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Effect of plastic deformation on charge motion in solid He^4

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The mobility of the charges in undeformed He^4 crystals with molar volumes 19.67 and 20.55 cm^3 was determined by two procedures: reduction of the diode current-voltage characteristics, and from the diode transient-current curves following application of a dc voltage. To justify the last procedure, the electrodynamic problem of the motion of the charge front in a plane-parallel diode is solved. The mobilities determined by the two methods differed by not more than 15%. Solid helium was plastically deformed in a gap between electrodes at three temperatures. Measurements of the time of flight in the deformed crystals have shown that the mobility changes by not more than 10%. The current-voltage characteristics are substantially altered: at low voltages the current is not proportional to the square of the voltage. The density of the immobile charge is estimated under the assumption that the change of the current is due to the pinning of the charges by the dislocations.

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Carrier mobility in crystalline helium had been investigated up to now in samples grown under the most favorable conditions: constant pressure and small temperature gradients. Keshishev[1] investigated the mobilities of positive and negative charges in such crystals (with molar volumes $V_{mol} = 19.5 - 20.9$ cm^3). The results obtained with crystals having the same molar volume were very well reproducible (see Ref. 1, Fig. 10). The observed nonlinear dependence of the ion velocity v on the electric field strength E was used to explain the deviation of the current-voltage characteristics from the relation $I \propto U^2$. The calculated current-voltage characteristics agreed well with the experimental data.

A different point of view was held by Dahm,[2] who assumed that the deviation from the $I \propto U^2$ law is due to partial release of the ions from the traps by a strong electromagnetic field. The traps can be formed by various crystal-structure defects, and in particular by dislocations. This effect was calculated by Murgatroyd.[3]

X-ray structure investigations show that samples grown at constant pressure and at a low rate have a high degree of monocrystallinity (see, e.g., Ref. 4). One cannot exclude, however, the presence of a large number of dislocations that are produced in the course of the crystal growth, as well as produced later when the crystal temperature is changed. There are presently no known direct measurements of the dislocation concentration Λ in He^4 crystals. Wanner, Iwasa, and Wales[5] cite an estimate $\Lambda \sim 3 \times 10^5$ cm^{-2} obtained indirectly from measurements of sound-velocity anomalies. This number of dislocations can produce, in principle, a large number of traps for the charges and influence significantly the current in solid helium.

We report here an experimental investigation of the effect of plastic deformation on the currents and charge mobility in solid helium.

EXPERIMENTAL PROCEDURE

Figure 1 shows the construction of the ampoule used in our experiments.

The crystal was grown at constant pressure at rates $(1 - 5) \times 10^{-4}$ cm/sec. It was first formed on the cold finger and then, as it grew, it filled the entire ampoule. The crystal growth procedure was described in detail by Shal'nikov.[6] In the present case the crystals were grown at two pressures, 32 atm ($V_{mol} = 20.55$ cm^3) and 49 atm ($V_{mol} = 19.67$ cm^3).

Plane-parallel electrodes 2 and 6, of 4 mm diameter and separated by $L = 0.28$ mm, formed the measuring diode cell. A molybdenum disk with a diameter 3 mm was pressed into the lower electrode and its outer sur-

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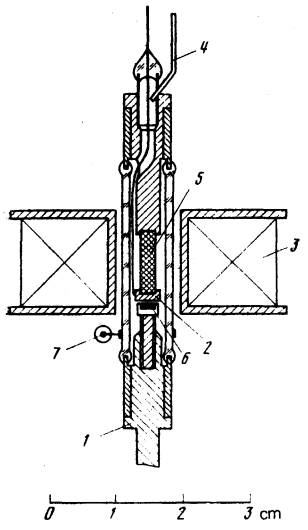

FIG. 1. Construction of the ampoule: 1—copper cold finger, 2—collector, 3—superconducting solenoid, 4—filling capillary, 5—magnetostrictor, 6—β-active electrode, 7—resistance thermometer.

face, coated with a titanium tritide layer, served as the source of the ionizing radiation. The saturation current of such a source was $I_s = 7.5 \cdot 10^{-10}$ A. Besides measuring the static characteristics, we plotted the current-transient curves following the switching-on of a dc voltage. In both cases, the current was measured with a "Takeda Riken-84 M" dc electrometer.

The helium sample between the electrodes was deformed with a magnetostrictor on which a collector was fastened. A large deformation could be obtained because terbium iron garnet was used as the magnetostriction material.[1)] Prior to assembling the ampoule, we measured the elongation of the magnetostrictor as a function of the applied field. In a magnetic field ~6 kOe the absolute elongation Δl in liquid helium was 2.5×10^{-4} cm at a magnetostrictor length 8.5 mm. This change of the magnetostrictor length decreases the gap between the electrodes and leads to a relative deformation compression $\varepsilon = 9 \times 10^{-3}$ of the solid helium. Such a deformation causes plastic flow of the solid helium in the gap (see our earlier results[7]). The absolute displacement of the collector, measured in liquid helium, practically coincided with the displacement in the crystal, since the rigidity of the magnetostrictor and of the ampoule body was ~100 times the rigidity of the solid helium in the gap.

A special unit enabled us to vary the solenoid current linearly in time. This made it possible to produce the deformation at approximately constant rate. The deviations of the deformation rate from a constant, due to the nonlinearity of the function $\Delta l(H)$, did not exceed 20%. In most experiments the rate of deformation of the solid helium was in the interval $\dot{\varepsilon} = (2 - 8) \times 10^{-7}$ sec^{-1}.

The helium-crystal temperature, measured with a carbon resistance thermometers, increased by not more than 0.02 K at these deformations. The sequence of the experiment was the following:

1. Prior to the start of the crystal growth, a current $I_c = 12$ A was made to flow in the superconducting solenoid for about a minute. The polarity of the current remained subsequently unchanged and its value did not exceed 12 A. The function $\Delta l(H)$ was well reproducible under these conditions.

2. The crystal was grown. The solenoid current was zero in the time of growth.

3. The temperature dependence of the current in the undeformed crystal was measured at a dc voltage (150 V). The current voltage characteristics and the current transient curves following the turning on of the voltage were measured at three temperature points ($T_1 > T_2 > T_3$). For the crystals with $V_{mol} = 20.55$ cm^3 these temperatures were $T_1 = 1.67$ K, $T_2 = 1.31$ K, $T_3 = 0.98$ K. At these temperatures, the yield points (see Ref. 7) were respectively $N_1 = 8 \cdot 10^3$, $N_2 = 2.8 \cdot 10^4$, and $N_3 = 4 \cdot 10^4$ dyn/cm^2. For the crystals with $V_{mol} = 19.67$ cm^3 the temperatures T_{1-3} were chosen such that the corresponding yield points remained unchanged. In this case the temperatures were $T_1 = 2.1$ K, $T_2 = 1.7$ K, and $T_3 = 1.43$ K.

4. The crystal temperature was set equal to T_1, after which the first deformation was carried out. The current through the solenoid increased at a constant rate from zero to 6.2 – 9 A ($\varepsilon \sim 8 \times 10^{-3}$), after which it was kept constant for ~15 min and then decreased at the same rate to zero. Measurements of the current-voltage characteristics of the crystal did not start until 15 minutes had elapsed after the vanishing of the current in the solenoid. The time 15 min was chosen on the basis of measurements of the relaxations of the mechanical stresses in the solid helium (see Ref. 7). During the time of the entire deformation process, the current flowing through the diode at a voltage 150 V was recorded with an automatic plotter.

5. We measured the current-voltage characteristics of the deformed crystal both with rising and with decreasing stresses, so as to estimate the hysteresis and the reproducibility of the results. The measurements were made down to the minimum voltages at which the hysteresis did not exceed 5% of the measured current I. The current transient following the turning-on the dc voltage was registered with the automatic plotter.

6. At temperatures T_2 and T_3 the deformations and the measurements were made in the same way as at the temperature T_1 (Items 4 and 5).

MEASUREMENT OF THE CURRENT IN THE COURSE OF THE DEFORMATION

The first stage of the deformation—elastic compression—takes place in our geometry with collector displacements not more than 3×10^{-6} cm. In this stage the current through the crystal changes by not more than 0.5%. Further increases of the magnetostrictor length led to a second deformation stage—plastic flow of the solid helium in the diode gap.

A typical plot of the flow of current through the crystal during the deformation is shown in Fig. 2. When the plastic flow sets in, the current increases. After the end of the deformation, the current assumes a constant value after a time characteristic of the relaxation of the mechanical stresses (see Ref. 7).

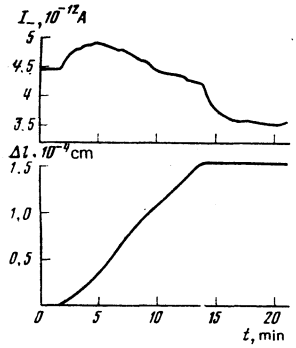

FIG. 2. Change of negative current in the course of plastic deformation in a crystal with $V_{mol} = 20.55$ cm^3 at a temperature 1.46 K.

RESULTS OF MEASUREMENT OF THE CURRENT TRANSIENT AFTER APPLYING THE DC VOLTAGE

The suggestion that the time of flight of the ions from the source to the collected be determined from the current transient curve was advanced by Efimov and Mezhov-Deglin.[8]

Figure 3 shows a typical experimental transient curve of the positive-ion current in the crystal with $V_{mol} = 19.67$ cm^3 at a temperature $T_2 = 1.7$ K. Curves 1*a* and 1*b* were obtained with the undeformed crystal after applying 203 and 73 volts, respectively. Curves 2a, 2b and 3a, 2b were obtained by applying the same voltages after plastic deformation of the crystal at $T = T_1 (T_1 = 2.1$ K) and $T = T_2$.

At the initial instant of time the experimental curves show a capacitive-current pulse that decreases exponentially with a time constant ~2 sec. The current then increases, goes through a maximum at an instant of time t_m, after which it approaches monotonically a constant value. Curves of this form were observed in all the investigated crystals at all temperatures and voltages.

Figure 4 (dark circles) shows a typical plot of t_m^{-1} against the applied voltage at two measurement temperatures for the positive-charge current. The crystal was grown at a pressure 49 atm ($V_{mol} = 19.67$ cm^3). It is seen from the figure that at low stresses t_m^{-1} is linear in the voltage; at a temperature $T_3 = 1.43$ K and voltage $U > 500$ V a deviation from linearity is observed.

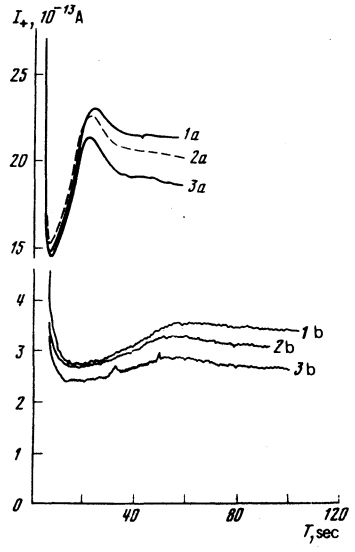

FIG. 3. Typical transient plots for positive current in the diode after application of a voltage 203 V—upper curves—or 73 V—lower curves—in a crystal with $V_{mol} = 19.67$ cm^3 at $T = 1.67$ K. Curves 1 were obtained for an undeformed crystal, curve 2 for a crystal subjected to deformation at $T_1 = 2.11$ K, and curves 3 after deformation at $T_2 = 1.67$ K.

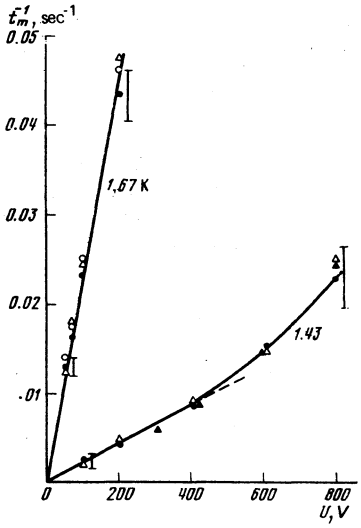

FIG. 4. Typical plots of $t_m^{-1}(U)$ for positive charges in a crystal with $V_{mol} = 19.67$ cm^3 before and after deformation: ●—undeformed crystal; ○—$T_1 = 2.11$ K, △—$T_2 = 1.67$ K, ▲—$T_3 = 1.43$ K. The vertical segments show the measurement error.

In a plastically deformed crystal, as seen from Fig. 3, the general form of the $I_+(t)$ curves remains the same, and the position of the maximum shifts by not more than 10%. The steady-state current is much less than the current in the undeformed crystal. Figure 4 shows plots of $t_m^{-1}(U)$ in a plastically deformed crystal ($V_{mol} = 19.67$ cm^3) at two temperatures. It is seen that at both temperatures and at all employed voltages the plastic deformation does not change the position of the maximum by more than 10%.

MEASUREMENT OF STATIONARY CURRENT-VOLTAGE CHARACTERISTICS

Figures 5 and 6 show typical current-voltage characteristics of crystals with $V_{mol} = 20.47$ and 19.67 cm^3.

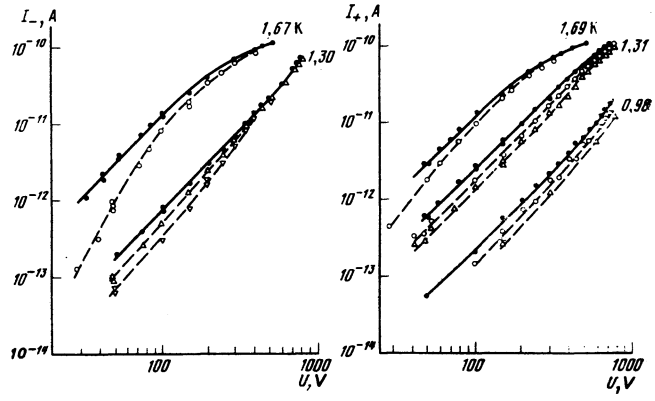

FIG. 5. Plots of positive and negative currents in a crystal with $V_{mol} = 20.55$ cm^3 before and after plastic deformation: ●—undeformed crystals; white circles—deformed crystal. In the left-hand diagram: ○—$T_1 = 1.67$ K, △—$T = 1.43$ K, ▽—$T_2 = 1.30$ K; in the right-hand diagram: $T_1 = 1.69$ K, △—$T_2 = 1.31$ K.

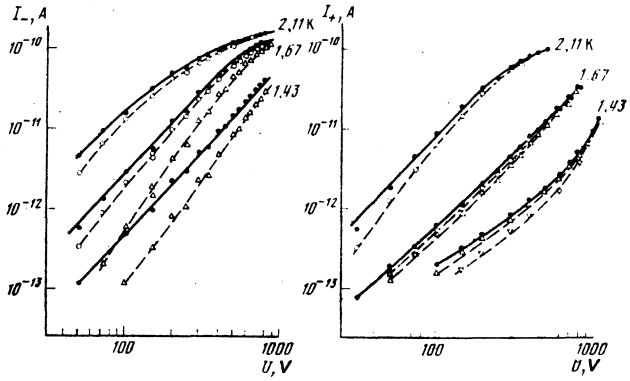

FIG. 6. Plots of positive and negative currents against the voltage in a crystal with $V_{mol} = 19.67$ cm^3 before and after plastic deformation: ●—undeformed crystal; ○—$T_1 = 2.11$ K, △—$T_2 = 1.67$ K, ▽—$T_3 = 1.43$ K.

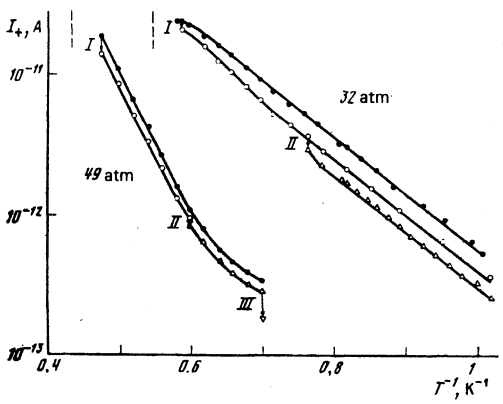

FIG. 7. Dependence of positive current on the temperature at a voltage 150 V before and after deformation at two pressures. The vertical dashed lines show the corresponding melting temperatures. The Roman numerals designate the sequence of the deformations.

In undeformed crystals at low voltages, the $I(U)$ functions are close to quadratic (see Fig. 5 for positive and negative currents and Fig. 6 for negative currents). The current-voltage characteristics measured on the crystal with $V_{mol} = 19.67$ cm^3 at temperatures <1.67 K and positive currents revealed no proportionality $I \propto U^2$ in a rather wide range.

In that voltage region where the relation $I \propto U^2$ does hold true it is possible to calculate the charge mobility by the Thomson formula[9] $\mu_{st} = 32\pi L^3 I/9U^2 S$, where S is the area of one of the electrodes. The mobilities obtained in this manner were compared with the mobilities from Ref. 1. For a comparison with a wider voltage interval, the current-voltage characteristics were reduced by the method proposed in Appendix I. This reduction was carried out for currents $I \ll I_s$ on crystals with a well defined $I \propto U^2$ dependence at low voltages. The calculated dependences of the charge drift velocity $v(E)$ were calculated with results of the measurement of this dependence in Ref. 1; the difference did not exceed ~20%.

In deformed crystals, the current decreased at a relative rate that depended on the voltage. The relation $I \propto U^2$ therefore no longer held at low voltages (see, e.g., Fig. 5 for negative currents at 1.67 and 1.30 K and for positive currents at 1.69 K). The general character of the variation of the current-voltage characteristics remained the same from crystal to crystal; all that changed was the size of the current drop.

Typical temperature dependences of the positive current, as measured at 150 V, are shown in Fig. 7. Plastic deformation does not alter substantially the slope of these curves, and leads only to their parallel shift towards weaker currents. The same is observed also on the temperature dependences of the negative currents.

DISCUSSION OF RESULTS

The quadratic dependence of the current on the applied voltage (the Thomson formula[9] $I_T = 9\mu U^2 S/32\pi L^3$) is derived under the following assumptions: a) $\delta/L \ll 1$ (δ is the thickness of the ionized layer), b) the entire diode space (with the exception of the ionized layer) contains only mobile charges of like sign, c) $I \ll I_s$, d) $\mu = v/E = \text{const}$.

Since the undeformed crystals had in most cases quadratic current-voltage characteristics at low voltages, it can be assumed that the conditions a)–d) are satisfied in these cases. A deviation from this regularity is observed for positive charges in the crystal with $V_{mol} = 19.67$ cm^3 at 1.43 K (see Fig. 6), despite the fact that the condition d) is satisfied, as follows from time-of-flight measurement results, up to ~300 V. The possible causes of this deviation are discussed in Ref. 1 and are at present not quite clear.

In our experiments we determined the mobility also from the plots of the current transient following application of the dc voltage. The determination of the time of flight and by the same token of the ion mobility from the transient current in the diode was proposed by Efimov and Mezhov-Deglin.[8] It is difficult, however, to determine the time of arrival of the charge front at the collector from the shape of the transient current (see Fig. 3). It is also clear that as the front moves in the diode gap the density of the space charge that is produced depends on the coordinate and on the time. The electric field in which the charges move also depend on the coordinate and on the time, and at a fixed instant of time the field changes from a value close to zero at the source to a value of the order of U/L near the collector. All these factors taken together make it impossible to relate a characteristic point on the curve—the position of the maximum—to a definite value of the electric field intensity. The electrodynamic problem of the buildup of the current in the diode was solved in Refs. 5. The applicability of the procedure was justified by comparing the mobilities calculated from the formula $\mu = L^2/t_m U$, with the results of measurements of the mobilities in a triode.[1]

To justify this procedure, we solved the problem of the propagation of a charge front in a diode following the switching on of a dc voltage. The formulation of the problem, the initial and boundary conditions and the results of the solution are given in Appendix II. The current transient curves calculated for our parameters

(see Fig. 10b below where $A = 20$ and when added) are of the same form as the experimental plots (Fig. 3). It follows also from the calculation that the time t_m corresponding to the maximum on the $I(t)$ curve is proportional to the time of flight t_f of a charge with mobility μ in an electric field $E_0 = U/L$: $t_m = 0.75 t_f$. In Appendix II we also determine the voltage interval for which the shape of the transient curves does not depend on the voltage. Our measurements were performed with these requirements satisfied.

Thus, from the position of the maximum it is possible to determine the time of flight in the field E_0 and then determine the dynamic mobility μ_d.

The mobilities of the ions in undeformed crystals, obtained from static and dynamic measurements, agree with each other and with the values given in Ref. 1 within 20%. For example, in a crystal with $V_{mol} = 19.67$ cm^3 at a temperature 1.72 K, the mobilities of the positive ions are $\mu_{st} = 2.64 \cdot 10^{-7}$ and $\mu_d = 2.4 \cdot 10^{-7}$ cm^2/V-sec (our measurements) and $\mu = 2.2 \cdot 10^{-7}$ cm^2/V-sec (data of Ref. 1). The good agreement between the mobilities obtained by different procedures is evidence of a small possible concentration ρ_0 of the charges pinned in traps. An estimate for undeformed crystals yields $\rho_0 < 0.7$ cgs esu/cm^3. This concentration of the pinned charges cannot greatly influence the current-voltage characteristics, as proposed in Dahm's paper.[2]

The uniaxial compression of the helium crystal between the electrodes, to a value sufficient to produce plastic flow of the crystal in the entire volume of the diode gap, results in a crystal with a large number of defects. The calculation of the residual mechanical stresses in the deformed crystal in our geometry is complicated matter. From simple considerations, knowing the yield point of solid helium at the deformation temperature, we can estimate the change Δp of the hydrostatic pressure in analogy with the procedure used in Ref. 7. An estimate shows that the maximum change of the hydrostatic pressure occurs at the center of the deformed crystal and that at $N = 4 \times 10^4$ dyn/cm^2 this change is $\Delta p \sim 0.5$ atm (the corresponding change is $\Delta V_{mol} \sim 0.025$ cm^3). This change in pressure causes a negligible change in the crystal density ($\Delta V_{mol}/V_{mol} \sim 1.3 \cdot 10^{-4}$). In undeformed crystals, the change ΔV_{mol} of the molar volume leads to a change of not more than 15% in the mobility in our temperature range.

In a deformed crystal with a large number of dislocations, an immobile space charge may be produced, due to the capture of charges by the dislocations. The immobile space charge can influence both the current transient curves and the steady-state current. To determine the character of this influence on the $I(t)$ curve, we solve in Appendix II the problem of the current transient in a diode whose gap contains pinned charges with a uniform density ρ_0 that does not depend on the time or on the electric field intensity. It follows from the solution (see Fig. 10c below) that the position of the maximum changes insignificantly, whereas the steady-state current decreases appreciably.

As seen from Fig. 3, it is precisely such a change in the current transient curve which is observed after plastic deformation of the helium crystal. It follows therefore that in plastically deformed helium the mobility changes apparently by not more than 10%. The fact that the slopes of the $I(T)$ plots at constant voltage do not change significantly after the crystal is deformed agrees with this conclusion.

The current-voltage characteristics measured in deformed crystals no longer reveal a $I \propto U^2$ dependence at low voltages. Since the conditions a) and c) are satisfied as before, and the condition d), as follows from dynamic measurements, also holds, the cause of the deviation of the current-voltage characteristics from the $I \propto U^2$ dependence is the violation of the condition b). Since the current is due as before to motion of charges of only one sign, the violation of condition b) may mean that immobile changes pinned by dislocations are present in the space between the electrodes.

On the basis of this assumption, we can estimate the density of the immobile pinned charges. An exact calculation of the relative change of the current as a function of the voltage cannot be carried out, since we do not know the spatial distribution of the traps with the charges, nor do we know the dependence of the trap population on the temperature and on the electric field. Assuming that the concentration of the pinned charges does not depend on the coordinate and on the electric field, we can integrate the Poisson equation in analogy with the Thomson calculations[9] and obtain a transcendental equation whose roots determine the $I(U)$ dependence in this simple model. Conditions a), c), and d) remain in force, while the condition b) is formulated as follows: in the entire diode space (with the exception of the ionized layer) the charges are of the same sign; they are either mobile with density ρ, or pinned with density ρ_0. The Poisson equation is integrated under the condition that the voltage between the electrodes is equal to U, the current density j is constant ($j = \mu\rho E$), and the field at the source $E(0)$ is zero. In a relative notation, the equation obtained as a result of the integration is of the form

$$12D\sqrt{D+F} - 8F \ln\left(1 + 3D\sqrt{D+F}/2F\right) - 9D^2 = 0,$$

where $F = j/j_T$, j_T is the current density determined from the Thomson formula, and $D = 32\pi\rho_0 L^2/9U$. We note that if the density ρ_0 is independent of temperature, then the relative decrease of the current F in this model is likewise independent of temperature.

Figure 8 shows the ratio of the current I in the deformed crystal to the current I_0 measured previously in the undeformed crystal at the same temperature, as a function of the voltage. As seen from the figure, in a number of cases the experimental points agree satisfactorily with the calculation on the basis of the described model (solid curve). In other cases there is no agreement. The possible causes are the simplifications made in the calculation. Since the quantity I/I_0 at fixed temperatures and fixed voltage changes from sample to sample by a factor of several times, we present estimates of the average density of the pinned charges, regardless of the sign of the charge, of the crystal temperature, of the molar volume, and of the temperature

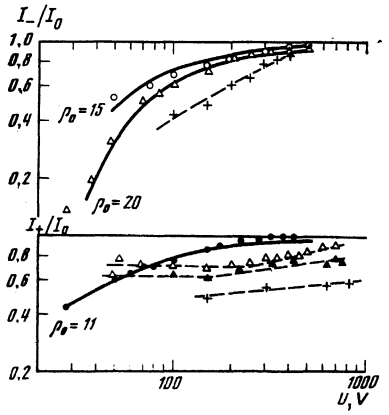

FIG. 8. Relative decrease of the current in a plastically deformed crystal with $V_{mol}=20.55$ cm^3 as a function of voltage in deformation temperature T_d. Upper diagram: △—$T_d=1.67$ K, $T=1.67$ K, ○—$T_d=1.67$ K, $T=1.30$ K; +—$T_d=1.30$ K, $T=1.30$ K. Lower diagram: ●—$T_d=1.69$ K, $T=1.69$ K; △—$T_d=1.69$ K, $T=1.31$ K; ▲—$T_d=1.69$ K, $T=0.98$ K; +—$T_d=1.31$ K, $T=0.98$ K. Solid curves—calculation of the dependence of the decrease of the current on the voltage in accordance with the model given in the text.

at which the plastic deformation was carried out. These values lie in the range $\rho_0=5-25$ cgs esu/cm^3.

We can therefore conclude the following from our results:

1. The density of the pinned charges in an undeformed crystal is low and cannot influence substantially the current-voltage characteristics.

2. The mobilities of the ions in deformed crystals differ from the mobilities of the ions in undeformed crystals by not more than 10%.

3. The most probable cause of the change of the current-voltage characteristics in deformed crystals is the presence, in the diode gap, of charges that are pinned on dislocations or on other capture centers.

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APPENDIX I

The mobilities of the charges in solid helium with the aid of the current-voltage characteristics had been determined up to now by reduction, using the Thomson formula,[9] of a section of the characteristic on which the relation $I\propto U^2$ holds true. In the derivation of the Thomson formula it was assumed that $v\propto E$. In sufficiently strong fields the ion drift velocity has a nonlinear dependence on the field (see Ref. 1), and this leads to a deviation of the current-voltage characteristics from the $I\propto U^2$ relation at high voltages. In this Appendix we solve the problem of determining the $v(E)$ dependence [the form of a function $v(E)$ was not specified] from the current-voltage characteristics under the following assumptions:

1) only mobile charges of one sign are present in the diode gap (with the exception of the ionized layer);

2) the field on the ionized layer is equal to zero;

3) the sign of the function $v(E)$ is constant and $v(0)=0$;

4) the current is much less than the saturation current, $I\ll I_s$.

The equations of motion of the charges and the diode are of the form

$$dE/dx=4\pi\rho,\quad j=v(E)\rho=\text{const} \tag{I.1}$$

with boundary conditions

$$\int_0^L E\,dx=U,\quad E(0)=0. \tag{I.2}$$

Since only charges of one sign are present in the diode gap, the maximum value of the field E_m is reached at the collector when $x=L$. Eliminating ρ from Eq(s). (1) and integrating from 0 to L, we get

$$\int_0^{E_m} v(E)\,dE=4\pi Lj. \tag{I.3}$$

The boundary condition (2) reduces, on going from integration with respect to dx to integration with respect to dE with the aid of (1), to the form

$$\int_0^{E_m} Ev(E)\,dE=4\pi jU. \tag{I.4}$$

Differentiating both halves of (3) and (4) with respect to E_m and combining the obtained expressions, we get

$$E_m=\frac{1}{L}\frac{d(jU)}{dj},\quad v(E_m)=4\pi L\frac{dj}{dE_m}, \tag{I.5}$$

and thus, the determination of $v(E)$ has been reduced to numerical differentiation.

Figure 9 shows the results of the calculation of $v(E)$ by the described method, for a comparison with the experimental data obtained by the time-of-flight procedure. The current-voltage characteristic was taken from Keshishev's paper,[1] Fig. 2; the experimental $v(E)$ is given in Fig. 12 of the same paper. The good agreement is evidence that the number of pinned charges in the helium crystal is small.

It should be noted that this calculation is followed only if it is know that the deviation of the current from a quadratic dependence is due precisely to the nonlinear-

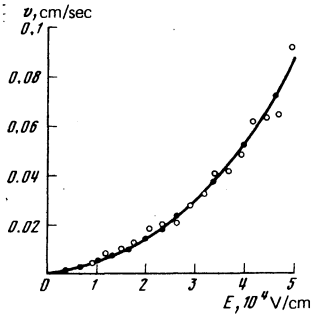

FIG. 9. Comparison of the function $v(E)$ for negative charges in a crystal with $V_{mol}=20.23$ cm^3 at $T=1.43$ K calculated from the current-voltage characteristics (○) with the results of time-of-flight measurements in a triode (●).

ity of $v(E)$. In this case the relation $I \propto U^2$ should be satisfied at low voltages.

APPENDIX II

Measurements of the transient current $I(t)$ in the diode following the switching on of the dc voltage make it possible in principle to determine the time of flight and consequently the mobility of the charges. To establish the concrete form of the $I(t)$ curves and of the influences exerted on them by pinned charges, the corresponding problem was solved under the following assumptions: the diode electrodes are infinite parallel plates separated by a distance L; the ionized layer is located at only one plate of the source and its thickness is $\delta \ll L$; inside this layer the intensity of the ionization is homogeneous and constant; outside this layer are located charges of one sign; the charge drift velocity is proportional to the field, i.e., $v = \mu E$.

The motion of the charges in the diode outside the ionized layer in the absence of pinned charges is described by the Poisson and continuity equations:

$$\partial E/\partial x = 4\pi\rho, \quad \partial j/\partial x + \partial\rho/\partial t = 0, \tag{II.1}$$

where $j = \mu\rho E$. The x axis is perpendicular to the electrodes and is directed towards the collector; the point $x = 0$ corresponds to the boundary of the ionized layer. We assume that at the instant $t = 0$ the voltage increases jumpwise from 0 to a constant value U_0. Then at $t > 0$ the condition

$$\int_0^L E(x, t)\,dx = U_0 \tag{II.2}$$

is satisfied and at the initial instant we have $E(x, 0) = E_0 = U_0/L$. The density of the current through the collector is determined in the general case by the expression

$$j_c(t) = \frac{1}{L}\int_0^L v\rho\,dx = \frac{\mu}{8\pi L}\left[E^2(L, t) - E^2(0, t)\right] \tag{A.3}$$

and is equal to zero at the initial instant: $j_c(0) = 0$.

The boundary condition on the ionized layer is obtained by equating the current through the boundary of the ionized layer j_i to the current in the gap at $x = 0$. As shown in Ref. 10, at small field intensities we have $j_i = \alpha E(0, t)$. Thus, we obtain

$$\left.\frac{\partial E}{\partial x}\right|_{x=0} = \frac{4\pi\alpha}{\mu}. \tag{II.4}$$

The conditions under which this relation is satisfied will be determined below. We note that this condition differs from the Thomson condition $E(0, t) = 0$,[9] but at large values of the derivative $\partial E/\partial x|_{x=0}$ the steady-state current determined with this boundary condition differs little from that calculated by the Thomson formula.

We now change to relative symbols: $z = x/L$, $t_{rel} = L^2/\mu U_0$, $\tau = t/t_{rel}$, $f = E/E_0$. Eliminating ρ from (II.1) and changing over to the new symbols, we obtain the equation

$$f\frac{\partial f}{\partial z} + \frac{\partial f}{\partial \tau} = \Phi(\tau) \tag{II.5}$$

with boundary and initial conditions

$$\int_0^1 f\,dz = 1, \quad f(z, 0) = 1, \quad \left.\frac{\partial f}{\partial z}\right|_{z=0} = \frac{4\pi\alpha L^2}{\mu U_0} = A, \tag{II.6}$$

and the expression for the current (II.3) takes the form

$$C(\tau) = j_c \Big/ \left(\frac{9\mu U_0^2}{32\pi L^3}\right) = \frac{4}{9}\left[f^2(1, \tau) - f^2(0, \tau)\right]. \tag{II.7}$$

To determine the latter from the initial conditions we note that the function $\Phi(\tau)$ is proportional to the current: $\Phi(\tau) = 9/8C(\tau)$. Then at the instant $\tau = 0$ the right-hand side of (A.5) is also equal to zero and

$$\left.\frac{\partial f}{\partial \tau}\right|_{\tau=0} = \begin{cases} 0, & 0 < z \leqslant 1 \\ -A, & z = 0 \end{cases}. \tag{II.8}$$

The results of a numerical solution of Eq. (II.5) are shown in Fig. 10a and 10b. The values $A \geqslant 20$ correspond to a "strong" source. This case was realized in our experiments. A characteristic feature of a "strong" source is that the stationary distribution of the electric field is close to the distribution $f(z) = 3/2\sqrt{z}$, i.e., the field is strongly inhomogeneous (see Fig. 10a for $A = 20$ and 100). The steady-state current differs little from that calculated by the Thomson formula. For example, at $A = 10$ we have $I/I_T = 0.984$. Small values of A correspond to a "weak" source; a condition of this kind was realized in Keshishev's studies[1] by introducing an auxiliary electrode—a grid. In this case the space charge is small and its field constitutes a small fraction of the external field (see Fig. 10a, $A = 0.25$). It follows from the calculations that under our assumptions $t_m = 0.75t_{rel}$.

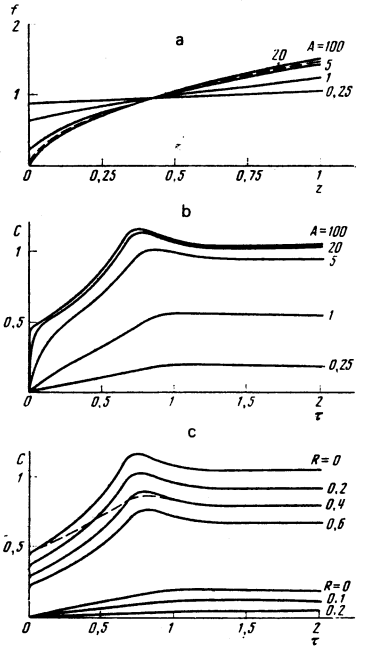

FIG. 10. Results of solution of the problem of the propagation of a chargefront in a diode: a) dependences of the relative electric field f on the coordinate z at the instant of time $\tau = 2$ in the absence of pinned charges; b) current transient curves in the absence of pinned charges; c) current transient curves with "strong" ($A = 100$, upper curves) and "weak" ($A = 0.25$, lower curves) source for a number of values of relative densities of the pinned charge R.

The next step was to solve the current transient problem when an immobile charge with density $\rho_0 = \text{const}$ is present in the diode gap. In this case a term $-Rf$ is added in the left-hand side of (II.5), where $R = 4\pi\rho_0 L^2/U_0$, and the expression for the current (II.7) is rewritten in the form

$$C(\tau) = {}^1/_2[f^2(1, \tau) - f^2(0, \tau) - 2R]. \quad \text{(II.9)}$$

The results of the solution of the equation with the parameter values $A = 100$ and 0.25 are shown for a number of values of R in Fig. 10c. It is seen from Fig. 10c that the pinned charges influence the current strongly, but the position of the maximum changes insignificantly. A similar phenomenon is observed for the steady-state current in a diode with a "weak" source.

It should be noted that this solution presupposes the presence of immobile charges prior to the turning on of the voltage. In our experiments such a distribution cannot be produced for a long time, since the field of the pinned charges, when applied to the source, will produce a charge current of opposite sign. Calculation shows that ~2/3 of all the pinned charges become electrically neutralized after a time $\sim 2t_{rel}$.

In the experiment, the front of the charges begins to move in the initially free space, and as they move behind the front, some of the charges are captured by traps. Therefore the experimental curve should start out as the curve corresponding to $I = 0$ and after a time of the order t_{rel} it should go over to a curve with the corresponding value of the parameter R (Fig. 10c, dashed line).

We now determine the limitation on the voltage U_0. It is seen from Fig. 10b that at parameter values $A > A_0 \approx 20$ the current transient curves are close, and the positions of the maxima differ by not more than 2%. From the condition $A \geq A_0$ it follows that

$$\alpha = j_i(E)/E(0, t) \geqslant \mu U_0 A_0/4\pi L^2. \quad \text{(II.10)}$$

The dependence of the current in the ionized layer on the electric field was derived under our conditions in Ref. 10. After transformations we obtain

$$U_0 \leqslant \left[8\pi I_s \delta L^2/\mu S \left[\left(1 + \frac{\delta}{L} A_0\right)^2 - 1\right]\right]^{1/2}. \quad \text{(II.11)}$$

for the considered geometry at $\delta = 10^{-3}$ cm, $I_s = 2.25$ cgs esu, $L = 2.8 \times 10^{-2}$ cm, $S = 7.1 \times 10^{-2}$ cm^2, and $A_0 = 20$ we get

$$U_0 \leqslant 0.31/\sqrt{\mu}, \quad \text{(II.12)}$$

where U_0 is in volts and μ is in cm^2/sec-V. For example, for positive charges in a crystal grown at $p = 49$ atm, at $T = 1.67$ K the condition (II.12) yields $U_0 \leqslant 980$ V.

[1)]The authors thank R. Z. Levitin for supplying the terbium iron garnet sample.

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