W. Let us point out that this value is smaller than the pump power by roughly five-seven orders of magnitude. Thus, $10^5 - 10^7$ resonators of the type under consideration can be placed one after another in one and the same pumping beam. Let us also note that, in the considered example, the luminous energy expendable in each of such resonators during the period $\tau_{ch}$ has a value of the order of $10^{-14}$ J.


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# Two-mode He–Ne laser in an axial magnetic field


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We consider the two-mode operation of a gas laser placed in an axial magnetic field. The dependences of the critical frequency, of the region of stable two-mode lasing, and of the frequency of the intermode beats on the magnetic field are investigated for different characters of polarizations of the fields of the interacting modes. It is observed that with increasing magnetic field the behavior of the critical frequency varies significantly with the character of the polarizations of the interacting-mode fields.


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Investigations, for example those reported in Refs. 1–3, have shown that an axial magnetic field affects strongly the emission characteristics of a single-mode gas laser. At the same time two-mode gas lasers with orthogonally polarized modes are presently offering great promise for use in a number of fundamental physical experiments and applications.[4] This explains the interest in the effect of the magnetic field on the characteristics of the emission of such lasers. In particular, even the very first investigations of a two-mode He–Ne/$CH_4$ laser in an axial magnetic field have shown that under certain conditions the amplitude of the output-power resonance increases substantially and the shift of the resonance peak as a result of the influence of the gain contour decreases. This improves greatly the characteristics of stabilized lasers of this type.

We report here, for the first time ever, the results of theoretical and experimental investigations of the influence of an axial magnetic field on the principal characteristics of two-mode gas lasers—the critical frequency, the range of continuous tuning of the intermode distance, the region of two-mode generation, and the frequency of the intermode beats at various polarizations of the interacting-mode fields.

## 1. EXPERIMENTAL RESULTS

The experiments were performed with an He–Ne laser operating on the $3s_2-3p_4$ transition of Ne ($j_a=2, j_b=1$, where $j_a$ and $j_b$ are the angular momenta of the lower and upper working levels, respectively). The distance $\omega_{12}$ between the axial modes was varied smoothly with a Fabry–Perot phase-anisotropy resonator in the form of two $\lambda/4$ plates.[5] The active medium was placed either between the phase plates or on the side of one of them. The magnitude and direction of the magnetic field were varied by changing the current flowing through the solenoid in which the discharge tube was placed.

By separating the emission of any mode with the aid

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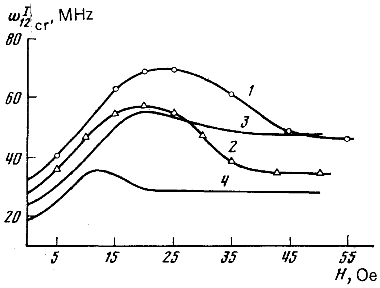


FIG. 1.a) Dependence of $\omega^{\rm I}_{12\rm cr}$ on $H$: curves 1, 2—experimental (1—$p=1.5$ Torr, 2—$p=1$ Torr); curves 3, 4—calculation (3—$p=2$ Torr, 4—$p=1$ Torr).

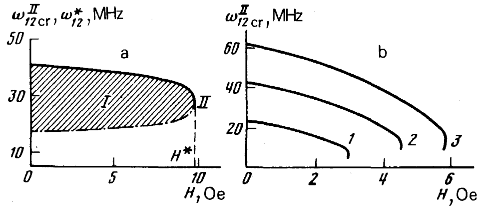


FIG. 2. Experimental plots of $\omega^{\rm II}_{12\rm cr}$ (solid line) and $\omega_{12}{}^*$ (dash—dot) against $H$ at $p=1.8$ Torr. I—single-mode generation region, II—stable two-mode generation region. b) Calculated plots of $\omega^{\rm II}_{12\rm cr}$ against $H$. Curve 1—$p=1.0$ Torr, 2—$p=1.5$ Torr, 3—$p=2.0$ Torr.

of the polarizer, it was possible to register the mode intensity $E_i^2$ ($i=1, 2$), the variation of $E_i^2$ with the tuning $\omega_{i0}=\omega_i-\omega_0$($\omega_i$ and $\omega_0$ are the respective frequencies of the lasing and of the center of the gain line), and the range $d$ of the two-mode generation ($d/2$ is the mode detuning, from the symmetrical position, at which stable two-mode generation breaks down). The intermode distance $\omega_{12}$ was measured by recording, with a spectrum analyzer, the beats between the modes separated with a fast-acting photodiode. A special electronic circuit made it possible to record the variation of the intermode beat frequency $\omega_{12}$ with the detuning.

Figure 1 shows plots of the critical frequency $\omega^I_{12\rm cr}$($\omega^I_{12\rm cr}$ is the minimal intermode distance when the modes are symmetrical with respect to the center of the gain line, at which the two-mode regime is still stable) as a function of the magnetic field $H$, at various pressures of the active medium, for the first resonator configuration (the active medium is located on one side of the phase plates). It is seen that the magnetic field always increases $\omega^I_{12\rm cr}$, i.e., it increases the coupling between the modes. The function $\omega^I_{12\rm cr}=f(H)$ exhibits a a certain resonance, wherein the maximum critical frequency $(\omega^{\rm I}_{12\rm cr})_{\max}$ increases and the maximum of the field dependence shifts towards larger $H$ with increasing $p$.

For the second resonator configuration (active medium located between the phase plates) it was shown earlier[5] that, in contrast to the preceding case, two-mode generation is realized not only at intermode distances exceeding some value, designated here $\omega^{\rm II}_{12\rm cr}$ ($\omega^{\rm II}_{12}>\omega_{12\rm cr}$), but also in the region of small intermode distances $\omega_{12\min}<\omega_{12}<\omega_{12}{}^*$ (in this case $\omega_{12\min}<\omega_{12}{}^*<\omega^{\rm II}_{12\rm cr}$), where $\omega^*_{12}$ has the meaning of the second critical intermode distance ($\omega^*_{12}\sim 10$ MHz). The mimimum value of $\omega_{12}$($\omega_{12\min}$) is determined in the latter case by the mode locking.[6] In the intermediate region $\omega^*_{12}<\omega^{\rm II}_{12\rm cr}$ a single-mode regime is realized. When the magnetic field $H$ is applied, as seen from Fig. 2a, the critical frequency $\omega^{\rm II}_{12\rm cr}$ decreases, and $\omega^*_{12}$ increases. The net result is that at $H>H^*$ the intermediate single-mode region vanishes. It was shown in Ref. 7 that stable two-mode generation takes place, with smooth variation of $\omega_{12}$, in a range from practically zero to $c/2L$ ($c$ is the speed of light and $L$ is the resonator length) on transitions with working-level angular momenta (1, 0) and (1, 1). It follows from the foregoing results that this problem can be solved with the aid of a weak magnetic field ($H\gtrsim H^*$) also for a transition with level angular-momenta (1, 2).

In the employed resonator configurations, the character of the field polarization of the interacting modes in the active medium is different.[5] It will be shown below that it is precisely this circumstance which leads to the substantial difference in the behavior of $\omega^{\rm I}_{12\rm cr}(H)$ and $\omega^{\rm II}_{12\,\rm cr}(H)$.

Figure 3 shows plots of the region of the two-mode generation $d$ against $H$ for different $\omega_{12}$ at the same resonator configurations. These plots agree with the behavior of the quantities $\omega^{\rm I,II}_{12\,\rm cr}(H)$. In fact, in the former case (Fig. 3a) at $\omega_{12}<(\omega^{\rm I}_{12\,\rm cr})_{\max}$ there exists a certain region of values of $H$ where $\omega_{12}<\omega^{\rm I}_{12\,\rm cr}$ (Fig. 1) and consequently there is no two-mode regime. When this strong-coupling region is approached, the value of $d$ decreases rapidly to zero (Fig. 3a, curve 1). If $\omega_{12}>(\omega^{\rm I}_{12\,\rm cr})_{\max}$, the two modes generate stable and $d\neq 0$ at all $H$, but in the region of resonant values $\omega^{\rm I}_{12\rm cr}$, where the critical frequency approaches $\omega_{12}$, a noticeable decrease of $d$ takes place (Fig. 3a, curve 2).

For the second resonator configuration, an increase of the magnetic field, in accordance with the behavior of $\omega^{\rm II}_{12\,\rm cr}=f(H)$ (Fig. 2), always leads to an increase of $d$ (Fig. 3b). If $\omega_{12}>\omega^{\rm II}_{12\,\rm cr}(H=0)$, then two-mode generation is realized for all $H$; if $\omega_{12}<\omega^{\rm II}_{12\,\rm cr}(H=0)$ it is realized for $H>H^*$, where $H^*$ is determined by the equation $\omega^{\rm II}_{12\,\rm cr}(H^*)=\omega_{12}$ (Fig. 3b, curve 2).

The magnetic field influences strongly also the frequency characteristic of the two-mode laser—$\omega_{12}$ changes when the mode spectrum is tuned along the gain contour. Figure 4a shows four characteristic plots of $\omega_{12}=f(\omega_{10})$. It is seen that an increase of $H$ causes reversal of the indicated dependence, and a definite $H=H_{\rm rev}$ the frequency of the intermode beats changes exceedingly weakly with the detuning (the slope of curve 2 in Fig. 4

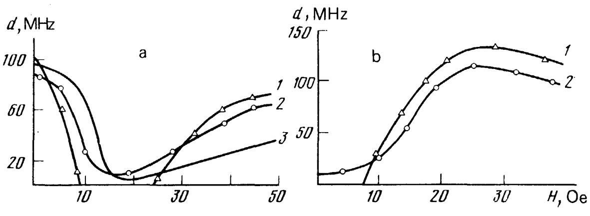


FIG. 3. a) Plots of $d(H)$ for resonator configuration I and $p=1.5$ Torr, curve 1—for $\omega^{\rm I}_{12}=40$ MHz, 2—for $\omega^{\rm I}_{12}=80$ MHz, 3—for $\omega^{\rm I}_{12}=45$ MHz (calculated dependence), b) $d(H)$ for configuration II of the resonator and $p=1.5$ Torr, curve 1—for $\omega^{\rm I}_{12}=35$ MHz, 2—for $\omega^{\rm I}_{12}=60$ MHz.

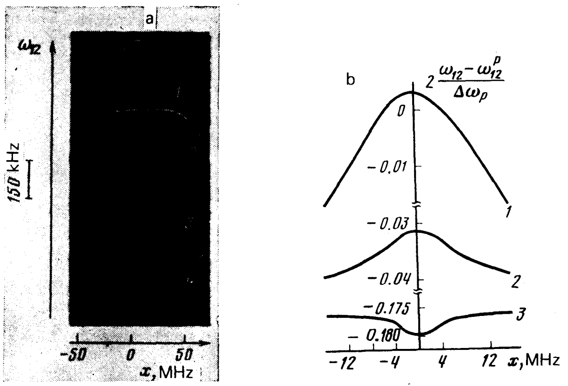


FIG. 4. a) Experimental plots of $\omega_{12}=f(x)$ for $\omega_{12}^{r}=55$ MHz, $p=1.5$ Torr, and resonator configuration II: 1) $H=0$, 2) $H=20$ Oe, 3) $H=28$ Oe, 4) $H=40$ Oe. b) Calculated plots of $\omega_{12}=f(x)$ for $\omega_{12}^{r}=35$ MHz and $p=1$ Torr ($\Delta\omega_r$ is the resonator bandwidth). Curve 1—for $H=0$, 2—for $H=10$ Oe, 3—for $H=25$ Oe.

is $\sim 10^{-5}$). Investigations have shown that a frequency characteristic with a small slope can be obtained for all $\omega_{12}$ and $p$, and the corresponding value of $H_{\text{rev}}$ increases with increasing $\omega_{12}$ and $p$. We note that in two-mode lasers without a magnetic field, the inversion of the frequency characteristic takes place only at a definite ratio of $\omega_{12}$ to $p$.[8]

## 2. INTERPRETATION AND ANALYSIS OF RESULTS

The experimental results have a simple physical explanation if the standing-wave field of the resonator modes is represented in the form of an equivalent set of circularly polarized traveling waves and the contribution to the nonlinear resonant interaction of the modes of each pair of these circular components is considered.

Waves with like polarization interact with one and the same $\sigma$ component of the Zeeman doublet. Its shift in a magnetic field is physically equivalent to a change, equal in magnitude and in sign, of the detunings of these waves relative to the center $\omega_0$ of the transition in the absence of a magnetic field. Oppositely polarized waves interact with different $\sigma$ components. The shift of the latter is equivalent to a change of equal magnitude but of opposite sign in the wave frequencies in a zero magnetic field. The singularities of the nonlinear interaction of two traveling waves are known.[9,10] By replacing the action of the magnetic field by an equivalent frequency shift and taking into account (in the case of inhomogeneous broadening of the working transition) the relative propagation direct in each pair of circular components of the mode field, it is easy to interpret the experimentally observed properties of the radiation of the two-mode laser. The fact that this approach does not take into account the spatial inhomogeneity of the saturation of the medium in the standing-wave field is of no significance for a qualitative analysis.

By way of example, we shall restrict ourselves to an explanation of the difference in the behavior of $\omega_{12\,\text{cr}}^{I}$ and $\omega_{12\,\text{cr}}^{II}$ (Figs. 1 and 2). It is known that realization of stable generation of two modes is firmly connected with the ratio of the saturations of the gain by the self-field of the mode $\omega_1$ (the coefficient $\beta_1$ in the standard notation) and by the cross saturation of the gain at the frequency $\omega_1$ by the field of the mode generating at the frequency $\omega_2$ (the coefficient $\theta_{12}$). In the case of symmetrical mode detuning ($\omega_{10}=-\omega_{20}$) at $\beta_1>\theta_{12}$, the generation of both modes is stable, at $\beta_1<\theta_{12}$ single-mode generation is realized at the frequency $\omega_2$, while the equality $\beta_1=\theta_{12}$ determines the critical frequency $\omega_{12\,\text{cr}}$. It is clear that to explain the $\omega_{12\,\text{cr}}(H)$ dependence it suffices to consider the behavior of the ratio $\beta_1/\theta_{12}$ in a magnetic field under the condition $\omega_{10}=-\omega_{20}$.

The simplest set of circular components corresponds to the modes of a resonator in which the active medium is placed between the phase plates and $\omega_{12}\gg\omega_{12\,\text{cr}}^{II}$ (Fig. 2a). According to Ref. 5, the standing wave of each mode can in this case be represented in the form of two opposing traveling waves with opposite circular polarizations. The components of the two neighboring modes propagating in the same direction are differently polarized:

$$E(z,t)=e_-E_{1-}\exp[i(k_1z-\omega_1t)]+e_+E_{1+}\exp[i(-k_1z-\omega_1t)]$$
$$+e_-E_{2-}\exp[i(-k_2z-\omega_2t)]+e_+E_{2+}\exp[i(k_2z-\omega_2t)]+\text{c.c.},\qquad(1)$$

where $e_\pm=(e_x\pm ie_y)/\sqrt{2}$, $E_{1\pm,2\pm}$ are the amplitudes of the waves with frequencies $\omega_{1,2}$ with right (+) or left(−) circular polarization, and $k_1\approx k_2=k$. We emphasize that at $\omega_{12}\geqslant\omega_{12\,\text{cr}}^{II}$ we have $E_{1+}\approx E_{1-}$, $E_{2+}\approx E_{2-}$.

For simplicity we assume that the transition broadening is essentially inhomogeneous, and the Zeeman sublevels are strongly intermixed by depolarizing collisions. In this limit we can neglect the contribution made to the saturation by the two-quantum processes in the interaction of the components of the field (1).[10] The resonant interaction reduces to pure population effects, which admit of a simple intuitive interpretation. Each of the four traveling waves in (1) is at resonance with atoms whose velocities are determined by the corresponding Doppler shifts:

$$kv_{1-}=\omega_{10}-\Omega,\quad kv_{1+}=-\omega_{10}-\Omega,\quad kv_{2-}=-\omega_{20}+\Omega,$$
$$kv_{2+}=\omega_{20}+\Omega,\qquad(2)$$

where $\Omega$ is the absolute magnitude of the shift of the $\sigma$ component, $\Omega=g\mu_BH$, $\mu_B$ is the Bohr magneton, and $g$ is the Landé factor (it is assumed that the $g$ factors of both levels are equal).

The standing wave of the mode $\omega_1$ is produced by two opposing waves, $E_{1-}$ and $E_{1+}$. In the absence of a magnetic field, the "holes" which they burn in the inversion are symmetrical about $v=0$. When a magnetic field is applied, these holes shift in like fashion in the velocity scale. The degree of their overlap does not change, and and therefore (provided that $\omega_{10}\pm\Omega\ll ku$, where $u$ is the average thermal velocity of the atoms), $\beta_1$ does not depend on the magnetic field.

In the symmetrical regime ($\omega_{10}=-\omega_{20}$) in the absence of a magnetic field, the wave pairs $E_{1-}$, $E_{2-}$ and $E_{1+}$, $E_{2+}$ interact with the same atoms (see Eq. (2)). The holes burnt out by the field of the mode $\omega_1$ are completely overlapped by the holes of mode $\omega_2$ (Fig. 5a). This corresponds to the maximum possible $\theta_{12}$ at the given $\omega_{10}$ and $\omega_{20}$. In a nonzero magnetic field the velocities

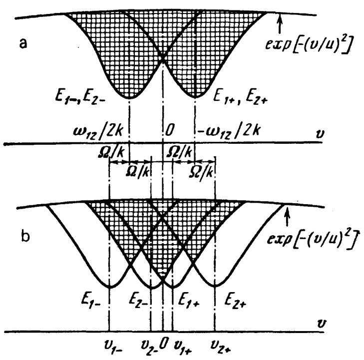

FIG. 5. Illustration of the behavior of $\omega_{12\,cr}^{II}(H)$, the cross hatching shows the overlap of the "holes" in a zero (a) and a nonzero (b) magnetic field.

of the resonant atoms change in the same manner for the circular components of each mode, but with different modes [see (2)]. As a result (Fig. 5b) the degree of overlap of the holes decreases, i.e., $\theta_{12}$ decreases with increasing $H$. If $\beta_1$ remains unchanged, this corresponds to a decrease of $\omega_{12\,cr}^{II}$, which agrees qualitatively with experiment (Fig. 2a).

If the active medium is located outside the phase plates, the neighboring modes are polarized linearly and orthogonally.[5] The field of the standing wave at the frequency of each mode is equivalent to four traveling waves (one pair of opposing waves for each polarization). The self-saturation of $\beta_1$ in this configuration, in contrast to (1), is determined by the overlap of four holes in the inversion, while that of $\theta_{12}$ is determined by eight holes. Omitting the obvious arguments and constructions, which are analogous to Fig. 5, we present the final result: an increase in $H$ results in a decrease of not only $\theta_{12}$, as for configuration (1), but also of $\beta_1$, and the decreases of the two quantities are similar. This corresponds to independence of $\omega_{12\,cr}^{I}$ of the magnetic field. Thus, even the pure population-dependent model results in a qualitative difference in the behavior of $\omega_{12\,cr}$ for the two configurations of the anisotropic resonator: $\omega_{12\,cr}^{II}$ decreases with increasing magnetic field, and $\omega_{12\,cr}^{I}$ remains at the most unchanged.

The nonmonotonic behavior of the experimental $\omega_{12\,cr}(H)$ curve (Fig. 1) becomes understandable when the two-photon processes are taken into account. It is known[9] that in the limit of inhomogeneous broadening two-photon resonance takes place only when waves propagating in the same direction interact. On the other hand, it was noted above that the influence of the magnetic field on the interaction of equally polarized waves is equivalent to their parallel shift relative to $\omega_0$. Assuming that $\omega_{10}, \omega_{20}, \Omega \ll ku$, the cross saturation of these waves can be regarded as independent of $H$. The change of $\beta_1/\theta_{12}$ is connected only with the change of the contribution of the two-photon interaction of equally directed but differently polarized circular components of the modes $\omega_1$ and $\omega_2$. We shall not list here the rather cumbersome set of all the possible pairings of these components. It suffices to consider two characteristic pairs, for example $E_{1+}$, $E_{2-}$ and $E_{1-}$, $E_{2+}$. If for one of these there is, say $E_{1+}, E_{2-}$ the shift of the corresponding $\sigma$ components in the magnetic field is equivalent to a decrease of $|\omega_{10}, \omega_{20}|$, then for the other, obviously, it is equivalent to a like increase. It is known[9] that the interaction of the first pair is enhanced in this case and that of the second is weakened. By virtue of the nonlinear singularities of the frequency shape of the two-photon resonance, the total contribution to $\theta_{12}$ increases, as is reflected in the increase of $\omega_{12\,cr}^{I}$ (Fig. 1). As the frequencies of the waves $E_{1-}$ and $E_{2+}$ corresponding to $\omega_{12\,cr}$ come closer to the centers of the $\sigma$ components, the coupling between them becomes maximal, a fact reflected by the maximum of the $\omega_{12\,cr}^{I}(H)$ curve in Fig. 1. With further increase of the magnetic field, this coupling becomes weaker, resulting in a decrease of $\omega_{12\,cr}^{I}$. With decreasing two-photon interaction of the differently polarized components of the mode fields, the fraction of the cross saturation of components with like polarization, which does not depend on the magnetic field, becomes increasingly predominant. This is reflected in the flattening of the $\omega_{12\,cr}^{I}(H)$ curve (Fig. 1).

For the other resonator geometry, allowance for the two-quantum processes does not introduce any substantial changes in the picture of the decrease of $\omega_{12\,cr}^{II}$ with increasing magnetic field (Fig. 2a). As to the increase of $\omega_{12}^{*}(H)$, it is apparently of the same nature as the behavior of $\omega^{I}_{12\,cr}$ (Fig. 1). In fact, according to Ref. 5, in the space between the phase plates the neighboring modes of the resonaot become deformed as $\omega_{12} \to 0$ into modes that are linearly polarized and are orthogonal to each other. Accurate to within a spatial shift, their field is the same as outside the phase plates. Therefore the analogy in the interpretation of the growth of both $\omega_{12}^{*}$ and $\omega_{12\,cr}^{I}$ with increasing magnetic field is perfectly legitimate.

It follows from all the foregoing that the differences between the characteristics of the two-mode generation regime in the magnetic field at different configurations of the phase anisotropy of the resonator are due to the differences in the character of the mode polarization.

For a quantitative analysis of the experimental results, using the standard procedure of expanding the density matrix in irreducible tensor operators,[11] we have obtained in third-order perturbation theory expressions for the polarization of the medium with allowance for the degeneracy of the working levels at an arbitrary ratio of the homogeneous and inhomogeneous broadenings, and for the presence of depolarizing collisions. Leaving out the rather cumbersome general formulas, we present expressions for $\beta_1$ and $\theta_{12}$ in the particular case of inhomogeneous broadening of the transition under symmetrical mode detuning ($\omega_{10} = -\omega_{20} = \omega_{12}/2$) and $\omega_{12}, \Omega \ll ku$.

In the configuration when the phase plates are on one side of the active medium we have

$$\beta_1{}^{I}(\omega_{10}) \sim \sum_{i,\varkappa} \Big\{ \frac{a_i^{\varkappa}}{\gamma_i^{(\varkappa)}} \Big[ 1 + \frac{\gamma^2}{\gamma^2 + (\omega_{10}+\Omega)^2} \Big]$$

$$+ \frac{b_i^{(\varkappa)}}{\gamma_i^{(\varkappa)}} \Big[ \frac{\gamma^2}{\gamma^2+\Omega^2} + \frac{\gamma^2}{\gamma^2+\omega_{10}{}^2} \Big] + \frac{c_i^{(\varkappa)}}{\gamma_i^{(\varkappa)2}+4\Omega^2} \Big[ \frac{\gamma\gamma_i^{(\varkappa)} - 2\Omega^2}{\gamma^2+\Omega^2}$$

$$+ \frac{\gamma\gamma_i^{(\varkappa)} - 2\Omega(\omega_{10}+\Omega)}{\gamma^2+(\omega_{10}+\Omega)^2} \Big] + \Big[ \begin{matrix}\text{the same but with} \\ \Omega \to -\Omega\end{matrix} \Big] \Big\}, \qquad (3)$$

$$\theta_{12}^{I}(\omega_{12}) \sim \sum_{i,\varkappa} \left\{ \frac{a_i^{(\varkappa)}}{\gamma_i^{(\varkappa)}} \left[ \frac{\gamma^2}{\gamma^2+\Omega^2} + \frac{4\gamma^2}{4\gamma^2+\omega_{12}^2} \right] + \frac{b_i^{(\varkappa)}}{\gamma_i^{(\varkappa)}} \left[ 1 + \frac{4\gamma^2}{4\gamma^2+(\omega_{12}+2\Omega)^2} \right] \right.$$
$$- \frac{c_i^{(\varkappa)}\gamma}{\gamma_i^{(\varkappa)2}+4\Omega^2} \left[ \frac{\gamma\gamma_i^{(\varkappa)}-2\Omega}{\gamma^2+\Omega^2} + \frac{4\gamma\gamma_i^{(\varkappa)}-4\Omega(\omega_{12}+2\Omega)}{4\gamma^2+(\omega_{12}+2\Omega)^2} \right]$$
$$+ \frac{2(a_i^{(\varkappa)}-b_i^{(\varkappa)}n_2/n_-)}{\gamma_i^{(\varkappa)2}+\omega_{12}^2} \frac{\gamma(2\gamma\gamma_i^{(\varkappa)}-\omega_{12}^2)}{4\gamma^2+\omega_{12}^2}$$
$$- \frac{2(b_i^{(\varkappa)}-a_i^{(\varkappa)}n_2/n_-)\gamma[2\gamma\gamma_i^{(\varkappa)}-\omega_{12}(\omega_{12}+2\Omega)]}{(\omega_{12}^2+\gamma_i^{(\varkappa)2})[4\gamma^2+(\omega_{12}+2\Omega)^2]}$$
$$\left. + \frac{2c_i^{(\varkappa)}(1+n_2/n_-)\gamma[2\gamma\gamma_i^{(\varkappa)}-(\omega_{12}+2\Omega)^2]}{[\gamma_i^{(\varkappa)2}+(\omega_{12}+2\Omega)^2][4\gamma^2+(\omega_{12}+2\Omega)^2]} + \left[ \begin{matrix}\text{the same but with}\\ \Omega\to-\Omega\end{matrix} \right] \right\}. \quad (4)$$

Here $\gamma$ is the homogeneous half-width of the transition, $i=a, b; \varkappa=0, 1, 2; \gamma_i^{(1,2)}=\gamma_i^{(0)}+\Gamma_i^{(1,2)}p$ are the probabilities of decay of the dipole magnetic and quadrupole electric moments of the level $i$, respectively, $\gamma_i^{(0)}$ is the radiative width of the $i$-th level, $\Gamma_i^{(1,2)}$ are the corresponding constants of the depolarizing collisions; for the transition $j_b=1\to j_a=2$ we have $a_a^{(0)}=1/45, a_a^{(1)}=1/40, a_a^{(2)}=7/1800, a_b^{(0)}=1/27, a_b^{(1)}=1/72, a_b^{(2)}=1/5400, b_i^{(0,2)}=a_i^{(0,2)}, b_i^{(1)}=-a_i^{(1)}, c_i^{(0,1)}=0, c_a^{(2)}=7/300, c_b^{(2)}=1/900$; $n_2/n_-$ is the ratio of the second spatial Fourier component of the inverted population to its mean value.[12] At $\Omega=0$ formulas (3) and (4) go over, accurate to within allowance for the dragging of the resonant radiation from the upper level $b$, into expressions (III–26) and III–27) of Ref. 12 for orthogonally polarized modes in the absence of a magnetic field.

If the active medium is placed between the phase plates, then in the region $\omega_{12}$, where the mode field is given by (1), we have

$$\beta_1^{II}(\omega_{10}) \sim 2\sum_{i,\varkappa} \left( \frac{a_i^{(\varkappa)}}{\gamma_i^{(\varkappa)}} + \frac{\gamma^2}{\gamma^2+\omega_{10}^2} \frac{b_i^{(\varkappa)}}{\gamma_i^{(\varkappa)}} \right), \quad (5)$$

$$\theta_{12}^{II}(\omega_{12}) \sim \sum_{i,\varkappa} \left\{ \frac{a_i^{(\varkappa)}}{\gamma_i^{(\varkappa)}} \frac{\gamma^2}{\gamma^2+\Omega^2} + \frac{b_i^{(\varkappa)}}{\gamma_i^{(\varkappa)}} \frac{4\gamma^2}{4\gamma^2+(\omega_{12}+2\Omega)^2} \right.$$
$$+ \frac{2c_i^{(\varkappa)}\gamma}{\gamma_i^{(\varkappa)2}+(\omega_{12}+2\Omega)^2} \left[ \frac{2\gamma\gamma_i^{(\varkappa)}-(\omega_{12}+2\Omega)^2}{4\gamma^2+(\omega_{12}+2\Omega)^2} \right.$$
$$\left. \left. + \frac{n_2}{n_-} \frac{2\gamma\gamma_i^{(\varkappa)}-\omega_{12}(\omega_{12}+2\Omega)}{4\gamma^2+\omega_{12}^2} \right] + \left[ \begin{matrix}\text{the same but with}\\ \Omega\to-\Omega\end{matrix} \right] \right\}. \quad (6)$$

It is easily seen that expressions (3)–(6) describe the regularities that govern the behavior of $\beta_1$ and $\theta_{12}$ in the magnetic field and were qualitatively discussed above. In particular, the dependence of $\beta_1^{II}$ on the magnetic field, obtained above from general considerations, is confirmed analytically by (5). In the region of small $\omega_{12}$, at the same resonator configuration, expression (1) no longer describes the mode fields. The character of their polarization changes abruptly with changing $\omega_{12}$ (Ref. 5). Therefore a theoretical analysis of the behavior of $\omega_{12}^{*}(H)$ (Fig. 2a) is quite difficult.

Using the foregoing expressions for the coefficients $\beta_1$ and $\theta_{12}$ we can obtain lucid analytic expressions for $\omega_{12\,cr}$, $d$, and $\omega_{12}=f(\omega_{10})$. It is convenient to carry out the analysis here for different values of $H$.

Thus, for example, in the region of weak magnetic fields $(\Omega\leqslant\gamma_a)$ we have

$$\omega_{12\,cr}^{I} \approx \omega_{12\,cr}^{I}(H=0)(1+8\Omega^2/\gamma_a^2)^{1/2}, \quad (7)$$

where $\omega_{12\,cr}^{I}(H=0)$ is the critical frequency in the absence of a magnetic field. For large $H(\Omega\geqslant\gamma)$ we have

$$\omega_{12\,cr}^{I} \approx \left[ 2\gamma\gamma_a \left( 1+\frac{\gamma^2}{\gamma^2+\Omega^2} \right) \right]^{1/2}. \quad (8)$$

It follows from (7) that $\omega_{12\,cr}^{I}$ in regions of small $H$ increases with increasing $H$, and in strong magnetic fields (8) decreases with increasing $H$ and tends in the limit (at $\Omega\gg\gamma$) to the value $(2\gamma\gamma_a)^{1/2}$, thus confirming the experimental results (Fig. 1, curves 1 and 2). An examination of the behavior of $\omega_{12\,cr}^{II}(H)$ and $d(H)$ has also confirmed the experimental behavior of $\omega_{12\,cr}^{II}(H)$ and of $d(H)$.

Together with the qualitative analysis of the behavior of $\omega_{12\,cr}(H)$ and $d(H)$, we obtained with a computer the dependences of the characteristics of a two-mode laser in a magnetic field for the ratios $\gamma/ku$ of the $3s_2-3p_4$ transition of Ne, which takes place at realistic pressures of the helium–neon mixture. The dragging of the resonant radiation from the $3s_2$ level was taken into account by renormalizing the lifetime of this level.[10] It is seen from Figs. 1–3 that the theoretical $\omega_{12\,cr}^{I}(H)$, $\omega_{12\,cr}^{II}(H)$, $d(H)$ curves are in qualitative agreement with the experimental ones. The quantitative difference is due apparently to the disparity between the third-order perturbation theory and the real field intensities of the generated modes.

We stop now to analyze the behavior of the frequencies of the intermode beats. Using the expressions for the generation frequencies $\omega_i$, the values of the coefficients $\sigma_i, \rho_i, \tau_{ij}$ with account taken of the magnetic field, and denoting by $x$ the mode detuning from the symmetrical position $(\omega_{10}=1/2\,\omega_{12}+x,\ \omega_{20}=-1/2\,\omega_{12}+x)$, we can obtain a rather simple expression for the frequency of the intermode beats as a function of the detuning. We assume here that the modes intensities are equal and remain constant at small detunings of the modes from the symmetrical position and that $x\ll\omega_{12}^{r}$, $\gamma_1\Omega$ (where $\omega_{12}^{r}$ is the distance between the modes of the empty resonator). Under these assumptions, the expression for the intermode beats (at $\Omega\gg\gamma$) takes the form

$$\omega_{12}(x) \sim -x^2 \left[ \frac{12\gamma^2-\omega_{12}^{p2}}{4} - \frac{3(80\gamma^4-40\omega_{12}^{p2}\gamma^2+\omega_{12}^{p4})\Omega^2}{(\omega_{12}^{p2}+4\gamma^2)^2} \right]. \quad (9)$$

The reversal condition, which is obtained from the condition that the frequency of the intermode beats be independent of $x$ at small detunings (i.e., that the coefficient of $x^2$ be equal to zero) is written for the experimental values $\omega_{12}^{r}<\gamma$ in the form

$$\Omega_{rev}^2 \approx (\gamma_{rev}^2+\omega_{12rev}^{r2})/5. \quad (10)$$

It is seen from (10) that, without changing the laser parameters $(\omega_{12}^{r}, \gamma)$ it is possible to obtain reversal by means of an external parameter—the magnetic field. We note here that the nature of the reversal of the frequency characteristic is connected mainly with the strong strong dependence of the coefficients $\rho_i$ on the resonator detuning $x$ and on the parameters $\omega_{12}^{r}$, $\gamma$, and $\Omega$.

The qualitative explanation of the behavior of $\omega_{12}(x)$ as a function of the magnetic field, presented above, was confirmed by a computer calculation (Fig. 4b). It is seen from the presented curves that, just as in the experiment, the shapes of the plots of $\omega_{12}=f(\omega_{10})$ change with increasing magnetic field.

Thus, our investigations of the characteristics of of two-mode He–Ne lasers with orthogonally polarized modes in an axial magnetic field have revealed a number of new effects that add greatly to our concepts concerining the physics of the interaction of modes and on the feasibility of lasers of a new type.

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# Stochastic mechanism of excitation of molecules that interact with their own radiation field

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Results are presented of a numerical experiment on the investigation of the interaction of a system of quantum nonlinear oscillators (molecules) with their own radiation field and with an external coherent field of laser radiation. The conditions under which a stochastic regime of excitation of high-lying levels sets in in such a system are obtained. It is shown that in the region of stochasticity of the motion the level population distribution function is substantially altered. An increase of the populations of the high-lying levels takes place simultaneously with generation of a self-consistent radiation field in the system. The character of the level population when several external fields that are resonant to various transitions of the nonlinear oscillator act on the system is investigated in the presence of stochastic instability in the system. The described phenomenon can be regarded as one of the possible mechanisms of stochastic excitation of polyatomic molecules into the region of high-lying levels of the vibrational spectrum.

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## 1. INTRODUCTION

Much attention is being paid presently to the interaction of coherent laser radiation with multilevel quantum systems. Particular interest attaches to the possibility of selective action of laser radiation on matter, to control of the rates of chemical reactions, and to collisionless dissociation of polyatomic molecules.[1-3] The main difficulties faced by a consistent theoretical interpretation of such results lie in the study of the possible mechanisms that lead to excitation of the molecule to high-lying levels of the vibrational spectrum under conditions of large anharmonicity. The process of vibrational excitation of a molecule was arbitrarily divided in Ref. 4 into two stages, the passage through the low levels and the subsequent excitation of the molecules into the region of large quantum numbers. Resonant character of the interaction was ensured there by a high density of the high-lying vibrational states. A mechanism of rotational compensation of the anharmonicity was proposed in Ref. 5 for low-lying levels. Tunneling in energy space was proposed[6] as a possible mechanism of passage through low-lying levels. Molecule excitation was considered in Ref. 7 as the consequence of the onset of stochastic instability due to the interaction of intermode resonances.

It should also be noted that under certain condition an important factor in the description of the behavior of systems of the indicated type is the allowance for the


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