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Theory of thermomagnetic phenomena in the intermediate-pressure range

V. D. Borman, S. Yu. Krylov, and B. I. Nikolaev

Moscow Engineering Physics Institute

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A kinetic theory of thermomagnetic phenomena occurring in polyatomic gases (the effect of fields on transport phenomena) in the pressure range where the mean free path, $\bar{l}$, of the molecules is comparable to the geometric dimension, L, is proposed. An integral kinetic equation for a gas with rotational degrees of freedom in a magnetic field is derived which takes into account both intermolecular collisions and nonspherical collisions of the molecules with the walls. The distribution function for such a gas confined between two surfaces in a magnetic field is found for pressures at which $\bar{l} \lesssim 0.1L$. The contribution of the various collision processes to the observable changes that occur in the macrofluxes in the field is investigated. The effect involving the appearance in a magnetic field of a pressure difference in a closed plane channel whose walls have different temperatures is considered in detail.

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1. INTRODUCTION

A wide class of phenomena connected with the effect of external fields on transport processes are known at present to occur in rarefied gases.[1] They are due to the precession of the molecules in the field and the nonspherical nature of the collisions between the gas molecules and between the molecules and the walls. These effects are as yet clearly understood only in the limiting cases $\bar{l} \ll L$ and $\bar{l} \gg L$ (l is the mean free path of the molecules, L is the characteristic dimension). The first limiting regime is characterized by the Senftleben-Beenakker effect,[1,2] which is connected with a change in the thermal conductivity and the viscosity of the gas in the field. A thermomagnetic effect whereby a magnetic field affects the heat flux between two surfaces is known to occur in a Knudsen gas ($\bar{l} \gg L$).[3,4] In the limiting cases of high and low gas pressures the kinetic phenomena are wholly determined respectively by the nature of the collisions between the molecules and the walls.

In the intermediate-pressure regime, when $\bar{l} \sim L$, the transport phenomena are determined by both the intermolecular collisions in the gas and the interaction of the molecules with the surface of the solid. As a result, there are observed more complex dependences of the macroscopic fluxes in the gas (e.g., the heat flux) on the field. Furthermore, there occur in the intermediate-pressure range specific effects that disappear at high pressures ($l/L \to 0$). To such phenomena pertain the recently discovered effect involving the appearance in a field of a thermomagnetic pressure difference in a closed plane channel whose walls have different temperatures,[5] the effect of the thermomagnetic force acting in the field on a body in an inhomogeneously heated gas,[6] and the Scott effect,[7,1] which consists in the rotation in a field of a heated cylinder located in a polyatomic gas.

There virtually does not exist at present a kinetic theory that describes the thermomagnetic phenomena occurring in the intermediate pressure region. Attempts made earlier to describe the thermomagnetic-pressure-difference,[8] the thermomagnetic-force,[9] and the Scott[10] effects only allowed a qualitative explanation of these phenomena. In essence, only Vestner's approach,[8] which is based on the solution of the equations for the moments of the distribution function with phenomenological boundary conditions (at the walls) introduced by the method of nonequilibrium thermodynamics, constitutes an attempt at the construction of a consistent theory for the intermediate-pressure region. However, in such an approach the choice of the moments connected with the rotational degrees of freedom is not validated. Furthermore, such a theory, which is more of a hydrodynamic theory than a kinetic one, does not allow us to determine the contribution to the effects of the various collision processes (the intermolecular collisions or the collisions of the molecules with the surface).

In the present paper we construct for the thermomagnetic effects occurring in the intermediate-pressure range a consistent theory based on the use of the integral kinetic equation. The constructed theory can be extended to the case of the effect of an electric field on transport phenomena in rarefied gases of polar molecules, since effects of this type are due to a single precession mechanism. As an application of the theory developed, we consider the thermomagnetic-pressure-difference effect.

Below, in Sec. 2 we formulate the integral kinetic equation for a polyatomic gas in a magnetic field. In Sec. 3 we find the distribution function for a gas confined between two surfaces for the pressure range where $\bar{l} \leqslant 9.1L$ (virtually all the available experimental data[5-7, 11-14] pertain to this regime). The solution constructed in the limit $\bar{l}/L \to 0$ reduces to well-known expressions for the thermal conductivity of a gas in a field (the Senftleben-Beenakker effect). Further (Sec. 4) we investigate the thermomagnetic-pressure-difference effect. The obtained expressions are compared with the experimental data.

2. THE INTEGRAL KINETIC EQUATION FOR A POLYATOMIC GAS IN A MAGNETIC FIELD

If the mean temperature is sufficiently high, so that the rotational degrees of freedom can be treated classically, the steady-state Boltzmann equation for a gas in a magnetic field, **H**, can be written in the form[2]

$$(\mathbf{v}\nabla)f+\gamma[\mathbf{MH}]\frac{\partial f}{\partial \mathbf{M}}=\iiint[f_1'f'W'-f_1fW]d\Gamma_1'\,d\Gamma_1\,d\Gamma'. \tag{2.1}$$

Here **v** and **M** are the velocity and the angular momentum of the molecule, $d\Gamma = d\mathbf{v}d\mathbf{M}$ is the phase volume, and γ is the gyromagnetic ratio of the molecule. The functions W and W' in (2.1) describe respectively the probabilities for the direct and inverse intermolecular (binary) collisions.

The interaction of the gas molecules with a wall is described by a boundary condition connecting the distribution functions of the molecules incident of (f_i), and reflected from (f_r), the surface[15]:

$$|\mathbf{vk}|f_r(\Gamma)=\int\limits_{\mathbf{v}'\mathbf{k}<0}|\mathbf{v}'\mathbf{k}|V(\Gamma'\to\Gamma,\mathbf{k})f_i(\Gamma')d\Gamma', \tag{2.2}$$

where $\Gamma=(\mathbf{v},\mathbf{M})$, **k** is the normal vector to the surface, and V is the probability density of the scattering on the wall.

At present the solution of the Boltzmann integro-differential equation with integral boundary conditions is an extremely complex problem even for the case of monatomic gases. Let us go over from the system of equations (2.1)–(2.2) to an integral kinetic equation that takes into account simultaneously the intermolecular collisions and the collisions with the walls. An integral kinetic equation has been proposed for monatomic gases by Vallander.[16] Let us consider the derivation of such an equation for a polyatomic gas confined between two infinite surfaces located in a magnetic field at a distance L from each other and having different temperatures (T_0 and T_L). Let us orient the coordinate system in such a way that the x-y plane coincides with one surface, while the second surface is described by the equation $z = L$,

The Boltzmann equation (2.1) can be rewritten in the form

$$v_z\frac{\partial f}{\partial z}+\omega\frac{\partial f}{\partial \tilde{\varphi}_{\mathbf{M}}}=I_c\{f\}, \tag{2.3}$$

where $\omega=\gamma H$ is the frequency of precession of the molecules in the field, $\tilde{\varphi}_{\mathbf{M}}$ is the polar angle of orientation of **M** relative to the direction of the field, and I_c is the collision integral.

Let us integrate Eq. (2.3) along the characteristic curves within the limits $0 \leqslant Z \leqslant L$. The system of characteristic curves can be written in the form

$$\frac{dz}{v_z}=\frac{d\tilde{\varphi}_{\mathbf{M}}}{\omega}=\frac{df}{I_c\{f\}}. \tag{2.4}$$

From the first equality we obtain the relation

$$\tilde{\varphi}_{\mathbf{M}}-(\tilde{\varphi}_{\mathbf{M}})_0=\omega(z-z_0)/v_z, \tag{2.5}$$

which gives the change in the orientation of the vector **M** when the molecule moves along the trajectory from the point z_0 to z. Let us split the collision integral into two parts:

$$I_c=I-f\Omega,$$
$$\Omega=\iiint f_1W\,d\Gamma_1'\,d\Gamma_1\,d\Gamma',\quad I=\iiint f_1'f'W'\,d\Gamma_1'\,d\Gamma_1\,d\Gamma'. \tag{2.6}$$

Assuming formally that the functionals Ω and I are known quantities, let us integrate the equation arising from (2.4) and (2.6):

$$v_z\frac{df}{dz}=I-f\Omega. \tag{2.7}$$

As a result, using the relations (2.2) and (2.5), we obtain the following integral equation:

$$\begin{aligned}f(\Gamma,z)=&\int\limits_{\substack{0(v_z>0)\\L(v_z<0)}}^{z}\frac{dz_1}{v_z}\exp\left\{-\int\limits_{z_1}^{z}\frac{dz_2}{v_z}\hat{P}(z\leftarrow z_2)\Omega\right\}\hat{P}(z\leftarrow z_1)\\
&\times\iiint f_1'f'W'd\Gamma_1'd\Gamma'd\Gamma_1+\Theta(v_z)\exp\left\{-\int\limits_0^z\frac{dz_2}{v_z}\hat{P}(z\leftarrow z_2)\Omega\right\}\hat{P}(z\leftarrow 0)\\
&\times\int\limits_{v_z'<0}\frac{|v_z'|}{v_z}V^{(1)}(\Gamma'\to\Gamma,T_0)f(\Gamma',0)d\Gamma'\\
&+\Theta(-v_z)\exp\left\{-\int\limits_L\frac{dz_2}{v}\hat{P}(z\leftarrow z_2)\Omega\right\}\hat{P}(z\leftarrow L)\\
&\times\int\limits_{v'_z>0}\frac{v_z'}{|v_z|}V^{(2)}(\Gamma'\to\Gamma,T_L)f(\Gamma',L)d\Gamma',\end{aligned} \tag{2.8}$$

where $\Theta(x)=1,\ x>0$ and $\Theta(x)=0,\ x\leqslant 0$; $V^{(1)}$ and $V^{(2)}$ are the probabilities of scattering on the lower ($z=0$) and upper ($z=L$) walls; and the functionals $\Omega=\Omega[f,\Gamma,z]$ and $I=I[f,\Gamma,z]$ are defined in (2.6). The operator $\hat{P}$ in (2.8) describes the periodic variation of the orientation of the angular momentum vector of the molecule as the molecule precesses in the magnetic field, and is determined by the relation (A is an arbitrary function of Γ and z_0)

$$\begin{aligned}\hat{P}(z\leftarrow z_0)A(\Gamma,z_0)=&\sum_{lmk\sigma}d_{m\sigma}^{(l)}\ (\theta_{\mathbf{H}})d_{mk}^{(l)}\ (\theta_{\mathbf{H}})Y_{lk}\left(\frac{\mathbf{M}}{M}\right)\\
&\times\exp\left\{i(\sigma-k)\varphi_{\mathbf{H}}+im\frac{\omega(z-z_0)}{v_z}\right\}\int Y_{l\sigma}^*\left(\frac{\mathbf{M}}{M}\right)A(\Gamma,z_0)do_{\mathbf{M}},\end{aligned} \tag{2.9}$$

where $\Theta_{\mathbf{H}}$, $\varphi_{\mathbf{H}}$ are the spherical angles of the field orientation and the functions $d_{mk}^{(1)}$ (the rotational transformation coefficients) are determined in Ref. 17. The integral kinetic equation (2.8) is closed with respect to f and correct for the entire interval $0<\bar{l}/L<\infty$.

In spite of its outward unwieldiness, the obtained equation has a clear physical meaning. Since $\Omega[f, \Gamma, z]$ is the nonequilibrium frequency of collision of the molecules in the state Γ, the exponential factors in all the three terms of Eq. (2.8) give the probabilities for a collisionless flight of the molecules to the point z from the point of the last collision (with a molecule or with a wall). The first term in (2.8) describes the distribution of the molecules that reach the point z after a collision (with a molecule) at any point in the space layer, $0 < z_1 < L$, occupied by the gas. The second and third terms in (2.8) give the distributions of the molecules that arrive respectively from the lower ($z = 0$) and upper ($z = L$) surfaces. These terms are appreciably different from zero only at distances of the order of $\bar{l}$ from the respective walls.

3. THE DISTRIBUTION FUNCTION OF A POLYATOMIC GAS IN A MAGNETIC FIELD IN THE REGIME $\bar{l} \lesssim 0.1L$

Below we shall be interested in the effects in a polyatomic gas at pressures when $\bar{l} \lesssim 0.1L$. In this case we can go over to the consideration of a system in which the gas occupies an infinite half-space ($z > 0$) bounded by the plane surface ($z = 0$) and the temperature of the gas at large distances from the wall is maintained at a value different from the wall temperature. The integral kinetic equation for this case can be obtained by dropping the third term in (2.8).

Let us first find the distribution function of the molecules under the assumption that there is maintained in the gas a temperature gradient having a constant value, $(\Delta T)_g$, at distances from the wall $z \gg \bar{l}$. We shall assume that $|T_0 - T_L| \ll T_0$ and, consequently, that the state of the system differs little from the equilibrium state. Assuming in this case that

$$f = f_M(1+\chi), \quad \chi \ll 1, \tag{3.1}$$

where f_M is the Maxwellian distribution with the wall temperature T_0, we can go over to a linearized kinetic equation for the function χ. The latter can be written in the form

$$\chi(\Gamma, z) = \hat{G}\chi + \hat{S}\chi, \tag{3.2}$$

$$\hat{G}\chi = \int\limits_{\substack{0\,(v_z>0)\\ \infty\,(v_z<0)}} \frac{dz_1}{v_z} \exp\left\{-\int\limits_{z_1}^{z} \frac{dz_2}{v_z} \hat{P}(z\leftarrow z_2)\Omega(\Gamma)\right\}\hat{P}(z\leftarrow z_1)$$

$$\times \iiint f_M(\Gamma_1)[\chi_1' + \chi' - \chi_1] W'\, d\Gamma_1'\, d\Gamma'\, d\Gamma_1,$$

$$\hat{S}\chi = \Theta(v_z)\exp\left\{-\int\limits_0 \frac{dz_2}{v}\hat{P}(z\leftarrow z_2)\Omega(\Gamma)\right\}\hat{P}(z\leftarrow 0)$$

$$\times \int\limits_{v_z'<0} \frac{|v_z'| f_M(\Gamma')}{v_z f_M} V(\Gamma'\rightarrow\Gamma)\chi(\Gamma', 0)\, d\Gamma'.$$

Here

$$\Omega(\Gamma) = \iiint f_M(\Gamma_1) W'\, d\Gamma_1'\, d\Gamma'\, d\Gamma_1. \tag{3.3}$$

Notice that, in contrast to Eq. (2.8), only the z-independent equilibrium collision frequency (3.3) figures in the linearized equation (3.2). It must be emphasized that the integral kinetic equation (3.2) is, in contrast to (2.8), not closed with respect to f, and requires an additional boundary condition for $z \rightarrow \infty$. As the latter, we shall give the temperature gradient in the gas at $z \gg \bar{l}$, regarding it temporarily as known.

We shall solve Eq. (3.2) by the method of iterations. As shown below, it is then possible to ascertain the extent to which the various collision processes contribute to make the distribution function and the macroscopic fluxes field dependent. In principle, the convergence of the iterative procedure for integral kinetic equations has been proved for any value of $\bar{l}/L$.[18] However, for small $\bar{l}/L$ the iterative method is not used in practice because of the necessity to carry out a large number of iterations (when a trivial initial function is used). At the same time the opinion is expressed in the literature that it is possible to obtain a rapid convergence even in the case of small $\bar{l}/L$ values if as the initial function we use a sufficiently correct solution.

As follows from the precession mechanism through which the field influences the kinetic phenomena in a gas (see below for details), to explain the dependence of the macrofluxes on H it is sufficient to consider the two nonspherical collisions (of a molecule with a molecule or with the wall). The integral operators $\hat{G}$ and $\hat{S}$ in (3.2) are nonlinear in the nonsphericity parameter of the intermolecular interaction, since the exponent of the exponential function contains the total scattering probability W. Therefore, when such a distribution function, which describes the macroscopic fluxes in the system in the absence of a field, is used as the initial function, the fluxes begin to depend on the field already after the first iteration (in the second-order approximation in the small nonsphericity parameters of the molecule-molecule and molecule-wall interactions).

Let us construct the solution in Eq. (3.2) in the following manner. Choosing some initial function $\chi^{(0)}$, we seek the more accurate solutions $\chi^{(1)}$, $\chi^{(2)}$... according to the scheme

$$\chi^{(n)} = \hat{G}\chi^{(n-1)} + \hat{S}\hat{G}\chi^{(n-1)}, \quad n = 1, 2, 3, \ldots \tag{3.4}$$

(n is the iteration number). As $n \rightarrow \infty$, the expression (3.4) goes over into an equation equivalent to Eq. (3.2). This is not difficult to show if we take into account the discontinuity of the distribution function in velocity space. The solution scheme (3.4) ensures the fulfilment of the requirement that there should be no gas pileup at the wall at any iteration step.

As the initial function $\chi^{(0)}$, it is advisable to choose a distribution function that correctly describes the heat flux in the gas in the absence of a field. Let us, allowing for the jump, δT, in temperature at the surface,[19] write the expression for $\chi^{(0)}$ in the form

$$\chi^{(0)} = \varphi\frac{\delta T}{T_0} + \frac{(\nabla T)_g}{T_0}\psi z - \frac{(\nabla T)_g}{T_0}\frac{\psi v_z}{\Omega_0}, \tag{3.5}$$

$$\Omega_0 = \iiint f_M(\Gamma_1) W_0'\, d\Gamma_1'\, d\Gamma_1\, d\Gamma', \tag{3.6}$$

where

$$\varphi = \frac{E}{T_0} - c_v, \quad \psi = \frac{E}{T_0} - c_p, \quad E = \frac{mv^2}{2} + \frac{M^2}{2I};$$

E is the energy of the molecule, c_v and c_p are the heat capacities of the gas, and W_0' is the M-independent part of collision probability. In the case when $\Omega_0 = \text{const}$, the expression (3.5) coincides with the solution obtainable for the problem of thermal conductivity by the Chapman-Enskog method (in the first approximation) for pseudo-Maxwellian molecules.[18]

The function $\chi^{(n)}$, in accordance with the relations (3.4)–(3.6), contains the temperature jump as an unknown parameter. Since in the steady-state regime the heat flux Q_z in the system under consideration does not depend on the coordinates, the parameter δT can be determined in any n-th approximation by computing with the aid of the function $\chi^{(n)}$ the flux, $Q_z^{(n)}$, corresponding to it and setting

$$Q_z^{(n)}|_{z=0}=Q_z^{(n)}|_{z\gg\bar{l}}.$$

Let us consider the expression that is obtained in accordance with (3.4) and (3.5) for the function $\chi^{(1)}$ after the first iteration. To simplify it, let us use the smallness of the non-spherical parts of the probabilities of scattering on a molecule and on the surface. As is well known,[2,4] the probabilities W and V can be represented in the form

$$W=W_0+\nu W_1, \quad V=V_0+\varepsilon V_1, \tag{3.7}$$

where W_0 and V_0 do not depend on the orientation of the angular momentum of the molecules; ν and ε are small nonsphericity parameter ($\nu^2, \varepsilon^2 \sim 10^{-3} - 10^{-2}$).

With allowance for (3.7), the expression for the function $\chi^{(1)}$ can be represented in the form of an expansion in powers of the nonsphericity parameters:

$$\chi^{(1)}=\chi_0^{(1)}+\chi_1^{(1)}+\chi_2^{(1)}+\ldots, \tag{3.8}$$

where the subscripts 0, 1, 2, ... correspond to the terms of zeroth first ($\sim\nu$ or ε), second ($\sim\nu^2$ or $\nu\varepsilon$), etc., order in the first ($\sim\nu$ or ε), second ($\sim\nu^2$ or $\nu\varepsilon$), etc., order in the non-sphericity of the interaction. The dependence of the macroscopic fluxes in the gas on the magnetic field is determined by the part of the function $\chi_2^{(1)}$ that is isotropic in **M**. The expression for $\chi_2^{(1)}$ is unwieldy and is given in the Appendix.

Computing with the aid of the found function $\chi^{(1)}$ the heat flux between the plates

$$Q_z^{(1)}=\int v_z E f_M \chi^{(1)} d\Gamma$$

and setting $Q_z^{(1)}|_{z=0}=Q_z^{(1)}|_{z\gg l}$, we can find the expression determining the parameter δT:

$$\delta T=B(\nabla T)_g, \tag{3.9}$$

where B is a quantity of the order of $\bar{l}$ and depending on W, V, and the magnetic field **H** (the expression for B is unwieldy, and is not given here). The expression thus found for the temperature jump can be represented in the form

$$\delta T=(\delta T)_0+(\delta T)_1+(\delta T)_2+\ldots, \tag{3.10}$$

where $(\delta T)_0$ is determined by the functions W_0 and V_0; $(\delta T)_1$ contains terms of first order in the nonsphericity ($\sim\nu, \varepsilon$); and $(\delta T)_2$ contains terms $\sim\nu^2$ and $\nu\varepsilon$. The quantity $(\delta T)_2$ and the terms of the next order in smallness in (3.10) depend on **H**. Let us note that in the case of specular-diffuse reflection of the molecules from the surface and for $\Omega_0 = \text{const}$ (the case of pseudo-Maxwellian molecules) the expression found for $(\delta T)_0$ coincides with the result obtained by the method of moments in the Navier-Stokes approximation.[20] This indicates that the temperature jump is taken account of sufficiently correctly in the constructed iterational solution.

Above we assumed that the temperature gradient in the gas far from a wall, $(\nabla T)_g$, is known. The ultimate aim of the computations is to determine the macroscopic fluxes for a given temperature difference, $\Delta T = T_0 - T_L$, between the walls. Thus, it is now necessary to relate $(\nabla T)_g$ with ΔT. Let us assume for simplicity that the two surfaces $z = 0$ and $z = L$ are identical. Since $|\Delta T| \ll T_0$, it is natural to assume then that the temperature jumps at the two surfaces are equal. In that case

$$(\nabla T)_g=(\Delta T-2\delta T)/L. \tag{3.11}$$

Eliminating δT from the expressions (3.9), (3.11), we obtain

$$(\nabla T)_g=\Delta T/(L+2B). \tag{3.12}$$

Let us consider the physical meaning of the solution. The iterational solution to Eq. (3.2) allows us to elucidate the physical nature of the effect of a field on transport phenomena in a gas. As can be verified, the terms $A_i^{(lmk\sigma)}$ ($i = 1, \ldots, 9$) in the expression for that part of the distribution function $\chi_2^{(1)}$, (A.7), which is isotropic in **M**, and which determines the dependence of the fluxes in the gas on the field, correspond to the contributions of the various processes. For convenience of exposition, let us use diagrams in which we shall represent: by a straight line the molecule distribution function that does not depend on **H** and is isotropic in **M**, by a wavy line the distribution function that is anisotropic in **M**, and describes the polarization of the molecules, and by a double line the distribution function that is isotropic in **M**, but dependent on **H**.

From the point of view of the precession mechanism the effect of a field on transport phenomena can be explained as follows. As a result of the nonspherical collisions (between the molecules and between the molecules and a wall), the molecules become polarized with respect to angular-momentum orientation. Because of the precession of the molecules when the field is switched on, the distribution of the polarized molecules becomes **H** dependent. After "repeated" nonspherical collisions the polarization of some of the molecules may disappear. As a result, that part of the distribution function which is isotropic in **M** and, consequently, the macrofluxes in the gas become **H** dependent.

Let us first consider the appearance of the molecular polarization. As can be seen from analysis of the structure of the operator $\hat{G}$, (3.2), two processes can occur as a result of the intermolecular collisions. First, polarized molecules are produced as a result of nonspherical collisions between the unpolarized molecules. This process is described by the factor

$$\iiint f_M(\Gamma_1)[\chi^{(0)}(\Gamma_1')+\chi^{(0)}(\Gamma')-\chi^{(0)}(\Gamma_1)]\nu W_1' \, d\Gamma_1' \, d\Gamma' \, d\Gamma_1$$

in the integrand for $\hat{G}\chi^{(0)}$. Let us represent this process by the diagram a) in Fig. 1. Second, since the

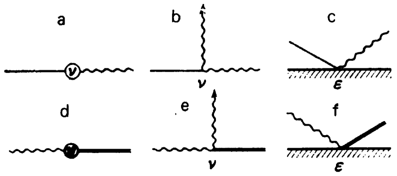

FIG. 1.

collision rate $\Omega(\Gamma)$ depends on **M**, there are removed from among the unpolarized molecules as a result of the collisions molecules largely with a definite orientation. Therefore, the remaining molecules will be polarized. This process is described by the linear term in the expansion of the probability for a collisionless flight (the exponential factors in (3.2) in powers of ν. Such a process is depicted by the diagram b) in Fig. 1. A third process that gives rise to polarized molecules is nonspherical reflection of the molecules from a surface (see Fig. 1, c)). This process is described by the probability εV_1 in $\hat{S}$, (3.2).

In analogy with the foregoing, we find that the inverse processes, i.e., the production of unpolarized molecules in collisions between polarized molecules, are depicted by the diagrams d)–f) in Fig. 1, and are also described by the corresponding expressions indicated above.

An analysis of the computations that lead to the expression for that part of the distribution function $\chi_2^{(1)}$, (A.7), which is isotropic in **M** and dependent on H, shows that the terms $A_i^{(lmk\sigma)}$ of this expression correspond to the processes depicted in Fig. 2.

The process depicted by the diagram d) in Fig. 1 does not figure in the diagrams shown in this figure. As follows from an analysis of the adopted solution scheme (3.4), such a process can be taken into account only after the second iteration. As a result of this, to the obtained eight diagrams will be added another group of eight (we can easily construct them in much the same way as the diagrams shown in Fig. 2, using in addition the process d) of Fig. 1). By simply sorting out the possible variants, it can easily be verified that these sixteen diagrams exhaust all the possible cases. In this way we convince ourselves that, to take account of all the possible causes of the appearance of a dependence of the macrofluxes in the gas on the field in the solution of Eq. (3.2) by the method of successive approximations, it is sufficient to carry out only two iterations. The subsequent iterations only lead to further refinement of the spherically symmetric part of the diagrams given above (the part depicted by a single straight line). This corroborates the assumption that the iterational solution converges rapidly from the standpoint of the description of the field dependences.

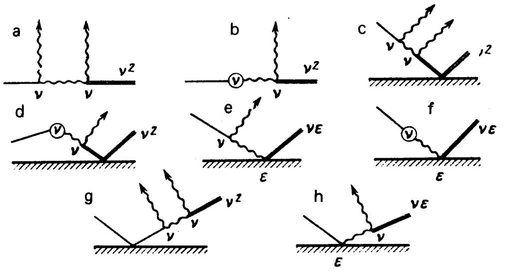

FIG. 2. Processes giving rise to the dependence of the macrofluxes in a gas on **H** in the solution obtained after the first iteration. The diagram a) corresponds to the terms A_1 and A_2; b) A_3; c) A_4; d) A_5; e) A_6; f) A_7; g) A_8; h) A_9 in (A.7).

Since the explicit form of the probabilities for nonspherical scattering of a molecule on a molecule, W_1, and on a wall, V_1, is not known in the general case, the inference of the necessity of allowance for one or another interaction process can be made only on the basis of a comparison with experiment. As the above-performed computations show, the observable characteristics of the thermomagnetic-pressure-difference and thermomagnetic-force effects can be described within the limits of experimental accuracy when the results of the first iteration are used, i.e., when the processes depicted in Fig. 2 are considered. Thus, we can restrict ourselves to the solution given above, a more accurate determination of the distribution function being unnecessary.

The constructed solution can be verified by using it to describe the effect of a field on the thermal conductivity of a gas (the Senftleben-Beenakker effect). Let us compute the heat flux, **Q**, between the surfaces in the limit $\bar{l}/L \to 0$. For this purpose, let us retain in the expression obtained above for the nonequilibrium distribution function $\chi_2^{(1)}$ only the terms of first order in $\bar{l}/L$, and set $z \gg \bar{l}$ (the solution has been constructed in such a way as to ensure the equality of the values of Q_z for $z=0$ and $z \gg \bar{l}$). As a result, it is not difficult to obtain for the field-dependent part of the heat flux the expression

$$Q_i(\mathbf{H}) = \int v_i E f_M \chi_2^{(1)} d\Gamma = B_i^{(1)} + B_i^{(2)}, \quad i=x,y,z;$$

$$B_i^{(1)} = \nu^2 \frac{(\nabla T)_z}{T_0} \int v_i E f_M a_2 \sum_{lmk\sigma} R_{mk-\sigma}^{(l)}\left(\frac{\mathbf{H}}{H}\right) \times \frac{1 - im\omega/\Omega_0}{1+(m\omega/\Omega_0)^2} \frac{\Omega_{1lk}\tilde{\Omega}_{1l\sigma}}{\Omega_0^2} d\Gamma, \tag{3.13}$$

$$B_i^{(2)} = -\nu^2 \frac{(\nabla T)_z}{T_0} \int v_i E f_M a_1 \sum_{lmk\sigma} R_{mk-\sigma}^{(l)}\left(\frac{\mathbf{H}}{H}\right) \times \frac{1}{1+(m\omega/\Omega_0)^2} \frac{\Omega_{1lk}\Omega_{1l\sigma}}{\Omega_0^2} d\Gamma,$$

where $(\nabla T)_z \approx \Delta T/L$. The quantities entering into (3.13) are determined by the expressions (A.1–6). The terms $B^{(1)}$ and $B^{(2)}$ in (3.13) are connected with the processes described respectively by the diagrams b) and a) in Fig. 2.

We shall further use the model expression proposed by Kagan and Maksimov[2] for the probability for nonspherical molecule-molecule scattering, when the dependence of W_1 on the orientations of the relative-velocity and angular-momentum vectors of the molecules is described by the Legendre polynomial P_2. In this case, as the above-performed calculations show, it is possible to have such types of molecular polarization as when the angular dependence of that part of the distribution function of the gas which is anisotropic in $\mathbf{M}(\chi_1^{(1)}$ in (3.8)) is described by the combinations $\ldots[\mathbf{v}][\mathbf{M}]^2$ and $\ldots[\mathbf{v}]^3[\mathbf{M}]^2$ ($[\mathbf{a}]^m$ is an irreducible tensor or rank m

constructed from the components of the vector **a**). A polarization of the first type corresponds to the process a) in Fig. 1; of the second type, to the process b). As is well known, it is sufficient for the description of the Senftleben-Beenakker effect as applied to thermal conductivity for diatomic gases to take only the $[\mathbf{v}][\mathbf{M}]^2$ polarization into account. In the expression (3.13) to such a molecular polarization corresponds the term $B^{(1)}$ (see the diagram b) in Fig. 2). Below we show that the expression (3.13) for $B^{(1)} \neq 0, B^{(2)} = 0$ allows us to account for the experimental data.

As follows from the relation (3.13), the dependence of $\mathbf{Q}$ on the field strength is determined by the relation $\omega/\Omega_0 \sim H/p$ (p is the pressure). Since $\Omega_0 = \Omega_0(v^2, M^2)$, the H/p dependence of $\mathbf{Q}$ is complex. Let us assume for simplicity that Ω_0 varies slowly as v and M vary near their thermal values. (Such an assumption is reasonable, since the model of pseudo-Maxwellian molecules, for which $\Omega_0 = \text{const}$ by definiton, provides a good qualitative description in the majority of the problems of the kinetics of rarefied gases.) Then the integration in (3.13) can be carried out approximately by taking out the quantity

$$[1+(m\omega/\Omega_0)^2]^{-1}$$

from under the integral sign and replacing in the process Ω_0 by some constant value $\bar{\Omega}_0^{(Q)}$.

Computing by the indicated method the thermal flux (3.13) for $B^{(2)} = 0$, we can find the expressions for the changes in the thermal conductivity of the gas in the fields $\mathbf{H} \| \nabla T (\Delta\lambda_{zz}^{\|})$ and $\mathbf{H} \perp \nabla T (\Delta\lambda_{zz}^{\perp})$, as well as for the transverse thermal conductivity (λ_{xz}^{tr}) arising in the field $\mathbf{H} = (0, H_z, 0)$:

$$\Delta\lambda_{zz}^{\|} = s_1 \frac{\xi^2}{1+\xi^2}, \quad \Delta\lambda_{zz}^{\perp} = \frac{s_1}{2}\left[\frac{\xi^2}{1+\xi^2} + 2\frac{4\xi^2}{1+4\xi^2}\right],$$
$$\lambda_{xz}^{tr} = \frac{s_1}{2}\left[\frac{\xi}{1+\xi^2} + 2\frac{2\xi}{1+4\xi^2}\right], \tag{3.14}$$

where $\xi = \omega/\bar{\Omega}_0^{(Q)}$; the constant s_1 ($|s_1| \sim \nu^2$) can be computed if the dependence of the probability W_1 on the quantities v^2 and M^2 is known. The expressions (3.14) coincide with the expressions found in Ref. 2, and describe the observed dependences for the majority of diatomic gases.[11]

It should be noted that the quantity $\bar{\Omega}_0^{(Q)}$ in (3.14) (in contrast to Ω_0) is not an equilibrium collision frequency. Its value is determined by the dependence of the functions $\tilde{\Omega}_{11\sigma}$ and Ω_{11k} in (3.13) on v and M^2; therefore, the parameter ξ in (3.14) is, as in the theory proposed earlier, connected with its own kind of "transport" collision frequency [see the expression (A.3)].

4. THE THERMOMAGNETIC-PRESSURE-DIFFERENCE EFFECT

As was recently discovered,[5,12,13] a pressure difference arises in a polyatomic gas ($\bar{l}/L = 10^{-3} - 10^{-1}$) contained in a closed plane channel whose walls have different temperatures upon the application of a magnetic field. This phenomenon is connected with gas-mass transport in a direction perpendicular to the magnetic field, $\mathbf{H}$, and to ∇T when the field is directed at an angle to ∇T (see Fig. 3). The pressure difference, δp, arising between the ends of the channel is an odd function of the field, is described at sufficiently high pressures by a function of the form

$$\delta p = \frac{\nabla T}{p} F\left(\frac{H}{p}\right)$$

(see Fig. 4), and vanishes in the limit of high pressures. Therefore we can conclude that the gas-mass transport arises only in a Knudsen layer (of thickness $\sim\bar{l}$) near the surfaces.

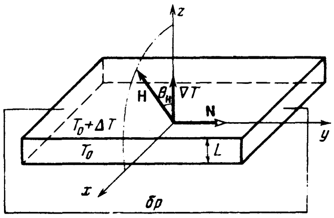

FIG. 3.

In order to understand the causes of the appearance of the thermomagnetic-pressure difference (TMPD), let us first consider the case of an open channel (it is assumed that the length and width of the channel are significantly greater than its height, L). Let the field vector lie in the x-z plane. Neglecting the terms of higher order in smallness in the collision nonsphericity, we can compute the thermomagnetic gas flux, N_y, near the lower wall from the formula

$$N_y = m\int v_y f d\Gamma = m\int v_y f_M \chi_2^{(1)} d\Gamma, \tag{4.1}$$

where $\chi_2^{(1)}$ is determined by the relation (A.7) and is the part of the nonequilibrium distribution function for the gas near the surface $z = 0$ that is $\mathbf{H}$ dependent and isotropic in $\mathbf{M}$. As is easy to verify, the quantity N_y, (4.1), is an even function of $\mathbf{k}$ ($\mathbf{k}$ is the normal vector to the surface). Therefore, if we assume that the two walls are identical and that $|\Delta T| \ll T_0$, then the gas flux is symmetric about the plane $z = L/2$. The gas flux averaged over the cross section can then be represented in the form

$$\bar{N}_y = \frac{1}{L}\int_0^L N_y(z)\,dz \approx \frac{2}{L}\int_0^\infty dz \int m v_y f_M \chi_2^{(1)}\,d\Gamma. \tag{4.2}$$

In the case of a closed channel the gas flux, $\tilde{N}_y$, that arises at the moment of application of the field is the same as the flux that arises in an infinite channel. How-

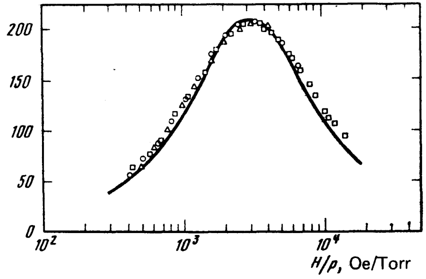

FIG. 4. Dependence of the pressure difference δp on H/p for $\mathbf{H} \perp \nabla T$ for the CO gas. The solid line corresponds to the expression (4.11) with $c_1/c_4 = 0.32$. The experimental points were taken from Ref. 12.

ever, later, under the action of $\bar{N}_y$ a pressure difference, δp, is established between the ends of the channel, and a gas counterflow, $N_{cf} = \Lambda\delta p$, arises. The coefficient Λ can be found from the solution of the problem of gas flow in the channel under the action of δp, and depends in the general case on the field. In the steady-state regime we evidently have

$$\bar{N}_y = -\bar{N}_{cf} = -\Lambda\delta p, \tag{4.3}$$

and there is no resultant gas flow in the channel. Since, as follows from (4.2), the flux $\bar{N}_y$ is quadratic in the nonsphericity, for the pressure difference in the steady state we have, neglecting the terms of higher order in smallness, the expression

$$\delta p = -\bar{N}_y/\Lambda_0, \tag{4.4}$$

where Λ_0 is field independent, and is determined by the spherically symmetric molecule-molecule and molecule-surface interactions.

According to the foregoing, to determine δp it is sufficient to find the mean thermomagnetic gas flux, N_y, in an open channel. Computing N_y from the formulas (4.2), (A.7), we obtain

$$\bar{N}_y = -\Lambda_0\delta p = \sum_{i=1}^{5} U_i, \quad U_i = \frac{[\Delta T - 2(\delta T)_0]m}{T_0 L^2} \sum_{lmk\sigma} R^{(l)}_{mk-\sigma} u_i^{(lmk\sigma)} \tag{4.5}$$

where

$$u_1^{(lmk\sigma)} = \nu^2 \int\limits_{v_z>0} \frac{v_y v_z f_M a_2}{\Omega_0^2} \frac{3\Omega_0 im\omega + im^3\omega^3}{(\Omega_0^2 + m^2\omega^2)^2} \Omega_{1lk} \tilde{\Omega}_{1l\sigma}\, d\Gamma,$$

$$u_2^{(lmk\sigma)} = -\nu^2 \int\limits_{v_z>0} \frac{v_y}{\Omega_0} d\Gamma \int\limits_{v_z'<0} \frac{|v_z'| f_M' a_2'}{\Omega_0'} \cdot V_0(\Gamma' \to \Gamma) \frac{im\omega}{\Omega_0'^2 + m^2\omega^2} \Omega_{1lk}' \tilde{\Omega}_{1l\sigma}'\, d\Gamma',$$

$$u_3^{(lmk\sigma)} = \nu\varepsilon \int\limits_{v_z>0} \frac{v_y}{\Omega_0} d\Gamma \int\limits_{v_z'<0} |v_z'| f_M' a_1' \frac{im\omega}{\Omega_0'^2 + m^2\omega^2} V_{lk00}(\Gamma' \to \Gamma) \Omega_{1l\sigma}'\, d\Gamma',$$

$$u_4^{(lmk\sigma)} = -\frac{\nu\varepsilon}{(4\pi)^{1/2}} \int\limits_{v_z>0} \frac{v_y}{\Omega_0} d\Gamma \int\limits_{v_z'<0} |v_z'| f_M' a_2' \times \frac{im\omega}{\Omega_0'^2 + m^2\omega^2} V_{lk00}(\Gamma' \to \Gamma) \tilde{\Omega}_{1l\sigma}'\, d\Gamma',$$

$$u_5^{(lmk\sigma)} = \frac{\nu\varepsilon}{(4\pi)^{1/2}} \int\limits_{v_z>0} \frac{v_y}{\Omega_0} d\Gamma \int\limits_{v_z'<0} |v_z'| f_M' \times \left\{ \frac{(\delta T)_0 L\varphi'}{\Delta T - 2(\delta T)_0} - a_1' \right\} \frac{im\omega}{\Omega_0'^2 + m^2\omega^2} \Omega_{1lk} V_{00l\sigma}(\Gamma' \to \Gamma)\, d\Gamma'.$$

Here, as above, under the quantities with primes (f_M', Ω_0', etc.) are meant the corresponding functions of $\bar{\Gamma}'$. The terms U_i, $i = 1, \ldots, 5$, in (4.5) are connected with the collision processes depicted respectively by the diagrams b), d), e), f), and h) in Fig. 2. The processes corresponding to the diagrams a), c), and g) cannot, as the calculations show, lead to effects that are odd in the field.

In the limit of sufficiently high gas pressures, we can neglect the quantity $2(\delta T)_0$ in comparison with ΔT in the expressions for U_i $(i = 1, \ldots, 5)$. In this case all the five terms of the expression (4.5) are of the order of

$$\frac{\Delta T}{T_0}\left(\frac{l}{L}\right)^2 mn\bar{v}$$

and reduce to the form $p^{-1}F(H/p)$, which agrees with experiment. The terms of the next order in smallness in $\bar{l}/L$ in (4.5) describe the deviation, observed at low pressures, of the TMPD from the law $p^{-1}F(H/p)$ ("Knudsen corrections" to the effect[12,13]).

As follows from (4.5), the TMPD effect is a composite phenomenon, connected with both the nonspherical collisions between the molecules and the nonspherical molecule-surface interaction. Let us emphasize that, although the term U_1 (the diagram b) in Fig. 2) is not formally connected with the collisons with the walls, it corresponds to the presence of a gas flux only near the surface, and arises because of the discontinuity of the distribution function of the gas (in velocity space) in the Knudsen layer. As shown by an analysis of the z dependence of the flux N_y, the processes d), e), and f) depicted in Fig. 2 (to them correspond the terms U_2, U_3, and U_4 in (4.5) give rise to a gas flux in the channel that is such that $N_y \neq 0$ at $z = 0$ and $z = L$, i.e., such that thermomagnetic slipping of the gas occurs at the wall. The processes b) and h) ($i = 1, 5$ in (4.5)) lead to the appearance of a gas flux only near the walls (at $z \neq 0$ and $z \neq L$).

The thermomagnetic gas flux connected with the spherically symmetric reflection of molecules from the walls (the diagram d) in Fig. 2) can apparently be neglected. Indeed, as follows from (4.5), the quantity $U_2 \neq 0$ only in the case when the probability V_0 contains terms proportional to $\sin(\varphi_v - \varphi_{v'})$ or $\cos(\varphi_v - \varphi_{v'})$. On account of the fact that V_0 is positive definite, such terms and, consequently, the quantity U_2 should be small. We shall therefore assume that $U_2 \approx 0$.

Let us consider the contributions of U_3 and U_4 to (4.5). As follows from the analysis of the Senftleben-Beenakker effect, a molecule-polarization process of the type b) in Fig. 1 can be neglected in comparison with the process a). Therefore, as can be seen from an analysis of the diagrams e) and f) in Fig. 2, the contribution of U_3 to (4.5) can be neglected. We shall, in accordance with the foregoing, set $U_3 \approx 0$.

Let us compute the quantity U_1 in (4.5). We shall carry out the $d\bar{\Gamma}$ integration approximately, in much the same way as was done above in the derivation of the expression, (3.14), for the thermal conductivity. The mean value, $\bar{\Omega}_0^{(N1)}$, of the collision frequency Ω_0 will then generally speaking differ from $\bar{\Omega}_0^{(Q)}$ in (3.14). As a result, it is not difficult to find (neglecting terms of the order of $(\bar{l}/L)^3$) that

$$U_1 = c_1 \frac{h_0}{p} \sin\theta_H \left[(1 + \cos^2\theta_H) \frac{3\xi_1 + \xi_1^3}{(1 + \xi_1^2)^2} + (4 + \sin^2\theta_H) \frac{3\xi_1 + 4\xi_1^3}{(1 + 4\xi_1^2)^2} \right],$$

$$h_0 = \frac{\Delta T m n_0^2 v_0^3}{L^2 [\bar{\Omega}_0^{(N_1)}]^2}, \quad \frac{v_0}{\bar{\Omega}_0^{(N_1)}} \sim \bar{l}, \quad v_0 = \left(\frac{2T_0}{m}\right)^{1/2}, \tag{4.6}$$

$$\xi_1 = \frac{\omega}{\bar{\Omega}_0^{(N_1)}} = c_1' \frac{H}{p}, \quad p = n_0 T_0$$

(n_0 is the equilibrium gas density). The parameters c_1 and c_1'($|c_1| \sim \nu^2$) can be computed if the dependence of the probability W_1 on v^2 and M^2 is known.

To compute the contributions of U_4 and U_5 to the thermomagnetic gas flux, it is necessary to choose an expression for the nonspherical part (V_1) of the probability for scattering of the molecules on a surface. For this purpose let us consider an expansion of the form[4]

$$V_1(\mathbf{v}', \mathbf{M}' \to \mathbf{v}, \mathbf{M}; \mathbf{k}) = |\mathbf{vk}| e^{-E/T} \sum_s \beta_s(v'^2, M'^2, v^2, M^2; T) A_s, \qquad (4.7)$$

where

$$s = \{(l_1'l_1)j_1, (l_2'l_2)j_2, l_3\},$$

$$A_s = [Y^{l_1'}(\mathbf{v}') \times Y^{l_1}(\mathbf{v})]^{j_1} [Y^{l_2'}(\mathbf{M}') \times Y^{l_2}(\mathbf{M})]^{j_2} Y^{l_3}(\mathbf{k}).$$

We shall construct a model expression for V_1 by retaining in (4.7) the necessary finite number of terms and determining the expansion parameters from a comparison with experiment. Notice that, according to (4.5) and (A.4), the thermomagnetic gas flux (as well as the heat flux in a Knudsen gas[4]) depends only on the terms of the expansion (4.7) with $l_2 = 0$ or $l_2' = 0$.

Unfortunately, in the experimental investigations published in Refs. 5, 12, and 13 the material from which the walls of the channel were made is not indicated. However, as investigations of the effect of a field on the heat flux in a Knudsen gas have shown,[4] the probability V_1 depends to a greater extent on the composition of the adsorbed layer on the surface than on the material of the backing. In particular, the probabilities of scattering of N_2 molecules on Au and Pt surfaces covered by chemisorbed O_2 layers have the general dominant terms $\{(11)1, (20)2, 2\}$ and $\{(11)1, (02)2, 2\}$, and differ only in that they have different supplementary terms [see the expression (4.7)]. Therefore, if we assume that in the experiments on TMPD,[5,12,13] just as in the indicated experiments in a Knudsen gas,[4] the surfaces were covered by chemisorbed oxygen,[1] then it is natural to assume that the terms

$$\{(11)1, (20)2, 2\}, \quad \{(11)1, (02)2, 2\} \qquad (4.8)$$

are also the dominant terms in the expansion (4.7) for the probability for scattering of the molecules N_2, CO_2, and CO on the surfaces in question. The computation of the fluxes U_4 and U_5, (4.5), with the use of the expression (4.8) for V_1 leads (upon the neglect of terms $\sim(\bar{l}/L)^3$) to the result

$$U_4 = c_4 \frac{h_0}{p} \sin\theta_H \left[\cos^2\theta_H \frac{\xi_4}{1+\xi_4^2} + \sin^2\theta_H \frac{\xi_4}{1+4\xi_4^2}\right], \qquad (4.9)$$

$$U_5 = c_5 \frac{h_0}{p} \sin^3\theta_H \left[\frac{\xi_5}{1+\xi_5^2} + \frac{\xi_5}{1+4\xi_5^2}\right], \qquad (4.10)$$

where

$$\xi_4 = \omega/\bar{\Omega}_0^{(N_4)} = c_4' H/p, \quad \xi_5 = \omega/\bar{\Omega}_0^{(N_5)} = c_5' H/p;$$

the parameters c_4, c_4', c_5, and c_5'($|c_4|$, $|c_5| \sim \nu\varepsilon$) can be computed if the dependence of W and V_1 on v^2 and M^2 is known.

The expressions (4.6), (4.9), and (4.10) have similar dependences on H/p. At the same time, U_1, U_4, and U_5 depend differently on the field orientation. Therefore, in order to compute the contribution of the processes b), h), and f) shown in Fig. 2, it is necessary to investigate the angular dependences of δp on θ_H. The θ_H dependences of $(\delta p)_{\max}$ and $(H/p)_{\max}$ obtained by Eggermont *et al.*[13] for the gas CO are presented in Fig. 5 (the experimental points). In the same figure are shown the dependences obtained under the assumption that $U_i = \Lambda_0 \delta p$ for $i = 1, 4, 5$ (the curves 1, 2, and 3, respectively). It can be seen from the figure that in the case of U_5 the quantity $(H/p)_{\max}$ does not depend on θ_H. On the other hand, it is clear that the experimental data can be accounted for if we use in (4.5) only a linear combination of the terms U_1 and U_4 (the curves 1 and 2 in the figure). It is therefore reasonable to assume that $U_5 \approx 0$ for CO. (This result follows from the specific dependence of W_1 and V_1 on the quantities v^2 and M^2.)

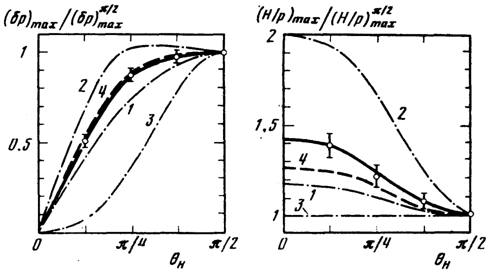

FIG. 5. Dependence of $(\delta p)_{\max}$ and $(H/p)_{\max}$ on the angle of orientation of the field for the CO gas. The curves 1, 2, 3 correspond to the expressions (4.6), (4.9), (4.10). The solid line corresponds to the expression (4.11) with $c_1/c_4 = 0.32$. The experimental points and the curves 4 (theory propounded in Ref. 8) were taken from Ref. 13.

There is another argument that leads to such a result. As the pressure is lowered, there is observed in all the investigated gases[5,12] a deviation from the law $\delta p = p^{-1}F(H/p)$, expressed in terms of a decrease in the quantity δp relative to the value predicted by this formula, as well as a shift of the dependence $F(H/p)$ along the H/p axis to the right (towards the region of higher H/p values). The decrease in the magnitude of the effect can be explained by the presence of the factor $\Delta T - 2(\delta T)_0$ in each of the terms U_i, $i = 1, \ldots, 5$, since the temperature jump increases with decreasing pressure. The shift along the H/p axis can be connected with only the term U_5 in (4.5) (its dependence on pressure is different from that of the other terms; therefore, the total function $F(H/p)$ varies as p is varied). Since in CO (in contrast to N_2, for example) the Knudsen shift is equal to zero,[12] it is natural to assume that $U_5 = 0$ for this gas and that the process depicted by the diagram h) in Fig. 2 does not occur. (Strictly speaking, the assertion is sufficient, but not necessary for the absence of the Knudsen shift.)

Thus, the TMPD effect in CO is determined by only the two terms U_1 and U_4 of the expression (4.5). We shall, for simplicity, assume that $\bar{\Omega}_0^{(N_1)} = \bar{\Omega}_0^{(N_4)}$ and, consequently, that $\xi_1 = \xi_4$. In accordance with (4.5), (4.6), and (4.9), we finally have

$$\delta p = -\frac{\bar{N}_y}{\Lambda_0} = c_1 \frac{h_0}{\Lambda_0 p} \sin\theta_H \left[(1+\cos^2\theta_H)\frac{3\xi_1+\xi_1^3}{(1+\xi_1^2)^2} + (4+\sin^2\theta_H)\frac{3\xi_1+4\xi_1^3}{(1+4\xi_1^2)^2}\right] + c_4 \frac{h_0}{\Lambda_0 p} \sin\theta_H \left[\cos^2\theta_H \frac{\xi_1}{1+\xi_1^2} + \sin^2\theta_H \frac{\xi_1}{1+4\xi_1^2}\right] \qquad (4.11)$$

($\xi_1 = \xi_4 = c_1' H/p$). The expression (4.11) contains three undetermined parameters: c_1, c_1', and c_4. Agreement of the angular dependences corresponding to (4.11) with experiment (the solid lines in Fig. 5) occurs when $c_1/c_4 \approx 0.32$. The parameters c_1', and c_4. Agreement mined by equating the values of $(\delta p)_{\max}$ and $(H/p)_{\max}$ resulting from (4.11) to the experimental values. The H/p dependence of δp obtained as a result in the case

$\mathbf{H}\perp\nabla T(\theta_H=\pi/2)$ is shown in Fig. 4.

The agreement with experiment, obtained with a small number of terms in the expression for the probability V_1, is not strange. As can easily be verified, further allowance for the terms $\{(11)0, (20), 2, 2\}$, $\{(11)0, (02)2, 2\}$, etc., entering into the expression for the probability for scattering of CO molecules[4] does not lead to a change in the expression (4.9) and, consequently, in (4.11).

It follows from (4.11) that the relative contribution of the terms U_1 and U_4 to the quantity $(\delta p)_{\max}$ is given by $U_1/U_4 \approx 2.35$. This means that 70% of the TMPD in CO is determined by the nonspherical collisions between the gas molecules near the walls (the process b) in Fig. 2) and the remaining 30% is connected with nonspherical reflection from the surfaces of polarized (as a result of collisions in the gas) molecules (the process f) in Fig. 2).

Let us briefly touch upon the distinctive features of the TMPD in other gases. In N_2 there is observed[12] at low pressures a small Knudsen shift of the dependence $\delta p(H/p)$ along the H/p axis. Therefore, here we should have that $U_5 \neq 0$. This implies that, besides the above-indicated two mechanisms by which a gas flux arises in a magnetic field, a third mechanism, connected with the polarization of the molecules reflected from a wall (the diagram h) in Fig. 2), also makes a small contribution in N_2. Since the angular dependence of the quantity U_5 (the curve 3 in Fig. 5) markedly differs from the analogous dependences for U_1 and U_4, in nitrogen, when the angle θ_H is decreased, the quantity $(\delta p)_{\max}$ should decrease somewhat more rapidly, while $(H/p)_{\max}$ should increase more slowly, than in CO. indeed, such a conclusion is corroborated by recent experiments.[14]

A different situation should, in accordance with the constructed theory, be observed in O_2 and NO (the TMPD effect has not been investigated in these gases). As shown in Ref. 24, the dominant terms of the expansion, (4.7), of the probability for scattering of these gases on a surface have $l_2'=1$, $l_2=0$ or $l_2'=0$, $l_2=1$. In this case, as follows from (4.5), $U_3=U_4=U_5=0$, and the cause of the TMPD can only be the nonspherical collisions between the molecules (the process b) in Fig. 2).

The calculations carried out reveal an interesting characteristic of the effect under consideration, to wit, an oscillating dependence of the gas flux N_y in a channel on the product Hz. Near a surface, at fixed H, the quantity N_y depends periodically on the distance from the wall, while at a given point z it depends periodically on H. These oscillations die down as we move away from the wall and as H increases. The number of peaks that the quantity N_y has in a Knudsen layer for a given H is approximately equal to $(H/p)/(H/p)_{\max}$. The profile of the flux $N_y(z)$ at fixed H in open and closed channels is shown (schematically) in Fig. 6 (the cases a) and b)).

It can be shown that a similar dependence on Hz obtains for the transverse heat flux $(Q_i,\ i=x,y)$ and stresses $(P_{ij},\ i,j=x,y,z)$ in a gas near walls, as well as for the profiles $T(z)$ and $n(z)$ near surfaces. This behavior is connected with the presence of the periodic-in Hz-factor $\exp\{-(\Omega_0-im\omega)z/v_z\}$ in the terms $A_i(i=1-3, 8, 9)$ of the expression for the nonequilibrium distribution function (A.7), and stems from the collision processes depicted by the diagrams a), b), g), and h) in Fig. 2. Physically, the periodic dependence of the macrofluxes in a Knudsen layer on Hz is similar to the periodic dependence of the macrofluxes in a Knudsen gas on HL[3,4]. Thus, for example, for the process h) we have the following. From a wall we have flying away molecules whose $\mathbf{v}$ and $\mathbf{M}$ vectors have some preferred relative orientation (the molecules are polarized). As a result of the precession of the molecules in the field and the dependence of the collision fre quency, Ω, on the orientation of $\mathbf{v}$ and $\mathbf{M}$, the collision probability for such molecules dpends periodically on $\omega t \sim$ Hz (t is the time of the motion). Then the distribution of the unpolarized molecules produced following collisions and, consequently, the macrofluxes in the gas will depend periodically on Hz. As we move away from the wall, or as H is increased, there occur as a result of repeated precessions of the molecules and the existing velocity distribution an averaging over all the possible orientations of $\mathbf{M}$ and the damping of the oscillations.

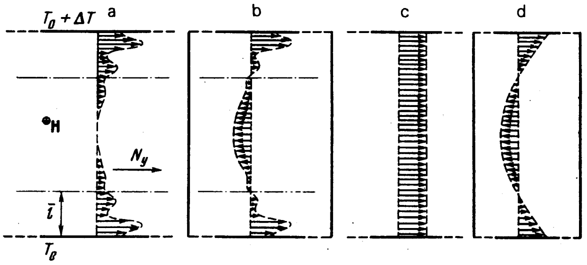

FIG. 6. Profiles of the thermomagnetic gas flux in open (a, c) and closed (b, d) channels. The profiles a) and b) correspond to the expressions (4.1) and (4.3). The profiles c) and d) were taken from Ref. 12.

By analogy with the above-constructed theory of the thermomagnetic-pressure-difference effect, we can, using the found distribution function for a gas at intermediate pressures in a magnetic field, analyze the thermomagnetic-force effect and the Knudsen corrections to the Senftleben-Beenakker effect as applied to thermal conductivity. A similar approach can also be used to construct a theory of the Scott effect.

In conclusion, let us note that the above-considered gas-pressure region, where $\bar{l}/L \leq 0.1$, together with the previously investigated[4] Knudsen regime $(\bar{l} \gg L)$, covers all the possible collision processes that lead to the appearance of field-dependent macrofluxes in a gas. Thus, the above-performed investigations allow us to obtain a complete physical of thermomagnetic phenomena for arbitrary values of $\bar{l}/L$.

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APPENDIX

Let us give the expression obtained for that part of the function $\chi_2^{(1)}$ which is isotropic in **M** after the first iterative procedure. Let first introduce a number of notations:

$$\Gamma=(\mathbf{v},M^2),\quad d\Gamma=4\pi d\mathbf{v}MdM, \tag{A.1}$$

$$\tilde{\Omega}_0(\Gamma)=\left(\frac{m}{2T_0}\right)^{1/2}\iiint f_M(\Gamma_1)[\psi_1'v_{z1}'+\psi'v_z'-\psi_1v_{z1}]W_0'\,d\Gamma_1'\,d\Gamma'\,d\Gamma_1,$$

$$\Omega_{1l\sigma}(\Gamma)=\int Y_{l\sigma}^*\left(\frac{\mathbf{M}}{M}\right)do_M\iiint f_M(\Gamma_1)W_1'd\Gamma_1'd\Gamma'd\Gamma_1, \tag{A.2}$$

$$\tilde{\Omega}_{1l\sigma}(\Gamma)=\left(\frac{m}{2T_0}\right)^{1/2}\int Y_{l\sigma}^*\left(\frac{\mathbf{M}}{M}\right)do_M\iiint f_M(\Gamma_1)(\psi_1'v_{z1}'+\psi'v_z'-\psi_1v_{z1})W_1'd\Gamma_1'd\Gamma'd\Gamma_1, \tag{A.3}$$

$$V_{lkjn}(\Gamma'\to\Gamma)=\iint Y_{lk}^*\left(\frac{\mathbf{M}'}{M'}\right)Y_{jn}^*\left(\frac{\mathbf{M}}{M}\right)V_1(\Gamma'\to\Gamma)do_{M'}do_M, \tag{A.4}$$

$$a_1(\Gamma)=\frac{\psi v_z}{\Omega_0(\Gamma)}+\left(\frac{2T_0}{m}\right)^{1/2}\frac{\tilde{\Omega}_0(\Gamma)}{\Omega_0^2(\Gamma)},\quad a_2(\Gamma)=\left(\frac{2T_0}{m}\right)^{1/2}\frac{1}{\Omega_0(\Gamma)}, \tag{A.5}$$

$$R_{mk\sigma}^{(l)}\left(\frac{\mathbf{H}}{H}\right)=\frac{(-1)^{l+\sigma}}{4\pi}d_{mk}^{(l)}(\theta_H)d_{m\sigma}^{(l)}(\theta_H)\exp[i(k-\sigma)\varphi_H]. \tag{A.6}$$

With allowance for (A.1–6) we can write the expression, computed in accordance with (3.4) and (3.8), for the part of the function $\chi_2^{(1)}$ that is isotropic in **M** in the form

$$\chi_2^{(1)}(\Gamma,z)=\sum_{lmk\sigma}R_{mk-\sigma}^{(l)}\left(\frac{\mathbf{H}}{H}\right)\left[\sum_{i=1}^{3}A_i^{(lmk\sigma)}+\Theta(v_z)\exp\left(-\frac{\Omega_0z}{v_z}\right)\sum_{i=4}^{9}A_i^{(lmk\sigma)}\right], \tag{A.7}$$

where

$$A_1^{(lmk\sigma)}=-v^2\Theta(v_z)\frac{\delta T}{T_0}\varphi\exp\left(-\frac{\Omega_0z}{v_z}\right)\frac{1}{m^2\omega^2}\times\left[1-\operatorname{Re}\exp\left(im\frac{\omega z}{v_z}\right)\right]\Omega_{1lk}\Omega_{1l\sigma},$$

$$A_2^{(lmk\sigma)}=-v^2\frac{(\nabla T)_r}{T_0}a_1\left\{\frac{1}{\Omega_0^2+m^2\omega^2}-\Theta(v_z)\exp\left(-\frac{\Omega_0z}{v_z}\right)\frac{1}{m^2\omega^2}\left[1-\operatorname{Re}\frac{\Omega_0\exp(im\omega z/v_z)}{\Omega_0-im\omega}\right]\right\}\Omega_{1lk}\Omega_{1l\sigma},$$

$$A_3^{(lmk\sigma)}=v^2\frac{(\nabla T)_r}{T_0}a_2\left[\frac{\Omega_0-im\omega}{\Omega_0(\Omega_0^2+m^2\omega^2)}+\Theta(v_z)\exp\left(-\frac{\Omega_0z}{v_z}\right)\frac{\Omega_0\exp(-im\omega z/v_z)-\Omega_0-im\omega}{\Omega_0im\omega(\Omega_0+im\omega)}\right]\Omega_{1lk}\tilde{\Omega}_{1l\sigma},$$

$$A_4^{(lmk\sigma)}=-v^2\frac{(\nabla T)_r}{T_0}\int\limits_{v_z'<0}\frac{|v_z'|f_M'}{v_zf_M}V_0(\Gamma'\to\Gamma)\times\frac{a_1'}{\Omega_0'^2+m^2\omega^2}\Omega_{1lk}'\Omega_{1l\sigma}'d\Gamma',$$

$$A_5^{(lmk\sigma)}=v^5\frac{(\nabla T)_r}{T_0}\int\limits_{v_z'<0}\frac{|v_z'|f_M'}{v_zf_M}V_0(\Gamma'\to\Gamma)a_2'\times\frac{\Omega_0'-im\omega}{\Omega_0'(\Omega_0'^2+m^2\omega^2)}\Omega_{1lk}'\tilde{\Omega}_{1l\sigma}'d\Gamma',$$

$$A_6^{(lmk\sigma)}=v\varepsilon\frac{(\nabla T)_r}{T_0}(4\pi)^{-1/2}\int\limits_{v_z'<0}\frac{|v_z'|f_M'}{v_zf_M}a_1'\times\frac{\Omega_0'+im\omega}{\Omega_0'^2+m^2\omega^2}V_{lk00}(\Gamma'\to\Gamma)\Omega_{1l\sigma}'d\Gamma',$$

$$A_7^{(lmk\sigma)}=-v\varepsilon\frac{(\nabla T)_r}{T_0}(4\pi)^{-1/2}\int\limits_{v_z'<0}\frac{|v_z'|f_M'}{v_zf_M}a_2'\times\frac{\Omega_0'+im\omega}{\Omega_0'^2+m^2\omega^2}V_{lk00}(\Gamma'\to\Gamma)\tilde{\Omega}_{1l\sigma}'d\Gamma',$$

$$A_8^{(lmk\sigma)}=v^2\left[\int\limits_{v_z'<0}\frac{|v_z'|f_M'}{v_zf_M}V_0(\Gamma'\to\Gamma)\left(\frac{\delta T}{T_0}\varphi'-\frac{(\nabla T)_g}{T_0}a_1'\right)d\Gamma'\right]\frac{1}{m^2\omega^2}\left(1-\operatorname{Re}\exp\left[im\frac{\omega z}{v_z}\right]\right)\Omega_{1lk}\Omega_{1l\sigma},$$

$$A_9^{(lmk\sigma)}=\frac{\varepsilon v}{(4\pi)^{1/2}}\int\limits_{v_z'<0}\frac{|v_z'|f_M'}{v_zf_M}\left(\frac{\delta T}{T_0}\varphi'-\frac{(\nabla T)_g}{T_0}a_1'\right)\times\frac{1}{im\omega}\left(\exp\left[-im\frac{\omega z}{v_z}\right]-1\right)\Omega_{1lk}V_{00l\sigma}(\Gamma'\to\Gamma)d\Gamma'.$$

Here under the quantities with primes are meant the corresponding functions of Γ'.

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[1] Such an assumption is natural since oxygen is chemisorbed on the majority of metals, and special measures were apparently not taken to clean the surfaces.

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