

lepton asymmetry is of the same order):

$$A \sim N_B/a^2 n^0 \sim C\alpha^2 \theta^3 \delta_a.$$

Our calculations contain too many uncertainties in order to be able to talk about agreement with experiment, which yields a value of $A \sim 10^{-8} - 10^{-9}$, however, our result is not in contradiction with experiment.

Thus, setting $\alpha = 10^{-2}$, $\vartheta = 0.5$, $\delta_a = 10^{-1}$, $C = 10^{-2}$ we obtain $A = 10^{-10}$.

The estimate is valid only if there exist hypothetical quarks with masses of the order of M_c . The additional small parameter which appears for the presently known quarks seems to exclude a possibility of agreement with experiment for the concrete model considered in § 4.

The result obtained in the present paper does not depend on the dimensionless parameter $k = 1/M_c G^{1/2}$, which determines the ratio of the duration of the "critical" phase for the process under consideration, $\Delta t \sim 1/G^{1/2} M_c^2$, to the characteristic reaction time for the mutual transformation of particles $\tau \sim 1/\alpha M_c$:

$$\Delta t/\tau \sim \alpha k.$$

Yoshimura² has obtained a formula which differs from ours, according to which the baryon asymmetry A is of the order of k , i. e., is proportional to the duration of the critical phase, Δt . This result is in contradiction with the absence of CP -violation in a stationary state. As was shown here (§ 3) small deviations from the equilibrium state, and thus from CP -symmetry, are proportional to $1/k$. An integration with respect to time

leads to a cancellation of the k -dependence. In the paper of Dimopoulos and Susskind⁴ it was assumed from the start that $k \sim 1$, and thus the dependence of the result on this parameter is not investigated.

In § 5 we have advanced arguments for the need of the assuming that the Universe is initially neutral, for the "multifoliated model of the Universe" with statistical characteristics which are repeated every cycle.

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¹A. D. Sakharov, ZhETF Pis. Red. 5, 32 (1967) [JETP Lett. 5, 24 (1967)]

²M. Yoshimura, Phys. Rev. Lett. 41, 281 (1978).

³H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32, 493 (1974).

⁴S. Dimopoulos and L. Susskind, SLAC-PUB-2126, June, 1978.

⁵F. Reines and M. F. Crouch, Phys. Rev. Lett. 32, 493 (1974).

⁶M. Kobayashi and T. Maskawa, Prog. Theor. Phys. (Kyoto) 49, 652 (1973).

⁷A. D. Sakharov, The multifoliated model of the Universe, Preprint of the Applied Mathematics Section of the V. A. Steklov Mathematical Institute., 1969.

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Theory of depolarization of positive muons in antiferromagnetic chromium

I. G. Ivanter and S. V. Fomichev

I. V. Kurchatov Institute of Atomic Energy

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Expressions are obtained for the time dependence of muon polarization in chromium at temperatures below the Néel point. It is shown that the muon method can yield information on the magnetic anisotropy. At low magnetic anisotropy, the depolarization rates in longitudinal and magnetic fields should be different in the region above the spin-flip temperature.

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1. We show here, with chromium as an example, how the muon method can yield information on some subtle features of magnetic ordering.

Experiments on the depolarization of μ^+ mesons in the antiferromagnetic phases of chromium have recently been reported.^{1,2} The magnetic structure of chromium is well known.³ Below the Néel point T_N , magnetic ordering of delocalized d electrons of the spin-density

wave (SDW) type occurs with a period equal to 27 periods of the crystal lattice, and with a wave vector directed along one of the edges of the cube. (We neglect hereafter the weak rhombicity of the crystal lattice and assume that chromium has a bcc structure.) Chromium has two magnetic sublattices, one over the corners of the cube, and the other over the centers. The directions of the magnetic moments are opposite at sites of different sublattices. At temperatures between

the Néel and spin-flip points ($T_{SF} < T < T_N$), the magnetic moments are perpendicular to the SDW wave vector, and in the absence of a sufficiently strong magnetic field they are directed along the edges of the cube. In a sufficiently strong external magnetic moments, the magnetic moments are rotated and they become perpendicular to the SDW and to the magnetic field (the spin-flop transition). Below the spin-flip temperature, the magnetic moments are directed along the SDW wave vector.

A study of the rate of depolarization of the μ^+ mesons in chromium shows that in a wide temperature range that includes both phase-transition points T_N and T_{SF} the muon experiences a rapid diffusion.^{1,2} The latter permits a study of the magnetic structure of chromium and, as will be shown here, makes it possible to determine the scale of the external magnetic fields at which rotation of the magnetic moments takes place.

2. There are two known possible positions where a muon can be located in a bcc crystal lattice: octapores (centers of the faces or midpoints of the edges), and tetrapores (shown in Fig. 1). It was experimentally established in Ref. 4 that deuterium occupies in chromium only octapore positions; arguments are advanced in the same reference that the proton can likewise be only in an octapore. We, however, will calculate the muon depolarization in chromium for both pore types.

Each cell has 12 tetrapores. The internal fields in the different tetrapores differ in magnitude and direction. If we disregard the variation of the magnetic moments along the SDW, then each unit cell contains three pairs of different magnetic-field vectors, and in each pair the fields are equal in magnitude and opposite in direction. In the octapores of each unit cell there are similar pairs of different magnetic-field vectors.

To determine the internal magnetic fields in different pores, we choose a system of internal unit vectors i, j, k along the cube edges. One of the axes of the external coordinate system will be directed along the external magnetic field—the x axis. The choice of the two other axes is arbitrary, but if the initial polarization is transverse to the field, we shall direct the y axis along this polarization. The orientation of the unit cell of the crystal lattice is completely determined by the orientation of the unit vectors i, j , and k . The direction i can be specified by two angles: the polar angle $\theta = (\mathbf{x} \cdot \mathbf{i})$ and the azimuthal angle φ . The orientation of the cube is then determined by the rotation angle α of

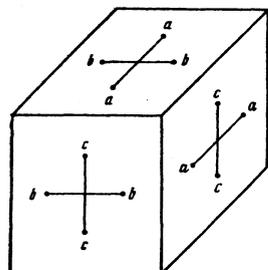


FIG. 1. Tetrapores in chromium. Identical letters designate pores having equal magnetic fields.

the cube about the i axis. This angle will be reckoned from the direction perpendicular to the axes i and x . Thus, we obtain the connection between the internal and external coordinate frames:

$$\begin{aligned} i &= n_x \cos \theta + n_y \sin \theta \cos \varphi + n_z \sin \theta \sin \varphi, \\ j &= -n_x \sin \theta \cos \alpha + n_y (-\sin \varphi \sin \alpha + \cos \theta \cos \varphi \cos \alpha) \\ &\quad + n_z (\cos \varphi \sin \alpha + \cos \theta \sin \varphi \cos \alpha), \\ k &= n_x \sin \theta \sin \alpha - n_y (\sin \varphi \cos \alpha + \cos \theta \cos \varphi \sin \alpha) \\ &\quad - n_z (-\cos \varphi \cos \alpha + \cos \theta \sin \varphi \sin \alpha). \end{aligned} \quad (1)$$

We introduce the characteristic internal magnetic field H^* . In the case of the tetrapore we have

$$H_{tetra}^* = 2^{1/2} \mu / r^3,$$

where μ is the maximum magnetic moment, $r = a\sqrt{5}/4$, and a is the period of the crystal lattice. In the case of the octapore

$$H_{octa}^* = 8(4 + \sqrt{2}) \mu / a^3.$$

We define the dimensionless magnetic field H^m in the following manner: the internal field at the muon is

$$H = H^* H^{(m)} \sin(qr),$$

where q is the SDW wave vector. The expressions for the dimensionless internal field (m is the number of the pore) in tetra- and octapores at $T > T_{SF}$, in the case when the sublattice magnetic moments are oriented perpendicular to the external magnetic field, are summarized in Tables I and II, respectively, where the following notation is used:

$$\begin{aligned} \cos \beta &= -\frac{\sin \theta \sin \alpha}{(1 - \sin^2 \theta \cos^2 \alpha)^{1/2}}, \quad \sin \beta = \frac{\cos \theta}{(1 - \sin^2 \theta \cos^2 \alpha)^{1/2}}, \\ \cos \gamma &= -\frac{\cos \theta}{(1 - \sin^2 \theta \sin^2 \alpha)^{1/2}}, \quad \sin \gamma = -\frac{\sin \theta \cos \alpha}{(1 - \sin^2 \theta \sin^2 \alpha)^{1/2}}. \end{aligned} \quad (2)$$

The dimensionless fields at $T < T_{SF}$ in the tetra- and octapores are summarized in Tables III and IV.

In the calculation of the fields in Tables I–IV account was taken of only the nearest neighbors. The dimensionless fields at $T > T_{SF}$ in the case when the moments of the sublattices are directed along the edges of the cube can be obtained from the expressions in Tables I and II by setting formally the angles α , β , and γ equal to zero or $\pi/2$.

3. Equations for the polarization $P(t)$ of an ensemble of muons in a medium with rapidly varying local fields were obtained in Ref. 3. In the situation considered by us, since the muon diffuses rapidly, the magnetic field is also rapidly changing and we can use the indicated equations:

$$\begin{aligned} dP_x/dt &= -P_x(G_{yy} + G_{zz}) + P_y G_{xy} + P_z G_{xz}, \\ dP_y/dt &= P_x G_{yx} - P_y(G_{xx} + G_{zz}) + P_z(G_{yz} - H_0), \\ dP_z/dt &= P_x G_{zx} + P_y(G_{zy} + H_0) - P_z(G_{zz} + G_{yy}). \end{aligned} \quad (3)$$

Here H_0 the external magnetic field, assumed to be much weaker than the characteristic local field;

TABLE I.

SDW direction	Number of tetrapore		
	1	2	3
i	$-j \sin \alpha + k \cos \alpha$	$-k \cos \alpha$	$j \sin \alpha$
j	$-k \sin \beta + i \cos \beta$	$-i \cos \beta$	$k \sin \beta$
k	$-i \sin \gamma + j \cos \gamma$	$-j \cos \gamma$	$i \sin \gamma$

TABLE II.

SDW direction	Number of octapore		
	1	2	3
i	$-j \frac{\sin \alpha}{2} + k \cos \alpha$	$-j \frac{\sin \alpha}{2} - k \frac{\cos \alpha}{2}$	$j \sin \alpha - k \frac{\cos \alpha}{2}$
j	$-k \frac{\sin \beta}{2} + i \cos \beta$	$-k \frac{\sin \beta}{2} - i \frac{\cos \beta}{2}$	$k \sin \beta - i \frac{\cos \beta}{2}$
k	$-i \frac{\sin \gamma}{2} + j \cos \gamma$	$-i \frac{\sin \gamma}{2} - j \frac{\cos \gamma}{2}$	$i \sin \gamma - j \frac{\cos \gamma}{2}$

$$G_{\alpha\beta} = t_c \overline{H_\alpha H_\beta} = \frac{t_c (H^*)^2}{6} \sum_{m=1}^3 H_\alpha^{(m)} H_\beta^{(m)}, \quad (4)$$

where the index m denotes the number of the pore, and t_c is the characteristic time between the hops of the diffusing muon. The averaging in (4) is over the positions of the diffusing muon, i. e., over the three tetra- (or octa-) pores as well as along the SDW.

We note that Eqs. (3) were obtained under the assumption $H^* t_c \ll 1$. In Eqs. (3) and below we refer the muon gyromagnetic ratio γ_μ directly to the field ($\mathbf{H} \rightarrow \gamma_\mu \mathbf{H}$), i. e., we put everywhere $\gamma_\mu = 1$. During its lifetime the muon does not manage to go beyond the limits of one domain, so that in the calculation of the tensor $G_{\alpha\beta}$ in Eq. (3) we must assume the direction of the SDW to be fixed, i. e., the averaging of the muon polarization over the three possible SDW directions at the given crystal orientation can be carried out only after solving Eqs. (3).

We now write down the solutions of (3) for three cases.

a) There is no external magnetic field ($H_0 = 0$). Calculation of $G_{\alpha\beta}(\alpha, \beta = x, y, z)$ for chromium under the assumption that the magnetic moments are directed along the edges of the cube shows that $G_{\alpha\beta}^2 = G_{\alpha\alpha} G_{\beta\beta}$ and that two principal values of the tensor are zero, while the third is, in the case of a tetrapore,

$$\Lambda_{\text{tetra}}^0 = (H_{\text{tetra}}^*)^2 t_c^{\text{tetra}} / 3,$$

and in the case of an octapore

$$\Lambda_{\text{octa}}^0 = (H_{\text{octa}}^*)^2 t_c^{\text{octa}} / 4.$$

The solutions take therefore the form

$$P_x(t) = 1 + C[\exp(-\Lambda^0 t) - 1],$$

$$P_y(t) = \frac{(C-1)G_{yz}}{G_{zz}} [\exp(-\Lambda^0 t) - 1], \quad (6)$$

$$P_z(t) = \frac{(C-1)G_{yz}}{G_{xy}} [\exp(-\Lambda^0 t) - 1], \quad (7)$$

where

$$C = (G_{yy} + G_{zz}) / \Lambda^0. \quad (8)$$

It is assumed in (5)–(8) that the initial polarization is directed along the x axis.

TABLE III.

SDW direction	Number of tetrapore		
	1	2	3
i	i	0	-i
j	j	0	-j
k	k	0	-k

TABLE IV.

SDW direction	Number of octapore		
	1	2	3
i	-i/2	i	-i/2
j	-j/2	j	-j/2
k	-k/2	k	-k/2

b) The external magnetic field $H_0 \gg \Lambda^0$ is directed along the x axis, and the initial polarization along the y axis. The solution for this case was obtained in Ref. 5:

$$P_y(t) + iP_z(t) = \exp\left\{ \left[iH_0 - \left(G_{xx} + \frac{G_{yy} + G_{zz}}{2} \right) \right] t \right\}, \quad (9)$$

$$P_x(t) = 0. \quad (10)$$

c) The external magnetic field $H_0 \gg 0$ as well as the initial muon polarization are directed along the x axis. According to Ref. 5,

$$P_x(t) = \exp\{-[G_{yy} + G_{zz}]t\}, \quad (11)$$

$$P_y(t) = P_z(t) = 0. \quad (12)$$

4. We consider now the muon depolarization in chromium single crystals. Calculations show that the tensors $G_{\alpha\beta}$ for the octa- and tetrapores are proportional to each other:

$$\frac{G_{\alpha\beta}^{\text{octa}}}{G_{\alpha\beta}^{\text{tetra}}} = \frac{3}{4} \frac{t_c^{\text{octa}}}{t_c^{\text{tetra}}} \left(\frac{H_{\text{octa}}^*}{H_{\text{tetra}}^*} \right)^2.$$

We shall therefore express hereafter all the results in terms of the quantities Λ^0 introduced above, with different values for the octa- and tetrapores. We leave out the expressions for $G_{\alpha\beta}$, which were obtained with the aid of formulas (1), (2), and (4) and Tables I–IV, and present directly the final expressions for the polarization.

We turn now to concrete cases.

A. We consider first the temperature region $T_{\text{SF}} < T < T_N$, in which a magnetic structure of the type of transverse spin-density wave is realized.

a) *No external magnetic field.* Although according to formulas (6) and (7) polarizations can occur in principle in directions perpendicular to the initial polarization, in the case of chromium the sum of the muon polarizations over the domains corresponding to the three SDW directions vanishes. For the polarization along the initial direction we obtain after summation the simple expression

$$P_x(t) = \frac{1}{3} \exp(-\Lambda^0 t). \quad (13)$$

b) An external magnetic field $H_0 \gg \Lambda^0$ is transverse to the initial polarization, and the magnetic moments are directed along the edges of the cube. On the basis of formula (9) we obtain after substituting the concrete expressions for $G_{\alpha\beta}$

$$P_y(t) + iP_z(t) = \frac{1}{3} \sum_{l=1}^3 \exp\{(-\Lambda_l^0 + iH_0) t\}, \quad (14)$$

where the index l corresponds to the three possible di-

rections, i , j , and k , of the magnetic moment along the edges of the cube. Here

$$\begin{aligned}\Lambda_{\perp}^i &= \frac{1}{2}\Lambda^0(1+\cos^2\theta), \\ \Lambda_{\perp}^j &= \frac{1}{2}\Lambda^0(1+\sin^2\theta\cos^2\alpha), \\ \Lambda_{\perp}^k &= \frac{1}{2}\Lambda^0(1+\sin^2\theta\sin^2\alpha).\end{aligned}\quad (15)$$

We note that

$$\bar{\Lambda}_{\perp} = \frac{1}{3} \sum_{i=1}^3 \Lambda_{\perp}^i = \frac{2}{3} \Lambda^0.$$

c) The external magnetic field $H_0 \gg \Lambda^0$ is directed along the initial polarization, and the magnetic moments are directed along the edges of the cube. According to (11) we get

$$P_x(t) = \frac{1}{3} \sum_{i=1}^3 \exp(-\Lambda_{\parallel}^i t), \quad (16)$$

where

$$\begin{aligned}\Lambda_{\parallel}^i &= \Lambda^0 \sin^2\theta, \\ \Lambda_{\parallel}^j &= \Lambda^0(1-\sin^2\theta\cos^2\alpha), \\ \Lambda_{\parallel}^k &= \Lambda^0(1-\sin^2\theta\sin^2\alpha).\end{aligned}\quad (17)$$

We note that

$$\bar{\Lambda}_{\parallel} = \frac{1}{3} \sum_{i=1}^3 \Lambda_{\parallel}^i = \frac{2}{3} \Lambda^0,$$

i. e., the mean values of the damping in the longitudinal and transverse fields are equal if the magnetic moments in the individual domains are directed along the cube edges.

d) The external magnetic field $H_0 \gg \Lambda^0$ is so strong that the magnetic moments have become perpendicular to the external magnetic field. Formulas (14) and (16) are then valid, with

$$\Lambda_{\perp}^i = \frac{1}{2}\Lambda^0(1+\frac{3}{4}\sin^2\theta\sin^2 2\alpha), \quad (18)$$

$$\Lambda_{\parallel}^i = \Lambda^0(1-\frac{3}{4}\sin^2\theta\sin^2 2\alpha) \quad (19)$$

(the indices i, j , and k corresponds here and below to the SDW direction),

$$\begin{aligned}\Lambda_{\perp}^j &= \frac{1}{2}\Lambda^0(1+\eta), & \Lambda_{\parallel}^j &= \Lambda^0(1-\eta), \\ \Lambda_{\perp}^k &= \frac{1}{2}\Lambda^0(1+\nu), & \Lambda_{\parallel}^k &= \Lambda^0(1-\nu),\end{aligned}\quad (20)$$

where

$$\eta = \frac{3\cos^2\theta\sin^2\theta\sin^2\alpha}{1-\sin^2\theta\cos^2\alpha}, \quad \nu = \frac{3\cos^2\theta\sin^2\theta\cos^2\alpha}{1-\sin^2\theta\sin^2\alpha}.$$

B. We consider now the temperature region $T < T_{SF}$, in which the magnetic structure of the chromium takes the form of a longitudinal SDW.

a) No external magnetic field. It is easily deduced from a comparison of Tables I and II with Tables III and IV that since the magnetic moments of the atoms are likewise directed along the cube edges, averaging over the different domains will yield precisely the same expression as (13).

b) External magnetic field $H_0 \gg \Lambda^0$. For the same reason as in the preceding paragraph, both cases of transverse and longitudinal field are described by for-

mulas (14), (15) and (16), (17).

5. All the known experiments were performed on chromium polycrystals.^{1,2} It is therefore important to determine how the expressions obtained in the preceding section are altered in the case of single crystals. To facilitate the calculations we choose the i axis to be henceforth that cube edge along which the magnetic moments of the atoms are directed if they are directed along cube edges. If, however, the magnetic moments of the atoms are not parallel to cube edges but are perpendicular to the external magnetic field, the i axis will be chosen to be the direction of the spin-density wave. We could not do so in the preceding section, since we were interested in the angular dependence of the polarization on the orientation of the crystal relative to the laboratory frame (x, y, z) . In a polycrystal, on the other hand, it is necessary to average over all the crystallites and over the domains in them. It suffices therefore to follow the orientation of one particular domain.

The time dependences of the polarization in a polycrystal become non-exponential, although in the zeroth approximation they can be described by exponentials with the average damping decrements cited in the preceding section. Averaging over the orientations, we obtain the following expressions for the time dependences of the polarizations for polycrystals:

a) There is no external field, $T < T_N$:

$$P_x(t) = \frac{1}{3} \exp(-\Lambda^0 t), \quad P_y(t) = P_z(t) = 0. \quad (21)$$

b) External field $H_0 \gg \Lambda^0$, sublattice magnetic moments directed along the edges of the cube, $T < T_N$. In this case, we obtain for an arbitrary field, using (17)

$$P_x(t) = \langle \exp[-\Lambda_{\parallel}^i(\theta, \alpha)t] \rangle = F_1(1, \frac{1}{2}, -\Lambda^0 t), \quad (22)$$

where $F_1(\alpha, \gamma, z)$ is a confluent hypergeometric function and the angle brackets denote averaging over the orientations. For a transverse field we get according to (15)

$$\begin{aligned}P_y(t) + iP_z(t) &= \langle \exp[-(\Lambda_{\perp}^i(\theta, \alpha) + iH_0)t] \rangle \\ &= \exp(iH_0 t) \left(\frac{\pi}{2\Lambda^0 t} \right)^{1/2} \Phi \left(\sqrt{\frac{\Lambda^0 t}{2}} \right) \exp\left(-\frac{\Lambda^0 t}{2}\right),\end{aligned}\quad (23)$$

where

$$\Phi(x) = \frac{2}{\pi^{1/2}} \int_0^{\infty} e^{-z^2} dz$$

is the probability integral.

c) External field $H_0 \gg \Lambda^0$, magnetic moment perpendicular to the external field, $T_{SF} < T < T_N$. In a longitudinal field, using (19), we get

$$P_x(t) = \langle \exp(-\Lambda_{\parallel}^i(\theta, \alpha)t) \rangle = \exp(-\Lambda^0 t) \sum_{n=0}^{\infty} \left(\frac{3\Lambda^0 t}{4} \right)^n \frac{1}{n!(2n+1)}. \quad (24)$$

In a transverse field we have according to (18)

$$\begin{aligned}P_y(t) + iP_z(t) &= \langle \exp[-(\Lambda_{\perp}^i(\theta, \alpha) + iH_0)t] \rangle \\ &= \exp\left(-\frac{\Lambda^0 t}{2}\right) \left(\frac{2\pi}{3\Lambda^0 t} \right)^{1/2} \Phi \left(\sqrt{\frac{3\Lambda^0 t}{8}} \right) e^{iH_0 t},\end{aligned}\quad (25)$$

where $\Phi(x)$ is the probability integral. We note that in case c) the mean depolarization rates are not equal in the longitudinal and transverse directions:

$$\langle \Lambda_{\parallel}^i(\theta, \alpha) \rangle = \langle \Lambda_{\parallel}^i(\theta, \alpha) \rangle = \langle \Lambda_{\parallel}^k(\theta, \alpha) \rangle = \frac{1}{2} \Lambda^0, \quad (26)$$

$$\langle \Lambda_{\perp}^i(\theta, \alpha) \rangle = \langle \Lambda_{\perp}^i(\theta, \alpha) \rangle = \langle \Lambda_{\perp}^k(\theta, \alpha) \rangle = \frac{1}{2} \Lambda^0. \quad (27)$$

6. It turns out thus that the muon method can, in principle, yield information on the degree of magnetic anisotropy of the chromium. If application of a magnetic field of given strength causes rotation of the magnetic moments, then the muon polarization damping in the polycrystal will depend on the direction of the external magnetic field relative to the initial muon momentum-longitudinal or transverse. At large values of the anisotropy, the average depolarization rate in a polycrystal should not depend strongly on the orientation of the external magnetic field relative to the initial polarization. This result can be explained in general form by the following arguments.

According to (9) and (11), the longitudinal and transverse depolarization rates are given by

$$\Lambda_{\perp} = G_{xx} + \frac{G_{yy} + G_{zz}}{2} = \frac{\Lambda^0}{2} + \frac{G_{xx}}{2}, \quad (28)$$

$$\Lambda_{\parallel} = G_{yy} + G_{zz} = \Lambda^0 - G_{xx}, \quad (29)$$

where $\Lambda^0 = \text{Tr } G_{\alpha\beta}$. For the mean values we have

$$\langle \Lambda_{\perp} \rangle = \frac{\Lambda^0}{2} + \frac{\langle G_{xx} \rangle}{2}, \quad \langle \Lambda_{\parallel} \rangle = \Lambda^0 - \langle G_{xx} \rangle.$$

If $\langle \Lambda_{\perp} \rangle = \langle \Lambda_{\parallel} \rangle$, then $\langle G_{xx} \rangle = \Lambda^0/3$, meaning that in a poly-

crystal all the directions of the internal magnetic fields are on a par. If $\langle G_{xx} \rangle < \Lambda^0/3$, i.e., the internal magnetic fields at the muon (and hence also the magnetic moments) are directed predominantly in a plane perpendicular to the external magnetic field, then $\langle \Lambda_{\parallel} \rangle < \langle \Lambda_{\perp} \rangle$. On the other hand if $\langle G_{xx} \rangle > \Lambda^0/3$, i.e., the internal magnetic fields are directed predominantly along the external field, then we should have $\langle \Lambda_{\parallel} \rangle > \langle \Lambda_{\perp} \rangle$.

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¹W. J. Kossler, A. T. Fiory, D. E. Murnick, C. E. Stronach, and W. F. Lankford, *Hyperfine interaction* 3, 287 (1977).

²V. G. Grebinnik, I. I. Gurevich, V. A. Zhukrov, I. G. Ivanter, A. L. Klimov, A. P. Marych, E. V. Mel'nikov, B. A. Nikol'skii, A. V. Pirogov, A. N. Ponomarev, V. I. Selivanov, V. A. Suetin, and S. V. Fomichev, *Pis'ma Zh. Eksp. Teor. Fiz.* 28, 456 (1978) [*JETP Lett.* 28, 423 (1978)].

³E. I. Kondorskii, *Zonnaya teoriya magnetizma* (Band Theory of Magnetism), part II, Izd. MGU, Moscow, 1977.

⁴S. A. Werner, A. Arrott, and H. Kendrick, *Phys. Rev.* 155, 528 (1967).

⁵I. G. Ivanter and S. V. Fomichev, *Teoriya depolyarizatsii μ^+ -mezonov redkozemel'nykh metallakh vblizi tochek Neelya* (Theory of Depolarization of mesons in Rare Earth Metals near the Neel Points), IAE-2999, Moskva, 1978.

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Investigation of second-harmonic generation in diffused LiNbO₃ waveguides

E. M. Zolotov, V. M. Pelekhatyi, A. M. Prokhorov, and V. A. Chernykh

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The conditions for efficient second-harmonic generation in diffused LiNbO₃ waveguides are considered. The overlap integrals of the interacting modes are computed as functions of the waveguide parameters. It is shown that the highest second-harmonic generation efficiency is attained in the case when the E_1 (1.06 μ) and H_2 (0.53 μ) modes interact. Second-harmonic generation is investigated experimentally in the 10^{-2} - 10^3 -W ($\lambda = 1.06 \mu$) pump-power range, and a comparison with the calculations is carried out. The highest nonlinear-conversion efficiency is 16% for a peak pump power of 1 kW.

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Integral optics opens up general possibilities for raising the effectiveness of nonlinear interactions. The use of optical waveguides allows us to obtain in a film of thickness of the order of the wavelength of high light intensities from relatively low-power sources, e.g., gas lasers. In contrast to the three-dimensional case, when the contraction of the luminous flux to small dimensions gives rise to its considerable diffraction divergence, in a waveguide the small cross section of

the flux (and, consequently, its high density) is preserved along its entire length. Another merit of thin-film waveguides is the possibility of obtaining in them phase matching of the interacting waves as a result of the dispersion of the modes. This allows the use of isotropic media possessing large nonlinear coefficients. As to anisotropic waveguides, they do not need temperature adjustment for obtaining ninety-degree phase matching. The latter is achieved by a proper choice of