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Splitting and coalescence of photons on a hydrogenlike atom

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At photon energies $\omega < m$ and in certain kinematic regions near $\omega \gtrsim m$, the splitting and coalescence of photons on an atom are due to their interaction with bound electrons. The amplitude, the angular and energy distributions, and the total cross section are calculated for all these cases. The calculations are performed for a hydrogenlike atom, but the principal estimates are valid for any atom.

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1. INTRODUCTION

Splitting of a photon into two photons can occur in the Coulomb field of a nucleus, and also when photons interact with bound electrons. If the final and initial states of the atom coincide, then the amplitude of the photon splitting on the atom can be represented in the form

$$F = F_0 + F_1, \tag{1}$$

where F_0 and F_1 are the nuclear and electronic parts of the amplitude. They are expressed by the Feynman diagrams shown in Figs. 1a and 1b, with all possible permutations of the photons. All the foregoing applies also to coalescence of two photons into one. These two processes will be considered together.

The splitting of photons in the field of a nucleus was first considered more than 40 years ago by Williams.[1] An exact expression for the amplitude of this process was derived by Shima.[2] Constantini *et al.*[3] investigated both the splitting and the coalescence of photons in the field of a nucleus. Various limiting cases were also investigated.[4-10]

In this paper we consider splitting and coalescence of photons on an atom. We calculate the amplitude and the cross sections for all the cases in which the main contribution to the amplitude is made by its electronic part F_1. It will be shown that if all the photon energies $\omega_i \ll m$ (m is the electron mass; $\hbar = c = 1$), then $F_0 \ll F_1$. At $\omega_i \gtrsim m$ the total splitting cross section is determined by the "nuclear" term F_0 of the amplitude. In certain kinematic regions, however, the amplitude F_1 makes the main contribution to the differential cross section of the splitting or to the total cross section of the coalescence of the photons.

All the calculations are made for the ground state of a hydrogenlike atom, but the main estimates are valid for any atom. It is assumed that the charge Z of the nucleus

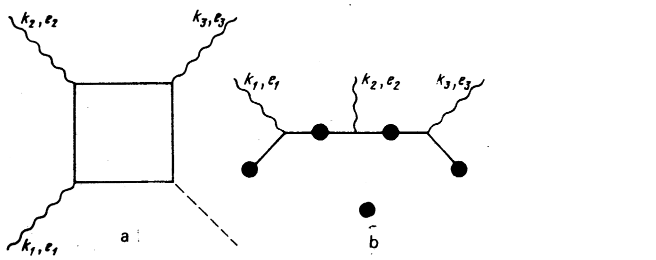

FIG. 1. Feynman diagrams of the amplitude of splitting of one photon into two and of coalescence of two photons into one on an atom. The dark blocks denote the Coulomb field. The dashed line corresponds to the Fourier component of the Coulomb field.

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satisfies the condition $\alpha Z \ll 1$.

We denote the energies (momenta) of the photons by $\omega_i(\mathbf{k}_i)$, and the momentum transferred to the nucleus by $\mathbf{q}$. Then

$$\omega_1=\omega_2+\omega_3, \quad \mathbf{q}=\mathbf{k}_1-\mathbf{k}_2-\mathbf{k}_3. \tag{2}$$

The average momentum of the bound electron is $\eta=m\alpha Z$, the energy of the ground state is $I=\eta^2/2m$. The dependence of the amplitude F_0 on the variables ω_i and q (Refs. 3, 11, 12) at $\omega_i \lesssim m$ is given by

$$F_0\sim\omega_1\omega_2\omega_3 q\alpha^{3/2}(\alpha Z)/m^6. \tag{3}$$

Both the splitting cross section and the "rate of the coalescence reaction"[10] $d\sigma'$ can be written in the form

$$d\sigma={}^1/_4\sum|F|^2\,d\Gamma, \tag{4}$$

where $\Sigma|F|^2$ denotes that $|F|^2$ is summed over the polarizations of the photons and is summed and averaged over the polarizations of the electrons. The factor $\frac{1}{4}$ is due to the identity of the photons and to averaging over the initial polarizations. The phase volume is

$$d\Gamma=\frac{2\pi}{2\omega_1}\delta(\omega_1-\omega_2-\omega_3)\frac{d^3k_2}{2\omega_2(2\pi)^3}\frac{d^3k_3}{2\omega_3(2\pi)^3}=\frac{\omega_2\omega_3}{8\omega_1}d\omega_2\frac{d\Omega_2\,d\Omega_3}{(2\pi)^5} \tag{5}$$

for the splitting and

$$d\Gamma'=\frac{2\pi}{2\omega_2 2\omega_3}\delta(\omega_1-\omega_2-\omega_3)\frac{d^3k_1}{2\omega_1(2\pi)^3}=\frac{\omega_1}{8\omega_2\omega_3}\frac{d\Omega}{(2\pi)^2} \tag{6}$$

in the case of coalescence. For simplicity we shall refer to $d\sigma'$ as well as to $d\sigma$ as cross sections, although $d\sigma'$ has a dimensionality m^{-5}.

We consider now different energy regions.

2. PHOTON ENERGY OF THE ORDER OF THE IONIZATION ENERGY ($\omega_i \sim I$)

At photon energies on the order of the ionization energy we have $q\sim m(\alpha Z)^2$. All the electron momenta on the diagrams of Fig. 1b are of the order of the average coupling momentum $\eta=m\alpha Z$. Since the Coulomb parameters of the intermediate electrons are $\xi_i=\eta/p_i\sim 1$, all these electrons should be described by Coulomb Green's functions. The contribution of the diagram on Fig. 1b to the amplitude is

$$F_1^B=(4\pi\alpha)^{3/2}\int\frac{d^3f_1\,d^3f_2\,d^3f_3}{(2\pi)^9}\langle\Psi_{1s}|\mathbf{f}_3-\mathbf{k}_3\rangle$$
$$\times A_3^*\langle\mathbf{f}_3|G(\omega_3-I)|\mathbf{f}_2-\mathbf{k}_2\rangle A_2^*\langle\mathbf{f}_2|G(\omega_1-I)|\mathbf{f}_1\rangle A_1\langle\mathbf{f}_1-\mathbf{k}_1|\Psi_{1s}\rangle, \tag{7}$$

where G is the Coulomb Green's function, and the operator A describes the interaction of the electron and photon.

The main contribution to (7) is made by values $f_i\sim\eta$. The order of magnitude of the Coulomb functions is $\langle\Psi_{1s}|\mathbf{f}\rangle\sim\eta^{-3/2}$, $\langle\mathbf{f}|G|\mathbf{f}_i\rangle\sim m\eta^{-5}$. A process with an odd (even) number of photons is allowed for transitions with (without) change of parity. The considered process with the 1s – 1s transition is therefore forbidden and contains an additional smallness $\omega\eta^{-1}$, i.e., the operator product $A_i\sim(\alpha Z)^2\omega\eta^{-1}$.

A quantity of the same order is obtained when account is taken of the relativistic correction $i\sigma\times\mathbf{k}\cdot\mathbf{e}/2m$ to the operator A_i. In the total amplitude, however, these terms add up to zero in the lowest order of the expansion in ω/η, as can be readily verified by representing the Green's function as an eigenfunction expansion

$$G=\sum\frac{|\Psi_n\rangle\langle\Psi_n|}{E-E_n}.$$

From (3) and (7) we get

$$F_1\sim\alpha^{3/2}\frac{\omega}{m\eta^2}\sim\frac{\alpha^{3/2}}{m^2}, \quad F_0\sim\alpha^{3/2}(\alpha Z)^9\frac{1}{m^2}\ll F_1; \tag{8}$$

$$\sigma\sim\alpha r_0^2(\alpha Z)^4, \; r_0=\alpha/m; \tag{9}$$

$$\sigma'\sim\alpha r_0^2/m\eta^2. \tag{10}$$

Describing the electrons by the nonrelativistic Coulomb functions taken from Refs. 13 and 14, and adding to diagrams 1b the "seagull" diagrams from Ref. 15, i.e., diagrams with two photon lines emerging from one point, we get:

$$F_1=\frac{(4\pi\alpha)^{3/2}}{m^2}\sum_{i=1}^{6}a_i\Psi_i; \tag{11}$$

$$\Psi_1=\Psi(\omega_1,\omega_2,\omega_3),\ \Psi_2=\Psi(\omega_1,\omega_3,\omega_2),$$
$$\Psi_3=\Psi(-\omega_2,-\omega_1,\omega_3),$$
$$\Psi_4=\Psi(-\omega_2,\omega_3,-\omega_1),\ \Psi_5=\Psi(-\omega_3,-\omega_1,\omega_2),$$
$$\Psi_6=\Psi(-\omega_3,\omega_2,-\omega_1),$$
$$a_1=(\mathbf{e}_1\mathbf{n}_2)(\mathbf{e}_2^*\mathbf{e}_3^*),\ a_2=(\mathbf{e}_1\mathbf{n}_3)(\mathbf{e}_2^*\mathbf{e}_3^*),$$
$$a_3=(\mathbf{e}_2^*\mathbf{n}_1)(\mathbf{e}_3^*\mathbf{e}_1),$$
$$a_4=(\mathbf{e}_2^*\mathbf{n}_3)(\mathbf{e}_3^*\mathbf{e}_1),\ a_5=(\mathbf{e}_3^*\mathbf{n}_1)(\mathbf{e}_1\mathbf{e}_2^*),$$
$$a_6=(\mathbf{e}_3^*\mathbf{n}_2)(\mathbf{e}_1\mathbf{e}_2^*);$$

$$\Psi(\omega_1,\omega_2,\omega_3)=-32im\omega_2\eta^5\int_0^1 x\,dx\int_0^1 y\,dy\{\Phi(\omega_1,\omega_2,\omega_3)-\Phi(-\omega_1,-\omega_2,-\omega_3)\}; \tag{12}$$

$$\Phi(\omega_1,\omega_2,\omega_3)=L(\omega_3,\omega_1)+xL(\omega_2,\omega_1)+xL(\omega_2,-\omega_3)+M(\omega_1),$$
$$L(\omega_i,\omega_k)=\frac{f(\omega_i)f'(\omega_k)}{\Lambda(\omega_i)[\Lambda(\omega_i)+\Lambda'(\omega_k)]^6}, \quad f(\omega)=\left(\frac{i\eta-p}{i\eta+p}\frac{\Lambda(\omega)+p}{\Lambda(\omega)-p}\right)^{i\xi};$$
$$M(\omega_i)=-2f(\omega_i)/i\eta\Lambda(\omega_i)[\Lambda(\omega_i)+i\eta]^5, \quad \Lambda(\omega)=[2m\omega(1-x)-\eta^2]^{1/2},$$
$$p=(2m\omega-\eta^2)^{1/2},\ \xi=\eta/p=(\omega/I-1)^{-1/2},\ I=\eta^2/2m,$$
$$f'(\omega)=f(\omega,x\to y), \quad \Lambda'(\omega)=\Lambda(\omega,x\to y).$$

The energy distribution of the photons after splitting is

$$d\sigma=\frac{1}{2}\alpha r_0^2\frac{\omega_2\omega_3}{m^2}\sum_{i,j}A_{ij}\Psi_i\Psi_j d\left(\frac{\omega_2}{\omega_1}\right). \tag{13}$$

The coalescence cross section is

$$\sigma'=4\pi^2\frac{\alpha r_0^2}{m^2}\frac{\omega_1}{\omega_2\omega_3}\sum_{i,j}B_{ij}\Psi_i\Psi_j, \tag{14}$$

where

$$A_{ij}=\int\overline{a_i a_j}\frac{d\Omega_2\,d\Omega_3}{(2\pi)^2}, \quad B_{ij}=\int\overline{a_i a_j}\frac{d\Omega_1}{2\pi}.$$

Here $\overline{a_i a_j}$ denotes that $a_i a_j$ are averaged and summed over the photon polarizations. The calculations yield

$$A_{ij}=\frac{16}{9}, \quad B_{11}=B_{22}=(1+\tau^2)/3, \quad B_{33}=B_{55}=(7-\tau^2)/15,$$
$$B_{44}=B_{66}={}^2/_3(1-\tau^2), \quad B_{12}=\tau(1+\tau^2)/3, \tag{15}$$
$$B_{14}=B_{26}=B_{46}=-\tau(1-\tau^2)/3, \quad B_{35}=(3\tau^2-1)/30,$$

and the remaining A_{ij} and B_{ij} are zero.

The results of the numerical calculations of the cross sections are given in Figs. 2 and 3.

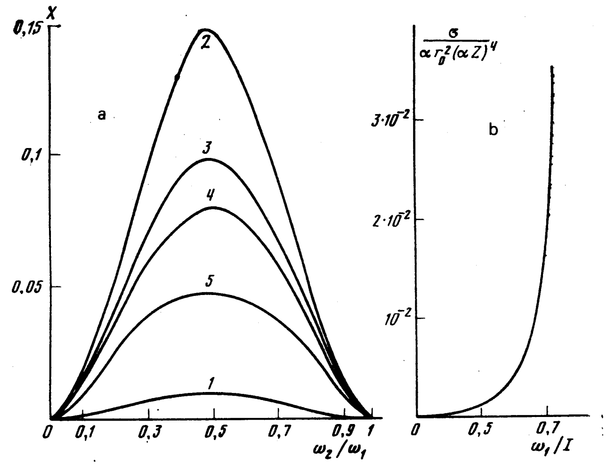

FIG. 2. a—Energy distribution of phonon splitting. Curves 1–5 corresponds to ω/I equal to 0.4, 0.5, 0.6, 0.65, and 0.7, for which n is equal to 10^3, 10^3, 2×10^2, 10, and 1;

$$X=n\frac{d\sigma}{d(\omega_2/\omega_1)}\frac{1}{\alpha r_0^2(\alpha Z)^4};$$

b—dependence of the splitting cross section on the energy ω_1.

3. THE INTERVAL $I\ll\omega_1\ll m(\omega_i\sim\eta)$

If all the ω_i are in the interval $I\ll\omega_1\ll m$, then the momenta of all the electrons are $p_i\sim(2m\omega_i)^{1/2}\gg\eta$. Therefore the interaction of the intermediate electrons with the nucleus can be taken into account by perturbation theory; the lowest-order term is the free propagator. The bound electrons are described, accurate to terms $\sim\alpha Z$, by nonrelativistic functions. The amplitude calculated with such functions and with free propagators, however, vanishes. In fact, this amplitude is proportional to the amplitude of the process without the nucleus, i.e., to the amplitude at $q=0$. But if $q=0$ the $k_1=\omega_i n$ and it is impossible to form a nonvanishing scalar out of the four vectors e_i and n. Thus the amplitude acquires an additional smallness $\sim q/m$:

$$F_1\sim\alpha^{3/2}\frac{1}{\omega_3 m}\frac{q}{m},\quad F_0\sim\alpha^{3/2}(\alpha Z)^5\frac{1}{m^2}\ll F_1. \tag{16}$$

These estimates are valid both for $\omega_3\sim\eta$ and for $\omega_3\sim I$. For the cross sections we have

$$\frac{d\sigma}{d(\omega_2/\omega_1)}\sim\alpha r_0^2(\alpha Z)^2\frac{\omega_1}{\omega_3},\qquad \sigma'\sim\alpha r_0^2(\alpha Z)^2\frac{1}{\eta^3}\left(\frac{\omega_1}{\omega_3}\right)^3. \tag{17}$$

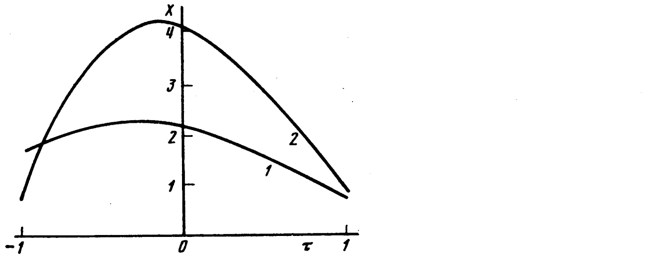

FIG. 3. Dependence of the coalescence cross section ω^1 on $\tau=(k_2\cdot k_3)/\omega_2\omega_3$ at $\omega_1/I=0.5$; $X=\sigma'(\alpha r_0^2/m\eta^2)^{-1}$. Curves 1 and 2 correspond to the cases $\omega_2/\omega_1=0.1$ and $\omega_2/\omega_1=0.5$.

At $\omega_3\ll I$ it is necessary to replace ω_3 in (16) and (17) by I, since the propagator denominator is $\sim(2m\omega_3+\eta^2)$.

We consider two cases.

a) The case $I\ll\omega_{1,2,3}\ll m$. The contribution to the terms linear in q/m is made by the relativistic corrections to the wave functions of the bound electrons and by the Coulomb corrections to the propagators. The latter have an additional smallness $\sim\alpha Z$ and need not be taken into account. The amplitude is given by the diagrams of Fig. 1b with free propagators and by the functions of the $1s$ electrons, which take corrections $\sim\alpha Z$ into account.[16] The obtained expressions for the amplitudes are

$$F=\frac{(4\pi\alpha)^{3/2}}{m^2}A\left\{\frac{(\mathbf{e}_1\mathbf{e}_2)(\mathbf{e}_3\mathbf{q})}{\omega_3}+\frac{(\mathbf{e}_1\mathbf{e}_3)(\mathbf{e}_2\mathbf{q})}{\omega_2}-\frac{(\mathbf{e}_2\mathbf{e}_3)(\mathbf{e}_1\mathbf{q})}{\omega_1}\right\}, \tag{18}$$

$$A=[\mu^2/(q^2+\mu^2)]^2,\quad \mu=2\eta; \tag{19}$$

$$\begin{aligned}\Sigma F^2=\frac{(4\pi\alpha)^3}{m^4}A^2\Bigg\{&\frac{q^2-(\mathbf{qn}_1)^2}{\omega_1^2}(1+\tau^2)+\frac{q^2-(\mathbf{qn}_2)^2}{\omega_2^2}(1+t_3^2)\\&+\frac{q^2-(\mathbf{qn}_3)^2}{\omega_3^2}(1+t_2^2)+\frac{2(\mathbf{qn}_1)(\mathbf{qn}_2)}{\omega_1\omega_2}\tau t_3\\&+2\frac{(\mathbf{qn}_1)(\mathbf{qn}_3)}{\omega_1\omega_3}\tau t_2-2\frac{(\mathbf{qn}_2)(\mathbf{qn}_3)}{\omega_2\omega_3}t_2t_3\Bigg\},\\&\mathbf{n}_i=\mathbf{k}_i/\omega_i,\ t_i=(\mathbf{n}_i\mathbf{n}_1),\ \tau=(\mathbf{n}_2\mathbf{n}_3).\end{aligned} \tag{20}$$

The results of the calculation of the cross sections are shown in Figs. 4 and 5. A simple analytic formula could be obtained only for the cross section of the coalescence of the collinear photons

$$\begin{aligned}&d\sigma=\alpha r_0^2A^2\pi^2\frac{\gamma(1-t^2)}{m^2\omega_1}\left\{\gamma^2-2\gamma+2\frac{\varkappa^2}{\omega_1^2}-2\frac{\varkappa}{\omega_1}\gamma t+\gamma^2t^2\right\},\\&\gamma=\frac{\omega_1^2}{\omega_2\omega_3}\qquad t=\frac{(\varkappa\mathbf{k}_1)}{\varkappa\omega_1},\qquad \varkappa=\mathbf{k}_2+\mathbf{k}_3;\end{aligned} \tag{21}$$

$\varkappa=\omega_2\pm\omega_3$ at $(\mathbf{k}_2\cdot\mathbf{k}_3)/\omega_2\omega_3=\pm1$ $(\omega_2>\omega_3)$. The total cross section is in this case

$$\begin{aligned}\sigma'=\frac{\alpha\pi^2r_0^2}{m^2\omega_1}\gamma(2\nu)^4\Bigg\{&\frac{1}{6}\frac{D+2B\lambda+C\lambda^2}{\lambda^2(1+\lambda)^2}-2C-\frac{(1+2\lambda)(B+C\lambda)}{\lambda(1+\lambda)}\\&+(C+2B+4C\lambda)\ln\frac{1+\lambda}{\lambda}\Bigg\};\end{aligned} \tag{22}$$

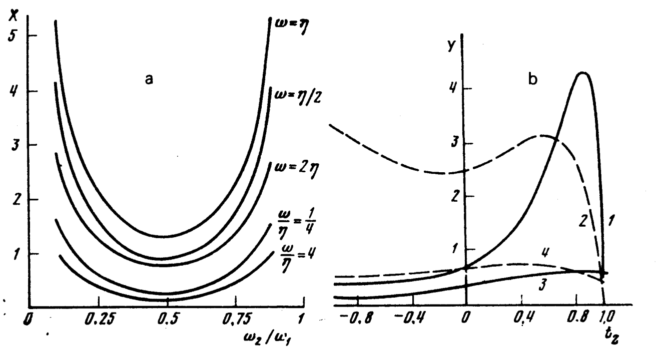

FIG. 4. Splitting of photons at $\omega_i\sim\eta$: a—energy distribution;

$$X=\frac{1}{\alpha r_0^2(\alpha Z)^2}\frac{d\sigma}{d(\omega_2/\omega_1)};$$

b—angular distribution at fixed energies. Curves 1 and 2 correspond to $\omega_2/\omega_1=0.1$ and curves 3 and 4 to $\omega_2/\omega_1=0.5$. Dashed lines—$\omega_1=\eta$; solid—$\omega_1=2\eta$;

$$Y=\frac{1}{\alpha r_0^2(\alpha Z)^2}\frac{d^2\sigma}{d(\omega_2/\omega_1)\,dt_2}.$$

$$\nu=\frac{\eta^2}{\varkappa\omega_1},\quad \lambda=\frac{(\omega_1-\varkappa)^2+4\eta^2}{4\varkappa\omega_1},\quad C=2\gamma^2,\quad B=\gamma\left(\gamma-\frac{\varkappa}{\omega_1}\right),$$
$$D=\left(\gamma-\frac{\varkappa}{\omega_1}\right)^2-\gamma\left(1-\frac{\varkappa}{\omega_1}\right).$$

At $\omega_2=\omega_3$ and $\tau=-1$, formula (22) simplifies to

$$\varkappa=0,\quad \sigma'=2^4\cdot\frac{7}{5}\frac{\pi\alpha}{m^2\omega_1}\sigma_0\left(\frac{4\eta^2}{4\eta^2+\omega_1^2}\right)^4; \tag{23}$$

$\sigma_0=(8\pi/3)r_0^2$ is the Thomson cross section.

b) The case $\omega_{1,2}\gg I$, $\omega_3\lesssim I$. If a photon of energy ω_3 is emitted by the final or initial electron, the corresponding intermediate electron should be described by a Coulomb propagator. On the other hand if this photon is emitted by the intermediate electrons, then we can disregard the interaction with the nucleus in all the propagators. Accordingly,

$$F_1=F_1'+F_1'',$$
$$F_1'=(4\pi\alpha)^{3/2}\frac{1}{\eta^2}\frac{(\mathbf{e}_3\mathbf{q})(\mathbf{e}_1\mathbf{e}_2)}{m}\Phi(\omega_3,q),$$
$$\Phi(\omega_3,q)=I(\omega_3,q)-I(-\omega_3,q),$$
$$I(\omega_3,q)=(4p)^3\int_0^{\xi}\left(\frac{1+x}{1-x}\right)^{\xi}\frac{(1-x^2)(p+x)[2p+x(p^2+1)]dx}{\{q_1^2(p+x)^2+[(p^2+1)x+2p]^2\}^3}. \tag{24}$$
$$q_1=q/\eta,\quad p=(1+\omega_3/I)^{1/2},\quad \xi=1/p,$$
$$F_1''=(4\pi\alpha)^{3/2}A[(\mathbf{e}_2\mathbf{q})(\mathbf{e}_1\mathbf{e}_3)-(\mathbf{e}_1\mathbf{q})(\mathbf{e}_2\mathbf{e}_3)]/m^2\omega_1.$$

For $\omega_3\sim I$ we have $\Phi(\omega_3,q)\sim I$, $F_1''\sim F_1'I/\omega_1\ll F_1'$. For $\omega_3\ll I$ we get $\Phi(\omega_3,q)\sim\omega_3/I$. If ω_3 is so small that $\omega_3\ll I(I/\omega_1)$, then we can neglect the contribution F_1', and $F_1=F_1''$. We obtain now the cross sections for the cases $I\gtrsim\omega_3\gg I(I/\omega_1)$ and $\omega_3\ll I/\omega_1$. In the former case, replacing the term dt_2 in the phase volume $d\Gamma$ by $dt_2=q\,dq/\omega_1^2$, where $q^2=2\omega_1^2(1-t_2)$, we have

$$\frac{d\sigma}{d\omega_3\,dq}=\alpha r_0^2\frac{2}{3}|\Phi(\omega_3,q)|^2\frac{\omega_3q^3[1+(1-q^2/2\omega_1^2)^2]}{\eta^4\omega_1^2}, \tag{25}$$
$$\frac{d\sigma'}{dq}=\alpha r_0^2\frac{\pi^2q}{2\omega_1^2\omega_3\eta^4}\left[1+\frac{q^2}{4\omega_1^2}+\tau^2\left(1-\frac{3}{4}\frac{q^2}{\omega_1^2}\right)\right]$$
$$\times\left[1+\left(1-\frac{q^2}{2\omega_1^2}\right)^2\right]|\Phi(\omega_3,q)|^2. \tag{26}$$

The calculation results are shown in Figs. 6 and 7. For the case $\omega_3\ll I(I/\omega_1)$ we have

$$\frac{d\sigma}{d\omega_3}=\alpha r_0^2\frac{\omega_3}{m^2}T(\nu),\quad \nu=\left(\frac{\eta}{\omega_1}\right)^2,$$
$$T(\nu)=(2\nu)^4\left\{\frac{2+\nu-2\nu^2}{9\nu^2(1+\nu)^3}-\frac{1+{}^4/_3\nu}{(1+\nu)^2}+\frac{2}{3}\ln\left(1+\frac{1}{\nu}\right)\right\}; \tag{27}$$

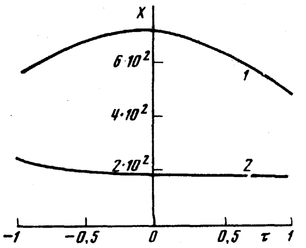

FIG. 5. Dependence of the coalescence cross section on $\tau=(\mathbf{k}_2\cdot\mathbf{k}_3)/\omega_2\omega_3$ at $\omega_1=\eta$. Curves 1 and 2 correspond to ω_2/ω_1 equal to 0.1 and 0.5. $X=\sigma'm^2\eta/\alpha r_0^2$ for curve 1 and $X=10\sigma'm\eta/\alpha r_0^2$ for curve 2.

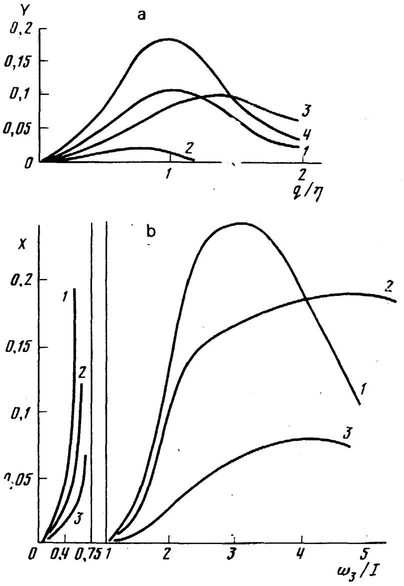

FIG. 6. Splitting of photon with energy $\omega_1\sim\eta$ into photons with energy $\omega_3=I$ and ω_2; a—doubly differential cross section at $\omega_1=\eta$, curves 1–4 show cases when ω_3/I is equal respectively to 0.6, 1.5, 2, and 3; b—energy distribution, curves 1–3 correspond to cases $\omega_1=\eta/2$, η, and 2η (curve 1 must be multiplied by 2);

$$Y=\frac{1}{\alpha r_0^2}\frac{d^2\sigma}{d(\omega_3/I)\,d(q/\eta)};\quad X=\frac{1}{\alpha r_0^2}\frac{d\sigma}{d(\omega_3/I)}$$

$$\sigma'=\frac{4\pi^2\alpha r_0^2}{m^2\omega_3}Q(\nu),\quad Q(\nu)=(2\nu)^4\left\{\frac{1+2\nu+2\nu^2}{24\nu^2(1+\nu)^2}+1-\right.$$
$$-\left(\frac{1}{2}+\nu\right)\ln\left(1+\frac{1}{\nu}\right)+3\tau^2\left[\frac{1-7\nu-10\nu^2}{72\nu^2(1+\nu)^3}\right. \tag{28}$$
$$\left.\left.-\frac{2+\nu}{6(1+\nu)^2}-1+\left(\frac{2}{3}+\nu\right)\ln\left(1+\frac{1}{\nu}\right)\right]\right\}.$$

4. ENERGIES $\omega_i\geqslant m$

If all the $\omega_i\gtrsim m$, then the cross sections σ and σ' are determined by the region of large momentum transfers $q\sim m$ to the nucleus. At these values of q, the main contribution to the amplitude is made by its "nuclear" part F_0, since $F_0\sim\alpha^{3/2}(\alpha Z)m^{-2}$, $F_1\sim\alpha^{3/2}(\alpha Z)^4m^{-2}$. With de-

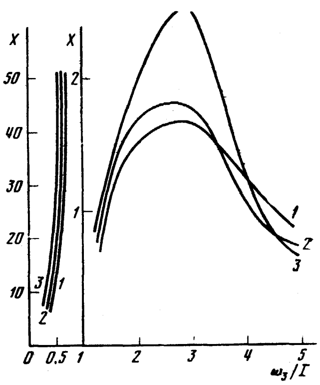

FIG. 7. Coalescence of photons with energies $\omega_3\sim I$ and $\omega_3=\eta$. Curves 1–3 correspond to the cases $\tau=0$, $\sqrt{2}/2$, and 1; $X=\sigma(\alpha r_0\eta^{-3})^{-1}$

crease in q or in one of the energies ω_i, the "nuclear" part of the amplitude F_0 decreases, and the "electronic" part F_1 increases. There exist kinematic regions where $F_0 \sim F_1$ or $F_0 \ll F_1$. Wherever $F_0 \sim F_1$, the interference between them must be taken into account; this is a separate and laborious task. We confine ourselves to an analysis of the process in those regions where $F_0 \ll F_1$.

If all $\omega_i \sim m$, then, as seen from (3) and (18), $F_0/F_1 = O(\alpha Z)$ at $q \lesssim \eta$. Therefore the cross sections $d\sigma/d\omega dq$ ($q \sim \eta$) and $d\sigma/dt$ ($1-t \sim \alpha Z$) are determined by the electronic diagrams and can be obtained with the aid of formula (2), in which we must put $t_i = 1$ and $\tau = 1$. With increasing ω_1, the momentum-transfer region where $F_0/F_1 \ll 1$ decreases, and if $\omega_1 \gg m/\alpha Z$, then $F_1 \ll F_0$ for all q.

If one of the energies is small enough ($\omega_3 \ll \omega_{1,2}$) the cross sections $d\sigma/d\omega_3$ and $\sigma'(\omega_1, \omega_2, \omega_3)$ are determined by the electron diagrams. These regions of ω_3 are respectively $\omega_3^4 \ll m^2\omega_1^2(\alpha Z)^3$ and $\omega_3^4 \ll m^2\omega_1^2(\alpha Z)^2$. If the condition $\omega_3 \gg I$ is also satisfied, then

$$\frac{d\sigma}{d(\omega_3/\omega_1)} = \frac{4}{3}\alpha r_0^2(\alpha Z)^2 \frac{\eta}{\omega_1}\lambda^3\left\{\frac{3+4\lambda^2}{1+\lambda^2} + \frac{1-4\lambda^2}{\lambda}\operatorname{arcctg}\lambda\right\}; \quad \lambda = \frac{\eta}{\omega_3}, \tag{29}$$

$$\sigma' = \frac{4}{3}\pi^2\alpha r_0^2(\alpha Z)^4\frac{1}{m^3}\left(\frac{m}{\omega_1}\right)^5\left(\frac{\omega_1}{\omega_3}\right)^3\frac{1+\tau^2+(1-\tau^2)4\beta x/(1+\beta x^2)}{(1+\beta x^2)^2},$$

$$x = 1-\tau, \quad \beta = (\omega_3/2\eta)^2. \tag{30}$$

The main contribution to (29) and (30) is made by $q \sim \eta$. If $\omega_3 \lesssim \eta$, then the main contributions from the region $q \gg \eta$ to the cross sections $d\sigma/d\omega_3$ and σ', which are corrections $\sim \alpha Z$ to (29) and (30), are made likewise by electronic diagrams, owing to the proximity of one of the intermediate electrons to the mass shell (Coulomb resonance on an electron[17,18]).

At $\omega_3 \ll I$ the valid formulas are (25)–(28), in which we must confine ourselves to the first terms of the expansion in powers of q^2/ω_1^2 and ω^2/ω_i^2.

5. DISCUSSION OF THE RESULTS

We have considered the splitting and coalescence of photons on atoms. All the calculations were made for hydrogenlike atoms, but the main estimates are valid in the general case. The amplitude of the processes consists of a nuclear part (Fig. 1a) and an electronic part (Fig. 1b). The nuclear term was investigated in many studies.[1-10] We calculated the contribution of the electron shell for all cases when it exceeds the contribution of the nuclear part of the amplitude.

The nuclear part of the amplitude can be neglected in the entire region $\omega_i \ll m$. At energies $\omega_i \sim I$ an estimate of the splitting and coalescence cross sections σ and σ', respectively, yields $\sigma \sim \alpha r_0^2(\alpha Z)^4$ and $\sigma' = \alpha r_0^2 m^{-1}\eta^{-2}$. At $\omega_i \sim \eta$ we have $\sigma \sim \alpha r_0^2(\alpha Z)^2$ and $\sigma' \sim \alpha r_0^2 m^{-2}\eta^{-1}$. At all $\omega_i \gtrsim m$ the total cross sections σ and σ' are determined by the nuclear part F_0 of the amplitude. In some kinematic regions, however, the main contribution to the amplitude is made by the electronic part F_1. At all $\omega_i \gtrsim m$ it makes the main contribution to the cross section $d\sigma/d\omega dq$ and $d\sigma'/dq$ at $q \sim \eta$. If one of the energies $\omega_3 \lesssim \eta$, then F_1 makes the main contributions to the cross sections $d\sigma/d(\omega_2/\omega_3) \sim \alpha r_0^2\omega_1\omega_3^{-1}$ and $\sigma' \sim \alpha r_0^2(\alpha Z)^4 m^2/\omega_1^2\omega_3^2$. If one of the energies $\omega_3 \sim I$ but $\omega_{1,2} \gg I$, then the amplitude is described by the diagrams of Fig. 1b, where the energy ω_3 is possessed by the "outermost" photon. When ω_3 is so small, however, that $\omega_3/I \ll I/\omega_1$, the energy ω_3 is possessed by the "middle" photon. This amplitude does not depend on ω_3, and $F \to$ const as $\omega_3 \to 0$.

The results of the numerical calculations are given in Figs. 2–7.

The differential cross section for photon splitting was measured by Adler and Cohen[19] and by Roberts and Liu[20] (see also the paper by Jarlskog *et al.*[21] and the interpretation of its results by Dzhilkibaev *et al.*[22]). Franken *et al.*[23] have reported observation of photon coalescence. Unfortunately, we can not compare our results with the experimental data, since the experiments in Refs. 19–21 were made in a kinematic region where the amplitude is determined by its nuclear part, and no quantitative results are given in Ref. 22.

We note in conclusion that at $\omega_1 \gg \eta$ the splitting and coalescence of photons on an atom, accompanied by ionization, are much more probable processes than those discussed by us. The cross sections of these processes are determined by the region $q \sim \eta$. At $q \sim \eta$ the amplitude is a product of the factor A defined in (18) and the amplitude of the double Compton effect on a free electron.[24] The boundaries of the kinematically allowed region of the latter process can be obtained from the results of Ref. 25.

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Theoretical pulse shapes for the photon (light) echo

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It is shown that the photon-echo pulse shape is governed both by the inhomogeneous broadening of the resonance level and by the parameters of the pump pulses. Short pulses of a strong external field produce a coherent echo response whose shape is wholly determined by inhomogeneous broadening. If the field is weak and the length of the first pulse is large in comparison with T^*_2, the profile of the echo is determined by the amplitude and shape of the pump pulses. In particular, if the "area" under the first pulse is less than $\pi/2$, there is a correlation between the shape of the first pulse and the echo. The results are then generalized to the case of approximate resonance.

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1. INTRODUCTION

Photon (light) echo is beginning to be widely used as a method of studying kinetic phenomena in solids and gases. It is well known that photon echo (PE) was originally discovered[1] in ruby as far back as 1964 and has since been used in this crystal to investigate relaxation processes in the case of the hyperfine interactions between chromium ions and the nuclei of aluminum atoms.[2-4] The range of materials in which the photon echo phenomenon has been observed and investigated has expanded considerably in recent years. In solids, the PE effect has been discovered, apart from $Cr^{3+}:Al_2O_3$ (Ref. 1), in $Pr^{3+}:LaF_3$ (Ref. 5), $Nd^{3+}:CaWO_4$ (Ref. 6), Nd^{3+}: YAG (Ref. 7), and so on. It has also been seen in certain organic crystals.[8,9] In gases, the PE effect has been investigated in SF_6 (Ref. 10), NH_2D (Ref. 11), $C^{13}H_3$ (Ref. 12), and SiF_4 (Ref. 13).

In most of these papers, the photon echo effect was used to determine the decay time T_2 of the PE signal, and this was then used as a source of information on the mechanism responsible for interatomic and intermolecular interactions. It is, however, important to emphasize that these characteristics are deduced from the PE pulse shape which, in general, is determined not only by the parameters of the medium (homogeneous and inhomogeneous broadening of the resonance level) but also by the shape, amplitude, and length of the pump pulses. In many cases (especially in the case of short intervals between the pump pulses), it has not been possible to establish whether the PE signal shape is, in the final analysis, determined by the resonance medium or by the envelope of the pump pulses. The solution of this problem is important not only from the point of view of determining the relaxation characteristics of the material under investigation from PE data, but also for establishing the possibility of using PE as a means of storage and subsequent reconstitution of the time structure of pump pulses.

In this paper, we develop a theory of the photon-echo pulse shape and establish criteria for the utilization of the PE effect either as a method of obtaining T_2 and T_2^* or as a method of recording, storage, and reconstitution of the envelope of the first pump pulse. A numerical experiment was used to investigate violations of these criteria, and the envelop of the PE signal under more complicated conditions was investigated.

2. BASIC ASSUMPTIONS AND INITIAL RELATIONSHIPS

The production of pulses of coherent radiation is investigated in this paper under a number of assumptions.

1. The pump pulses are plane waves propagating in the z direction with phase velocity v in a medium with constant refractive index. The slow field and polariza-

