confined ourselves to application of the general results to two particular cases. In the case of low pump intensities we have shown that all the results agree with the known ones. On the other hand, to illustrate the application of the general methods to cases that are not described by the usual theory, we have chosen only that class of systems for which all the resonance detunings become small simultaneously. It must also be emphasized that our procedure is applicable to the analysis of experiments at arbitrary ratios of the resonance detunings, matrix elements, and other parameters.

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Photon echo from small-area exciting pulses

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A theoretical investigation is reported of photon echo formed in a gas medium, by two linearly polarized exciting pulses of small area. The intensity and polarization are obtained for the echo on an optically allowed transition with arbitrary angular momenta of the levels and with arbitrary ratio of the durations of the exciting pulses and the time of the reversible Doppler relaxation. Analytic expressions are obtained for the form of the echo pulse for both a narrow and a broad spectral line. The results of the theoretical investigations point to the need of performing new experiments on photon echo in gases for the purpose of identifying the resonant transitions.

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With the advent of lasers, various methods of investigating the characteristics of matter, based on nonlinear interaction between the matter and electromagnetic radiation, have found extensive applications. One such method is the photon echo, which uses the nonlinear response of a medium after the passage of two exciting pulses. The photon-echo method yields information on the relaxation characteristics of matter, and permits identification of the corresponding resonant transition. To reduce the experimental results by this method, theoretical relations are necessary between the experimentally recorded quantities, such as the echo intensity and its polarization, on the one hand, and the characteristics of the medium on the other.

The first photon-echo experiment was performed in ruby.[1] Subsequently it came into extensive use also for the investigation of gases. In a gas medium, the resonant levels on which the echo is formed are usually degenerate. This adds to the complexity of the theoretical analysis of the photon echo in gases. We note that up to now there were no calculations for the intensity and polarization of the echo at arbitrary values of the angular momenta of the resonant-transition levels.

An important role in the theoretical investigation of the photon echo phenomenon is played by the relation between the durations T_1 and T_2 of the exciting light pulses and the time of the reversible Doppler relaxation T_0. As a rule[2-4] the calculations are performed within the limits of a narrow spectral line ($1/T_0 \ll 1/T_i$; $i=1, 2$). At the same time, most experiments on photon echo in gases have been performed either for small values of the angular momenta of the levels,[13] or with the degeneracy neglected completely.[14] It follows from the results of Refs. 13 and 14 that on a broad spectral line the echo pulse has a complicated shape, and its polarization on the transitions $j \to j (j>1)$ and $j \rightleftarrows j\pm 1 (j>1/2)$ depends on the intensities of the exciting pulses and on the ratio of T_0 and T_i. On a narrow spectral line, as follows from Refs. 2–4, the echo polarization on these same transitions also depends on the intensities of the exciting pulses.

It is established in the present paper that in the limit of small areas of the exciting pulses the calculation of

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the intensity and polarization of the photon echo in a gas can be carried out for both a narrow and a broad spectral line at arbitrary values of the angular momenta of the resonant-transition levels. The final results are simple in form and are convenient for the reduction of the experimental data. We emphasize that even in the case of a broad spectral line, the shape of the echo pulse in the considered limit is described by a simple expression that shows explicitly the instant of its maximum. It is also found that the polarization properties of the echo at small areas of the exciting pulses are independent on the type of line (broad or narrow).

It follows from the results that experiments on photon echo for the purpose of identifying transitions must be performed with exciting pulses of small areas. By way of example of the feasibility of a photon echo with small exciting-pulse are, we point out the experiments of Baer and Abella,[15] where the corresponding areas were of the order of 0.1. To ascertain which exciting-pulse areas can be regarded as small (in a given particular experiment), a series of measurements must be made with successive decrease of these area. The experimental results suitably extrapolated can then be reduced by using the formulas of the present article.

1. BASIC EQUATIONS AND RELATIONS

We consider the formation of a photon echo in a gas under the influence of two linearly polarized exciting light pulses at angle ψ with each other and with electric field intensities

$$\mathbf{E}_1=(\mathbf{l}_x\sin\psi+\mathbf{l}_z\cos\psi)e^{(1)}\exp[i(\omega t-ky+\Phi_1)]+\text{c.c.},\qquad 0\leqslant t-y/c\leqslant T_1; \tag{1}$$

$$\mathbf{E}_2=\mathbf{l}_z e^{(2)}\exp[i(\omega t-ky+\Phi_2)]+\text{c.c.},\qquad \tau_s+T_1\leqslant t-y/c\leqslant \tau_s+T_1+T_2. \tag{2}$$

The amplitudes $e^{(1)}$ and $e^{(2)}$ and the phase shifts Φ_1 and Φ_2 are constant, and the carrier frequency ω is at resonance with the frequency ω_0 of the atomic (molecular) transition with a total angular momentum change $j_a\to j_b$; $\mathbf{l}_x$ and $\mathbf{l}_z$ are unit vectors of the corresponding Cartesian axes.

To find the intensity of the electric field of the photon echo, we use the d'Alambert equation

$$\Box\mathbf{E}=\frac{4\pi}{c^2}\frac{\partial^2}{\partial t^2}\int\mathbf{P}dv \tag{3}$$

and the quantum-mechanical equation for the density matrix, with account taken of the gas atoms (molecules) with the electromagnetic field, the irreversible relaxation, and also the radiative arrival at the lower resonance level on account of the spontaneous emission on the upper one. The medium-polarization vector $\mathbf{P}$ in (3), which pertains to a group of atoms moving with velocity $\mathbf{v}$, is connected with the density-matrix component $\rho^{(ba)}_{\mu m}$, which describes the transitions between the considered levels, by the relation

$$\mathbf{P}=\sum_{\mu,m}\rho^{(ba)}_{\mu m}\mathbf{d}_{m\mu}+\text{c.c.},$$

where $\mathbf{d}_{m\mu}$ are the matrix elements of the dipole-moment operator of the atom, and correspond to a transition between the upper state a and the lower state b with total angular momenta j_a and j_b and with total-angular momentum projections m and μ.

The solution of the system of equations for the components of the density matrix is best obtained by expanding the solution in irreducible tensor operators[16]:

$$\rho^{(ba)}_{\mu m}=(-1)^{j_b-\mu}(2j_b+1)^{-1/2}\sum_{\varkappa,q}(2\varkappa+1)\begin{pmatrix} j_b & j_a & \varkappa\\ \mu & -m & q\end{pmatrix}\Psi^{(\varkappa)}_q,$$

and the connection of $\rho^{(aa)}_{m'm}$ with $f^{(\varkappa)}_q$ and of $\rho^{(bb)}_{\mu\mu'}$ with $\varphi^{(\varkappa)}_q$ is obtained from this relation by the respective substitutions

$$j_b\to j_a,\ \mu\to m' \text{ and } j_a\to j_b,\ m\to\mu'.$$

We separate the rapidly oscillating factors of the sought functions

$$\mathbf{E}=\mathbf{e}\exp[i(\omega t-ky+\Phi)]+\text{c.c.},$$
$$\Psi^{(\varkappa)}_q=\psi^{(\varkappa)}_q\exp[i(\omega t-ky+\Phi)],$$

where $\mathbf{e}$ and $\psi^{(\varkappa)}_q$ are the slowly varying amplitudes and Φ is a constant phase shift. In the resonant approximation we get from (3) and from the equations for $\Psi^{(\varkappa)}_q, f^{(\varkappa)}_q$ and $\varphi^{(\varkappa)}_q$ (Ref. 17) the following equations for the slow functions:

$$\left[\frac{\partial}{\partial t}+c\frac{\partial}{\partial y}\right]e_q=-2i\pi\omega\int p_q dv, \tag{4}$$

$$\left[\frac{\partial}{\partial t}+i(\Delta\omega-kv)+\gamma^{(\varkappa)}-i(\Delta^{(\varkappa)}-\Delta^{(1)})\right]\psi^{(\varkappa)}_q=\frac{i}{\hbar}(2j_a+1)^{-1/2}d^*\sum_{\varkappa',q',q_1}e_{-q_1}[S^{\varkappa\varkappa'}_{qq'q_1}f^{(\varkappa')}_{q'}+(-1)^{\varkappa+\varkappa'}R^{\varkappa\varkappa'}_{qq'q_1}\varphi^{(\varkappa')}_{q'}], \tag{5}$$

$$\partial_a f^{(\varkappa)}_q=\frac{i}{\hbar}(2j_b+1)^{-1/2}\sum_{\varkappa',q',q_1}C^{\varkappa\varkappa'}_{qq'q_1}T^{\varkappa\varkappa'}_{q'q_1}, \tag{6}$$

$$\partial_b\varphi^{(\varkappa)}_q=\frac{i}{\hbar}(2j_b+1)^{-1/2}\sum_{\varkappa',q',q_1}(-1)^{\varkappa+\varkappa'}B^{\varkappa\varkappa'}_{qq'q_1}T^{\varkappa\varkappa'}_{q'q_1}+\Gamma_\varkappa f^{(\varkappa)}_q, \tag{7}$$

where

$$p_q=(-1)^{j_a-j_b+q}(2j_b+1)^{-1/2}d\psi^{(1)}_q,\quad \Delta\omega=\omega-\omega_0-\Delta^{(1)},$$
$$\partial_{a,b}=\frac{\partial}{\partial t}+\gamma^{(0)}_{a,b}+\Gamma^{(\varkappa)}_{a,b},\quad \gamma^{(\varkappa)}={}^1/_2(\gamma^{(0)}_a+\gamma^{(0)}_b)+\Gamma^{(\varkappa)}.$$

Here e_q and p_q are the cyclic components of the corresponding vectors, d is the reduced matrix element of the dipole-moment operator of the $j_a\to j_b$ transition, and v denotes the projection of the velocity on the Y axis.

The quantity S in (5) is obtained from

$$C^{\varkappa\varkappa'}_{qq'q_1}=(-1)^{2j_b+q'}(2j_a+1)^{1/2}(2\varkappa'+1)\begin{pmatrix}\varkappa' & 1 & \varkappa\\ -q' & q_1 & q\end{pmatrix}\begin{Bmatrix}\varkappa' & 1 & \varkappa\\ j_a & j_a & j_b\end{Bmatrix}$$

by substituting $j_a\rightleftarrows j_b$ everywhere except in the second column of the $6j$-symbol. The quantities B and R are obtained in turn from C and S respectively by the redesignation $j_a\rightleftarrows j_b$.

In (6) and (7) the term

$$T^{\varkappa\varkappa'}_{q'q_1}=d\psi^{(\varkappa')}_q e^*_{-q_1}+(-1)^{q'+\varkappa+\varkappa'}e_{-q_1}d^*(\psi^{(\varkappa')}_{-q'})^*$$

describes the change of the components of the density matrix because of the interaction with the electromagnetic field. The second term in the right-hand side of (7) takes into account the arrival at the lower level on account of spontaneous emission on the upper one. In this case

$$\Gamma_{\varkappa}=(-1)^{1+\varkappa+j_a+j_b}\gamma[(2j_a+1)(2j_b+1)]^{1/2}\begin{Bmatrix} j_a & j_a & \varkappa \\ j_b & j_b & 1 \end{Bmatrix},$$

and $1/\gamma$ is the lifetime of the state a relative to the spontaneous decay to the state b. Next, the spectroscopic parameters $1/\gamma_a^{(0)}$ and $1/\gamma_b^{(0)}$ are the relaxation times of the states a and b on account of gaskinetic inelastic collisions and radiative decay, and $\Gamma_a^{(\varkappa)}$ and $\Gamma_b^{(\varkappa)}$ describe the collisional relaxation of the upper and lower levels, due to elastic depolarizing collisions. Finally $\Gamma^{(\varkappa)}$ and $\Delta^{(\varkappa)}$ characterize the relaxation of the irreducible component of the optical-coherence matrix, due to the action of such collisions. We note also that the designations of the $3j$ and $6j$ symbols are the conventional ones.

At the initial instant of time, $t-y/c=0$, prior to the incidence of the first exciting pulse on the medium, the quantities

$$\psi_q^{(\varkappa)}=\psi_q^{(\varkappa)}(v,t-y/c),$$
$$f_q^{(\varkappa)}=f_q^{(\varkappa)}(v,t-y/c),\quad \varphi_q^{(\varkappa)}=\varphi_q^{(\varkappa)}(v,t-y/c)$$

are given by

$$\psi_q^{(\varkappa)}(v,0)=0,\quad f_q^{(\varkappa)}(v,0)=(2j_a+1)n_af(v)\delta_{\varkappa,0}\delta_{q,0},$$
$$\varphi_q^{(\varkappa)}(v,0)=(2j_b+1)n_bf(v)\delta_{\varkappa,0}\delta_{q,0}, \tag{8}$$

where $(2j_a+1)n_a$ and $(2j_b+1)n_b$ are the densities of the atoms on the upper and lower levels at $t-y/c=0$, and $f(v)$ describes the Maxwellian distribution of the atom velocities.

When solving Eqs. (4)–(7) we shall neglect the reaction of the medium on the exciting light pulses. The equations then become linear and enable us to determine the electric field intensity of the photon echo in the approximation in which the field (1) or (2) is given. To linearize Eq. (4)–(7) we must satisfy the condition

$$\omega|d|^2L|N_0|T/\hbar c\ll 1,$$

where T is the shortest of the times T_1, T_2, and T_0; $N_0=n_a-n_b$; L is the length of the gas medium, and the time T_0 is expressed in terms of the average thermal velocity u of the gas atoms, $T_0=1/ku$. In addition, we assume that the durations T_1 and T_2 of the exciting light pulses are small compared with the time interval τ_s between them and the times of the irreversible relaxations.

We shall solve Eqs. (4)–(7) assuming small areas of the exciting pulses:

$$|d|e^{(i)}T_i/(2j_a+1)^{1/2}\hbar\ll 1;\quad i=1,2. \tag{9}$$

We then obtain from (4)–(7), taking the initial conditions (8) into account, $\Psi_q^{(\varkappa)}(t-y/c)$ at the instant of time $t=T_1+y/c$, when the first exciting pulse leaves the point y of the gas medium, in the form

$$\Psi_q^{(\varkappa)}(T_1)=(-1)^{1+j_b-j_a}(2j_b+1)^{1/2}(p^{(1)}/d)C_q^{(\varkappa)}(\psi)\exp[i(\omega T_1+\Phi_1)],$$
$$C_q^{(\varkappa)}(\psi)=[\cos\psi\delta_{q,0}+2^{-1/2}\sin\psi(\delta_{q,1}-\delta_{q,-1})]\delta_{\varkappa,1}, \tag{10}$$
$$p^{(1)}=-\frac{i|d|^2}{3\hbar kv}e^{(1)}N_0f(v)[\sin kvT_1+i(1-\cos kvT_1)].$$

Formula (10) was written for the case of exact resonance $\Delta\omega=0$. It follows from (10) that the polarization vector P of the medium is directed at the instant of time $t=T_1+y/c$ along the polarization vector of the exciting pulse, this being a consequence of the given-field approximation. We note that in (10) we have retained terms of first order of smallness in the parameter (9).

In the region $T_1\leqslant t-y/c\leqslant\tau_s+T_1$, after the passage of the first exciting pulse, we get $\Psi_q^{(\varkappa)}(t-T_1-y/c)$ from Eq. (5) with (10) taken into account:

$$\Psi_q^{(\varkappa)}(t-T_1-y/c)=(-1)^{1+j_b-j_a}(2j_b+1)^{1/2}(p^{(1)}/d)C_q^{(\varkappa)}(\psi)$$
$$\times\exp\{-\gamma^{(\varkappa)}(t-T_1-y/c)+i[\omega(t-y/c)+\Phi_1+(kv+\Delta^{(\varkappa)}-\Delta^{(1)})(t-T_1-y/c)]\}. \tag{11}$$

During the time $\tau_s+T_1\leqslant t-y/c\leqslant\tau_s+T_1+T_2$ the second exciting pulse (2) will pass through the point y of the medium. The linearized equations (4)–(7) are solved, as in the preceding case of passage of the first pulse. The initial conditions are taken to be the solutions of the system (4)–(7), taken at the instant of time $t=\tau_s+T_1+y/c$. One of the initial conditions is obtained from (11) at $t=\tau_s+T_1+y/c$. We note that it is just only this initial condition which contributes to the intensity of the electric field of the photon echo.

At the instant of time $t=\tau_s+T_1+T_2+y/c$, when the second pulse (2) leaves the point y of the gas medium, the part $\Psi_q^{(\varkappa)}(T_2)$, that contributes to the echo is of the form

$$\Psi_q^{(1)}(T_2)=(-1)^{1+j_b-j_a}9(2j_b+1)^{1/2}\frac{p^{(1)*}p^{(2)}}{2d}\Big[\frac{1}{3}\delta_{q,0}$$
$$\times A(j_a,j_b)\cos\psi+\frac{1}{2\sqrt{2}}(\delta_{q,1}-\delta_{q,-1})B(j_a,j_b)\sin\psi\Big]$$
$$\times\exp\{-\gamma^{(1)}\tau_s+i[\omega(\tau_s+T_1+T_2)-kv\tau_s+2\Phi_2-\Phi_1]\}, \tag{12}$$
$$p^{(2)}=\frac{2|d|^2e^{(2)2}}{3\hbar^2(kv)^2}(1-\cos kvT_2),$$

$$A(j,j)=\frac{2(3j^2+3j-1)}{5j(j+1)(2j+1)},\quad B(j,j)=\frac{4(j-1)(j+2)}{15j(j+1)(2j+1)}, \tag{13}$$

$$A(j+1,j)=A(j,j+1)=\frac{2(4j^2+8j+5)}{5(j+1)(2j+1)(2j+3)},$$
$$B(j+1,j)=B(j,j+1)=-\frac{8j(j+2)}{15(j+1)(2j+1)(2j+3)}. \tag{14}$$

We emphasize that expressions (12)–(14) were obtained in the first nonvanishing approximation in the parameter (9).

The quantity (12) serves as the initial condition for the solution of Eqs. (4) and (5) in the region $\tau_s+T_1+T_2\leqslant t-y/c$ after passage of the second exciting pulse. Leaving out the intermediate calculations, we write down the final expression for the intensity of the electric field of the photon echo:

$$\mathbf{E}^e=-\frac{\pi}{\hbar^3}\omega\frac{L}{c}|d|^4e^{(1)}T_1e^{(2)2}T_2^2N_0I\exp[-\gamma^{(1)}(t'$$
$$+\tau_s)]\mathbf{e}^e\exp[i(\omega t-ky+2\Phi_2-\Phi_1)]+\text{c.c.}, \tag{15}$$

$$e_x^e={}^1/_2B(j_a,j_b)\sin\psi,\quad e_y^e=0,\quad e_z^e={}^1/_3A(j_a,j_b)\cos\psi, \tag{16}$$

$$I=\frac{2}{T_1T_2^2}\int\frac{1}{(kv)^3}\{\sin(kvT_1)\cos[kv(t'-\tau_s)]$$
$$+[1-\cos(kvT_1)]\sin[kv(t'-\tau_s)]\}[1-\cos kvT_2]f(v)dv, \tag{17}$$
$$t'=t-\tau_s-T_1-T_2-y/c.$$

2. DISCUSSION OF RESULTS

Formulas (13)–(17) describe the electric-field intensity of the photon echo for both the transitions $j \to j$ and the transitions $j \rightleftarrows j+1$ at an arbitrary value of j. They are valid for any ratio of the durations of the exciting pulses and time of reversible Doppler relaxation. We emphasize a characteristic feature of the result: the polarization properties of the photon echo are independent in the considered limit of the type of line (narrow or broad). It follows from (13)–(17) that the echo polarization vector makes an angle θ_e with the polarization vector of the second exciting pulse. This angle is determined from the equation

$$\operatorname{tg}\theta_e=[3B(j_a,j_b)\operatorname{tg}\psi]/2A(j_a,j_b). \quad (18)$$

In particular, for transitions with small level angular momenta $0 \rightleftarrows 1, 1/2 \to 1/2, 1 \to 1$ and $1/2 \rightleftarrows 3/2$, the polarization properties of the photon echo, which follow from (18), coincide with those obtained without the restriction (9) in Refs. 2–4 for a narrow spectral line and in Ref. 13 for a broad one.

Thus, for transitions with small angular momenta of the levels, the results are valid also if the inequality (9) does not hold.

It follows that for the transitions $j \to j (j \geq 3/2)$ the echo polarization vector lies inside the angle between the polarization vectors of the exciting pulses while for the transitions $j \rightleftarrows j+1 (j \geq 1/2)$ it lies outside this angle.

For experiments in molecular gases, the case of large angular momenta ($j \gg 1$) is particularly important. In this case we have from (13), (14), and (18)

$$\operatorname{tg}\theta_e={}^1/_3\operatorname{tg}\psi \text{ For the transitions } j\to j, \quad (19)$$

$$\operatorname{tg}\theta_e=-{}^1/_2\operatorname{tg}\psi \text{ For the transitions } j\rightleftarrows j+1. \quad (20)$$

Expression (18) makes it possible to obtain the angular momenta of the transition from experimental measurements of the angle θ_e as a function of ψ. A reliable determination of these quantities is possible apparently only if they are relatively small. On the other hand if j is large, then formulas (19) and (20) make it possible to identify the type of transition, $j \to j$ or $j \rightleftarrows j\pm 1$, in which the echo is formed. This identification of the type of transition is relatively easy, as is seen from the relation $\theta_e=\theta_e(\psi)$ plotted in the figure for the case $j \gg 1$. Curve 1 of this figure pertains to the transitions $j \rightleftarrows j+1$, and the second to the transitions $j \to j$. Curves 1 and 2 are respectively plots of Eqs. (20) and (19) of the present paper.

Curve 3 is a plot of the theoretical result of Ref. 3, where the transition $j \to j$ is considered at $j \gg 1$ for the case of a narrow spectral line. This curve was plotted for the particular case of the values of the areas of the exciting pulses $\pi/2$ and π, respectively. We note that at large areas of the exciting pulses, depending on their ratio, the curve 3 can have also a more complicated form. We emphasize that the difference between our result, obtained in the limit of small areas of exciting pulses (curve 2), and the result of Ref. 3, does not exceed 5–10% starting with areas of the exciting pulses of the order of 1. Curve 4 of the figure is based on the experimental results of Alimpiev and Karlov[6,7] for pho-

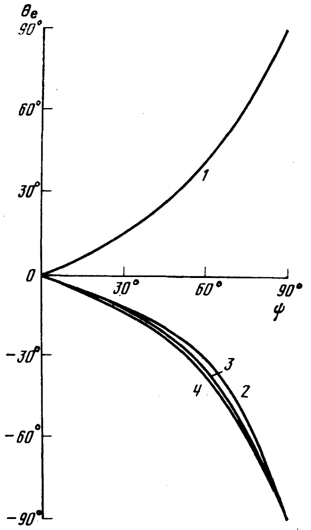

FIG. 1. Dependence of the angle θ_e on ψ. The angle θ_e is positive or negative, depending on whether the echo polarization echo is located outside or inside the angle ψ.

ton echo produced in SF_6 by CO_2 laser pulses at a frequency 947.73 cm^{-1} (P16).

When comparing the results of the polarization experiments with the theory one must bear in mind the possibility of echo formation not in one but in several resonant transitions, and furthermore differing in type: $j \to j$ and $j \rightleftarrows j\pm 1$. Cases when the echo is formed on several transitions, say two, with one common level, call for a special analysis. But if the echo is formed on independent transitions, then in addition to the cases already considered (curves 1 and 2), it is possible to have in the small-area limit an even more complicated dependence of θ_e on ψ, when the echo is formed, for example, on two transitions, one with $j_1 \to j_1$ and the other with $j_2 \to j_2+1$.

When the angle ψ between the polarizations of the exciting pulses is varied, the intensity of the echo signal also changes. In the limit $j \gg 1$ we obtain for the dimensionless intensity from (13)–(17)

$$I_e(\psi)/I_e(0)={}^1/_9(5+4\cos 2\psi), \quad j\to j,$$

$$I_e(\psi)/I_e(0)={}^1/_8(5+3\cos 2\psi), \quad j\rightleftarrows j+1.$$

It follows from these expressions that the curves start out from 1 at $\psi=0$, and then diverge and are equal at $\psi=\pi/2$ to 1/9 and 1/4, respectively. Thus, this dependence can also be used to identify the type of transition.

We discuss now the echo-pulse shape that follows from (17). We consider two limiting cases, narrow and broad spectral line. In the case of the narrow line ($1/T_0 \ll 1/T_i; i=1,2$), confining ourselves to the first nonvanishing term, we have

$$I=\exp[-(t'-\tau_s)^2/4T_0^2]. \quad (21)$$

It follows from (21) than when the echo is formed on the narrow spectral line the maximum of the echo amplitude occurs at the instant of time $t=2\tau_s+T_1+T_2+y/c$, and the echo-pulse width is of the order of the Doppler width of the spectral line. This result agrees with the corresponding earlier results, e.g., Ref. 2.

In the case of a broad spectral line ($1/T_0 \gg 1/T_i$), integrating in (17), we obtain an answer whose analytic form depends on the ratio of T_1 and T_2. For simplicity we write this answer in the particular case $T_1 = T_2 = T_p$, which is usually realized in the experiments:

$$I=\sqrt{\pi}\frac{T_0}{T_p}\{(a+1)^2[\theta(a+1)-\theta(a)]-(2a^2-2a-1)[\theta(a)-\theta(a-1)]+(a-2)^2[\theta(a-1)-\theta(a-2)]\}, \tag{22}$$

where

$$a=(t'-\tau_s)/T_p, \quad \theta(x)=\begin{cases}1, & x\geq 0\\ 0, & x<0\end{cases}.$$

Investigating (18) for a broad spectral line, we conclude that the maximum of the echo is shifted relative to the instant of time $t=2\tau_s+T_1+T_2+y/c$ by 0.5 T_1, and the duration of the echo pulse is of the order of the duration of the first exciting pulse. The echo amplitude on the broad spectral line decreases compared with the echo amplitude on the narrow line by a factor T_0/T_1. We note that for $T_1 = T_2 = T_p$ these laws are clearly seen from (22).

The delay of the maximum of the echo pulse relative to the instant of time $t=2\tau_s+T_1+T_2+y/c$ was first observed in the experiment of Abella, Kurnit, and Hartmann[1] on a broad spectral line in ruby. Such a delay was observed also in the experiment of Ref. 18. A similar effect in the investigation of optical induction on a broad spectral line was deduced in a theoretical analysis by Zakharov and Manykin.[19] We note that in all the theoretical studies[13,14,19] the delay of the maximum was the result of numerical calculations, whereas in the present paper it was determined analytically. We emphasize that the shift 0.5 T_1 obtained by us agrees with the experimental values[1,18] and with the numerical calculations.[13,14]

In conclusion, we discuss the question of the applicability of the obtained expression for the reduction of the results of concrete experiments. As follows from (15)–(17), the ratio of the echo-signal intensity I_e to the product of the intensity I_1 of the first exciting pulse by the square of the intensity I_2 of the second is likewise independent of I_1 and I_2. Therefore, by investigating the behavior of this ratio as the intensities of the exciting pulses are successively decreased in an experiment, we can find its limiting value by extrapolation into the region $I_1 I_2^2 \to 0$. It is precisely these extrapolated results that must be reduced by the formulas of the present article. However, as already indicated in the analysis of the polarization properties of the echo, formulas (15)–(17) can sometimes be used even starting with exciting-pulse areas of the order of 1.

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