $\rho\approx10^{11}$, $\mu\approx5$, $\beta\approx10^{-5}$.

It will be shown below that the parameter ρ determines the character of the solutions in an asymptotically strong field, the parameter μ characterizes the conditions of the transition to the saturation regime, and the parameter β determines small quantum corrections which we shall neglect.

We shall also make extensive use from now on of the stationary, i.e., quasi-energetic, solutions of Eq. (9): $a_n(t)=e^{-i\gamma t}b_n$, where γ is the quasi-energy and b_n are constant coefficients that satisfy the equations

$$\left[\gamma-n\left(q_0-\frac{m^2\omega}{2\varepsilon^2}\right)-n^2\frac{\omega^2m^2}{2\varepsilon^3}\right]b_n=\frac{e^2E_0B_0}{2q_0\omega\varepsilon}(b_{n-1}+b_{n+1}). \tag{16}$$

3. PERTURBATION THEORY

If the field intensities E_0 and B_0 are small enough, then the right-hand sides of (9) and (16) can be treated be perturbation theory. Obviously, in first order the quantities that differ from zero, besides $a_0(t)\approx1$, are only the coefficients $a_{\mp1}(t)$, which can be easily shown to be of the form

$$a_{\mp1}=\frac{-i}{\omega-\omega_{e,a}}\frac{e^2E_0B_0\varepsilon}{q_0\omega m^2}\left\{1-\exp\left[-i\frac{m^2}{2\varepsilon^2}(\omega-\omega_{e,a})t\right]\right\}, \tag{17}$$

where the frequencies $\omega_{e,a}$ are defined by Eqs. (12). Taking into account the relative smallness of the difference $\omega_e-\omega_a$ with the aid of the definitions (13) and (14) we can easily obtain from (17) the total gain (per pass):

$$G=\frac{8^{1/2}\pi n_eB_0^2e^4q_0^{1/2}t^3}{m^3\omega^{5/2}}\frac{\partial}{\partial u}\frac{\sin^2u}{u^2},$$

$$u=\frac{tm^2}{4\varepsilon^2}\left(\omega-2q_0\frac{\varepsilon^2}{m^2}\right)\approx-\frac{m^2\omega t}{2\varepsilon^2}\Delta. \tag{18}$$

This expression, as expected, coincides with the result of the direct quantum electrodynamic calculation by perturbation theory for the case of an abrupt boundary of the interaction region.[15] Formula (15) is valid for a monoenergetic beam or, more accurately for a beam with an energy distribution width $\Delta\varepsilon\ll\varepsilon/n=2\pi\varepsilon/q_0t$, where n is the number of periods of the magnetic field. In the opposite case $\Delta\varepsilon\gg\varepsilon/n$, the expression $\sin^2u/u^2$

in (18) can be replaced by $\pi\delta(u)$ and the gain G must be averaged over the electron distribution function $f(\varepsilon)$, which also leads to one of the results of Ref. 15:

$$G=\frac{2^{1/2}\pi^2e^4B_0^2t}{mq_0^{5/2}\omega^{1/2}}f'(\varepsilon), \tag{19}$$

where $\varepsilon=m\sqrt{\omega/2q_0}$.

We now estimate the limits of applicability of the calculations by perturbation theory. It is necessary to recognize here that under the conditions of the experiment[2] the parameter $\mathscr{E}_{\text{anh}}t(\sim10^{-5})$ was very small. For this reason, the reciprocal duration of the interaction, $1/t$, must be regarded as the effective width of the levels, which at small $|n|$ greatly exceeds the anharmonicity energy. Under these conditions, the only criterion for the applicability of perturbation theory is smallness of the interaction energy compared with the effective widths of the levels:

$$\mathscr{E}_{\text{int}}t\ll1. \tag{20}$$

Under the conditions of the experiment[2] this requirement is not satisfied, since $\mathscr{E}_{\text{int}}t\sim10^6\gg1$. This means that in a wide range of parameters the criteria for the applicability of the calculations in the lowest order of quantum-mechanical perturbation theory are not satisfied. In the model of the equivalent anharmonic oscillator, this result means that we cannot confine ourselves to allowance for transitions between the levels $n=0$ and $n=\pm1$ but, on the contrary, we must solve the problem of the excitation of a multilevel system in accordance with Eq. (11).

The fact that perturbation theory cannot be used to calculate the amplitudes of probability $a_n(t)$ does not mean automatically that the formulas obtained within the framework of perturbation theory for the gain are no longer correct. In Sec. 5, on the basis of an analysis of the excitation of a multilevel system, it will be shown that formula (18) remains in force in fact even if the inequality (20) is violated. The only limitation on the applicability of formula (18) is the condition $\mu<1$. We note that in the μ scale the limits of applicability of perturbation theory $\mathscr{E}_{\text{int}}t\sim1$ corresponds to $\mu\sim3\cdot10^{-3}$ at the values of all the parameters that correspond to the conditions of Ref. 2, with the exception of the field intensity E_0.

4. QUASIENERGY SOLUTIONS

We consider the stationary equation (16) for the quasi-energy wave functions of the electron. For its solution we use a method similar to that developed previously for the solution of the problem of excitation of an anharmonic oscillator by a strong resonant field in diatomic molecules.[22] The idea of the method is to multiply the coefficients b_n by functions $\varphi_n(u)$ of a certain basis (u is an auxiliary variable) and sum over n, thereby going from a system of equations for b_n to a homogeneous differential equation of the type of the stationary Schrödinger equation. Its solution can then be obtained by the standard methods of quantum mechanics. In the case of Eq. (16) it is convenient to choose $\varphi_n(u)=e^{inu}$, $0<u<2\pi$, which yields for the function

$$\psi(u)=\sum_n b_ne^{inu} \tag{21}$$

the expression

$$\frac{d^2\psi}{du^2}+2i\frac{\Delta}{\omega}\frac{d\psi}{du}-2\rho\cos u\psi+\frac{2\varepsilon^3\gamma}{m^2\omega^2}\psi=0. \tag{22}$$

The boundary condition for (22) is the periodicity condition $\psi(0)=\psi(2\pi)$. The transformation inverse to (21) is obviously of the form

$$b_n=\frac{1}{2\pi}\int du\psi(u)e^{-inu}.$$

After replacing the unknown function

$$\psi(u)=\exp\left(-i\frac{\Delta}{\omega}u\right)\tilde{\psi}(u)$$

Eq. (22) takes the form of the Schrödinger equation

$$\left(-\frac{d^2}{du^2}+U(u)-\bar{\gamma}\right)\tilde{\psi}(u)=0, \tag{23}$$

where

$$U(u)=2\rho\cos u,\quad \bar{\gamma}=\frac{\Delta^2}{\omega^2}+2\frac{\varepsilon^3\gamma}{m^2\omega^2}. \tag{24}$$

Equation (23) with the potential energy $U(u)$ (24) is the well known Mathieu equation. However, the use of its exact solutions is difficult, since we are interest not in the eigenfunctions of this equation themselves, but in the physical quantities $\Delta\mathscr{E}$ and G [(13), (14)] that are expressed in their terms. As will be shown below, to calculate $\Delta\mathscr{E}$ and G it is necessary to perform a large number of operators on the solutions of Eq. (23)—summation and integration, something hardly possible to do in analytic form when Mathieu functions are used. We consider therefore below simpler approximations of the solution of Eqs. (22) and (23).

We estimate the relative role of the different terms in Eqs. (9), (11), (16), and (22). Obviously, the maximum number of levels $n_{\max}$ that are substantially perturbed by the external action is $n_{\max}\sim(\mathscr{E}_{\text{int}}/\mathscr{E}_{\text{anh}})^{1/2}=\sqrt{\rho}$. The time of excitation of the levels with $n=\pm1$ is of the order of $\mathscr{E}_{\text{int}}^{-1}$, the time of excitation of n levels is $\sim n\,\mathscr{E}_{\text{int}}^{-1}$, and the number of levels excited within a time t is obviously $t\mathscr{E}_{\text{int}}$. If $t\mathscr{E}_{\text{int}}<n_{\max}$ (i.e., if $\mu<1$), then the influence of the anharmonicity is weak, (the term $n^2\mathscr{E}_{\text{anh}}\sim(n/n_{\max})^2\mathscr{E}_{\text{int}}$ in Eqs. (11) and (16) is small in this case). This makes it possible to take into account the anharmonicity in the region $\mu<1$ as a small perturbation, which is equivalent to taking into account the term $d^2\psi/du^2$ in (22) by perturbation theory. It is easy to verify that when the periodicity condition is taken into account, the solutions of (22) take the form

$$\gamma_n=\frac{m^2\omega}{\varepsilon^3}\left(n\Delta+n^2\frac{\omega}{2}\right),\quad n=0,\ \pm1,\ \pm2,\ldots,$$

$$\psi_n=\frac{1}{\sqrt{2\pi}}\exp\left(i\left(nu-\rho\frac{\omega}{\Delta}\sin u\right)\right)$$

$$\times\left\{1+\frac{\rho\omega^2}{\Delta^2}\left[\left(n+\frac{1}{2}\right)e^{iu}+\left(n-\frac{1}{2}\right)e^{-iu}\right]\right\}. \tag{25}$$

In formulas (25), the perturbation $-d^2/du^2$ was taken into account in first order. It follows from calculations that this must be done at because in general, if no account is taken of the anharmonicity, the radiation and the absorption cancel each other completely, as a result of which the gain becomes equal to zero.

In the region $\mu>1$, the anharmonicity plays an essen-

tial role for a large number of excited levels, and therefore must be taken into account exactly. In this case it is possible to formulate another approximation, which corresponds to the asymptotic strong-field approach, in which it is possible to obtain simple solutions of (23).

According to (23), the motion of an electron (in the space u) is bounded by potential curve $U(u)$ (24), which has a deep minimum at $u=\pi$. The depth of the minimum, equal to $2\rho \gg 1$, greatly exceeds the distance between the lower levels in the unperturbed system. It follows therefore that the wave functions of a large number of levels near the bottom of the well are localized at distances Δu much smaller than 2π. This makes it possible to expand $U(u)$ in a series about the value $u=\pi$:

$$U(u)=2\rho\left(1-\frac{(u-\pi)^2}{2}+\frac{(u-\pi)^4}{24}\right)$$

and write down for $\psi(u)$ oscillator solutions. Without dwelling on the details of the transformations, we write down the final expressions for the quasi-energies γ_n (with account taken of the anharmonicity) and for the corresponding functions $\psi_n(u)$:

$$\gamma_n=\frac{m^2\omega^2}{2\varepsilon^3}\left\{\frac{\Delta^2}{\omega^2}-2\rho+\sqrt{\rho}(2n+1)-\frac{1}{16}(2n^2+2n+1)\right\};\quad n=0,1,2,\ldots,$$
$$\psi_n(u)=\exp\left(-i\frac{\Delta}{\omega}u\right)\chi_n(\rho^{1/4}(u-\pi));\qquad (26)$$
$$\chi_n(\xi)=e^{-\xi^2/2}H_n(\xi)/(2^n n!\sqrt{\pi})^{1/2},$$

where $\chi_n(\xi)$ are the ordinary oscillator functions,[21] and $H_n(\xi)$ are Hermite polynomials.

We note that the anharmonic corrections to the quasi-energy can arise not only as a result of the expansion of $U(u)$ up to $(u-\pi)^4$, but also as a result of allowance for the higher derivatives in (23), which arise when account is taken of the higher powers of n in the expansion (10) of the energy ε_n. It is easy, however, to verify that the contribution made to the anharmonicity γ_n of the matrix elements from the higher derivatives d^3/du^3 and d^4/du^4 is small compared with the accounted-for contribution from $(u-\pi)^4$ in terms of the parameter $\rho\omega^2/\varepsilon^2$, which is equal to 10^{-5} under the conditions of the experiment of Ref. 2.

It follows from (26) that with increasing interaction energy the quasi-energies γ_n increase in proportion to ρ. The system of levels is nearly equidistant. The distance between the neighboring levels (the oscillator frequency) increases like $\sqrt{\rho}$. The anharmonic correction to the quasi-energies γ_n does not depend on ρ and is therefore relatively small. The wave functions of the lower levels $\psi_n(u)$ are localized according to (26) over a distance $\Delta u \sim \rho^{-1/4}$. At the boundaries of the region where they are defined, at $u=0$ and $u=2\pi$, the functions $\psi_n(u)$ are exponentially small, i.e., they practically vanish, therefore the boundary conditions (the periodicity conditions) are automatically satisfied.

5. TIME EVOLUTION OF THE SYSTEM. THE GAIN

Starting from (11), we can obtain an equation that describes the time evolution of the function

$$\psi(u,t)=\sum_n a_n(t)e^{inu},\qquad (27)$$

in the form

$$i\frac{\partial\psi(u,t)}{\partial t}=\frac{m^2\omega^2}{2\varepsilon^3}\left[\frac{\partial^2\psi(u,t)}{\partial u^2}+2i\frac{\Delta}{\omega}\frac{\partial\psi(u,t)}{\partial u}-2\rho\cos u\psi(u,t)\right].\qquad (28)$$

A transformation inverse to (27)

$$a_n(t)=\frac{1}{2\pi}\int_0^{2\pi}\psi(u,t)e^{-inu}\,du,\qquad (29)$$

makes it possible to express in terms of $\psi(u,t)$ the coefficients $a_n(t)$, and consequently also the radiated energy $\Delta\mathscr{E}$ [and the gain G which is connected with it by Eq. (14)]:

$$\Delta\mathscr{E}=-\frac{i\omega}{2\pi}\int_0^{2\pi}du\frac{\partial\psi^*(u,t)}{\partial u}\psi(u,t).\qquad (30)$$

To solve (28) we can use the quasi-energy solutions $\{\psi_n(u),\gamma_n\}$ (25), (26) which were obtained in the preceding section. The solution method and the final result depend very substantially on the manner in which the interaction is turned on: adiabatically or instantaneously. The conditions of the experiment of Ref. 2 correspond to the amount of instantaneous turning-on, since the time to turn on the interaction $\Delta t \lesssim \lambda_0=2\pi/q_0$ is small compared with the period of the natural oscillations of the system $\sim(\gamma_{n+1}-\gamma_n)^{-1}$ [see (25) and (26)]: $\Delta t(\gamma_{n+1}-\gamma_n)^{-1}\lesssim\Delta/\varepsilon\ll 1$. Since $a_n(0)=\delta_{n,0}$, it is obvious that $\psi(u,0)=1$. Let the interaction be turned on instantaneously at $t=0$. The expansion of $\psi(u,0)\equiv 1$ in the complete system of functions $\psi_n(u)$ (25) or (26) makes it possible to represent the energy $\Delta\mathscr{E}$ radiated by the electron per pass in the form

$$\Delta\mathscr{E}=-\frac{i\omega}{2\pi}\sum_{n,m}\exp[i(\gamma_n-\gamma_m)t]\int du_1\psi_n(u_1)$$
$$\times\int du_2\psi_m^*(u_2)\int du_3\frac{\partial\psi_n^*(u_3)}{\partial u_3}\psi_m(u_3).\qquad (31)$$

In accordance with the results of the preceding section, we now examine separately the regions $\mu<1$ and $\mu>1$.

1. *Weak fields*, $\mu<1$. Substituting in (31) the quasi-energies γ_n and the functions $\psi_n(u)$ (25) and integrating with respect to the u_3, we transform (31) into

$$\Delta\mathscr{E}=-\frac{\omega}{2\pi}\sum_{n>-\infty}^{+\infty}n|d_n|^2+\frac{\rho\omega^2}{4\pi\Delta}\sum_n d_n$$
$$\times\left[d_{n+1}^*\exp\left\{-i\frac{m^2\omega}{\varepsilon^3}(\Delta+n\omega)t\right\}+d_{n-1}^*\exp\left\{i\frac{m^2\omega}{\varepsilon^3}(\Delta+n\omega)t\right\}\right],\qquad (32)$$

where

$$d_n=\int du\psi_n(u)=\sqrt{2\pi}\left[J_n\left(\frac{\rho\omega}{\Delta}\right)+\frac{\rho\omega^2}{\Delta^2}\left(\left(n+\frac{1}{2}\right)J_{n+1}\left(\frac{\rho\omega}{\Delta}\right)\right.\right.$$
$$\left.\left.-\left(n-\frac{1}{2}\right)J_{n-1}\left(\frac{\rho\omega}{\Delta}\right)\right)\right],\qquad (33)$$

and $J_n(u)$ are Bessel functions.

Calculating the sums over n in accordance with the standard formulas,[23] we again obtain for the gain G formula (18), which is valid in the weak-signal approximation. This result shows that the region of applicability of formula (18) is much wider than the region of applicability of perturbation theory, and is determined only by the condition of the smallness of the parameter μ. The number of levels of the equivalent anharmonic oscillator which are effectively excited by the external field can be easily estimated from formula (23): $n\sim\rho\omega/$

Δ, which at $\Delta \sim \varepsilon^3/m^2\omega t$ yields, as expected,

$$n \sim \rho \mathscr{E}_{\text{anh}} t = \mathscr{E}_{\text{int}} t \sim 10^6.$$

The fact that perturbation theory does not hold for the solution of Eq. (11), simultaneously with the possibility of using formula (18) obtained in the lowest order of perturbation theory, means that a very substantial cancellation of the contributions of the higher orders to the different amplitudes $a_n(t)$ takes place when the probabilities of finding the electrons at different levels, which determine the values of $\Delta\mathscr{E}$ and G [(13), (14)] are summed. We note that a perfectly analogous cancellation effect takes place also in the theory of induced inverse bremsstrahlung when nonrelativistic electrons are scattered by Coulomb centers.[24] The nonlinearity of the cross sections of the multiphoton radiation and absorption is determined in this case by the quantum parameter $\gamma = ev_0E_0/\hbar\omega^2$, whereas the nonlinearity of the absorption coefficient is determined by the much weaker classical parameter $\gamma\hbar\omega/mv_0^2 = eE_0/m\omega v_0$, where $\hbar\omega \ll mv_0^2$.

2. We consider now the region of strong fields $\mu > 1$.

Substituting in (31) the quasi-energy solutions (26), we integrate with respect to the variable u_3, using the localization of the functions $\psi_{n,m}(u_3)$, and letting the limits of integration go to $\pm\infty$. This yields

$$\Delta\mathscr{E} = \Delta - \frac{i\omega}{4\pi^{3/2}} \int_{-q}^{q} d\xi_1 \int_{-q}^{q} d\xi_2 \exp\left\{-\frac{\xi_1^2+\xi_2^2}{2} + i\left(\frac{\beta}{2} - \frac{\Delta}{\omega\rho^{1/4}}(\xi_1-\xi_2)\right)\right\} \sum_n \frac{H_n(\xi_1)}{2^n n!} \{H_n'(\xi_2) e^{i(\mu-\beta n)} + (H_n'(\xi_2) - 2\xi_2 H_n(\xi_2)) e^{i(\beta n-\mu)}\}; \quad q \equiv \pi\rho^{1/4}. \tag{34}$$

Summation over n in (34) is realized with the well known Möller formula,[25] as a result of which we get

$$\Delta\mathscr{E} = \Delta - \frac{\omega}{\pi^{3/2}} \operatorname{Re}\left\{ i \int_{-q}^{q} d\xi_1 \int_{-q}^{q} d\xi_2 \exp\left\{-\frac{\xi_1^2+\xi_2^2}{2} + i\left(\mu - \frac{\beta}{2} + \frac{\Delta}{\omega\rho^{1/4}}(\xi_2-\xi_1)\right)\right\} (\xi_1 - e^{-i\beta}\xi_2) \times (1-e^{-2i\beta})^{-3/2} \exp\left(\frac{2\xi_1\xi_2 e^{-i\beta} - (\xi_1^2+\xi_2^2)e^{-2i\beta}}{1-e^{-2i\beta}}\right)\right\}. \tag{35}$$

The integration in (35) with respect to one of the variables (for example, with respect to ξ_2) is carried out with account taken of the smallness of the parameter β. The integrand in (35) is localized with respect to the variable ξ_2 on the interval $\sim\sqrt{\beta} \ll 1 \ll \rho^{1/4}$. This makes it possible to replace again in the integral with respect to ξ_2 the limits of integration by $\pm\infty$, which yields ultimately in the lowest order in the small parameter β

$$\Delta\mathscr{E} = \Delta\left\{1 - (2\pi\rho^{1/4})^{-1} \operatorname{Re}\left\{\int_{-q}^{q} d\xi \exp\left[-i\frac{\beta\xi^2}{2} + i\mu\left(1 - \frac{\Delta^2}{16\omega^2\rho}\right)\right]\right\}\right\}. \tag{36}$$

In the integral (36) the integrand is no longer necessarily localized, and therefore the limits of integration in this formula cannot be made infinite. Equation (36) can be expressed formally in terms of elliptic integrals[23]:

$$\Delta\mathscr{E} = \Delta\left\{1 - \frac{2\sqrt{2}}{\sqrt{\pi\mu}}\left[\cos\mu\left(1 - \frac{\Delta^2}{16\omega^2\rho}\right) C\left(\frac{\pi\sqrt{\mu}}{4}\right) + \sin\mu\left(1 - \frac{\Delta^2}{16\omega^2\rho}\right) S\left(\frac{\pi\sqrt{\mu}}{4}\right)\right]\right\}. \tag{37}$$

Actually, however, the extension of formula (37) to the region of small values of μ is not justified. The reason is that in the initial formula (31) the integrand is obviously localized with respect to all three variables: ξ_1, ξ_2, and ξ_3. Therefore in the final expression (36) the integrand should also be localized, and this corresponds to the condition $\mu > 1$. The appearance of a nonlocalized integrand in the case $\mu < 1$ means that in the sum over n an important role is played here by too large a number of terms, as a result of which the assumption of small anharmonicity of the potential energy $E(x)$ (24) is not justified. On the contrary, localization of the integrand in (36) in the region $\mu > 1$ confirms the validity of all the approximations made above. Changing over in formula (37) to the asymptotic form of large μ, we obtain

$$\Delta\mathscr{E} = \Delta\left\{1 - \sqrt{\frac{2}{\pi\mu}}\left[\cos\mu\left(1 - \frac{\Delta^2}{16\omega^2\rho}\right) + \sin\mu\left(1 - \frac{\Delta^2}{16\omega^2\rho}\right)\right]\right\}. \tag{38}$$

with increasing parameter μ the energy $\Delta\mathscr{E}$ radiated by the electron in one pass changes in oscillating fashion, tending to a constant value Δ as $\mu \to \infty$ (Fig. 1).

One of the main premises in the derivation of this asymptotic formula is the assumption of the smallness of the anharmonic corrections to the expansion of the quasi-energy $\Delta\mathscr{E}$ (26). The expansion parameter in this case is the quantity $n/\sqrt{\rho}$. The characteristic values n can be easily estimated by calculating with the aid of (26) the function $\psi(x, t)$ and the coefficients $a_n(t)$ which are given, apart from a phase factor, by

$$a_n \propto \sqrt{2\pi}\rho^{1/4}\chi_n\left(\frac{\Delta}{\omega\rho^{1/4}}\right). \tag{39}$$

At large n and ξ, the oscillator functions $\chi_n(\xi)$ are maximal at $\sqrt{n} \approx \xi$, and it is this which determines the characteristic values of n: $n \approx \Delta^2/\omega^2\sqrt{\rho}$. Consequently, the expansion parameter of γ_n (26) is $\Delta^2/\omega^2\rho$, and the condition for the validity of this expansion limits the detuning $|\Delta| < \Delta_{\max}$, where

$$\Delta_{\max} = \omega\sqrt{\rho} = \omega\left(\frac{\mathscr{E}_{\text{int}}}{\mathscr{E}_{\text{anh}}}\right)^{1/2} = \frac{e\varepsilon}{m}\left(\frac{E_0B_0}{\omega q_0}\right)^{1/2} = \left(\frac{e}{q_0}\right)\left(\frac{E_0B_0}{2}\right)^{1/2}. \tag{40}$$

At $|\Delta| \sim \Delta_{\max}$ the detuning, the anharmonicity, and the energy of the interaction with the field in (11) turn out to be of the same order. At $|\Delta| > \Delta_{\max}$ the anharmonicity

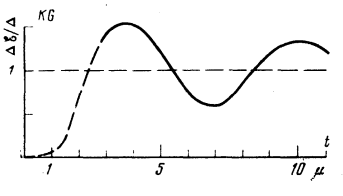

FIG. 1. Normalized plot of $\Delta\mathscr{E}$ (μ); the same curve represents the normalized plot of $G(t)$ at constant E_0 ($K = E_0^2/4m_e\Delta$). Dashed—region intermediate between the regions of applicability of the weak-signal approximation ($\mu < 1$) and the strong-field asymptotic limit ($\mu > 1$).

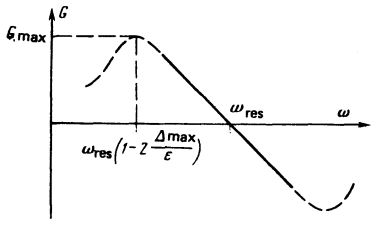

FIG. 2. Spectral dependence of the gain $G(\omega)$ in the asymptotic strong-field limit ($\mu > 1$).

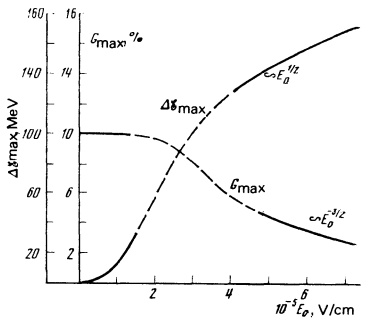

FIG. 3. Qualitiative plots of the maximum (over the spectrum) gain G_{max} and of the maximum radiated energy $\Delta\mathscr{E}_{max}$ against E_0.

is relatively small, so that we can again consider the second derivative in (22) as a small perturbation, and we arrive at the results of Item 1. Consequently, the quantities $\Delta\mathscr{E}$ and G in the region $|\Delta| > \Delta_{max}$ decrease with increasing $|\Delta|$ (Fig. 2). Therefore the quantity Δ_{max} can be regarded as the width of the resonant function $G(\Delta)$. We note that at $\mu < 1$ the width of the resonance in the formula (18) is the quantity $\omega/\mathscr{E}_{anh}t$, which at $\mu > 1$ is replaced by Δ_{max}(40). According to (40), the width of the resonance at $\mu > 1$ does not depend on t and with increases like $\sqrt{E_0B_0}$ increasing E_0 or B_0.

Formulas (38) and (40) enable us to estimate at $\mu \gg 1$ the maximum radiated energy $\Delta\mathscr{E}_{max}$ and the maximum gain G_{max}

$$\Delta\mathscr{E}_{max}\approx\Delta_{max};\quad G_{max}=G(\Delta=\Delta_{max})=\frac{4\pi n_e\Delta_{max}}{E_0^2}=\frac{2\sqrt{2}\pi n_e e\sqrt{B_0}}{q_0E_0^{3/2}}. \tag{41}$$

According to (40) and (41), the maximum energy radiated by the electron per pass, even under the saturation condition $\mu \gg 1$ increases with increasing E_0 like $\sqrt{E_0}$. The gain G_{max} decreases in this case like $E_0^{-3/2}$ (Fig. 3). Neither $\Delta\mathscr{E}_{max}$ nor the gain G_{max} depends in this case on the duration of the interaction t (i.e., on the length L of the magnet). The frequency ω at which the gain is maximal shifts with increasing E_0 into the infrared region

$$\omega=2q_0\left(\frac{\varepsilon}{m}\right)^2\left(1-2,\frac{\Delta_{max}}{\varepsilon}\right)=\omega_{res}\left(1-2\frac{\Delta_{max}}{\varepsilon}\right). \tag{42}$$

The relative shift under the conditions of experiment of Ref. 2 is 3×10^{-3}. The value of the parameter μ under these conditions is $\mu_0 \approx 5$, which corresponds to the start of the saturation region. The estimate of the gain G by formula (41) yields in this case $G \sim 10\%$.

6. CONCLUSION

Thus, the principal new results of this paper are the determination of the saturation parameter μ (15), the derivation of the asymptotic formula (38), and the estimate of the functional dependences and of the values of the maximum gain (41) and of the spectral width and shift of the resonance (40), (42) at $\mu > 1$.

Formulas (38) and (41) for $\Delta\mathscr{E}$ and G, as well as the experessions for the parameters μ (15) and Δ_{max} (40), are classical. This does not mean, of course, that the electron behavior on the whole is in any way classical. For example, the distribution of the scattered electrons in energy can be essentially quantum-mechanical. An initially monoenergetic beam can break up into an assembly of satellites separated by $n\hbar\omega$, $n = \pm1, \pm2$ etc., and the distribution over which is characterized by quantum formulas of the type (33) or (39). The observation of this effect is possible in principle,[26] but calls for a beam that is highly monoenergetic $\Delta\varepsilon < \omega$, a condition not satisfied in the experiment of Ref. 2.

As to the gain G, it is classical in a wide range of variation of the parameters of the problem. It is therefore obvious that there should exist also another method of deriving the asymptotic formulas (38) and (40)–(42), based on a classical approach. At the present time there are no results of this type in the literature. In this connection we can compare our conclusions only with the results of the numerical solution of the classical equations in Ref. 11.

The saturation parameter (Ωt) used in Ref. 11 coincides with our parameter μ (15). In Ref. 11 they plotted the function $\Delta\mathscr{E}(t)$ at different values of the other parameters. Some of these curves are qualitatively similar to the curve shown in Fig. 1. Qualitatively we can trace also the decrease of the gain with increasing E_0. On the whole, however, the comparison of the analytic and numerical results is difficult, since the numerical calculations were not performed with such a comparison in mind, and do not reflect the entire multiparametric dependence of the physical quantities. In Ref. 11 they did not investigate the spectral properties of the gain. Some of the results of that reference cannot be understood. Thus, for example, the estimate of the maximum field energy reached in an ideal resonator is not understood. According to our results, even in the saturation region $\mu > 1$, the quantity $\Delta\mathscr{E}_{max}$ increases with increasing E_0 [(40), (41)]. Under ideal conditions the gain can continue for an arbitrarily long time with increasing number of passes. Actually the gain limit can be determined either by the number of passes or by the real losses that can become comparable with the gain G, which decreases with increasing E_0.

There is undoubted interest in an analytic solution of the classical equations of motion in the asymptotic limit of a strong field, and in a comparison of the results of the classical and quantum description. The analytic solutions, in contrast to the numerical solutions, make it possible to trace the functional dependences of the physical quantities and provide a better insight into the physics of the phenomenon.

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Kinetics of Doppler-spectrum saturation

A. G. Kofman and A. I. Burshteĭn

Institute of Chemical Kinetics and Combustion, Siberian Division, USSR Academy of Sciences
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The influence of spectral migration on the saturation kinetics is investigated. It is assumed that the radiation is monochromatic and that its interaction with the medium can be described in the balance approximation. A dependence of the spectral migration and of the method of realizing the latter on the frequency ν is observed at not too large and not too small field powers and of ν (migration-accelerated stage). The width of the saturation-induced dip in the population spectrum is either constant in time if the frequency jumps are comparable with the width of the spectrum, or else increases monotonically if the jumps are so small that they result additively in spectral diffusion.

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1. INTRODUCTION

It is known that when a powerful coherent field interacts with an ensemble of inhomogeneously broadened two-level systems the first to be saturated are those transitions whose frequency detunings are within the limits of the homogeneous width. This leads to the appearance of a dip in the density of the distribution of the population difference of the two-level systems with respect to the transition frequencies—a phenomenon which serves as the basis of numerous nonlinear spectroscopy effects.[1,2] On account of absorption on the wing of the inhomogeneous line, and also because of spectral migration in the course of time, the transitions farther away from resonance become successively saturated in the course of time. The width of the resultant dip increases until the interaction with the medium, which tends to produce an equilibrium distribution of the populations over the frequencies, establishes a stationary form of the dip.

Nonlinear spectroscopy methods can be used to study all the factors on which the shape of the dip depends: the characteristics of the saturating field itself, as well as the processes of spectral migration and relaxation due to the interaction with the medium. The study of the latter is important not only from the point of view of population of quantum generators, but also because they yield valuable information on the irradiated sample, information difficult or impossible to obtain by other methods.

However, whereas the influence of relaxation on the formation of a dip can in most cases of practical importance be adequately described by introducing into the theory the relaxation constants, the role of the spectral migration is a much more difficult to evaluate.

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