Quasi-one-dimensional generalization of the Tomonaga-Luttinger model

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We investigate the manner whereby the ordinary "three-dimensional" properties of a system are restored with increasing probability of electron tunneling from string to string in a quasi-one-dimensional system made up of one-dimensional metallic strings with long-range interaction between the electrons (the Tomonaga-Luttinger model). In the case of weakly corrugated Fermi surfaces, modified parquet equations are obtained for the vertex. The critical temperature is calculated and the corresponding susceptibilities and the particle momentum distribution functions are obtained. The results differ from those obtained by the mean-field treatment of the transverse kinetic coupling of the strings.

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One of the basic models in the physics of one-dimensional Fermi systems is the Tomonaga-Luttinger (TL) model.[1] Because of the relative simplicity of the interaction (only long-range interaction is taken into account), it was one of the first many-particle models for which an exact solution could be obtained, and this made it possible to reveal a number of important consequences pertaining to one-dimensional Fermi systems. One of them is the special role of fluctuations of the electron density in one-dimensional systems and their quantum character. It is precisely the presence of such long-wave low-lying excitations which prevents the establishment of long-range order in a one-dimensional system. Single-particle states turn out to be unstable to decay into such collective excitations.[2] A manifestation of this effect is the absence of a Fermi step in the particle momentum distribution function.[1]

The TL model was subsequently expanded to account for short-range interaction. Considerable progress could be made in this case by using for the fermion operator the "Bose" representation introduced by Luther, Peshel, and Mattis,[3] a representation first established also within the framework of the TL model. As a result, the correlation functions[4] and the excitation spectrum[5] were calculated for a one-dimensional Fermi gas with a most general form of the interaction.

Since the experimentally investigated objects are more readily quasi-one-dimensional than one-dimensional, it is of interest to study the generalized TL model for the quasi-one-dimensional case. It was established that the long-range interaction between electrons located on different strings does not change the main results obtained within the framework of the TL one-dimensional model.[2,4,6] The critical temperature turned out to be equal to zero, as before. However, allowance for the direct or induced four-fermion interaction, in the course of which the electrons go from one string to another, is natural and can lead to a finite transition temperature. This can be a transition into a ferromagnetic state or a transition into a state of a Peierls or anti-Peierls dielectric.[6]

We focus our attention below on the quasi-one-dimensional generalization of the TL model to include the case of transverse kinetic linkage (tunneling of the electrons from string to string). In this case the system is close to being three-dimensional and strongly anisotropic, but in contrast to the latter it is still subject to the strong effects because it is nearly one-dimensional. Recent attempts of this kind of generalization were based on the "bosonization" method.[6] The transverse kinetic coupling is taken into account by the average-field method,[7] according to which the influence of the transverse motion reduces, in the macroscopic calculation of various correlators, to allowance for matched transitions of a pair of electrons or of a pair made up of an electron and a hole from one string to another. This scheme of action presupposes the presence of strong correlations between the two particles making up this pair. Strictly speaking, such particles must be paired, and their binding energy Δ should be much larger than the transverse resonance integral w ($w \ll \Delta$ is the condition of weak coupling in the transverse linkage). For two limiting cases (quasi-one-dimensional system of thick strings and a system of one-dimensional strings with extremely strong interaction on each of the strings[7]) this situation can be realized. One can also hope that for other quasi-one-dimensional systems, in which strong linkage sets in at low temperatures (regions II–IV of Fig. 2 of Ref. 4) the weak-coupling condition in the limit of w will be satisfied with respect to the transverse linkage.

In the TL model, however, the parameter $\Delta = 0$, and therefore the average-field method cannot be used here. The influence of the transverse kinetic coupling will be more appreciable since the use of the "bosonization" method at $w \neq 0$ is also doubtful.

We carry out the analysis of the present model by summing the most essential diagrams. In the case of the one-dimensional TL model this analysis yields approximation a qualitatively correct result even in the parquet. The value of the critical exponent obtained in this approximation is the first term of the expansion of the exact expression[4,8] in the interaction constant. In the case of a quasi-one-dimensional TL system, the parquet approximation modified for the case $w \neq 0$ also makes possible a detailed analysis whose result differs

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from the average-field method when it comes to transverse kinetic linkage.

1. THE MODEL

It is most convenient to carry out the investigation within the framework of the following simple model of quasi-one-dimensional metal. We represented as a system of parallel metallic strings packed in a planar lattice with a distance a between the strings. The character of the motion of the electrons along the string corresponds to the weak-coupling limit, and the motion across the strings corresponds to the strong-coupling limit, i.e.,

$$\varepsilon(p)=p_\parallel^2/2m-w\varphi(p_\perp). \tag{1}$$

The Fermi surface of such a system constitutes two corrugated planes described by the equation

$$p_\parallel=\pm p_0(p_\perp)=\pm p_0\left(1+\frac{w}{\varepsilon_F}\varphi(p_\perp)\right)^{1/2}, \quad p_0=(2m\varepsilon_F)^{1/2}. \tag{2}$$

The quantity w characterizes the amplitude of the corrugation. At $w<\varepsilon_F$ the Fermi surfaces turn out to be open. The function $\varphi(p_\perp)$ specifies the function of the corrugated Fermi surface.

With respect to $\varphi(p_\perp)$ we shall assume that $\varphi(p_\perp-q_\perp^0)=-\varphi(p_\perp)$, where $q_\perp^0=(\pi\hbar/a,\pi\hbar/a)$ is half the reciprocal-lattice vector. In this case we can expect in the system to observe strong fluctuations of the dielectric type. It is known that in a one-dimensional systems, besides the instability to electron pairing of the BCS type there always exists an instability with respect to electron-hole pairing due to the degenerate form of the electron spectrum in the one-dimensional case[11]:

$$\varepsilon(p)-\varepsilon_F=-[\varepsilon(p-q^0)-\varepsilon_F], \tag{3}$$

where $q^0=2p_0$. In a quasi-one-dimensional system the condition of dielectric instability (3) is approximate. It follows from (2) that it is satisfied accurate to quantities of order w^2/ε_F for $q^2=(2p_0,q_\perp^0)$. As a result, the corresponding phase transition appears at w higher than a definite value $w_c=(T_p\varepsilon_F)^{1/2}$, where T_p is the temperature of the phase transition calculated without suppression effect.

We investigate henceforth the behavior of the system as a function of w and of the form of $\varphi(p_\perp)$. An important factor in this case is the behavior of $\varphi(p_\perp)$ near $p_\perp=\pm\frac{1}{2}q_\perp^0$. In the vicinity of these points, the condition for the superconducting the dielectric enstabilities is satisfied. In the concrete calculations we shall use below $\varphi(p_\perp)$ in the form

$$\varphi(p_\perp)=\frac{1}{4}\left\{\cos\frac{ap_x}{\hbar}+\cos\frac{ap_y}{\hbar}\right\}. \tag{4}$$

The character of the evolution of the indicated instabilities is determined by the interaction. In the present problem, an important role is played by the interaction of electrons belonging to different sections of the Fermi surface. It is subdivided into processes with large transfer of the longitudinal momentum, of the order of $2p_0$ (the constant g_1), and a part with a small, compared with p_0, transfer of the angular momentum (the constant g_2)[9]:

$$\Gamma_{\alpha\beta\gamma\delta}=g_1\delta_{\alpha\gamma}\delta_{\beta\delta}-g_2\delta_{\alpha\delta}\delta_{\beta\gamma}. \tag{5}$$

In accordance with the definition of the TL model, to which we confine ourselves in the present article, we have $g_1=0, g_2=g\neq 0$, i.e., we are considering interaction with only a small transfer of the longitudinal momentum. The bare constant g is assumed to be different from zero only for electrons pertaining to different pieces of the Fermi surface and located in an energy strip with a width of the order of $\omega_D\ll\varepsilon_F$ near the corresponding Fermi surface.

Near the Fermi surface, the electron spectrum can be linearized with respect to the longitudinal momentum:

$$\varepsilon(p)-\varepsilon_F=v(p_\perp)(|p_\parallel|-p_0(p_\perp)), \tag{6}$$

where $v(p_\perp)=p_0(p_\perp)/m$ is the longitudinal component of the electron velocity on the Fermi surface. The function $f(p_\perp)$ characterizes the dependence of the state density and the Fermi surface. In Ref. 10 they investigated the effect of $v(p_\perp)$ in the absence of corrugation (this corresponds to a "rinsed" Fermi surface). The resultant situation is known as "fast parquet" wherein there is a fast dependence on the transverse momentum and a weak logarithmic dependence on the longitudinal momentum. In our problem the main three-dimensionality effect is the corrugation. The situation is the opposite of the fast parquet. The dependence n the transverse momentum turns out to be weaker than the logarithmic dependence on the longitudinal momentum. In (6) the dependence of v on $p_\perp$ can be neglected in terms of the small parameter ω_D/ε_F. We therefore use finally a spectrum in the form

$$\varepsilon(p)-\varepsilon_F=v(|p_\parallel|-p_0)-w\varphi(p_\perp). \tag{7}$$

The effects of suppression of the dielectric transition, which are due to the next terms of the expansion of $p_0(p_\perp)$, will be taken into account by means of the considerations advanced above.

2. PRINCIPAL LOGARITHMIC APPROXIMATION

Many properties of a system of many particles, including the presence of thermodynamic instabilities in the system, can be established by investigating the scattering amplitude. The calculation of the corresponding diagrams lead to logarithmically large terms. The appearance of diagrams that produce the highest order of the logarithm in each order of perturbation theory (the so-called parquet effects) is due to the presence in the system of two anomalous scattering channels, the Cooper and Peierls channels. The general scheme for summing diagrams of the parquet type is well known.[11,12] We shall therefore discuss only those singularities which arise when the scheme is applied to a quasi-one-dimensional system. We note first that in contrast to the one-dimensional situation, in the quasi-one-dimensional system the elementary "bricks" of which the total scattering amplitude is constructed depend on the transverse momentum. Thus, for example, the Cooper diagram (Fig. 1), after intergrating with the respective frequency

FIG. 1.

and longitudinal momentum, turns out to be

$$C=-\frac{g^2}{4\pi v\hbar}\int\frac{d^2l_\perp}{(2\pi\hbar)^2}\ln\{[[v k_\parallel+w[\varphi(k_\perp-l_\perp)-\varphi(l_\perp)]]^2-\omega^2-i0]$$
$$\times[[\omega_D-|v k_\parallel+w[\varphi(k_\perp-l_\perp)-\varphi(l_\perp)]|]^2-\omega^2-i0]^{-1}\}. \quad (8)$$

Here ω and k are the summary frequency and momentum in the Cooper channel. It is seen from (8) that in the case of a continuous function $\varphi(l_\perp)$ the expression for C can diverge if $k_\perp = 0$. With logarithmic accuracy, C can be represented in the region of small $\omega, vk_\parallel, wk_x a/\pi\hbar, wk_y a/\pi\hbar \ll \omega_D$ in the form $C = -\frac{1}{4}g^2 N_0 \xi$, where $N_0 = 2/\pi v\hbar a^2$ is the state density on the Fermi surface and

$$\xi=\ln\max\left\{\frac{|\omega|}{\omega_D};\frac{v|k_\parallel|}{\omega_D};\frac{w}{\omega_D}\left|\frac{k_x a}{\pi\hbar}\right|;\frac{w}{\omega_D}\left|\frac{k_y a}{\pi\hbar}\right|\right\}. \quad (9)$$

We have left out above the imaginary part of ξ, which is equal to $-i\pi/2$ and appears at $\xi = \ln(|\omega|/\omega_D)$. If the calculations are carried out at finite temperature, the temperature T enters in the definition of ξ under the logarithm sign in the curly brackets. A similar result takes place also for the Peierls diagram. A singularity appears in it if the transverse component of the momentum transfer turns out to be equal to $q_\perp = q_\perp^0$. Near this value it equals, with logarithmic accuracy, $\frac{1}{4}g^2 N\eta$, where

$$\eta=\ln\max\left\{\frac{|\Omega|}{\omega_D};\frac{v|q_\parallel|-2p_0|}{\omega_D};\frac{w}{\omega_D}\left\|\frac{q_x a}{\pi\hbar}\right|-1\right|;\frac{w}{\omega_D}\left\|\frac{q_y a}{\pi\hbar}\right|-1\right|;\frac{T}{\omega_D}\right\}, \quad (10)$$

Ω and q are the transferred frequency and the momentum in the Peierls channel.

The appearance of a dependence on the transverse momentum in ξ and η at sufficiently large $\xi, \eta < \beta = \ln(w/\omega_D)$ (it is assumed that $w \leq \omega_D$) is a reflection of the fact that in a quasi-one-dimensional system a singularity in any particular channel is realized only at a definite value of the transported transverse momentum, in contrast to a one-dimensional system, where the singularity takes place at an arbitrary value of the transverse momentum. As a result, in diagrams of mixed type at $\xi < \beta$ and $\eta < \beta$, the phase volume of the transverse momentum in which both instabilities are simultaneously realized will decrease at large ξ and η. We illustrate this circumstance using as an example the diagram of Fig. 2. It is a Peierls loop inserted in one of the vertices of the Cooper diagram. After integrating with respect to frequency and the longitudinal momentum, the result can be represented in the form

$$C_p=-\frac{1}{2}g^3\left(\frac{N_0}{4}\right)^2 a^4\int\frac{d^2l_1}{(2\pi\hbar)^2}\int\frac{d^2l_2}{(2\pi\hbar)^2}$$
$$\times\left\{\ln^2\frac{\tau}{\omega_D}-2\ln\frac{\tau}{\omega_D}\ln\frac{\tau}{|\omega|}-i\pi\ln\frac{\tau}{\omega_D}\right\}. \quad (11)$$

Here $\tau = \max\{\sigma, w|f|\}$, where $\sigma = \max\{|\omega|, |Q|\}$ and $f = \frac{1}{4}\{\varphi(l_1) - \varphi(l_1 + l_2 - \frac{1}{2}q_\perp^0) - \varphi(l_2)\}$. If we introduce a function $N(Z)$ defined by

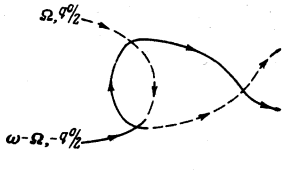

FIG. 2.

$$N(z)=a^4\int_0^{2E_b}dz'\int\frac{d^2l_1}{(2\pi\hbar)^2}\int\frac{d^2l_2}{(2\pi\hbar)^2}\delta(z'-|f|), \quad (12)$$

where $E_b = \max|f|$ [for $\varphi(p_\perp)$ in the form (4) we have $E_b = 3^{3/2}/4$], then expression (11) can be represented in the form

$$C_p=-\frac{1}{2}g^3\left(\frac{N_0}{4}\right)^2\left\{2\left(\ln\frac{|\omega|}{\omega_D}-i\frac{\pi}{2}\right)\ln\frac{wE_b}{\omega_D}\right.$$
$$\left.-\ln^2\frac{wE_b}{\omega_D}-2\left(\ln\frac{|\omega|}{wE_b}-i\frac{\pi}{2}\right)I_1\left(\frac{|\Omega|}{wE_b}\right)+I_2\left(\frac{|\Omega|}{wE_b}\right)\right\}, \quad (13)$$

where

$$I_1(x)=\int_x^1\frac{N(z)}{z}dz,\quad I_2(x)=2\int_x^1\frac{N(z)}{z}\ln z\,dz. \quad (14)$$

It is assumed in (13) that $|\omega| < |\Omega| < w$.

The first two terms in (13) can be combined with C and correspond to the corrections to the interaction constant in the Cooper channel. The next two terms describe the connection between the Cooper and the Peierls instabilities in the energy region $|\omega|, |\Omega| < w$ and should become equalized with the terms of the type $g^3(\xi-\beta)^2$ and $g^3(\eta-\beta)^2$, which appear in the same order of perturbation theory as a result of grafts of pure Peierls and Cooper type. The degree of divergence of the last two terms of (13) depends substantially on the behavior of the function $N(z)$ at $z \ll 1$, which depends in turn on the type of the function $\varphi(p_\perp)$ that describes the corrugation of the Fermi surface. By definition $N(z)$ has the meaning of the phase volume over the transverse part of the momenta, in which Cooper and Peierls instabilities are simultaneously realized, i.e., the one-dimensional result is preserved. If $N(z)$ is a slowly decreasing function of z, for example, if it tends to a certain constant limit d as $z \to 0$ (this corresponds to the presence of flat sections on the Fermi surface), then $C_p \approx dg^3(\xi-\beta)(\eta-\beta)$ and such terms must be taken into account, at not too small values of d, on a par with the terms of the type $g^3(\xi-\beta)^2$ and $g^3(\eta-\beta)^2$. If the function $N(z)$ is a rapidly decreasing function of z, so that $I_1(x)$ reaches rapidly its constant value $I_1(0)$, then such corrections can be combined with the first two terms, thus leading effectively to a renormalization of the three-dimensionality parameter w. At small $I_1(0) \ll 1$ this renormalization is of no importance and the corresponding corrections can be left out. However, when more complicated diagrams are considered, when for example, instead of one Peierls loop one inserts in the vertex of the Cooper diagram a ladder consisting of such loops, then the rapid decrease of $N(z)$ at small z can be compensated by the presence of a singularity in this ladder. Therefore allowance for this type of corrections may turn out to be substantial also in this case. Since such corrections are due to diagrams of the parquet type, their summation can also be carried out within the framework of the parquet scheme.

In the derivation of the corresponding equations we shall use below the principal ideas on which the Sudakov scheme is based. As applied to our case, the two-particle cross section in each of the diagrams is so separated that the longitudinal components of the momenta are closer to the Fermi surface than in other cross sections. The integration over the transverse part of the

momentum is in this case over the entire reciprocal lattice. As a result we obtain for the scattering amplitude the following nonlinear integral equation:

$$\gamma(\xi,\eta,\zeta)=g-\int_{\xi}^{0}dt\int_{-\infty}^{\beta}ds\,\rho(s)\gamma(t,\{t,s,\eta\},[s,\eta])\gamma(t,\{t,s,\zeta\},[s,\zeta])$$

$$+\int_{\eta}^{0}dt\int_{-\infty}^{\beta}ds\,\rho(s)\gamma(\{t,s,\xi\},t,[s,\xi])\gamma(\{t,s,\zeta\},t,[s,\zeta]). \quad (15)$$

We have changed over above to the dimensionless amplitude and to g, multiplying them by $N_0/4$. The curly and square brackets in (15) stand for $\{t,s,\eta\}=\max(t,s,\eta)$ and $[s,\eta]=\min(s,\eta)$. The logarithmic variable ζ in (15) is defined as $\zeta=[\zeta_1,\zeta_2]$, where

$$\zeta=\ln\max\left\{\frac{\varepsilon}{\omega_D};\frac{v\|p_\|\|-p_0|}{\omega_D};\frac{w}{\omega_D}\left\|\frac{p_x a}{\pi\hbar}\right|-\frac{1}{2}\Big|;\frac{w}{\omega_D}\left\|\frac{p_y a}{\pi\hbar}\right|-\frac{1}{2}\Big|;\frac{T}{\omega_D}\right\}. \quad (16)$$

Here $\varepsilon=\varepsilon_{1,2}$ and $p=p_{1,2}$ are the energy and momentum of the outer ends of the amplitude, corresponding to one of the scattered particles. We note that by virtue of the conservation laws the following equality holds: $\{\zeta_1,\zeta_2\}=\{\xi,\eta\}$.[12]

In the integration with respect to the transverse momentum in (15) we have likewise changed over to a logarithmic variable s, since the dependence on the transverse momentum in the amplitude appears under a logarithm sign [see (9) and (10)]. The substantial region of integration with respect to the transverse momentum is a circle near $l_\perp=\pm\frac{1}{2}q_\perp^0$, where the conditions for the superconducting and dielectric instabilities are simultaneously satisfied. Therefore

$$s-\beta=\tfrac{1}{3}\ln|\varphi(l_\perp)-\nabla\varphi(\tfrac{1}{2}q_\perp^0)(l_\perp-q_\perp^0)|. \quad (17)$$

We have taken into account above the fact that the term linear in $|l_\perp-q_\perp^0|$ enters in the definition of t. The function $\rho(s)$ in (15) is connected with the change of variables indicated above and is equal to

$$\rho(s)=a^2\int\frac{d^2l_\perp}{(2\pi\hbar)^2}\delta\left(\beta+\frac{1}{3}\ln\left|\varphi(l_\perp)-\nabla\varphi\left(\frac{1}{2}q_\perp^0\right)(l_\perp-q_\perp^0)\right|-s\right).$$

It will be convenient in what follows to change over to another function with the aid of the relation

$$R(s)=\int_{-\infty}^{s}ds\,\rho(s)$$

and use subsequently just this function. In analogy with the previously introduced function $N(\varepsilon)$ [see (12)], $R(s)$ characterizes the value of the phase volume with respect to the transverse part of the momenta in which the one-dimensional situation is preserved; that is to say, for electrons with energy ε, reckoned from the Fermi surface, the curvature of the Fermi surface, can be neglected in a certain region of the reciprocal lattice, amounting to a fraction $R[\ln(\varepsilon/\omega_D)]$ of its total area. The function $R(s)$ is defined essentially by the form of the corrguation of the Fermi surface. In the case of a flat Fermi surface $R(s)=1$ [the function of $\varphi(p_\perp)$ is zero practically everywhere], Eq. (15) coincides with the corresponding equations for the one-dimensional case. The transition to the one-dimensional case can also be carried out by assuming in (15) a corrugation amplitude $w=0$. If

$$\varphi(p_\perp)=\frac{1}{2}\left\{1-\left|\frac{p_x a}{\pi\hbar}\right|-\left|\frac{p_y a}{\pi\hbar}\right|\right\},$$

then the surface has flat section parallel to one another. The function R at small ε then turns out to be constant and equal to $\frac{1}{8}$. For the function $\varphi(p_\perp)$ of type (4) the essential region of integration with respect to the transverse momentum in (15) is the vicinity near $\pm\frac{1}{2}q_\perp^0$ and the corresponding phase volume, which is accumulated precisely on account of this region, decrease in power-law fashion at small ε. This corresponds to an exponential decrease in the logarithmic variables. When the normalization $R(\beta)=1$ is taken into account, the function $R(s)$ in the case of the spectrum (4)–(7) can be approximated in the following manner:

$$R(s)=\exp\{2(s-\beta)\}. \quad (18)$$

This expression is approximate in the region $s\approx\beta$, but here $R(s)\approx1$. Therefore the presence of this region reduces effectively to a redefinition of the value of the parameter w (of the corrugation amplitude).

With the aid of $R(s)$ we can rewrite (15) in the following integro-differential form

$$d\gamma(\xi,\xi)/d\xi=\gamma^2(\xi,\{\xi,\beta\})-\gamma^2(\{\xi,\beta\},\xi)$$
$$-\int_{\xi}^{(\xi,\beta)}ds\,R(s)\frac{\partial}{\partial s}\{\gamma^2(\xi,s)-\gamma^2(s,\xi)\},$$
$$\partial\gamma(\xi,\eta,\zeta)/\partial\xi=\gamma(\xi,\{\eta,\beta\},[\eta,\beta])\gamma(\xi,\{\zeta,\beta,\xi\},[\zeta,\beta])$$
$$-\int_{\xi}^{(\xi,\beta)}ds\,R(s)\frac{\partial}{\partial s}\gamma(\xi,\{s,\eta\},[s,\eta])\gamma(\xi,\{s,\zeta\},[s,\zeta]), \quad (19)$$
$$\partial\gamma(\xi,\eta,\zeta)/\partial\eta=-\gamma(\{\xi,\beta\},\eta,[\xi,\beta])\gamma(\{\zeta,\eta,\beta\},\eta,[\zeta,\beta])$$
$$+\int_{\eta}^{(\eta,\beta)}ds\,R(s)\frac{\partial}{\partial s}\gamma(\{s,\xi\},\eta,[s,\xi])\gamma(\{s,\zeta\},\eta,[s,\zeta]).$$

The last two equations pertain respectively to the regions $\xi<\eta$ and $\eta<\xi$. In (19) we have taken account of the fact that at $\eta\leqslant[\xi,\eta]$ there is no dependence of $\gamma(\xi,\eta,\zeta)$ on ζ, and therefore $\gamma(\xi,\eta,\zeta)=\gamma(\xi,\eta)$ [this can be verified by integrating (19) or (15)]. In the region $\zeta\geqslant[\xi,\eta]$, in contrast to the one-dimensional case, the function $\gamma(\xi,\eta,\zeta)$ is no longer expressed in terms of $\gamma(\xi,\eta)$ and to determine this function it is necessary to solve equations (19) with boundary conditions $\gamma(\xi,\eta,[\xi,\eta])=\gamma(\xi,\eta)$.

If $R(s)$ is a rapidly decreasing function (no slower than $1/s$), then the terms that contain $R(s)$ in the series that can be obtained for the amplitude in g by iterating (19), will have an additional smallness. Therefore in the approximation of the principal logarithm (which in our case corresponds to summation of the terms that give in each order of perturbation theory the highest powers of β, ξ, and η), we must put $R(s)$ equal to zero. As seen from (19), the total amplitude in the region $\xi\leqslant\beta,\eta\leqslant\beta$ takes the form of two ladders corresponding to independent summation of diagrams of the Cooper and of the Peierls type. Allowance for $R(s)$, meaning a departure beyond the framework of the principal logarithmic approximation, makes it possible to describe smoothly the manner in which the parquet degenerates into a sum of two ladders when the correlation between the fluctuations of the Cooper and Peierls types weakens with decreasing temperature. In the general case, since $R(s)=0$ only as $s\to-\infty$, and $R(\beta)=1$, we can expect this to lead to a substantial change of the results obtained at $R(s)=0$. For our model, however, allowance for $R(s)$

is not important, even in the case of a weakly decreasing function $R(s)$. To verify this, we approximate $R(s)$ in the following manner:

$$R(s)=\begin{cases}1, & s\geqslant\beta\\ \mu, & s<\beta\end{cases}. \tag{20}$$

Replacement of $R(s)$ in (19) by a constant is valid near the singularity of the amplitude, and consequently, the present approach makes it possible to investigate the behavior of the amplitude near the critical point. The parameter μ can be assumed to be of the order of $\mu=R(\xi_c)=1$, where ξ_c is the position of the amplitude singularity calculated with allowance for (20). The system (19) with $R(s)$ taken from (20) is now rewritten in the form

$$\begin{gathered}\partial\gamma(\xi,\eta,\zeta)/\partial\xi=(1-\mu)\gamma(\xi,\{\eta,\beta\},[\eta,\beta])\\ \times\gamma(\xi,\{\xi,\beta,\xi\},[\xi,\beta])+\mu\gamma(\xi,\eta)\gamma(\xi,\{\xi,\zeta\}),\\ d\gamma(\xi,\xi)/d\xi=(1-\mu)[\gamma^2(\xi,\{\xi,\beta\})-\gamma^2(\{\xi,\beta\},\xi)],\\ \partial\gamma(\xi,\eta,\zeta)/\partial\eta=-(1-\mu)\gamma(\{\xi,\beta\},\eta,[\xi,\beta])\gamma(\{\zeta,\beta,\eta\},\eta,[\zeta,\beta])\\ -\mu\gamma(\xi,\eta)\gamma(\{\eta,\xi\},\eta).\end{gathered} \tag{21}$$

It follows from (21) that $\gamma(\xi,\eta,\zeta)$ at arbitrary ξ, η, and ζ can be obtained if we know $\gamma(\xi,\xi)$ at arbitrary ξ and $\gamma(\xi,\beta)$, $\gamma(\xi,\beta,\beta)$ and $\gamma(\beta,\xi),\gamma(\beta,\xi,\beta)$ at $\xi\leqslant\beta$. To determine the last ones, in accordance with (21), we have ($\xi\leqslant\beta$)

$$\begin{gathered}\partial\gamma(\xi,\xi)/\partial\xi=(1-\mu)[\gamma^2(\xi,\beta)-\gamma^2(\beta,\xi)],\\ \partial\gamma(\xi,\beta)/\partial\xi=(1-\mu)\gamma(\xi,\beta,\beta)\gamma(\xi,\beta)+\mu\gamma(\xi,\beta)\gamma(\xi,\xi),\\ \partial\gamma(\xi,\beta,\beta)/\partial\xi=(1-\mu)\gamma^2(\xi,\beta,\beta)+\mu\gamma^2(\xi,\beta),\\ \partial\gamma(\beta,\xi)/\partial\xi=-(1-\mu)\gamma(\beta,\xi,\beta)\gamma(\beta,\xi)-\mu\gamma(\beta,\xi)\gamma(\xi,\xi),\\ \partial\gamma(\beta,\xi,\beta)/\partial\xi=-(1-\mu)\gamma^2(\beta,\xi,\beta)-\mu\gamma^2(\beta,\xi).\end{gathered} \tag{22}$$

In the region $\xi\leqslant\beta$ according to (21) we have $\gamma(\xi,\xi)=g$ and consequently the initial condition for (22) is the equality $\gamma(\beta,\beta)=\gamma(\beta,\beta,\beta)=g$.

If $g<0$, then it follows from (22) that $\gamma(\xi,\xi)$, $\gamma(\xi\beta)$, $\gamma(\xi,\beta,\beta)$ increase in absolute magnitude in the region of large ξ whereas $\gamma(\beta,\xi)$ and $\gamma(\beta,\xi,\beta)$ decrease. This result corresponds to the fact that in this model the dielectric and superconducting transitions exclude each other. Therefore the increase of fluctuations of one type leads to a suppression of fluctuations of the other type, and as a result the reaction turns out to be weakened. On the basis of these considerations we can leave out at $g<0$ in the right-hand side of the equation for $\gamma(\xi,\xi)$ the quantity $\gamma(\beta,\xi)$. Near the critical point, the sought amplitudes can be represented in the form

$$\begin{gathered}\gamma(\xi,\xi)=-\frac{1}{\mu}\frac{\theta(x)}{\xi-\xi_c},\quad \gamma(\xi,\beta)=-\frac{1}{[\mu(1-\mu)]^{1/2}}\frac{u^{1/2}(x)}{\xi-\xi_c},\\ \gamma(\xi,\beta,\beta)=-\frac{1}{1-\mu}\frac{1+v(x)}{\xi-\xi_c},\end{gathered} \tag{23}$$

where

$$x=\left(\ln\frac{\beta-\xi_c}{\xi-\xi_c}+\frac{1}{x_0^2}\right)^{-1/2}.$$

To determine θ, u, and v we have the following system of equations:

$$\begin{gathered}-x^3\theta'=2(u-\theta),\quad -x^3u'=u(\theta+v),\\ -x^3v'=2(v+v^2+u).\end{gathered} \tag{24}$$

From (24) we easily obtain the first terms of the asymptotic series for θ, u, and v at $x\ll1$:

$$\begin{gathered}\theta=x-x^2+(1-c)x^3+O(x^4),\\ u=x-x^2+(1/2-c)x^3+O(x^4),\quad v=-x+cx^3+O(x^4).\end{gathered} \tag{25}$$

It was assumed above that $0<\mu<1$. If we put in (22) $\mu=1$, then the system (22) coincides with the system for the one-dimensional case. On the other hand at $\mu=0$ we have for the amplitude the average-field result

$$\gamma(\xi,\beta)=\gamma(\xi,\beta,\beta)=-(\xi-\xi_c)^{-1}, \tag{26}$$

where ξ_c satisfies the equation

$$|g|(\beta-\xi_c)=1. \tag{27}$$

Thus, allowance for $R(s)$ ($\mu\neq0$) can lead to an insignificant logarithmic weakening of the pole singularity in the amplitude. Using (25) and the initial conditioning, we can obtain the constants ξ_c, x_0 and c. In the region of small μ the corresponding equation for the determination ation of ξ_c will differ from (27) in the right-hand side by an amount on the order of μ. If it is recognized that $\mu\approx R(\xi_c)=\exp[-2|g|]$, then Eq. (27) must be regarded as valid in the region of small interaction constants even when $R(s)$ is taken into account. At $g<0$ the expression for the critical temperature, which follows from (27),

$$T_{cr}=w\exp\left(-\frac{1}{|g|}\right)=T_{cr}^0\frac{w}{\omega_D},\quad T_{cr}^0=\omega_D\exp\left(-\frac{1}{|g|}\right), \tag{28}$$

corresponds to the critical temperature of the superconducting transition. The amplitude $\gamma(\xi,\beta,\beta)$ corresponds to a sum of diagrams of the Cooper type.

In the case of a repulsion interaction ($g>0$) Eq. (27) determines the position of the singularity in the Peierls channel, and (28) yields the value of the temperature of the dielectric transition. It must be borne in mind here that (28) is valid if $T_{cr}^0\leqslant w^2/\varepsilon_F$. At $w>w_c$, where

$$w_c=T_{cr}^0\frac{\varepsilon_F}{\omega_D}, \tag{29}$$

the Peierls transition is suppressed. Expression (28) was obtained under the condition that $w<w_D$. If $w>w_D$, then in the right sides of (15)–(27) we must put $\beta\to0$, and T_{cr} turns out to be T_{cr}^0, and if $T_{cr}^0\geqslant\omega_D^2/\varepsilon_F$, then $w_c=(T_{cr}^0\varepsilon_F)^{1/2}$.

Knowing the solution (22) and using (21), we can calculate the total amplitude $\gamma(\xi,\eta,\zeta)$ with the aid of which we can calculate the different susceptibilities. Thus, for example, the propagator of the fluctuation Cooper pairs is expressed in the following manner in terms of $\gamma(\xi,\eta,\zeta)$:

$$\begin{gathered}C(\xi)=-\xi-\int_t^0 dt_1\int_{-\infty}^{\beta}ds_1\int_t^0 dt_2\int_{-\infty}^{\beta}ds_2\\ \times\rho(s_1)\rho(s_2)\gamma(\xi,\{t_1,s_1,t_2,s_2\},[t_1,s_1,t_2,s_2]).\end{gathered} \tag{30}$$

At $\xi>\beta$ the behavior of $C(\xi)$ coincides with the one-dimensional case:

$$C(\xi)=\frac{1}{2|g|}\{e^{-2|g|\xi}-1\}. \tag{31}$$

Near the critical point $\xi\approx\xi_c$ we can obtain the singular part $C(\xi)$, namely

$$C(\xi)\approx\frac{e^{2g\beta}}{g^2}\frac{1}{\xi-\xi_c}\left\{1+\frac{x}{2}+O(x^2)\right\}. \tag{32}$$

As already mentioned, allowance for $R(s)$ when the latter decreases rapidly takes us outside the framework of the principal logarithmic approximation in the region $\xi,\eta>\beta$, and at small g, as we have seen, it does not lead in our physical consideration to a substantial

change of the results of the preceding approximation. Corrections of this type, however, can appear also in the region $\xi, \eta > \beta$. Here they have a purely one-dimensional nature and turn out to be essential for the determination of the critical temperature. In the next section we proceed directly to the analysis of these corrections.

3. DEPARTURE FROM THE FRAMEWORK OF THE PRINCIPAL LOGARITHMIC APPROXIMATION

The results obtained in the preceding section can be understood in the following manner; in the region of high temperatures the quasi-one-dimensional system behaves like a one-dimensional one, but at low temperatures there come into play three-dimensional effects and behavior of the system is more reminiscent of a three-dimensional one. The region of energies ($T \gg w$) in which the behavior of the system turns out to be one-dimensional was described by us in the parquet approximation, while the three-dimensional region ($T \ll w$) was described in the ladder approximation. The temperature at which the one-dimensional behavior gives way to three-dimensional is equal in this case to w.

In the present section we attempt to go outside the framework of the parquet approximation, and take into account corrections to the mass operator and the first unpaired diagrams for the vertex function. The diagram of lowest order in the interaction constant, which describes the renormalization of the Green's function, is shown in Fig. 3. After integrating with respect to frequency and to the longitudinal angular momentum, the result of the calculation of this diagram takes the form

$$\Sigma(\omega, p) = g^2 \frac{a^4}{2} \int \frac{d^2 l_1}{(2\pi\hbar)^2} \int \frac{d^2 l_2}{(2\pi\hbar)^2} (\omega - f) \ln \frac{f^2 - \omega^2 - i0}{\omega_D^2},$$

where

$$f = v(p_\parallel - p_0) + w[\varphi(l_1) - \varphi(l_2) - \varphi(l_1 + l_2 - p_\perp)], \quad p_\parallel > 0.$$

Integration with respect to transverse part of the momenta is best carried out with the aid of the function $\rho(z)$ defined by

$$\rho(z) = a^4 \int \frac{d^2 l_1}{(2\pi\hbar)^2} \int \frac{d^2 l_2}{(2\pi\hbar)^2} \delta(z + \varphi(l_2) - \varphi(l_1) + \varphi(l_1 + l_2 - p_\perp)).$$

The expression for $\sum(\omega, p)$ then takes the form

$$\Sigma(\omega, p) = g^2 \Big\{ (\omega - y) \ln \frac{w}{\omega_D} + \int_{E_2}^{E_1} dz\, \rho(z)\, (\omega - y - wz) \ln \Big| z + \frac{y}{w} \Big| + \int_{(-y-|\omega|)/w}^{(-y+|\omega|)/w} dz\, \rho(z)\, (\omega - y - wz) \Big[\ln \frac{\omega}{w} - i\frac{\pi}{2} - \ln \Big| z + \frac{y}{w} \Big| \Big] \Big\}, \quad (33)$$

where $y = v(p_\parallel - p_0)$, and E_1 and E_2 are the limiting values of c, between which the function $\rho(z)$ is different from zero. We note that $\sum(\omega, p)$ depends on the transverse momentum only via $\rho(z)$. It was assumed above that $|y|, |\omega| \ll wE_1 \approx wE_2$. In the case of the opposite relation we have, with logarithmic accuracy,

$$\Sigma(\omega, p) = g^2 (\omega - y) \Big\{ \ln \frac{\max(|\omega|, |y|)}{\omega_D} - i\frac{\pi}{2} \theta(|\omega| - |y|) \Big\}.$$

This result, which corresponds to the one-dimensional case, also follows from (33) if we put $\rho(z) = \delta(z)$. For $\varphi(p_\perp)$ in the form (4) (this is precisely the case considered from now on) $\rho(z)$ has at zero a weak singularity ($z^{-1/3}$), and therefore the first term in (33) constitutes the main contribution to the mass operator. For example,

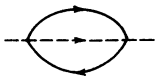

FIG. 3.

$$\Sigma\Big(\omega, p = \frac{1}{2} q^0\Big) = g^2 \omega \Big\{ \ln \frac{wE_b}{\omega} - i\frac{\pi}{2} N\Big(\frac{|\omega|}{wE_b}\Big) - I_1\Big(\frac{\omega}{wE_b}\Big) \Big\},$$

where

$$N(z) = 2 \int_0^{zE_b} dz'\, \rho(z'), \quad I_1(x) = \int_x^1 \frac{dz}{z} N(z), \quad E_1 = -E_2 = E_b = \frac{3^{3/4}}{4}$$

[cf. (12) and (14)]. At $\omega \approx y \ll wE_b$ we have

$$\Sigma\Big(\omega, p_\parallel, \frac{1}{2} q_\perp^0\Big) = g^2 \Big\{ (\omega - y) \Big(\ln \frac{wE_b}{\omega_D} - I_1(0) \Big) - y + O\Big(\frac{|\omega||y|}{w}\Big) \Big\},$$

where $I_1(0) \approx 1.8$.

Thus, the obtained corrections constitute a renormalization of the residue at the pole of the Green's function and the values of the width of the band in the transverse w direction, and also of the regular corrections to the velocity on the Fermi surface. Leaving out the corrections of the last type, since they are small in the interaction constant, we can express the total result of the calculation of $\sum(\omega, p)$ in the form

$$\Sigma(\omega, p) = g^2 (\omega - v(|p_\parallel| - p_0)) \times \Big\{ \ln \frac{\max(|\omega|, v\|p_\parallel| - p_0|, w)}{\omega_D} - i\frac{\pi}{2} \theta(|\omega| - \max(v\|p_\parallel| - p_0|, w)) \Big\}. \quad (34)$$

A similar result is obtained also for the first non-parquet diagrams of the third order in the interaction constant. We present here the result of the calculation of the vertex function by perturbation theory up to third order in the interaction constant:

$$\Gamma = 1 + g\Big[\ln\frac{|\omega|}{\omega_D} - i\frac{\pi}{2}\Big] - g\Big[\ln\frac{|\Omega|}{\omega_D} - i\frac{\pi}{2}\Big] + g^2\Big[\ln\frac{|\omega|}{\omega_D} - i\frac{\pi}{2}\Big]^2 + g^2\Big[\ln\frac{|\Omega|}{\omega_D} - i\frac{\pi}{2}\Big]^2 - g^2\Big[\ln^2\frac{s}{\omega_D} + 2\ln\frac{s}{\omega_D}\ln\frac{|\omega|}{s} - i\pi\ln\frac{s}{\omega_D}\Big] - g^2\Big[\ln^2\frac{s}{\omega_D} + \ln\frac{|\Omega|}{s}\ln\frac{s}{\omega_D} - i\pi\ln\frac{s}{\omega_D}\Big] - 2g^2\ln\frac{s}{\omega_D}. \quad (35)$$

The external momenta of the vertex function Γ are set equal to $\pm\frac{1}{2}q^0$. In (35), s stands for $s = \max(|\varepsilon|, w)$ under the condition that $|\varepsilon| > |\omega|, |\Omega|$, where ε is the frequency transferred in the third channel.

It is seen from these calculation results that the corrections to the parquet approximation in the high-energy region coincide with those for the one-dimensional system,[9] and that at energies above a certain characteristic value they cease to operate (they turn out to be temperature-independent). It is therefore best to take these corrections into account by using the renormalization-group method previously proposed for the one-dimensional system.[9] The Green's function in accordance with the character of the obtained corrections must be represented in the form

$$G(\omega, p) = \frac{d}{G_0^{-1}(\omega, p_\parallel) + W\varphi(p_\perp)}, \quad W = wd, \quad G_0^{-1} = \omega - v(|p_\parallel| - p_0), \quad (36)$$

and then the function d is connected with the mass operator Σ in the same manner as in the case of the one-dimensional system:

$$d=[1-G_0(\omega,\ p_\parallel)\Sigma(\omega,\ p)]^{-1}. \tag{37}$$

The functional relations of the group of multiplicative renormalizations must in this case be supplemented by one more relation for w [see (39) below]

$$d\left(\frac{\omega}{\omega_D'},g'\right)=z_1 d\left(\frac{\omega}{\omega_D},g\right),\quad \Gamma\left(\frac{\omega}{\omega_D'},g'\right)=z_2^{-1}\Gamma\left(\frac{\omega}{\omega_D},g\right), \tag{38}$$

$$g'=z_1^{-2}z_2 g,\quad w'=z_1^{-1}w. \tag{39}$$

Here z_1 and z_1 are renormalization factors that depend on the size of the scale transformation, and g' is an invariant charge having the meaning of the effective interaction constant at an energy of the order of ω_D.

Generally speaking, the left- and right-hand sides of (38) and (39) contain besides ω the parameter W, which remains invariant under scale transformation. According to (36) and (39), its value is

$$W=wd\left(\frac{\omega}{\omega_D},g\right)=w'\left(\frac{\omega}{\omega_D},g\right) \tag{40}$$

and it has the meaning of the effective width of the band in the transverse direction at an energy of the order of ω. Calculating the diagrams corresponding to the first orders of perturbation theory, we have verified that in the region $\omega > W$ they do not depend on W. Extending this result to higher-order made up of the considered diagrams, we can state that in the considered approximation (second order in the renormalization-group method) the influence of the transverse motion can be neglected in the region $\omega < W$, and consequently the parameter W can be left out of (39). Taking (40) into account, we can rewrite the condition $\omega > W$ in the form $\omega > T_1$, where T_1 satisfies the equation

$$T_1=wd\left(\frac{T_1}{\omega_D};g\right)=w'\left(\frac{T_1}{\omega_D};g\right). \tag{41}$$

The quantity T_1 determines the limiting value of the energy, above which the behavior of the system turns out to be one-dimensional, and below which a transition takes place from the one-dimensional to the three-dimensional behavior. As follows from (41), it can differ from w.

Thus, in the region $\omega > T_1$ the situation turns out to be similar to the one-dimensional one. In the TL model, the invariant charge in the second order of the renormalization-group method, just as in first order (the first order of the renormalization-group method corresponds to the parquet approximation), remains equal to the unrenormalized value. The reason is that the corrections to the mass operator and the vertex function in the function of Gell-Mann and Low cancel each other. For the function d, which describes the renormalization of the residue at the pole of the Green's function, we obtain in this approximation the following expression:

$$d\left(\frac{\omega}{\omega_D};g\right)=\left(\frac{\omega}{\omega_D}\right)^{g^2} \tag{42}$$

and consequently

$$W\left(\frac{\omega}{\omega_D};g\right)=w\left(\frac{\omega}{\omega_D}\right)^{g^2},\quad T_1=w\left(\frac{w}{\omega_D}\right)^{g^2}. \tag{43}$$

The total scattering amplitude in the symmetrical point $\xi=\eta=\zeta$ is connected with $d(\xi)$ and with the invariant charge $g(\xi)$ [$\xi=\ln(|\omega|/\omega_D)$] by the relation

$$\gamma(\xi)=g(\xi)d^{-2}(\xi), \tag{44}$$

which enables us to obtain $\gamma(\xi)$ at $\omega > T_1$:

$$\gamma(\xi)=g\exp\{-2g^2\xi\}. \tag{45}$$

In the region $\omega < T_1$ or $\omega < W$ the lower-order diagrams, which contribute to the second order of the renormalization-group method, do not depend on ω and "freeze" their value at $\omega = T_1$. Generalizing this result to diagrams of higher order in g, we find that in the region $\omega < W$ the functions d and W remain constant and equal to

$$d=(w/\omega_D)^{g^2},\quad W=w(w/\omega_D)^{g^2}. \tag{46}$$

The relation (44) makes it possible to obtain the total scattering amplitude also in the region $\omega < T_1$. It must be borne in mind here that it suffices to calculate the function $g(\xi)$ of (44) in the principal-logarithm approximation, since the next-approximation diagrams do not depend on ξ. In addition, in the approximation of the principal logarithm, the parquet diagrams of the mixed type, as shown in the preceding section, are also of no importance in the region $\omega < W$. The corresponding invariant charge was in fact calculated in the preceding section in a more general form at $\xi \neq \eta$. If we use the results of the preceding section, then we have for the scattering amplitude $\gamma(\xi,\eta,\zeta)$ in the region $\xi,\eta,\zeta \leq \delta = \ln(T_1/\omega_D)$,

$$\gamma(\xi,\eta,\zeta)=\gamma(\xi,\eta)=\frac{g}{2}e^{-2g^2\delta}\left\{\frac{1-g(\eta-\delta)}{1+g(\eta-\delta)}+\frac{1+g(\xi-\delta)}{1-g(\xi-\delta)}\right\}. \tag{47}$$

We have neglected here the influence of the transition region, and have put $\mu = 0$ in (21).

The present results corresponds to the next scheme for summing the perturbation-theory series for the scattering amplitude. We carry out first a partial summation of the series terms that contain the parameter w. In the parquet approximation these are the terms $g(g\beta)^n$, and in the next higher approximation (second order in the renormalization group), these are the terms of the type $g(g\beta)^n(g^2\beta)^m$. This is followed by gathering together the rearranged series, the general term of which takes the form $\tilde{g}[\tilde{g}(\eta-\tilde{\beta})]^n$ or $\tilde{g}[\tilde{g}(\xi-\tilde{\beta})]^n$, where $\tilde{g}$ is the invariant contribution (equal to the number of normalized contribution in our model) and $\tilde{\beta}=\delta$. Within the framework of Ref. 13, the proposed approach would correspond to scheme III. There is, however, a substantial deviation from the complete scheme III. On going from the problem with two "cutoff" parameters ω_D and w to the problem with a single parameter, the renormalization of the interaction constant is accompanied also by renormalization of the lower cutoff parameter, namely $w \to T_1$. Generally speaking, if ω_D in Ref. 13 is regarded not as a formally specified cutoff parameter, but as a physical quantity to be determined in the problem (for example, the Debye frequency in the phonon subsystem), then on going from the problem with two "cutoff" parameters ε_F and ω_D to the

problem with one parameter w_D we get besides the renormalization of the interaction constant also a change in the lower parameter, i.e., $\omega_D \to \omega_D'$, which causes a softening of the phonon spectrum. It is important here that $g_1 \neq 0$. In the present model $g_1 = 0$ and there is no renormalization of ω_D.

Expression (36) can be obtained with the average-field or similar approximation from the transverse kinetic linkage. Here, however, this pertains only to the single-particle Green's function, whereas in Ref. 6 such an approximation is used to calculate two-particle correlation functions. In addition, in contrast to Ref. 6, an important role is played in the present problem by corrections to the results of the field averaged over w. The region of applicability of (36) is limited by the condition $\omega > T_1$. At energies below T_1 the influence of w reduces to a "freezing" of the one-dimensional effects. As a result, in this region the system is more reminiscent in its properties of a three-dimensional system. The critical temperature, defined as the position of the pole singularity of the amplitude with respect to ω, turns out to be[1)]

$$T_{cr} = T_{cr}^0 \left(\frac{w}{\omega_D}\right)^{1+g^2} = T_{cr}^0 \frac{T_1}{\omega_D}. \quad (48)$$

Its change compared with (28) is due to renormalization of the transverse width of the band from the value w to T_1. At $g_1 > 0$ Eq. (48) determines the temperature of the dielectric transition, and at $g_1 < 0$ it determines the temperature of the superconducting transition. In the case of the dielectric transition it must be kept in mind that (48) is valid for $w < w_c$, where w_c, as before, is obtained from (29). At $w \geq w_c$, the dielectric transition is suppressed.

It is known that in a one-dimensional Fermi system the particle momentum distribution function does not have the Fermi step. In a quasi-one-dimensional system the step is restored and is equal to

$$\Delta n = d(\omega = 0) = (w/\omega_D)^{g^2}. \quad (49)$$

The result obtained in Ref. 14 corresponds to the first two terms of the expansion in the interaction constant (49).

In the investigation of the three-dimensional region we made use of the ladder approximation, which from the point of view of the phenomenological theory of phase transitions is an approximation of the average-field type. It is therefore valid only outside a certain vicinity of the phase transition point, in which the fluctuations of the corresponding order parameter are already significant. To determine this region we can use the known formulas for the estimate of the fluctuations of the order parameter from the phenomenological Ginzburg-Landau theory for an anisotropic system. As a result we find that the region of the essential singularity near the phase-transition point extends over a temperature interval[11] ΔT_{cr}, equal to

$$\Delta T_{cr} = T_{cr} (T_{cr}/T_1)^4, \quad (50)$$

where T_c depends on w in accordance with (48). Substituting (48) we find that the fluctuation region around the transition point is quite narrow: at $w \leq \omega_D$

$$\Delta T_{cr}/T_c = (T_{cr}^0/\omega_D)^4$$

and at $w \geq \omega_D$

$$\Delta T_{cr}/T_{cr}^0 = (T_{cr}^0/w)^4,$$

and it is this which allows us to speak of a critical temperature in the entire range of variation of w.

4. CONCLUDING REMARK

We have confined ourselves above to consideration of a quasi-one-dimensional system, in which there is no short-range interaction ($g_1 = 0$), the so-called TL model. It is known that the one-dimensional TL model is equivalent to the problem considered in Ref. 12. It was shown there that in the parquet approximation the corrections to the scattering amplitude determined at a symmetrical point cancel out in each order of perturbation theory, on account of the diagrams of the mixed type [this property can be noted directly in (35)], i.e., the correlation between the fluctuations of the two types is substantial in our problem. From the foregoing analysis it follows that the fluctuations corresponding to different types of instability suppress each other. The cause of this effect is that for one sign of the interaction constant, in the absence of correlation, fluctuations of one type increase whereas the fluctuations of the other type are suppressed. As a result, even in the parquet approximation, the critical temperature in the purely one-dimensional TL model turns out to be equal to zero.

The situation is entirely different in the presence of a short-range interaction $g_1 \neq 0$. At $g_1 < 0$ the fluctuations of both the Cooper and the Peierls type can grow, and allowance for the correlation between them leads to their mutual enhancement and to the appearance, in the one-dimensional system in the parquet approximation, of a new state in which Cooper and Peierls pairing coexists.[11] However, in order to obtain in the one-dimensional case at $g_1 \neq 0$ a zero critical temperature it is necessary to go outside the framework of the parquet approximation. Therefore, in a quasi-one-dimensional generalization of the model with two types of interaction, which can be carried out within the framework of the parquet approach, we can expect new singularities[15] to appear in the behavior of the system at a function of w, for example an increase of T_c in the region of small but nonzero w.

[1)]According to the average-field method with respect to w (Ref. 6), the critical temperature for this model is equal to $T_{cr} = w\,(w/\omega_D)^{-|g|}$.

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Role of slow relaxation processes in the formation of the Kapitza jump on the boundary between a superconductor and a dielectric

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The effect of slow relaxation of excitons in a superconductor on the thermal Kapitza resistance between a superconductor and a dielectric is considered. It is shown that the presence of a hierarchy of relaxation length in the superconductor leads to the existence near the boundary of a region that is in strong disequilibrium. The observed Kapitza jump depends on the size of the energy gap. The results of the theory agree in order of magnitude with experiment.

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When heat passes through a boundary between two media, a temperature jump is produced, called a Kapitza jump. The theory of this phenomenon on the boundary between superfluid He II and a solid was constructed by Khalatnikov.[1,2] Little[3] extended this theory to the case of a contact between two bodies with different acoustic properties (densities and sound velocities).

The conclusions of the theory[1-3] reduce to the following. Only phonons transport the energy through the boundary. By virtue of the difference between the acoustic properties of the two media, the phonons have a definite probability of being reflected from the boundary. As a result, the energy flux I_ε the boundaries is connected with the temperatures in the two media on the boundary by the relation

$$I_\varepsilon = A(T_0^4 - T_1^4). \quad (1)$$

Here A is a coefficient proportional to the probability of the passage of the phonons through the boundary. In the derivation of (1) it is assumed also that the phonon distribution functions in both media remain in equilibrium up to the boundary, with temperatures T_0 and T_1.

There exist, however, experimental facts which have so far not been explained within the framework of the theory of acoustic mismatch.[1-3] One of them is that the Kapitza jump between a superconductor and a dielectric depends on the energy gap in the superconductor, which is varied in the experiments with the aid of a magnetic field (see, e.g., Refs. 4 and 5). In the superconducting state the Kapitza jump turns out to be larger than in the normal metal at the same temperature. The change in the Kapitza jump fluctuates in various experiments from a fraction of several hundreths to a factor of several times.

This fact does not agree with the theory,[1-3] since neither the sound velocity nor the coefficient of the phonon reflection from the boundary is dependent in practice on the energy gap in the superconductor. Andreev[6] and Little[7] considered the influence of the conduction electrons on the phonon transmission coefficient through the boundary. However, allowance for this mechanism does not lead to a substantial improvement of the agreement between experiment and theory.

On the other hand, the assumption that the distribution functions retain their equilibrium form all the way to the boundary in generally speaking incorrect. Equilibrium distributions set in only at distances from the boundary that are larger than the characteristic relaxation lengths. In the immediate vicinity of the boundary the distribution functions of the excitations are not in equilibrium, and their form depends on the relaxation mechanisms.

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