The optical isotropy of space and its relation to the equivalence principle

O. N. Naĭda and Ya. A. Smorodinskiĭ

I. V. Kurchatov Atomic Energy Institute
(Submitted 5 October 1978)
Zh. Eksp. Teor. Fiz. **76**, 1153–1161 (April 1979)

It is shown that the optical isotropy of cosmic space can be verified with great accuracy (one part in 10^{26}) by polarization studies of reflection nebulae. This, in turn, makes possible a high-precision verification of the equivalence principle as applied to electromagnetic waves.

PACS numbers: 98.40.Kb, 04.80. + z, 98.90. + s

1. THE OPTICAL ISOTROPY OF COSMIC SPACE

There exists a very sensitive astrophysical effect that can be used, among other things, to verify the equivalence principle. To do this one must set in correspondence the following two facts:

a) There exist astrophysical objects (reflection nebulae) whose polarization patterns (that is, the way the polarization of the light reflected to the Earth depends on the point from which it leaves the object) are quite simple.

b) In theory the light arrives at the Earth with the same polarization it had on leaving the nebula.

The importance of the effect is that the reflection nebulae are located at cosmic distances from the Earth ($l \sim 300\,\text{ps} = 10^{21}$ cm). If intragalactic space possessed even an extremely small birefringence (such as, for example, from the Faraday effect or of the crystallographic type), say with $\Delta n = \lambda/2\pi l \sim 10^{-26}$ ($\lambda \sim 5000$ Å $= 5 \times 10^{-5}$ cm), the Earth-bound observer would receive a highly distorted polarization pattern from such objects.

The polarization pattern from a reflection nebula is determined by the laws for scattering of light, according to which the polarization (the electric field vector) should be perpendicular to the direction from the observer to the illuminating star.[1] In any case, this radial polarization pattern should be observed in a sufficiently close neighborhood of the illuminating star in at least the brightest nebulae, when the distortion of the polarization pattern by the interstellar medium can be neglected. Such a pattern, actually obtained by Elvius and Hall,[2] is shown on the left side of the Fig. 1.[1)] The right side shows what the polarization pattern would have been if the space between the Earth and the nebula had been birefringent. The diagram illustrates only the simplest examples of such a possible effect, but it is clear from this example that since the observed polarization pattern (that is, the one on the left of the diagram) is not substantially distorted, one can obtain an upper limit for the difference in indices of refraction (and therefore for the optical anisotropy) of space along the incoming ray. This upper limit is $\Delta n \lesssim \lambda/2\pi l$.

Many polarization studies have been made of several reflection nebulae (essentially of the brightest ones), and they show that these nebulae do indeed exhibit this radial pattern of polarization, especially pronounced in the neighborhood of the illuminating stars. This is true, for example for the nebulae IC 349,[2,3] NGC 1976,[4-6] M78,[7,8] NGC 2261,[9,10] NGC 7023,[11,8] the nebula in the vicinity of the star VYCMa,[12,13] and the main (southern) part of the Trifid nebula (M20)[14]. (A detailed description of these and of all the other reflection nebulae is given by Rozhgovskiĭ and Kurchakov.[15]) Thus, at least for the nebulae named, the polarization pattern is observed without significant distortion. If we now take into account the fact that the distances to these nebulae are $\gtrsim 300$ ps[15-17] (except for IC 349, for which the distance is $\simeq 130$ ps), we obtain the following limit for the difference in the index of refraction:

$$\Delta n < 10^{-26}. \tag{1}$$

This estimate could be improved significantly if one

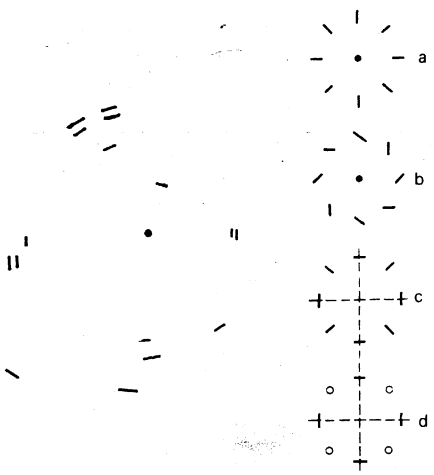

FIG. 1. On the left: Schematic polarization pattern of the reflection nebula IC 349 located near Merope in the Pleiades[2]: The dot at the center is Merope, the illuminating star; the lines show the direction of the electric vector (i.e., the direction of polarization); the lengths of the lines are proportional to the degree of polarization. It is easily seen that the electric vectors tend to lie in concentric circles about the illuminating star. On the right: What the schematic pattern on the left would look like if cosmic space were birefrigent in several possible ways. (a) Birefrigence of the type of the Faraday effect with an optical path difference $d = \lambda/4$. (b) The same, with $d = \lambda/8$. (c) Birefringence of the optical crystal type with $d = \lambda/2$. The dotted lines represent the symmetry axes and show the directions of polarization of the normal waves. (d) The same, with $d = \lambda/4$. The circles represent circular polarization.

 0038-5646/79/040585-05$02.40

were to use extragalactic reflective objects. Unfortunately, however, few of them are at present known.[18]

We see thus that polarization studies of reflective nebulae permit an enormously accurate verification of optical isotropy of cosmic space. Unfortunately this accuracy (one part in 10^{26}) is not sufficient to detect the vacuum birefringence resulting from the Galactic magnetic field ($\sim 10^{-5}$ gauss[17]), which is predicted by quantum electrodynamics. This effect would lead to a Δn of only 10^{-42}. On the other hand, the sensitivity of one part in 10^{26} turns out to be quite sufficient to verify the equivalence principle by using the isotropy of cosmic space, as will be shown in Sec. 2.

Without the equivalence principle, the tensor character of the gravitational field would lead in such a field (by analogy with propagation of light in crystals), to a birefringent effect connected with differences in the propagation of waves with polarizations corresponding to the different principal direction of the gravitational field tensor. No such effect can occur, however, if the equivalence principle is to hold. According to this principle, the gravitational field cannot have any effect inside a falling elevator, and in particular it cannot lead to birefringence; but since the existence or nonexistence of birefringence cannot depend on the choice of coordinate frame, it cannot occur in general. It requires, of course, more than Galileo's law to arrive at this conclusion; what is needed is Einstein's equivalence principle in its complete generality.

The formal mathematical explanation of the lack of gravitational birefringence within the general theory of relativity (GTR) is straightforward[21,23]. The point is that for any $g_{\mu\nu}$ the electrodynamic equations of GTR can be written as the electrodynamic equations for a moving medium (in the absence of gravity) in the form

$$\partial(s^{\alpha\beta\gamma\delta}F_{\gamma\delta})/\partial x^{\beta}=0, \quad e^{\alpha\beta\gamma\delta}\partial F_{\gamma\delta}/\partial x^{\beta}=0, \tag{2}$$

where $s^{\alpha\beta\gamma\delta}$ is the Tamm tensor[23] which is expressed through the $g_{\mu\nu}$ by the equation

$$s^{\alpha\beta\gamma\delta}_{\mathrm{GTR}}=\sqrt{-g}g^{\alpha\gamma}g^{\beta\delta}. \tag{3}$$

In the simplest case, when $g_{0m}=0$, Eq. (2) can be written in "$\varepsilon-\mu$ form," and from (3) one obtains

$$\hat{\varepsilon}_{\mathrm{GTR}}=\hat{\mu}_{\mathrm{GTR}}. \tag{4}$$

Proportionality of the $\hat{\varepsilon}$ and $\hat{\mu}$ tensors, as shown by Federov,[24] is the necessary and sufficient condition for the absence of birefringence in any stationary medium. A similar, although somewhat more complicated, interpretation of Eqs. (2) and (3) can be made also when $g_{0m}\neq 0$.

Therefore an experimental proof of the absence of gravitational birefringence would constitute a clear verification of the equivalence principle. Because of the weakness of the gravitational interaction, of course, such a test is in practice impossible under the conditions that exist on Earth. It turns out, however (see Sec. 2), that this test can be performed on a cosmic scale, that is, by using Eq. (1).

2. SOME QUANTITATIVE ESTIMATES

Let us see how small the upper limit on Δn must be in order for it to constitute a verification of the equivalence principle. It should be clear that in testing for any negative effect, an appropriate "standard of accuracy" can be obtained only by starting from some model in which the effect is violated. The need for such a "test model" is easily seen in the example of the well-known Braginskiĭ–Panov experiment[25] and the earlier one of Dicke and co-workers.[26] If one started out by excluding *a priori* the possibility that the equivalence principle is violated, all of these experiments would be pointless. They begin to look useful when one allows, for example,[27] the hypothetical possibility that the equivalence principle may be violated to first order in the gravitational potential and to first order in the atomic mass defect (i.e., to first order in the kinetic energy of the nucleons which make up the nucleus). Then one might expect that the ratio m_g/m_{in} of the gravitational to the inertial mass for different substances could differ, for different substances, by an amount

$$\Delta(m_g/m_{in})\sim\varphi v^2/c^2, \tag{5}$$

where v is a characteristic velocity inside the nucleus.

When we put $v^2/c^2\sim 10^{-3}$ and $\varphi\sim 10^{-8}$ (the Sun's potential at the Earth) into Eq. (5), we obtain $\Delta(m_g/m_{in})\sim 10^{-11}$ within the framework of our test model. This is just the value obtained by Lightman and Lee[27] as an acceptable "standard of accuracy" in measuring $\Delta(m_g/m_{in})$, and it is precisely from this point of view that the above mentioned experiments[25,26] are of interest, since they lie within this standard. It should be mentioned, however, that it would make more sense to choose φ in (5) as the gravitational potential of the Galaxy in the neighborhood of the Sun,[17] namely $\varphi^G\sim 0.7\times 10^{-6}$. This would raise the standard of accuracy to 10^{-9}. This, of course, increases the value of the experiments[25,26]: one need not worry that the difference between Braginskiĭ and Panov's value of 10^{-12} and our estimate of 10^{-9} obtained from (5) results from the neglect of some chance small factor (such as $1/4\pi^2$) which might arise when the calculation is performed using some concrete theoretical model. (The calculations used to illustrate the success of this scheme were in fact carried out by Lightman and Lee[25] within the framework of the theory developed by Belinfante and Swichart[28] and by Capella.[29])

Now let us obtain the standard of accuracy for Δn in a similar way. Let us assume that the GTR equations for the electromagnetic field hold only to first order in φ (i.e., that the bending of rays is as given by GTR). Let us assume further that to the next order in φ the equations for the electromagnetic field, violating the equivalence principle are sensitive not only to $\nabla\varphi$, but also to φ itself; in particular, let there be a gravitational birefringence, arising from the anisotropy of the gravitational potential, which is of second order in φ. We shall call the gravitational potential $\psi_{\mu\nu}$, implying that $\psi=1/2\psi_{00}$. Let us also assume that for a point source the anisotropic components of the potential, ψ_{0m} or $\psi_{mn}-1/3\delta_{mn}$, are of the same order as is ψ_{00}. Then one might expect that an estimate for Δn would be

$$\Delta n \sim \varphi^2, \tag{6}$$

which yields $\Delta n \sim 10^{-16}$ even for the Sun's potential at the Earth ($\varphi \simeq 10^{-8}$), a value much greater than that of (1). For distances of the order of the Earth's orbit diameter ($\sim 3\times 10^{13}$ cm) this would give an optical path difference of about 3×10^{-3} cm, which is much bigger than the wavelengths of the visible region. Thus the observed optical isotropy of cosmic space immediately allows one to reject the (relatively rough) model we have presented for violation of the equivalence principle.

It turns out that the estimate of (1) is sufficiently good to reject a more delicate test model for violating the equivalence principle. Let us assume that the gravitational potential of a stationary point source is spacially isotropic (i.e., that $\psi_{0m}=0$, $\psi_{mn}=\delta_{mn}\psi_{11}$) and therefore in itself leads to no gravitational birefringence (precisely the kind of model used by Lightman and Lee in analyzing the Braginskiĭ–Panov experiment and others), so that spacial anisotropy can arise only as a result of relative motion of the massive bodies which create the potential $\psi_{\mu\nu}$. Then the estimate of Eq. (6) is replaced by

$$\Delta n \sim \varphi^2 v^2/c^2, \tag{7}$$

where v is a characteristic velocity of regular relative motion of the massive bodies, such as the velocity of rotational motion of the field source or the relative velocity of translational motion of its parts.

In this more subtle model, the Sun's gravitational field is found to be insufficient to lead to an observable path difference along the ray. The gravitational field of the Galaxy is, however, sufficient when one includes its rate of rotation. Putting $\varphi=\varphi^G\simeq 0.7\times 10^{-6}$ and $v=v^G \simeq 250$ km/sec (the rotational velocity of the Galaxy at the Sun[17]) into (7), we arrive at

$$\Delta n \sim 10^{-19}. \tag{8}$$

This is the desired standard of accuracy for measuring Δn which is required for verifying the equivalence principle. By comparing (8) and (1) we see that the observed optical isotropy of cosmic space allows a reliable verification of the equivalence principle for electromagnetic waves.

Of course the estimate of (7), as well as that of (5), can be obtained also by calculations within the framework of specific noneinsteinian theories. We illustrate this in Sec. 3.

3. CALCULATION OF GRAVITATIONAL BIREFRINGENCE WITHIN THE FRAMEWORK OF A "TRIAL" THEORY

We choose the "trial" equations to be the GTR equations in which we change the electromagnetic GTR equations in second order in the quantity

$$\psi_{\mu\nu}=g_{\mu\nu}-\eta_{\mu\nu} \quad (\eta_{\mu\nu}=2\delta_{0\mu}\delta_{0\nu}-\delta_{\mu\nu}). \tag{9}$$

For example, Eqs. (2) and (3) can be linearized with respect to the $\psi_{\mu\nu}$. Then (2) does not change its form, and (3) is replaced by

$$s^{\alpha\beta\gamma\delta}=\eta^{\alpha\gamma}\eta^{\beta\delta}(1+{}^1/_2\psi)-\psi^{\beta\delta}\eta^{\alpha\gamma}-\psi^{\alpha\gamma}\eta^{\beta\delta}. \tag{10}$$

Here and in the sequel, indices are raised and lowered with the aid of $\eta^{\mu\nu}$. Greek indices run over the values 0, 1, 2, 3, and Latin ones over 1, 2, 3. We use the summation convention.

When $\psi_{0m}=0$, linearizing $s^{\alpha\beta\gamma\delta}_{\mathrm{GTR}}$ with respect to the $\psi_{\mu\nu}$ implies also that $\hat{\varepsilon}_{\mathrm{GTR}}$ and $\hat{\mu}^{-1}_{\mathrm{GTR}}$ are linearized with respect to the $\psi_{\mu\nu}$, for their components are contained in the form of linear combinations in $s^{\alpha\beta\gamma\delta}_{\mathrm{GTR}}$. The $\hat{\varepsilon}$ and $\hat{\mu}$ tensors will, on the other hand, satisfy not (4) but an equation of the form

$$\hat{\varepsilon}=\hat{\mu}+O(\psi_{\mu\nu}^2).$$

Thus if $\psi_{0m}=0$ and the ψ_{mn} tensor is spatially anisotropic, one may expect that the proportionality of $\hat{\varepsilon}$ to $\hat{\mu}$ is broken in second order in the $\psi_{\mu\nu}$, and therefore that gravitational birefringence is also of the same order. Of course, the same can be expected [within the framework of Eqs. (2) and (10)] also if $\psi_{0m}\neq 0$. The Tamm material tensor (10) corresponds to an optical-crystal type of birefringence.

Since, unlike Eqs. (2) and (3), the set of Eqs. (2) and (10) is not generally covariant, its solution depends significantly on the coordinate representation of metric tensor $g_{\mu\nu}$ which appears in Eq. (9). In order, therefore, to complete the trial model we formulated above, we would have to add to the gravitational equations some coordinate conditions. Since, however, our goal here is merely to illustrate a possible mechanism for gravitational birefringence, we shall not try to complete the model, but shall restrict our choice to such representations of the $g_{\mu\nu}$ as will lead [in (2), (10)] to minimum birefringence.

Let us set

$$F_{\mu\nu}(x)=f_{\mu\nu}(x)\exp[i\zeta(x)],$$

in (2), and then let us look for the eikonal $\zeta(x)$ and the amplitudes $f_{\mu\nu}(x)$ in the usual geometrical-optical manner (see, for example, Chapter 6 of Courant[30]). Then we find that the $F_{\mu\nu}$ wave breaks up (as in crystal optics) into two linearly polarized waves $F^{\mathrm{I}}_{\mu\nu}(x)$ $F^{\mathrm{II}}_{\mu\nu}(x)$, each with its own eikonal $\zeta^{\mathrm{I}}(x)$ and $\zeta^{\mathrm{II}}(x)$ and with its own slowly varying amplitudes $f^{\mathrm{I}}_{\mu\nu}(x)$ and $f^{\mathrm{II}}_{\mu\nu}(x)$. It is also easy to calculate the difference Δn between the indices of refraction corresponding to these two waves, namely

$$\begin{aligned}(\Delta n)^2={}^1/_4[&(\psi_{22}-\psi_{11})(\psi_{00}+\psi_{33}+2\psi_{03})+(\psi_{10}+\psi_{13})^2\\ &-(\psi_{20}+\psi_{23})^2]^2+[\psi_{12}(\psi_{00}+\psi_{33}+2\psi_{03})-(\psi_{01}+\psi_{13})(\psi_{20}+\psi_{23})]^2,\end{aligned} \tag{11}$$

where we have dropped terms of higher order in the $\psi_{\mu\nu}$, and the indices 1, 2, 3 correspond to a choice of Cartesian coordinates in which the x^3 axis is parallel to the wave vector.

When one rewrites (11) in an arbitrary system of coordinates and averages over all directions of the wave vector, one obtains the positive definite expression

$$\overline{(\Delta n)}^2={}^1/_5(A_{mn}A_{mn}+B_{mn}B_{mn}), \tag{12}$$

where

$$A_{mn}=\chi_{00}\chi_{mn}-\psi_{0m}\psi_{0n}+{}^1/_3\delta_{mn}(\psi_{0p}\psi_{0p})-{}^1/_3\chi_{mp}\chi_{pn}-{}^1/_3e_{mpq}e_{nij}\chi_{pi}\chi_{qj},$$
$$B_{mn}=\psi_{0p}(e_{pqm}\chi_{qn}+e_{pqn}\chi_{qm}) \tag{13}$$

and where

$$\chi_{00}=\psi_{00}+{}^1/_3\psi_{mm}=g_{00}+{}^1/_3g_{mm},$$
$$\chi_{ij}=\psi_{ij}-{}^1/_3\delta_{ij}\psi_{mm}=g_{ij}-{}^1/_3\delta_{ij}g_{mm}, \tag{14}$$

that is, χ_{ij} is the anisotropic part of the tensor potential.

When the $g_{\mu\nu}$ in Eqs. (12)–(14) are taken to be those given by the Schwarzschild metric, i.e., when

$$g_{00}=1-\frac{r_g}{r},\quad g_{0i}=0,\quad g_{ij}=-\delta_{ij}-\left(\frac{r}{r_g}-1\right)^{-1}\frac{x^ix^j}{r^2} \tag{15}$$

then the desired parameter of gravitational birefringence becomes

$$\overline{(\Delta n)^2}=\frac{2}{15}\left(\frac{r_g}{r}\right)^4+O\left(\frac{r_g}{r}\right)^5, \tag{16}$$

which is in good agreement with the estimate of (6), obtained earlier for a similar situation.

Of course Δn vanishes if one replaces (15) by the conformally Euclidean Schwarzschild representation

$$g_{00}=\left(1-\frac{r_g}{4r}\right)^2\left(1+\frac{r_g}{4r}\right)^{-2},\quad g_{0i}=0,\quad g_{ij}=-\left(1+\frac{r_g}{4r}\right)^4\delta_{ij}. \tag{17}$$

If, on the other hand, the gravitational field is created by a rotating source, then as is well known,[31] anisotropic increments appear in (17) of the form

$$\delta g_{00}=\delta g_{ij}=0,\quad \delta g_{0m}=-\frac{2\gamma}{c^3r^3}e_{mij}M^j, \tag{18}$$

where γ is the Newtonian gravitational constant and M^j is the j-th component of the angular momentum vector **M** of the source; we are assuming a large distance from the source and that the field is weak ($r_g/r\ll 1$), and we have dropped terms of second order in γ. If now the $g_{\mu\nu}$ are replaced by these $g_{\mu\nu}+\delta g_{\mu\nu}$ in Eqs. (12)–(14), Eq. (16) becomes

$$\overline{(\Delta n)^2}=\frac{2}{15}\delta g_{0m}\delta g_{0m}=\frac{8}{15}\left(\frac{\gamma}{c^3r^3}\right)^2[\mathbf{rM}]^2. \tag{19}$$

It is easily verified that although this formula was obtained for the gravitational field of a distant source, it will nevertheless give (within the framework of our "trial model") the correct order of magnitude also for light propagating close to the source of the gravitational field. On the other hand, it is easily seen that Eq. (19) is in agreement with the estimate of (7). We have thus illustrated the mechanism by which this estimate appears in a concrete noneinsteinian model.

It can be shown that the estimate of (7) appears also in the theories of Belinfante and Swichart and of Capella, for the electromagnetic field equations of those theories are identical with (2) and (10). For simplicity, however, we have restricted our considerations to the above model.

The results of Secs. 1–3 show that the optical isotropy of cosmic space is a negative effect which may be taken as a highly sensitive instrument for studying the basic laws of nature. In particular, it successfully extends the methods for establishing the equivalence principle, based on the experiments of Eotvos, Dicke, and Braginskiĭ.

The authors express the hope that this work will stimulate the search for further gravitational experiments.

The authors are grateful to B. P. Artamonov for discussions of this work.

APPENDIX

We have discussed using reflection nebulae to estimate optical anisotropy. It would be helpful to compare the resolving power of this method with some others. We present a very simple variant of such a comparison.

Suppose that a source of electromagnetic waves with frequency ω is approaching or receding from an observer with velocity v. If the medium through which the source is moving is birefringent, the radiation will be broken up into the ordinary and extraordinary waves, and the observer will receive them with a beat frequency given by $\Omega=\omega(v/c)\Delta n$, where Δn is the difference between the two indices of refraction (the beats result from the relative motion of the source and observer). Thus if $(v/c)\sim 10^{-3}$ (the velocity of orbital motion at the surface of the Sun), to record a value of $\Delta n\sim 10^{-3}$ corresponding to the hypothetical violation of the equivalence principle one would need an oscillator whose frequency was stable to $\Delta f/f\sim\Omega/\omega\sim 10^{-21}$. This is four orders of magnitude higher than the present experimental limit (10^{-17}), as the authors were kindly told by B. V. Braginskiĭ. Reflection nebulae, on the other hand can be used to record birefringence seven orders of magnitude weaker ($\Delta n\sim 10^{-26}$), as described in Sec. 1.

[1)]Elvius and Hall[2] drew the lines (the results of photoelectric polarization measurements) right on a photograph of the nebula IC 349. Similar diagrams or tables will be found in all of Refs. 2–14, 18.

---

[1]S. A. Kaplan, S. B. Pikel'ner, Mezhzvezdnaya sreda (The interstellar medium), Fizmatgiz, M. 1963.
[2]A. Elvius, J. S. Hall, Lowell Obs. Bull., **7**, 17, 1967.
[3]B. P. Artamanov, Yu. S. Efimov, Astron. Zh. **44**, 755 (1967) [Sov. Astron. **11**, 607 (1967)].
[4]R. C. Hall, J. Opt. Soc. Amer., **41**, 963, 1951.
[5]V. A. Dombrovskiĭ, Dokl. Akad. Nauk SSSR **102**, 907 (1955).
[6]B. M. Gabdullin, D. I. Gorodetskiĭ, Tr. Astrof. in-ta KazSSR **26**, 3 (1974).
[7]Yu. I. Glushkov, Tr. Astrof. in-ta KazSSR **5**, 277 (1965).
[8]A. Elvius, J. S. Hall, Lowell Obs. Bull., **6**, 257, 1966.
[9]E. E. Khachikyan, Soobshcheniya Byurokanskoĭ obs. **25**, 67 (1958).
[10]R. C. Hall, Astrophys. J., **139**, 759, 1964.
[11]M.-T. Martel, Ann. d'Aph. Suppl., **7**, 1958.
[12]K. Serkowski, Astrophys. J. (Lett.), **158**, L107, 1969.
[13]G. H. Herbig, Astrophys. J., **172**, 375, 1972.
[14]D. A. Rozhkovskiĭ, Izv. Astrof. in-ta KazSSR **12**, 21 (1961).
[15]D. A. Rozhkovskiĭ, A. V. Kurchakov, Tr. Astrof. in ta KazSSR **11**, 3 (1968).
[16]A. Becvar, Atlas Coeli II—Katalog 1950.0, Praga, 1959.
[17]C. W. Allen, Astrophysical Quantities, Athlone Press, 1970.
[18]A. B. Solinger, T. Market, Astrophys. J., **197**, 309, 1975.
[19]S. L. Adler, Ann. of Phys., **67**, 599, 1971.
[20]Yu. N. Gnedin, G. G. Pavlov, and Yu. A. Shibanov, Pis'ma

Astron. Zh. 4, 214 (1978) [Sov. Astron. Lett. 4, 117 (1978)].
[21]O. N Naida, Pis'ma Zh. Eksp. Teor. Fiz. 8, 110 (1968) [JETP Lett. 8, 68 (1968)]; Izv. vuzov, Radiofizika 12, 569 (1969)].
[22]A. M. Volkov, A. A. Izmest'ev, G. V. Skrotskiĭ, Zh. Eksp. Teor. Fiz. 59, 1254 (1970 [Sov. Phys. JETP 32, 686 (1971)].
[23]I. Tamm, ZhRFKhO (chast' fizicheskaya) 56, 248 (1924).
[24]F. I. Fedorov, Optika anizotropnykh sred (Optics of anisotropic media) Iz. ANBSSR, 1958.
[25]V. B. Braginskiĭ, V. I. Panov, Zh. Eksp. Teor. Fiz. 61, 875 (1971) [Sov. Phys. JETP 34, 463 (1972)].
[26]P. G. Roll, R. Krotkov, R. H. Dicke, Ann. of Phys., 26, 442 (1964).
[27]A. P. Lightman, D. L. Lee, Phys. Rev. D 8, 364 (1973).
[28]F. G. Belinfante, J. C. Swichart, Ann. of Phys., 1, 168 (1957).
[29]A. Capella, Nuovo Cim., 42, 321 (1966).
[30]R. Courant, Partial Differential Equations (Russ. Transl. Mir, M. 1964).
[31]L. D. Landau, E. M. Lifshitz, Teoriya polya, Nauka, M. 1967 Classical Theory of Fields, Pergamon, 1968.

Translated by E. Saletan

Electromagnetic and scalar fields around an infinite filament and around other bare singularities of the Kasner type

S. L. Parnovskiĭ

L. D. Landau Institute of Theoretical Physics, USSR Academy of Sciences
(Submitted 26 October 1978)
Zh. Eksp. Teor. Fiz. 76, 1162–1171 (April 1979)

It is shown that the intensity of the electric field of an infinite charged filament increases to infinity with increasing distance from it, as a result of self-gravitation, and forms an inhomogeneous singularity. An object of this kind with $H_\varphi \neq 0$ and $H_z \neq 0$ cannot exist in general relativity theory. The scalar field of a filament is considered and it is shown that in contrast to the electromagnetic field such a field is capable of upsetting the oscillatory approach to bare singularities and replacing it by a power-law approach for which a formula is given. The effect of such fields on other bare singularities of the Kasner type is also investigated.

PACS numbers: 12.20. – m, 12.90. + b

1. INTRODUCTION

This paper is devoted to a study of the influence of electromagnetic and scalar fields on bare singularities of the Kasner type and primarily on the simplest among them, described by a Kasner spatial metric [Eq. (10) at $c=0$]. This metric describes the gravitational field around an infinitely long and thin filament with mass[1] (see also Ref. 2). A situation is investigated wherein this filament is a source of an electromagnetic or a scalar field. Thus, in the next section we consider the classical electrostatic problem of finding the electric field intensity around an infinite charged filament. Its solution within the framework of general relativity theory differs substantially from the result obtained in the Newtonian approximation. With increasing distance to the source, the field intensity first decreases, but later the gravitational interaction of the electric field with the filament and with itself causes the field to increase and to tend to infinity at a finite distance from the source. This distance is the limit for the given model, and a position at a greater distance from the filament is impossible. This phenomenon cannot be avoided without resorting to a source with a negative and infinite "nonrenormalized" linear mass density, a situation having hardly any physical meaning.

In Sec. 3 we consider the effect exerted on the spatial Kasner matrix by an electric or magnetic field that depends on one variable. It turns out that no object that might be described as an infinitely long and thin charged filament with finite positive linear mass density, surrounded by a magnetic field with nonzero components H_z and H_φ, can exist within the framework of general relativity.

In Sec. 5 we consider the scalar field around a linear source. For a zero-mass field, a metric is obtained at arbitrary distance from the filament, similar to that obtained by V.A. Belinskiĭ and I.M. Khalatnikov for singularities attainable on spacelike hypersurfaces in the presence of a scalar zero-mass field.[3] For a scalar field with mass, asymptotic expressions can be obtained for the metric near the singularity, where it coincides with the solution for the zero-mass field, and far from the axis, where the field attenuates exponentially and does not influence the metric. It is proved that a phenomenon analogous to the increase of the electric field intensity on account of self-gravitation does not exist for a scalar field. It is shown in the same section that in the presence of a scalar field near a bare singularity the metric cannot have an oscillatory character similar to the oscillatory regime of V.A. Belinskiĭ, E.M. Lifshitz, and I.M. Khalatnikov (BLKh). This oscillatory form can be possessed by the metric near bare singularities of general type in the absence of nongrav-

 0038-5646/79/040589-06$02.40