collectors (see Ref. 1) the first transfers were carried out after evaporation of each eight-liter portion of water at a filter temperature 50°C (each portion to a new collector). The second transfers were made with accumulation of the residual salts after evaporation of ~40 liters of water (filter and crucible temperature 200°C). Then the salt was removed from the crucible, powdered, and studied as a new material with the complete scheme. In order to reduce the contamination of the collectors by vapors of the salts, during the last stage the crucible was heated only to 600°C.

EXPERIMENTAL RESULTS

In the measurements with the technique described above, which as previously[1] were designed for quark ions of both signs of charge, in none of the tests did we observe effects which would be explained by the existence of the desired particles, and as a rule the intensity of light did not exceed the intrinsic background of the heated source. For this reason in estimates of the upper limits of the concentration of fractionally charged particles (quarks) in the materials studied, we considered all particles recorded in the measurements to be quarks. The results of the estimates are given in the table. In comparison with the results of Ref. 1 the values of the lower limits for quark concentration for water and the clay-silts are lower, roughly in accordance with the increase in sample size, and for the concretions they are lower by almost two orders of magnitude—the latter being due to the unfavorable background situation in the previous experiments with concretions.[1] Unfortunately, as a result of the lack of material, we were unable to study large samples of volcanic lava, for which in the previous work[1] we observed weakly expressed effects similar to those sought.

TABLE I.

Material studied	Upper limits of concentration, (quarks/nucleon)			
	In detection of negative particles		In detection of positive particles	
	$T_s = 25°$ C	$T_s = 25-470°$ C	$T_s = 25°$ C	$T_s = 25-280°$ C
Sea water (84 liters)	$8\cdot10^{-29}$	$6\cdot10^{-28}$	$1\cdot10^{-28}$	$5\cdot10^{-28}$
Finely divided clays and silts (23 kg)	$4\cdot10^{-28}$	$2\cdot10^{-27}$	$3\cdot10^{-28}$	$4\cdot10^{-27}$
Concretions (16 kg)	$4\cdot10^{-28}$	$4\cdot10^{-27}$	$5\cdot10^{-28}$	$3\cdot10^{-27}$

Thus, the result of the search is negative also in this series of measurements. However, taking into account the success of the quark model (which has again brought to life the question of the reality of quarks), one can with a certain amount of optimism consider our results as a further proof of the unobservability of free quarks.[2]

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Translated by Clark S. Robinson

Wave function with spin on a light front

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A method is developed for constructing relativistic wave functions with spin on a light front. The spin structure of the wave function of a deuteron in the relativistic region is obtained. The calculation procedures are illustrated by a determination of the *pd*-scattering cross section. The described construction is equivalent to solving the problem of allowance for the spins and angular momenta in the parton wave functions in a system with infinite momentum.

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1. INTRODUCTION

The author has previously[1] developed a formalism of wave functions (WF) on a light front, which describe relativistic systems consisting of zero-spin particles and having zero total angular momentum. The need for developing a covariant formalism, convenient for use in practice, for the description of nuclei when the relative momenta of the nucleons are of the order of their masses and of elementary particles within the framework of the composite models, is brought about by modern experimental data. Wave functions in relativistic coordinate space were discussed in the papers by Shapiro.[2] A review of the experimental situation in relativistic nuclear physics, of the theoretical problems that are raised, and of the existing approaches is presented in Ref. 3.

The wave fronts on the light front are the components of a Fock column of the wave vector of state in the

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invariant Schrödinger representation (ISR), which differs from the ordinary Schrödinger representation at $t=0$ in the fact that the state vector in the invariant Schrödinger representation is defined on an arbitrary flat space-like hypersurface, which is conveniently chosen to be the hypersurface of the light front $\omega x=\omega_0 t-\boldsymbol{\omega}\mathbf{x}=0$, where ω is a four-vector: $\omega=(\omega_0, \boldsymbol{\omega})$, $\omega^2=0$, $\omega_0>0$.

The advantage of covariant wave functions on a light front lies in the fact that they have a probabilistic interpretation, they depend on three-dimensional vectors, and they are connected in simple fashion with the vertex parts of a special diagram technique that makes it possible to express the amplitudes of the processes in terms of these wave functions. Problems that arise when the amplitudes are expressed in terms of wave functions on the light front were investigated in Ref. 4. The diagram technique is based on a three-dimensional formulation of field theory developed by Kadyshevskiĭ.[5] For the case of the light front, it is an invariant generalization of old perturbation theory in a system with infinite momentum (SIM), and the wave functions on the light front can be regarded as a covariant generalization of the equal-time Fock components in the SIM. We note that wave functions in a system with infinite momentum have found extensive use in parton models.

In the present paper we consider wave functions on the light front for a system with spin. Because of their connection with the wave functions in the system with infinite momentum, the construction of such wave functions is equivalent to automatic solution of the problem of taking the spins and angular momenta into account in parton wave functions. The need for solving this theoretical problem was noted by Feynman.[6]

In Sec. 2 we recall briefly the principal results of Ref. 1, which pertain to wave functions on a light front for the case of zero-spin particles. In Sec. 3 we construct the wave functions with spin. In Sec. 4, the developed formalism is illustrated by using as an example the calculation of the cross section of the elastic pd scattering within the framework of the mechanism of single-nucleon exchange.

2. WAVE FUNCTION OF ZERO-SPIN SYSTEM

The wave function $\psi(x_1, x_2, p, \omega)$ on a light front is a coefficient in the expansion of the state vector $\Phi(p)$, defined on the hypersurface $\omega x=0$, in terms of the states of the free particles:

$$\Phi(p)=\int\psi(x_1, x_2, p, \omega)\varphi^{(+)}(x_1)\varphi^{(+)}(x_2)|0\rangle \times\delta(\omega x_1)d^4x_1\delta(\omega x_2)d^4x_2+\ldots, \tag{1}$$

where

$$\varphi^+(x)=\int e^{ikx}c^+(\mathbf{k})\frac{d^3k}{(2\varepsilon_k)^{1/2}}, \tag{2}$$

and $c^+(\mathbf{k})$ is the creation operator. The state vector is determined by an aggregate of components corresponding to states with different numbers of particles. Their contribution to (1) is designated by the three dots. For simplicity we consider below only the two-particle component. In momentum space, taking (2) and the condition $\omega x=0$ into account, we have

$$\Psi(k_1, k_2, p, \omega)=\int\psi(x_1, x_2, p, \omega)\exp(ik_1x_1+ik_2x_2) \times\delta(\omega x_1)d^4x_1\delta(\omega x_2)d^4x_2, \tag{3}$$

where $k_1^2=k_2^2=m^2$, $p^2=M^2$. Let us ascertain the limitations that are imposed on the wave function by the law governing its transformation under translations, Lorentz transformations, and rotations.

We begin with translations. We assume that we have separated from the wave function $\psi(x_1, x_2, p, \omega)$ the dependence on the "oblique" time (along the ω direction), analogous to the e^{-iEt} dependence in the nonrelativistic wave function: $e^{-iEt}\psi(\mathbf{r})$. Therefore $\psi(x_1, x_2, p, \omega)$ does not change when the arguments x_1 and x_2 are shifted along ω. Under translation in an arbitrary direction μ orthogonal to ω, we obtain the usual phase factor

$$\psi(x_1+\mu, x_2+\mu, p, \omega)=\exp(-ip\mu)\psi(x_1, x_2, p, \omega).$$

At the same time, substituting in the integral of (3) the wave function $\psi(x_1+\mu, x_2+\mu, p, \omega)$ and making the change of variable $x_{1,2}+\mu=x_{1,2}$, we obtain the phase factor $\exp(-i(k_1+k_2)\mu)$. Consequently, the projections of the four-vectors p and (k_1+k_2) on the hypersurface of the light front should be equal: $p\mu=(k_1+k_2)\mu$. Separating in Ψ the δ function that takes into account the equality of these projections:

$$\Psi(k_1, k_2, p, \omega)=\int\psi(k_1, k_2, p, \omega\tau)\delta^{(4)}(k_1+k_2-p-\omega\tau)d\tau, \tag{4}$$

we find that translational invariance leads to the following relation[1)] between the four-momenta in $\psi(k_1, k_2, p, \omega\tau)$:

$$k_1+k_2=p+\omega\tau. \tag{5}$$

This enables us to express the wave function $\psi(k_1, k_2, p, \omega\tau)$ in the form of a four-point diagram (Fig. 1). With a changeover to particles with spin in view, we have indicated the spin indices in Fig. 1. For a zero-spin system the wave function $\psi(k_1, k_2, p, \omega\tau)$ is invariant to rotations and Lorentz transformations. Therefore, in analogy with the amplitude of the reaction $1+2\to3+4$, the wave function depends on the scalar products of the four-vectors it contains. Introducing the variables

$$s_1=(k_1+k_2)^2,\quad t_1=(p-k_1)^2,\quad u_1=(p-k_2)^2, \qquad s_1+t_1+u_1=2m^2+M^2, \tag{6}$$

we obtain $\psi=\psi(s_1, t_1)$.

It is convenient to introduce variables that have the meaning of the momenta of the particles in the c.m.s. of the "reaction" shown in Fig. 1. Putting[2)]

$$Q=m(k_1+k_2)/s_1^{1/2}, \tag{7}$$

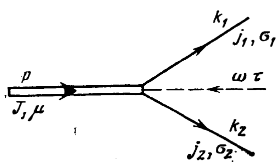

FIG. 1.

we have

$$\mathbf{q}=L^{-1}(Q)\mathbf{k}_1=\mathbf{k}_1-\frac{\mathbf{Q}}{m}\left[k_{10}-\frac{\mathbf{k}_1\mathbf{Q}}{m+\varepsilon(\mathbf{Q})}\right] \tag{8}$$

$$\mathbf{n}=L^{-1}(Q)\omega/|L^{-1}(Q)\omega|=s_1^{1/2}L^{-1}(Q)\omega/(\omega p), \tag{9}$$

where

$$k_{10}=\varepsilon(\mathbf{k}_1)=(\mathbf{k}_1^2+m^2)^{1/2},\quad \varepsilon(\mathbf{Q})=(\mathbf{Q}^2+m^2)^{1/2},$$

$L^{-1}(Q)$ is the Lorentz transformation that "extinguishes" the velocity $\mathbf{v}=\mathbf{Q}/Q_0$. The vector $\mathbf{q}$ has the meaning of the momentum of particle 1 in the c.m.s. of the "reaction" of Fig. 1, and $\mathbf{n}$ is a unit vector in the direction of ω in the c.m.s. We note that when a Lorentz transformation or a rotation g is applied to all the four-vectors, the vectors $\mathbf{q}$ and $\mathbf{n}$ undergo only rotations:

$$\mathbf{q}'=R(g,Q)\mathbf{q},\quad \mathbf{n}'=R(g,Q)\mathbf{n}, \tag{10}$$

where

$$R(g,Q)=L^{-1}(gQ)gL(Q). \tag{11}$$

Therefore $\mathbf{q}^2$ and $\mathbf{n}\cdot\mathbf{q}$ are invariants and can be expressed in terms of s_1, t_1, and u_1:

$$\mathbf{q}^2=s_1/4-m^2,\quad \mathbf{nq}=(u_1-t_1)s_1^{1/2}/2(s_1-M^2). \tag{12}$$

Thus, the wave function of a spinless relativistic system depends on two variables, $\mathbf{q}^2$ and $\mathbf{n}\cdot\mathbf{q}$:

$$\psi=\psi(\mathbf{q}^2,\mathbf{nq}). \tag{13}$$

The physical reason for the dependence of the relativistic wave function on the variable $\mathbf{n}$ is the impossibility of separating the motion of the mass center in the system of the interacting particles. For the simultaneous wave function $\psi(\mathbf{k}_1,\mathbf{k}_2,\mathbf{p})$ this means that after introducing a relativistic relative momentum $\mathbf{q}$, the wave function remains dependent on the total momentum of the system $\mathbf{p}$: $\psi=\psi(\mathbf{q},\mathbf{p})$. In a system with infinite momentum the dependence on the modulus of $\mathbf{p}$ vanishes, but a dependence remains on the unit vector $\mathbf{n}=\mathbf{p}/|\mathbf{p}|$.

An examination of the dynamic models shows that the characteristic parameter that determines the dependence of the nuclear wave function on the variable $\mathbf{n}\cdot\mathbf{q}$, which does not appear in the nonrelativistic case, is the nucleon mass. Therefore at $\mathbf{q}^2\ll m^2$ the dependence of the wave function on the variable $\mathbf{n}\cdot\mathbf{q}$ becomes inessential and we return to the nonrelativistic wave function that depends on the single variable $\mathbf{q}^2$.

The wave function on the light front, which is not simultaneous in an arbitrary system, becomes simultaneous in an infinite-momentum system moving along ω. It coincides therefore with the simultaneous wave function $\psi(\mathbf{k}_1,\mathbf{k}_2,\mathbf{p})$, defined in the system with infinite momentum, i.e., as $\mathbf{p}\to\infty$. This wave function in the infinite-momentum system is usually parametrized with the aid of the variables $\mathbf{R}_\perp$ and x: $\psi=\psi(\mathbf{R}_\perp^2,x)$. Here x is the fraction of the momentum of particle 1 relative to the infinite momentum of the entire system, and $\mathbf{R}_\perp$ is the projection of the momentum of the particle 1 in a direction perpendicular to the infinite momentum:

$$x=(\mathbf{k}_1\mathbf{p})/\mathbf{p}^2, \tag{14}$$

$$\mathbf{R}_\perp^2=\mathbf{k}_1^2-(\mathbf{k}_1\mathbf{p})^2/\mathbf{p}^2. \tag{15}$$

We now connect the variables x and $R_\perp^2$ with $\mathbf{q}^2$ and $\mathbf{n}\cdot\mathbf{q}$. To this end we first consider the wave function $\psi(k_1,k_2,p,\lambda\tau)$; defined on the hyperplane $\lambda x=0(\lambda^2=1)$, which is simultaneous in the system where $\boldsymbol{\lambda}=0$, i.e., in a system moving with velocity $\mathbf{v}=\boldsymbol{\lambda}c/(1+\boldsymbol{\lambda}^2)^{1/2}$. The variable x, which goes over into expression (14) in this system, is

$$x=\frac{(k_1\lambda)(p\lambda)-(k_1p)}{(p\lambda)^2-M^2}. \tag{16}$$

The transition to a wave function that is simultaneous in the system with infinite momentum is effected by taking the limit as $\boldsymbol{\lambda}\to\infty$. Letting $\boldsymbol{\lambda}$ go to infinity in (16), neglecting the difference between $|\boldsymbol{\lambda}|$ and λ_0, and replacing λ by ω, we obtain

$$x=\frac{\omega k_1}{\omega p}=\frac{1}{2}\left(1-\frac{\mathbf{nq}}{\varepsilon(\mathbf{q})}\right). \tag{17}$$

For $\mathbf{R}_\perp^2$ we get similarly

$$\mathbf{R}_\perp^2=\mathbf{q}^2-(\mathbf{nq})^2. \tag{18}$$

The variables $\mathbf{q}$ and $\mathbf{n}$ are more convenient than $\mathbf{R}_\perp$ and x for two reasons. First, in a wave function that depends on $\mathbf{q}$ and $\mathbf{n}$ it is convenient to go to the nonrelativistic limit (neglecting the dependence of the wave function on $\mathbf{n}$). In terms of the variables $\mathbf{R}_\perp$ and x, the nonrelativistic wave function depends on the following combination of these variables:

$$\mathbf{q}^2=\frac{\mathbf{R}_\perp^2+m^2}{4x(1-x)}-m^2, \tag{19}$$

which arises when the system of equations (17) and (18) is solved relative to $\mathbf{q}^2$. Second, the simple transformational properties of the variables $\mathbf{q}$ and $\mathbf{n}$ make it easy to construct states with spin.

3. ALLOWANCE FOR THE SPIN

We consider first the case of a system with spin and parity $J^P=1/2^+$, consisting of particles with spins 0^+ and $1/2^+$, and then change over to the general case. We operate in a representation in which the particles have definite spin projections on the z axis in their own rest systems. The state vector of the system under consideration is given by

$$\Phi_\mu(p)=\sum_\sigma\int\psi_\sigma^\mu(k_1,k_2,p,\omega\tau)a_\sigma^+(\mathbf{k}_1)c^+(\mathbf{k}_2)|0\rangle$$
$$\times\delta^{(4)}(k_1+k_2-p-\omega\tau)d\tau\frac{d^3k_1}{(2\varepsilon_1)^{1/2}}\frac{d^3k_2}{(2\varepsilon_2)^{1/2}}. \tag{20}$$

In the construction of the wave functions with spin we start from the fact that, in accordance with the definition of the spin, the state vector of a system with definite spin is transformed by the change of coordinate $x\to x'=gx$ in accordance with the law

$$\Phi_\mu(p)\to\Phi_\mu'(gp)=U\Phi_\mu(p)=\sum_{\mu'}D^{1/2}_{\mu'\mu}\{R(g,p)\}\Phi_{\mu'}(gp), \tag{21}$$

where $D^{1/2}_{\mu'\mu}$ are the matrix elements of the rotation group and $R(g,p)=L^{-1}(gp)gL(p)$.

We emphasize that the transformation law (21) is determined exclusively by the group properties of the spin. It does not depend on the concrete representation in field theory, and is the same both for the Schrödinger representation in the plane $t=0$ and for the invariant Schrödinger representation on the light front. The concrete representation governs the law that follows from (21) for the Fock-component transformation and the form of the generators that determine the operator U. The explicit form of these generators in the invariant Schrödinger representation and the group-theoretical questions connected with the transformation of a state vector in the invariant Schrödinger representation were elucidated by the author with I. S. Shapiro and will be published elsewhere. In the present paper we do not need the explicit form of the operator U.

Let us see how the wave function $\psi_\sigma{}^\mu(k_1, k_2, p, \omega\tau)$ is changed by the Lorentz transformation and by the rotation g of the arguments. The state vector $\Phi_\mu(p)$ in the invariant Schrödinger representation is defined in the different reference frames on one and the same hyperplane, in contrast to the state vector in the Schrödinger representation at $t=0$. The latter is defined in the systems A and A' on different hypersurfaces $t=0$ and $t'=0$. Therefore the law governing the transformation of the state vector in the invariant Schrödinger representation is purely kinematic, and the operator U does not contain any interaction and does not change the number of particles. Consequently, the given Fock component is transformed in terms of itself. For a zero-spin particle this means invariance of the wave function. The presence of particle spins leads, obviously, to the appearance of D functions that correspond to the rotations of the spins. Thus, we obtain

$$\psi_\sigma{}^\mu(gk_1, gk_2, gp, g\omega\tau) = \sum D^{1/2*}_{\mu\mu'}\{R(g,p)\} D^{1/2}_{\sigma\sigma'}\{R(g,k_1)\}\psi_{\sigma'}^{\mu'}(k_1, k_2, p, \omega\tau). \tag{22}$$

In exactly the same manner we transform the amplitude of the reaction $1/2+0 \to 1/2+0$. Therefore the problem of constructing the wave function with spin reduces to a known problem—expansion of the amplitude of the reaction $1/2+0 \to 1/2+0$ in terms of invariant amplitudes. This expansion takes the form

$$\psi_\sigma{}^\mu = F_1 \bar{u}_\sigma(k_1) u_\mu(p) + F_2 \bar{u}_\sigma(k_1)(\hat{k}_1 + \hat{k}_2) u_\mu(p), \tag{23}$$

where $\hat{k}_1 + \hat{k}_2 = (k_1+k_2)_\alpha \gamma_\alpha$, F_1 and F_2 are functions of the invariant variables s_1 and t_1 or $\mathbf{q}^2$ and $\mathbf{n}\cdot\mathbf{q}$, and $u_\sigma(k_1)$, $u_\mu(p)$ are spinors. For example, the spinor $u_\mu(p)$ takes the form

$$u_\mu(p) = \begin{pmatrix} (\varepsilon(\mathbf{p})+M)^{1/2} w_\mu \\ (\varepsilon(\mathbf{p})-M)^{1/2} \dfrac{(\mathbf{p}\sigma)}{|\mathbf{p}|} w_\mu \end{pmatrix}, \quad w_{1/2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad w_{-1/2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{24}$$

We recall that the spinor is transformed in the following manner:

$$u_\mu'(gp) = \Lambda(g) u_\mu(p) = \sum_{\mu'} D^{1/2}_{\mu'\mu}\{R(g,p)\} u_{\mu'}(gp), \tag{25}$$

as a result of which the wave function (23) is indeed transformed in accordance with the law (22).

It is convenient to transform the expansion (23) into a form that comes closest to being nonrelativistic. For this purpose we use the variables $\mathbf{q}$ and $\mathbf{n}$ to build up the spinor structures. We change over from the wave function ψ_σ^μ to a wave function $\tilde{\psi}_\sigma^\mu$ that transforms in accordance with the spin indices with the aid of D functions that depend on the same rotation $R(g,Q) = L^{-1}(gQ)gL(Q)$ that determines the laws governing the transformation of the variables $\mathbf{q}$ and $\mathbf{n}$ [see formulas (10)]. We shall show that the connection between $\psi_\sigma{}^\mu$ and $\tilde{\psi}_\sigma{}^\mu$ is given by the following relations

$$\tilde{\psi}_\sigma{}^\mu = \sum D^{1/2}_{\sigma\sigma_1}\{R(L^{-1}(Q), k_1)\} D^{1/2*}_{\mu\mu_1}\{R(L^{-1}(Q), p)\}\psi_{\sigma_1}{}^{\mu_1}, \tag{26}$$

$$\psi_\sigma{}^\mu = \sum D^{1/2*}_{\sigma_1\sigma}\{R(L^{-1}(Q), k_1)\} D^{1/2}_{\mu_1\mu}\{R(L^{-1}(Q), p)\}\tilde{\psi}_{\sigma_1}{}^{\mu_1}, \tag{27}$$

where, in accordance with the definition (11),

$$R(L^{-1}(Q), k_1) = L^{-1}(L^{-1}(Q)k_1) L^{-1}(Q) L(k_1), \qquad Q = m(k_1+k_2)/s_1^{1/2}, \quad s_1 = (k_1+k_2)^2; \tag{28}$$

$L^{-1}(Q)$ and $L(k_1)$ are as usual the direct and inverse Lorentz transformations with parameters Q and k_1, while $L^{-1}[L^{-1}(Q)k_1]$ is the Lorentz transformation L^{-1} with parameters obtained from k_1 by the Lorentz transformation $L^{-1}(Q)$.

We note that the following equality holds[7]:

$$D^{1/2}\{R(L^{-1}(Q), k)\} = \frac{(k_0+m)(Q_0+m) - (\sigma\mathbf{k})(\sigma\mathbf{Q})}{[2(k_0+m)(Q_0+m)(k_0Q_0 - \mathbf{k}\mathbf{Q} + m^2)]^{1/2}}. \tag{29}$$

When the parameters Q and k_1 are subjected to the transformation g, the quantity $R[L^{-1}(Q)k_1]$ is transformed in the following manner:

$$R(L^{-1}(gQ), gk_1) = R(g, Q) R(L^{-1}(Q), k_1) R^{-1}(g, k_1). \tag{30}$$

To check on this formula, we rewrite it in explicit form

$$L^{-1}(L^{-1}(gQ)gk_1) L^{-1}(gQ) L(gk_1) = L^{-1}(gQ) g L(Q) L^{-1}(L^{-1}(Q)k_1) L^{-1}(Q) L(k_1) L^{-1}(k_1) g^{-1} L(gk_1). \tag{31}$$

Bearing in mind that $L^{-1}(gQ)g\mathbf{k}_1 = R(g,Q)\mathbf{q}$, $\mathbf{q} = L^{-1}(Q)\mathbf{k}_1$, we obtain after simple transformations from (31):

$$L^{-1}(R(g,Q)\mathbf{q}) = R(g,Q) L^{-1}(\mathbf{q}) R^{-1}(g,Q).$$

Denoting the rotation by $R(g,Q) \equiv R$, we have

$$RL(\mathbf{q}) = L(R\mathbf{q})R. \tag{32}$$

The validity of this relation is obvious, hence the validity of the identity (30). From (26), (22), and (30), taking into account the equality $D(R_1R_2) = D(R_1)D(R_2)$, we obtain the law governing the transformation of the function $\tilde{\psi}_\sigma{}^\mu$:

$$\tilde{\psi}_\sigma{}^\mu(gk_1, gk_2, gp, g\omega\tau) = \sum D^{1/2*}_{\mu\mu_1}\{R(g,Q)\} D^{1/2}_{\sigma\sigma_1}\{R(g,Q)\}\tilde{\psi}_{\sigma_1}{}^{\mu_1}(k_1, k_2, p, \omega\tau). \tag{33}$$

Thus, by "additional turning" the spins in accordance with (26), we have obtained a function $\tilde{\psi}_\sigma{}^\mu$ which is transformed with respect to the spin indices with the aid of D functions containing the rotation $R(g,Q)$ that transforms the variables $\mathbf{n}$ and $\mathbf{q}$. It is therefore easy to construct, in terms of the variables $\mathbf{q}$ and $\mathbf{n}$, a function $\tilde{\psi}_\sigma{}^\mu$ that transforms in accordance with (33):

$$\tilde{\psi}_\sigma{}^\mu(\mathbf{q},\mathbf{n}) = f_1(\mathbf{q}^2, \mathbf{n}\mathbf{q})\delta_{\mu\sigma} + f_2(\mathbf{q}^2, \mathbf{n}\mathbf{q}) C^{1/2\mu}_{1/2\sigma 1m} C^{1m}_{1m_1 1m_2} Y^*_{1m_1}\left(\frac{\mathbf{q}}{|\mathbf{q}|}\right) Y^*_{1m_2}(\mathbf{n}) \tag{34}$$

where $C^{1/2\mu}_{1/2\sigma 1m}$ and $C^{1m}_{1m_1 1m_2}$ are Clebsch–Gordan coefficients. With the aid of Pauli matrices, this function takes the form

$$\tilde{\Phi}(\mathbf{q},\mathbf{n})=f_1(\mathbf{q}^2,\mathbf{nq})+\frac{i\sqrt{6}}{8\pi}f_2(\mathbf{q}^2,\mathbf{nq})\frac{\sigma[\mathbf{n}\times\mathbf{q}]}{|\mathbf{q}|}. \tag{35}$$

Comparing the expansions (35) and (23) in a system where $\mathbf{k}_1+\mathbf{k}_2=0$ (in this system all the D functions in (26) and (27) turn into unit matrices), we obtain the relations between the coefficients f_1, f_2, and F_1, F_2:

$$\begin{gathered} f_1=N^{1/2}(F_1+s_1^{1/2}F_2)+N^{-1/2}(F_1-s_1^{1/2}F_2)(\mathbf{qn})|\mathbf{q}|,\\ \frac{\sqrt{6}}{8\pi}f_2=N^{-1/2}(s_1^{1/2}F_2-F_1)\mathbf{q}^2, \end{gathered} \tag{36}$$

where

$$N=[(\mathbf{q}^2+m^2)^{1/2}+m][(\mathbf{q}^2+M^2)^{1/2}+M],\quad s_1=(k_1+k_2)^2,$$

and m is the mass of the particle with spin 1/2.

In the nonrelativistic case, only zero orbital angular momentum is possible in the wave function of the system in question. This means that at $|\mathbf{q}|\ll m$ the coefficient f_1 goes over into the nonrelativistic S-wave function (and ceases to depend on $\mathbf{n}\cdot\mathbf{q}$), while the coefficient f_2 vanishes. Of course, there is no automatic vanishing of the coefficient f_2 in (34) and (35), since the nonrelativistic character of the system manifests itself in the dynamic properties of the invariant functions f_1 and f_2, i.e., in the character of their dependences on the variables q^2 and $\mathbf{n}\cdot\mathbf{q}$. The coincidence of the relativistic wave functions on the light front with the nonrelativistic wave functions at $|\mathbf{q}|\ll m$ is guaranteed by the fact that the relativistic dynamics which determines the relativistic wave functions and their dependence on $\mathbf{n}$ leads in the nonrelativistic limit to the Schrödinger equation, and consequently to the vanishing of those functions whose appearance is due to the existence of spin structures constructed with the aid of the vector $\mathbf{n}$.

It is easy to generalize the described method of constructing the relativistic wave functions with spin to include the case of arbitrary spins. For a system with total angular momentum J, consisting of particles with spins j_1 and j_2, it is necessary to change over from the wave function $\psi^{\mu}_{\sigma 1\sigma 2}$, which is a coefficient in an expansion similar to (20), to the function $\tilde{\psi}^{\mu}_{\sigma 1\sigma 2}$ in accordance with the formula

$$\begin{aligned} \tilde{\psi}^{\mu}_{\sigma_1\sigma_2}=\sum D^{J*}_{\mu\mu'}\{R(L^{-1}(Q),p)\}D^{j_1}_{\sigma_1\sigma_1'}\{R(L^{-1}(Q),k_1)\}\\ \times D^{j_2}_{\sigma_2\sigma_2'}\{R(L^{-1}(Q),k_2)\}\psi^{\mu'}_{\sigma_1'\sigma_2'}. \end{aligned} \tag{37}$$

The function $\tilde{\psi}^{\mu}_{\sigma 1\sigma 2}$ is transformed with the aid of D functions that contain the rotation $R(g,Q)$, and is constructed in accordance with the known rules for expanding the nonrelativistic amplitude of the reaction $J+0\to j_1+j_2$ into invariant amplitudes[8] (with replacement of the nonrelativistic relative momenta by $\mathbf{q}$ and $\mathbf{n}$). The wave function $\tilde{\psi}^{\mu}_{\sigma_1\sigma_2}$ takes the form

$$\begin{aligned} \tilde{\psi}^{\mu}_{\sigma_1\sigma_2}=\sum_{j_{12}L\lambda} f(j_{12},L,\lambda)\sum C^{j_{12}\sigma_{12}}_{j_1\sigma_1 j_2\sigma_2}C^{J\mu}_{j_{12}\sigma_{12}LM}\\ \times C^{LM}_{r-\lambda,m_1,r+\lambda,m_2}Y^{*}_{r-\lambda,m_1}(\mathbf{n})Y^{*}_{r+\lambda,m_2}(\mathbf{q}/|\mathbf{q}|), \end{aligned} \tag{38}$$

where $r=1/2[L+1/2(1+\pi\pi_1\pi_2(-1)^{L+1})]$, λ runs through the values from $-r$ to r; π,π_1,π_2 are the intrinsic parities of the particles. The functions $f(j_{12},L,\lambda)$ depend on $\mathbf{q}^2$ and $\mathbf{n}\cdot\mathbf{q}$. The number of invariant functions $f(j_{12},L,\lambda)$, with account taken of the conservation of spatial parity for the case of one integer and two half-integer spins, is $N=(2J+1)(2j_1+1)(2j_2+1)/2$. If all the spins are integers, then

$$N=[(2J+1)(2j_1+1)(2j_2+1)+\pi\pi_1\pi_2(-1)^{J+j_1+j_2}]/2.$$

Thus, for the deuteron we have $N=6$ as against $N=2$ in the nonrelativistic case.

It is clear from (38) that the angular-momentum operator takes in the representation chosen by us the form

$$\hat{\mathbf{J}}=\frac{1}{i}\left[\mathbf{q}\times\frac{\partial}{\partial\mathbf{q}}\right]+\frac{1}{i}\left[\mathbf{n}\times\frac{\partial}{\partial\mathbf{n}}\right]+\hat{\mathbf{j}}_1+\hat{\mathbf{j}}_2, \tag{39}$$

where j_1 and j_2 are the operators of the spins of particles 1 and 2. This expression can also be obtained by starting from the Pauli-Lubansky vector, made up of the generators that transform the state vector in the invariant Schrödinger representation. For a spinless system the action of this operator on the wave function $\psi(\mathbf{q}^2,\mathbf{n}\cdot\mathbf{q})$ considered in Sec. 2 and corresponding to zero total angular momentum does in fact yield zero. In the nonrelativistic limit, the action of the operator $\mathbf{n}\times\partial/\partial\mathbf{n}$ on a wave function that does not depend on $\mathbf{n}$ leads to a zero result, and we return to the nonrelativistic expression for the operator of the total angular momentum.

To construct the N-particle wave function $\psi^{\mu}_{\sigma 1\ldots\sigma N}$ we must use a formula similar to (37) to change from $\psi^{\mu}_{\sigma 1\ldots\sigma_N}$ to $\tilde{\psi}^{\mu}_{\sigma 1\ldots\sigma_N}$, and to construct $\tilde{\psi}^{\mu}_{\sigma 1\ldots\sigma N}$ we must use the formulas obtained by Kolybasov for the expansion of the nonrelativistic amplitude of the reaction $J+0\to j_1+\ldots+j_N$ into invariant amplitudes,[9] where the nonrelativistic relative momenta must be replaced by n and $\mathbf{q}_i=L^{-1}(Q_i)\mathbf{k}_i$, $Q_i=m_i(p+\omega\tau)/[(p+\omega\tau)^2]^{1/2}$, $i=1,\ldots,N$.

Instead of expanding the function $\tilde{\psi}$ with the aid of Clebsch–Gordan coefficients and spherical functions, we can, in the case of arbitrary spins, write down a direct expansion of the function ψ in the bispinor formalism, similar to the expansion (23). To this end we must go over from the amplitude $M^{\sigma 3\sigma 4}_{\sigma 1\sigma 2}$ to $\tilde{M}^{\sigma 3\sigma 4}_{\sigma 1\sigma 2}$:

$$\begin{aligned} \langle p_4\sigma_4,p_3\sigma_3|M|p_1\sigma_1,p_2\sigma_2\rangle=\sum D^{j_4*}_{\sigma_4'\sigma_4}\{R_4\}D^{j_3*}_{\sigma_3'\sigma_3}\{R_3\}\\ \times D^{j_2}_{\sigma_2'\sigma_2}\{R_2\}D^{j_1}_{\sigma_1'\sigma_1}\{R_1\}\langle p_4\sigma_4',p_3\sigma_3'|\tilde{M}|p_1\sigma_1',p_2\sigma_2'\rangle, \end{aligned} \tag{40}$$

where

$$R_i=R(L^{-1}(Q_i),p_i),\quad Q_i=m_i(p_1+p_2)/[(p_1+p_2)^2]^{1/2},\quad i=1,2,3,4.$$

To construct $\tilde{M}$ it is necessary to use formulas analogous to (38) for the nonrelativistic amplitudes.[8] This method of expansion into invariant amplitudes is similar to that developed in Ref. 11. The invariant amplitudes in $\tilde{M}^{\sigma 3\sigma 4}_{\sigma 1\sigma 2}$ will have, generally speaking, kinematic singularities, a fact illustrated by formulas (36), in which f_1 and f_2 have square-root singularities when $|\mathbf{q}|$ and $\mathbf{n}\cdot\mathbf{q}$ are expressed in terms of the variables s_1 and t_1. In problems for which the presence of kinematic singularities is immaterial, this expansion may be useful. In the nonrelativistic limit, the amplitude

$\tilde{M}^{\sigma 3\sigma 3}_{\sigma 1\sigma 2}$ does not change its form, but the relativistic relative momenta change into nonrelativistic ones, while the D functions that relate M and $\tilde{M}$ turn into unit matrices.

We note that the choice of the vector $Q=m(k_1+k_2)/s_1^{1/2}$ in formulas (37) which connect ψ and $\tilde{\psi}$ [or in the analogous formulas (40) for the connection between M and $\tilde{M}$] was made for the sake of convenience. If we choose in (40) $Q=p_3$ in the D functions "associated" with the indices σ_1', σ_3', and $Q=p_4$ in the D functions associated with the indices σ_2', σ_4', we arrive at the representation in which Skachkov[12] obtained expressions for the amplitudes in the model of the one-boson exchange.

In concluding this section, we present an expression for the relativistic wave function of the deuteron on the light front:

$$\begin{aligned}\tilde{\psi}^{\mu}_{\sigma_1\sigma_2}(\mathbf{q},\mathbf{n}) &= G_1 C^{1\mu}_{1/2\sigma_1 1/2\sigma_2} \\ &+ G_2 C^{1\sigma}_{1/2\sigma_1 1/2\sigma_2} C^{1\mu}_{1\sigma 1m} C^{1m}_{1m_1 1m_2} Y^{*}_{1m_1}(\mathbf{n}) Y^{*}_{1m_2}(\mathbf{q}/|\mathbf{q}|) \\ &+ \sum_{\lambda=-1,0,+1} G_{4+\lambda} C^{1\sigma}_{1/2\sigma_1 1/2\sigma_2} C^{1\mu}_{1\sigma 2m} C^{2m}_{1-\lambda,m_1,1+\lambda,m_2} Y^{*}_{1-\lambda,m_1}(\mathbf{n}) Y^{*}_{1+\lambda,m_2}(\mathbf{q}/|\mathbf{q}|) \\ &+ G_6 C^{00}_{1/2\sigma_1 1/2\sigma_2} C^{1\mu}_{1m_1 1m_2} Y^{*}_{1m_1}(\mathbf{n}) Y^{*}_{1m_2}(\mathbf{q}/|\mathbf{q}|). \qquad (41)\end{aligned}$$

Allowance for the Pauli principle leads to

$$\begin{aligned} G_{1,3,5,6}(\mathbf{q}^2, \mathbf{nq}) &= G_{1,3,5,6}(\mathbf{q}^2, -\mathbf{nq}), \\ G_{2,4}(\mathbf{q}^2, \mathbf{nq}) &= -G_{2,4}(\mathbf{q}^2, -\mathbf{nq}). \end{aligned} \qquad (42)$$

In the nonrelativistic limit we are left with the functions G_1 (S wave) and G_5 (D wave).

In the bispinor formalism, the wave function of the deuteron $\psi^{\alpha}_{\sigma 1\sigma 2}$ takes the form

$$\begin{aligned}\psi^{\alpha}_{\sigma_1\sigma_2} &= F_1 \bar{u}_1 U_c \bar{u}_2 e_{\alpha}^{-} + F_2 \bar{u}_1 (\gamma_{\alpha} - p_{\alpha}\hat{p}/M^2) U_c \bar{u}_2 \\ &+ F_3 \bar{u}_1 \gamma_5 U_c \bar{u}_2 e_{1\alpha} + F_4 \bar{u}_1 \hat{p} U_c \bar{u}_2 e_{\alpha}^{+} + F_5 \bar{u}_1 U_c \bar{u}_2 e_{\alpha}^{+} + F_6 \bar{u}_1 \hat{p} U_c \bar{u}_2 e_{\alpha}^{-}, \qquad (43)\end{aligned}$$

where $U_c=\gamma_2\gamma_0$ is the charge-conjugation matrix

$$\bar{u}_{1,2}=\bar{u}_{\sigma_{1,2}}(k_{1,2}), \quad e^{-}=\left(\frac{k_1}{pk_1}-\frac{k_2}{pk_2}\right)\Big/\left[-\left(\frac{k_1}{pk_1}-\frac{k_2}{pk_2}\right)^2\right]^{1/2},$$

$$e^{+}=\left(p-\frac{M^2}{p\omega}\omega-\frac{M^2(\omega e^{-})}{p\omega}e^{-}\right)\Big/M\left[1-\frac{M^2(\omega e^{-})^2}{p\omega}\right]^{1/2},$$

$e_{1\alpha}=\varepsilon_{\alpha\nu\rho\gamma}p_{\nu}/M\, e_{\rho}^{-}e_{\gamma}^{+}$, $(e^{-})^2=(e^{+})^2=(e_1)^2=-1$, and the four-vectors $e^{-}, e^{+}, e_1, p/M$ are mutually orthogonal. In this representation, ψ^{α} is a four-dimensional pseudovector [in formula (43) the index α runs through the values 0, 1, 2, 3]. The wave function ψ^{α} satisfies the condition $p_{\alpha}\psi^{\alpha}=0$. It follows from the Pauli principle that the functions $F_{1,2,3,4}$ do not reverse sign when $\mathbf{q}\to-\mathbf{q}$, while $F_{5,6}$ reverse sign when $\mathbf{q}\to-\mathbf{q}$.

In the nonrelativistic limit, the functions F_1 and F_2 survive, and the spatial components of the four-vector ψ^{α} go over into the well known nonrelativistic wave function of the deutron:

$$\psi(\mathbf{q})=w_1^{*}\left[\frac{1}{\sqrt{2}}u_S\sigma+\frac{1}{2}u_D\left(\frac{3(\sigma\mathbf{q})\mathbf{q}}{\mathbf{q}^2}-\sigma\right)\right]\sigma_y w_2^{*}, \qquad (44)$$

where

$$u_S=\frac{4\sqrt{2}}{3}|\mathbf{q}|F_1-2\sqrt{2}mF_2, \quad u_D=\frac{4}{3}|\mathbf{q}|F_1,$$

u_S and u_D are S- and D-wave functions.

4. CROSS SECTION OF *pd* SCATTERING WITH ALLOWANCE FOR SPIN

We illustrate the method developed above for taking the spin into account, using as an example the cross section for backward pd scattering within the framework of the single-nucleon exchange mechanism. The diagram-technique rules for zero-spin particles in the invariant Schrödinger representation on the plane $\lambda x=0$, which make it possible to express the amplitudes of the processes in terms of the wave functions, were formulated in Ref. 5, and for the case of a light front they were formulated in Ref. 1. For particles with spin in the invariant Schrödinger representation on the plane $\lambda x=0(\lambda^2=1)$, these rules were developed in Ref. 13, and for the case of the light front in Ref. 14. The diagram technique for the invariant Schrödinger representation on the light front is an invariant generalization of the old perturbation technique for infinite-momentum systems. According to the rules of this technique,[13,14] the amplitude of the pd scattering within the framework of the mechanism of the single-nucleon exchange is set in correspondence with the diagram of Fig. 2. The vertices Γ of the diagram of Fig. 2 are represented in the same manner as the wave functions (Fig. 1), and are connected with the wave functions by the formula (see Ref. 1)

$$\psi=\left(\frac{2\pi}{m}\right)^{1/2}\frac{\Gamma}{s_1-M^2}.$$

The intermediate dashed line in Fig. 2 corresponds to the factor $1/2\pi\cdot(\tau-i0)^{-1}$, while the intermediate neutron corresponds to the propagator $(\hat{p}_n+m)\cdot\theta(\omega p_n)\delta(p_n^2-m^2)$, where the neutron four-momentum is $p_n=p-k'+\omega\tau$. The integration with respect to $d\tau$ is between infinite limits. The external particle lines correspond to the same spinors as in the Feynman diagram technique. The backward pd scattering reaction was considered within the framework of the invariant Schrödinger representation in Ref. 15, where a study was made of the qualitative consequences of the dependence of the deuteron wave function on the variable $\mathbf{n}$. The following expression was obtained for the cross section (disregarding spin):

$$\frac{d\sigma}{d\Omega}=\frac{m^2(m^2-u)^2}{64\pi^2 s(1-\beta)^4}\psi^4(\mathbf{q}^2,\mathbf{nq}), \qquad (45)$$

where $s=(p+k)^2, u=(p-k')^2$,

$$\beta=\frac{\varepsilon_p(\mathbf{k})-|\mathbf{k}|\cos(\theta/2)}{\varepsilon_d(\mathbf{k})+|\mathbf{k}|\cos(\theta/2)} \qquad (45a)$$

$$\mathbf{q}^2=\frac{m^2-u}{4(1-\beta)}+\frac{M^2}{4}-m^2, \qquad (45b)$$

$$\mathbf{nq}=\varepsilon(\mathbf{q})(1-2\beta), \qquad (45c)$$

$\varepsilon(\mathbf{q})=(\mathbf{q}^2+m^2)^{1/2}$; m and M are the masses of the nucleon and deuteron; $|\mathbf{k}|$ and θ are the momentum and scattering angle in the c.m.s. of the reaction.

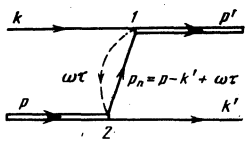

FIG. 2.

Formula (45) without allowance for the dependence of the wave function of the deuteron on the variable n was obtained also by Kondratyuk and Shevchenko.[16] In the spinless case, the vertices 1 and 2 of Fig. 2 correspond to product $\Gamma_1\Gamma_2$ of the vertex parts. After taking into account the spin, the product becomes

$$\bar{u}_{\alpha}^{\mu'}(p')\Gamma_{\beta\gamma}^{\alpha}(1)u_{\beta}^{\sigma_p}(k)(\hat{p}_n+m)_{\gamma\gamma'}\bar{u}_{\beta'}^{\sigma_{p'}}(k')\Gamma_{\beta'\gamma'}^{\alpha'}(2)u_{\alpha'}^{\mu}(p),$$

where $p_n = p - k' + \omega\tau$ is the neutron four-momentum.

Representing $(\hat{p}_n + m)$ in the form

$$(\hat{p}_n+m)_{\gamma\gamma'}=\sum_{\sigma_n}\bar{u}_{\gamma'}^{\sigma_n}(p_n)u_{\gamma}^{\sigma_n}(p_n) \qquad (46)$$

(we recall that $p_n^2 = m^2, p_{0n} > 0$), we find that ψ^2 in the amplitude for the zero-spin particles goes over into

$$\sum_{\sigma_n}\psi_{\sigma_p\sigma_n}^{T\mu'}(1)\psi_{\sigma_p'\sigma_n}^{\mu}(2). \qquad (47)$$

The sum over σ_n which appears here means that in the assumed representation the propagator is a multiple of the unit matrix.

The wave function $\psi_{\sigma_p\sigma_n}^{T\mu}$ is connected with the time-reversed vertex part that describes the process $NN \to d$, and is expressed in terms of $\psi_{\sigma_p\sigma_n}^{\mu}$ with the aid of the relation

$$\psi_{\sigma_p\sigma_n}^{T\mu}=(-1)^{\mu+\sigma_p+\sigma_n}\psi_{-\sigma_p,-\sigma_n}^{-\mu} \qquad (48)$$

where the spatial components of the four-momenta, on which the wave function in the right and left sides of (48) depend, have opposite signs. It is easy to change over from the function ψ in (47), by using (37), to the function $\tilde{\psi}$ defined by (41). We then obtain in place of (47) an expression that contains the wave function $\tilde{\psi}$ and D functions that depend on the spin indices of both the external particles and of the intermediate neutron. After averaging the square of this expression over the initial spin projections and summing over the final ones, the D functions corresponding to the external particles vanish by virtue of the orthogonality.

We note that the D functions corresponding to the neutron depend on the rotations $R[L^{-1}(Q'), p_n]$ and $R[L^{-1}(Q')p_n]$, where

$$Q=m(p+\omega\tau)/s_1^{1/2},\quad Q'=m(p'+\omega\tau)/\sqrt{s_1'},\quad s_1=(p+\omega\tau)^2,$$
$$s_1'=(p'+\omega\tau)^2.$$

It is therefore convenient to continue the calculations in the rest system of the intermediate neutron, i.e., in a system where $\mathbf{p}_n = 0$. Such a system exists, inasmuch as $p_n^2 = m^2 > 0$. In this system, the D functions corresponding to the neutron turn into the unit matrices, as is seen, for example, from formula (29). As a result we find that the cross section is determined by the quantity

$$\bar{\psi}^4=\frac{1}{6}\sum\tilde{\psi}_{\sigma_p\sigma_n}^{T\mu'}(\mathbf{q}',\mathbf{n}')\tilde{\psi}_{\sigma_p'\sigma_n}^{\mu}(\mathbf{q},\mathbf{n})$$
$$\times\tilde{\psi}_{\sigma_p\sigma_n'}^{*T\mu'}(\mathbf{q}',\mathbf{n}')\tilde{\psi}_{\sigma_p'\sigma_n'}^{*\mu}(\mathbf{q},\mathbf{n}), \qquad (49)$$

where

$$\mathbf{q}=L^{-1}(Q)\mathbf{p}_n=-\mathbf{Q},\quad \mathbf{q}'=L^{-1}(Q')\mathbf{p}_n=-\mathbf{Q}',$$
$$\mathbf{n}=s_1^{1/2}L^{-1}(Q)\omega/(\omega p),\quad \mathbf{n}'=\sqrt{s_1'}L^{-1}(Q')\omega/(\omega p').$$

The function $\tilde{\psi}^T$ is determined by formula (41) without complex conjugation in the spherical functions, a fact easily obtained with the aid of formulas (48) and the relations (37) between ψ and $\tilde{\psi}$.

The product $\sum\tilde{\psi}_{\sigma'_p\sigma'_n}^{*\mu}\tilde{\psi}_{\sigma'_p\sigma_n}^{\mu}$ in (49) can be represented in the form

$$\sum_{\mu,\sigma_p'}\tilde{\psi}_{\sigma_p'\sigma_n'}^{*\mu}(\mathbf{q},\mathbf{n})\tilde{\psi}_{\sigma_p'\sigma_n}^{\mu}(\mathbf{q},\mathbf{n})=A\delta_{\sigma_n\sigma_n'}+B(\boldsymbol{\sigma}[\mathbf{nq}])_{\sigma_n'\sigma_n}. \qquad (50)$$

The coefficients A and B are real. Their expressions in terms of the functions $G_1, \ldots, G_6$, which determine the wave function of the deuteron (41), are given in the Appendix. The summation in (49) reduces to the calculation of the trace of the Pauli matrix, which yields

$$\bar{\psi}^4={}^1/_3\{AA'+BB'[(\mathbf{nn}')(\mathbf{qq}')-(\mathbf{nq}')(\mathbf{n}'\mathbf{q})]\}, \qquad (51)$$

where A, and B and A', B' depend respectively on n, q and n', q'.

The problem of choosing the four-vector ω, on which $\bar{\psi}^4$ depends via the variables q, n, q', and n', was discussed in Refs. 4 and 15. It was shown that the value of the four-vector ω must be chosen such that the ratios $y = \omega k/\omega p'$ and $y' = \omega k'/\omega p$ be equal: β: $y = y' = \beta$, where β is given by (45a). At $y = y'$ the expression $(\mathbf{n}\cdot\mathbf{n}')(\mathbf{q}\cdot\mathbf{q}') - (\mathbf{n}'\cdot\mathbf{q})(\mathbf{n}\cdot\mathbf{q}')$ is transformed into $\omega^2(\mathbf{Q}\cdot\mathbf{Q}') - (\omega\cdot\mathbf{Q})(\omega\cdot\mathbf{Q}')$, a fact that can be demonstrated by using the explicit expressions for the variables $\mathbf{q},\mathbf{n},\mathbf{q}',\mathbf{n}'$ in terms of p_n, ω, Q, Q' [see formula (8)]. Writing down this expression, defined in the rest system of the neutron, in invariant form with the aid of the relations

$$\omega^2=\frac{1}{m^2}(\omega p_n)^2,\quad (\mathbf{QQ}')=\frac{1}{m^2}(Qp_n)(Q'p_n)-(QQ')$$

etc., and recognizing that at $y = y'$ we have $\mathbf{q}^2 = \mathbf{q}'^2, \mathbf{n}\cdot\mathbf{q} = \mathbf{n}'\cdot\mathbf{q}'$, we get

$$\bar{\psi}^4={}^1/_3\{A^2(\mathbf{q}^2,\mathbf{nq})+B^2(\mathbf{q}^2,\mathbf{nq})[\beta(1-\beta)m^2-(1-\beta)M^2$$
$$+{}^1/_2(1-\beta)^2s+{}^1/_2(1+\beta^2)u]\}. \qquad (52)$$

The expressions of the variables $\beta, \mathbf{q}^2$, and $\mathbf{n}\cdot\mathbf{q}$ in terms of the energy and the scattering angle are given by formulas (45a)–(45c). In the nonrelativistic limit, formula (52) leads to the known result:

$$\bar{\psi}^4={}^3/_4(\psi_S^2(q)+\psi_D^2(q))^2,$$

where ψ_S, ψ_D are the S and D wave functions, $\psi_S = G_1, \psi_D = G_5/4\pi$, and the normalization is

$$\int(\psi_S^2(q)+\psi_D^2(q))\frac{d^3q}{(2\pi)^3}=1.$$

The dependence of $\bar{\psi}^4$ on $\mathbf{n}\cdot\mathbf{q}$ leads to a qualitative change in the behavior of the pd-scattering cross section as a function of the scattering angle near 180°.[15]

5. CONCLUSION

The construction of the wave function with spin on the light front, which was presented above and which is needed for the investigation of nuclei in the relativistic region, solves the problem of taking the spins and angular momenta into account in the parton wave functions. This construction was carried out using variables different from those customarily employed for the parametrization of the parton wave functions ($\mathbf{R}_\perp$ and x). Although it is possible to change in the expression from the cross section from the variables **q** and **n** to $\mathbf{R}_\perp$ and x, in practice there is no need for this. Thus, in the case of pd scattering the variables $\mathbf{q}^2$ and $\mathbf{n}\cdot\mathbf{q}$ are simply expressed in terms of the energy and the scattering angle [see formulas (45a)–(45c)].

The wave functions on the light front, with account taken of the spin within the framework of the "relativistic quantum mechanical approach" with fixed number of particles, was constructed for a two-particle system by Terent'ev[17] and turned out to be independent of **n**. We note that this approach is not based on field theory and does not have such an essential property as retardation of the interaction. Namely, the phenomenon of relativistic retardation leads in final analysis to the impossibility of separating the motion of the mass center in a system of relativistic interacting particles and to a dependence of the wave function on the argument **n**.

The appearance in the relativistic wave function of spin structures connected with the dependence of the wave function on **n** may lead to qualitative polarization phenomena in nuclear reactions with large momentum transfer. Experiments that are sensitive to the dependence of the relativistic wave functions on the variable **n**, and experimental observation and investigation of the character of this dependence, are of considerable interest.

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APPENDIX

We present the expressions for the coefficients A and B that determine the pd-scattering cross section:

$$A(\mathbf{q}^2,\mathbf{nq})=\frac{1}{2}\sum \bar{\psi}^{\mu}_{\sigma_p\sigma_n}\varphi^{*\mu}_{\sigma_p\sigma_n}$$

$$=\frac{3}{2^5\pi^2}\sum_{jL\lambda\lambda' l} f(j,L,\lambda)f^*(j,L,\lambda')(-1)^{L+\lambda+\lambda'}N_{r+\lambda}N_{r-\lambda}N_{r+\lambda'}N_{r-\lambda'}$$

$$\times C^{l0}_{r-\lambda,0,r-\lambda',0}C^{l0}_{r+\lambda,0,r+\lambda',0}\begin{Bmatrix} r-\lambda' & l & r-\lambda \\ r+\lambda & L & r+\lambda' \end{Bmatrix}P_l(\cos\widehat{\mathbf{nq}}), \tag{53}$$

$$(\mathbf{q}^2-(\mathbf{nq})^2)B(\mathbf{q}^2,\mathbf{nq})=i\frac{4\pi}{\sqrt{6}}|\mathbf{q}|\sum \bar{\psi}^{\mu}_{\sigma_p\sigma_n}\bar{\psi}^{*\mu}_{\sigma_p\sigma'_n}$$

$$\times C^{1/2\,\sigma_n'}_{1/2\,\sigma_n 1m}C^{1m}_{1m_1 1m_2}Y_{1m_1}\left(\frac{\mathbf{q}}{|\mathbf{q}|}\right)Y_{1m_2}(\mathbf{n})$$

$$=i\frac{3^2}{2^4\pi^2}|\mathbf{q}|\sum f(j,L,\lambda)f^*(j',L',\lambda')(-1)^{j+j'+L+a'}$$

$$\times N_jN_{j'}N_LN_{L'}N_{r+\lambda}N_{r'+\lambda'}N_{r-\lambda}N_{r'-\lambda'}N_aN_{a'}$$

$$\times C^{a0}_{r-\lambda,0,r'-\lambda',0}C^{a'0}_{r+\lambda,0,r'+\lambda',0}\begin{Bmatrix} 1 & j & j' \\ 1/2 & 1/2 & 1/2 \end{Bmatrix}\begin{Bmatrix} L & L' & 1 \\ j' & j & 1 \end{Bmatrix}$$

$$\times\begin{Bmatrix} a' & r+\lambda & r+\lambda' \\ 1 & L & L' \\ a & r-\lambda & r'-\lambda' \end{Bmatrix}\begin{Bmatrix} 1 & 1 & 1 \\ a' & a & l \end{Bmatrix}C^{l0}_{a010}C^{l0}_{a'010}P_l(\cos\widehat{\mathbf{nq}}), \tag{54}$$

where

$$N_j=(2j+1)^{1/2},\quad N_{r+\lambda}=[2(r+\lambda)+1]^{1/2},\ldots .$$

The functions $f(j,L,\lambda)$ in (53) and (54) are connected with the functions G_i in formula (41) in the following manner: $G_1=f(1,0,0)/4\pi$, $G_2=f(1,1,0)$, $G_{4+\lambda}=f(1,2,\lambda)$ $(\lambda=-1,0,1)$, $G_6=f(0,1,0)$.

To obtain (53) and (54) it is convenient to use graphic methods for the summation of Clebsch–Gordan coefficients.[18] It is easily seen from (53) and (54) that A and B are real functions.

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1)Stricting speaking, this analysis is not strictly correct, inasmuch as the four-vector ω is "orthogonal to itself" $(\omega\cdot\omega=0)$. Relation (5) can be obtained by considering the translational properties of a wave function defined on an arbitrary flat hypersurface $\lambda x=0[\lambda=(\lambda_0,\boldsymbol{\lambda}),\ \lambda^2=1]$, and then change over to a wave function on the light front with the aid of the limit $\boldsymbol{\lambda}\to\infty$.

2)The notation in formulas (7)–(9) differs from that used in Ref. 1.

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