must know the molecular field at the impurity, namely the sum of $\chi(r)$ over the lattice, starting at least from the nearest neighbor. The oscillating character of the susceptibility at short distances makes all the quantities containing sums over the lattice highly sensitive to the discreteness and parameters of the lattice. This fact must be taken into account in the analysis of magnetic ordering.

The expression obtained by us for the nonlocal susceptibility can be used also in the study of the question of the line width of paramagnetic resonance on localized magnetic moments along with the study of the Korringa mechanism of relaxation in a superconductor.[4,10]

[1)]Here and below nonlocality is understood in the sense of spatial dispersion.
[2)]The RKKY potential in a pure superconductor was also obtained in Ref. 7, but it differs from (4.3) and does not satisfy the condition that the homogeneous susceptibility vanish at $T = 0$.
[3)]We note that it differs from the numerical solution by several percent when l_p, l_s and ω vary in a wide range.

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Fluctuation diamagnetism of "dirty" superconductors

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The diamagnetic properties of "dirty" superconducting metals above the transition temperature is systematically investigated. The dependence of the fluctuation-induced increment to the magnetization on the temperature and on the magnetic field of type-II superconductors is calculated under conditions close to surface superconductivity. It is shown that the magnetization varies strongly near $T_{c3}(H)$ and near the third critical field $H_{c3}(T)$. The conditions under which the contribution made to the magnetization by surface-type superconducting nucleation centers is larger than the contribution of the volume centers are determined.

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1. INTRODUCTION

The change produced in the diamagnetic susceptibility of a superconductor by fluctuations of the superconducting phase was observed in a large range of temperatures above T_c. Gollub *et al.*[1] observed diamagnetism due to electronic pair fluctuations in strong magnetic fields $[H \sim H_{c2}(0)]$ and at high temperatures $(T \sim 2T_{c0})$ in bulky samples. The fluctuation-induced mechanism was investigated by Schmidt[2] and by Prange[3] on the basis of the Ginzburg–Landau Theory. This theory describes correctly only the behavior of long-wave fluctuations, so that the theoretical results of Schmidt and Prange are not in good agreements with the experiments of Gollub *et al.*[1] at high temperatures and in strong magnetic fields, when the short-wave fluctuations must be correctly accounted for. Corrections for the short-wave fluctuations were introduced in the calculations of the magnetization of superconductors above T_c on the basis of the Gor'kov theory in a number of papers.[4-6] In these papers, the dependence of the magnetization on the temperature and on the applied magnetic field was determined by the procedure developed in Schmid's paper,[2] namely, by calculating the free energy of the system. The results agree well in the main with the experimental data.

In other experiments, Gollub *et al.*[7] measured the fluctuation-induced magnetization of bulky type-II superconductors under conditions close to surface superconductivity. If a stationary magnetic field is applied parallel to the sample surface, fluctuation-induced surface nu-

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cleation centers are produced in a narrow layer near the surface at a fixed magnetic field H near a temperature $T \gtrsim T_{c3}(H)$, or else at a fixed temperature near the third critical field $H \gtrsim H_{c3}(T)$, whereas the number of these centers in the interior of the sample is still small [$T_{c3}(H)$ and $H_{c3}(T)$ will be defined below]. Under these conditions it may turn out that the contribution from the surface superconducting nucleation centers to the magnitization is more substantial than from those in the volume.

The present paper is devoted to an investigation of the role of the fluctuation surface centers in the change of the diamagnetic susceptibility of "dirty" superconducting metals, and to a comparison of the results with the experimental results.[7]

2. BASIC FORMULAS

To calculate the diamagnetic susceptibility of metals we use the formula that connects the diamagnetic current density with the magnetization of the system

$$\mathbf{j}=\operatorname{rot}\mathbf{M}. \tag{1}$$

Naturally, the paramagnetism of the free electrons in the metal will not be taken into account in our calculations. We use hereafter a system of units in which $c=\hbar=k_B=1$. We express the magnetic moment per unit of the system in the form

$$\mathbf{M}=M(H)\mathbf{h}=\chi(H)\mathbf{H}, \tag{2}$$

$\mathbf{h}$ is a unit vector along the magnetic field $\mathbf{H}=\operatorname{curl}\mathbf{A}$. The vector-potential is of the form

$$\mathbf{A}=\mathbf{A}_0+\mathbf{A}_1,$$

where $\mathbf{A}_0$ is the vector potential corresponding to the applied stationary magnetic field, and $\mathbf{A}_1$ is an artificially introduced increment

$$\mathbf{A}_1=\mathbf{a}\exp\{i\mathbf{kr}\} \tag{3}$$

with a gauge $\mathbf{a}\cdot\mathbf{k}=0$. After simple transformations we obtain in an approximation linear in the small increment

$$\mathbf{j}=\chi(H)\operatorname{rot}\operatorname{rot}\mathbf{A}=\chi(H)k^2\mathbf{A}_1. \tag{4}$$

Thus, if we calculate the response of the system to a small perturbation (3), we can determine the formulas (2) and (4) the magnetization of the system.

The current density is calculated from the known formula

$$\mathbf{j}=\frac{e}{m}(\nabla_r-\nabla_{r'})G^+(x,x')|_{x=x'}+2i\frac{e}{m}\mathbf{A}(x)G^+(x,x), \tag{5}$$

G^+ is the Green's function introduced by Keldysh,[8] and $x=(\mathbf{r},t)$ is a 4-vector.

Far from the transition temperature (above T_c) and in the absence of the stationary magnetic field, we readily obtain from (5) in first order in the small perturbation

$$\mathbf{j}=-i\frac{e^2}{m^2}\int\frac{d\mathbf{p}\,d\varepsilon}{(2\pi)^4}\mathbf{p}(\mathbf{pA}_1)\left(1-\operatorname{th}\frac{\varepsilon}{2T}\right)\{G_\omega^R(\mathbf{p}_+)G_\omega^R(\mathbf{p}_-)$$
$$-G_\omega^A(\mathbf{p}_+)G_\omega^A(\mathbf{p}_-)\}-i\frac{e^2\mathbf{A}_1}{m}\int\frac{d\mathbf{p}\,d\varepsilon}{(2\pi)^4}\left(1-\operatorname{th}\frac{\varepsilon}{2T}\right)\{G_\omega^R(\mathbf{p})-G_\omega^A(\mathbf{p})\}, \tag{6}$$

where

$$\mathbf{p}_\pm=\mathbf{p}\pm\frac{\mathbf{k}}{2},\quad G_\omega^{R(A)}(\mathbf{p})=\left(\varepsilon-\frac{p^2}{2m}+\varepsilon_F\pm i\delta\right)^{-1}.$$

The integration in (6) can be easily carried out at $T=0$, and yields

$$\mathbf{j}=-\frac{e^2p_F}{12\pi^2m}k^2\mathbf{A}_1.$$

Comparing this expression with (4), we obtain the well known expression for the Landau diamagnetic susceptibility:

$$\chi_A=-e^2p_F/12\pi^2m. \tag{7}$$

The fluctuation increment to the diamagnetic susceptibility is given by the Green's function corresponding to the graph in the figure. The solid lines are the Green's functions of the electron in the normal state. The dashed line is the interaction of the electron with the external electromagnetic field, and corresponds to the expression $em^{-1}(\mathbf{p}\cdot\mathbf{A}_1)$. The wavy line is the Cooper vertex function, with corresponding retarded and advanced functions in the form

$$K_\omega^{R(A)}(q)=\nu^{-1}\left[\ln\frac{T}{T_{c0}}+\psi\left(\frac{1}{2}+\frac{Dq^2}{4\pi T}\mp\frac{i\omega}{4\pi T}\right)-\psi\left(\frac{1}{2}\right)\right]^{-1}, \tag{8}$$

$\nu=mp_F/2\pi^2$ is the electron state density, $D=f_Vl/3$ is the diffusion coefficient, l is the mean free path, and $\psi(x)$ is the logarithmic derivative of the gamma function. Expression (8) is valid in the "dirty" limit $l\ll\xi(T)$, where $\xi(T)=[\pi D/8(T_{c0}-T)]^{1/2}$ is the coherence length. From (8) we obtain the Ginzburg–Landau limit if we confine ourselves to low-energy fluctuations $\{Dq^2,\omega\}=T_{c0}$. If an inorganic dirty superconductor is located in a permanent magnetic field, it is necessary to make in (8) the substitution $q^2\to q_z^2+2eH(2n+1)$, where n is the Landau quantum number.

The current density corresponding to the graph in the figure was calculated in Ref. 9. We can write similarly[1)]

$$\mathbf{j}_\Phi=\frac{i}{2}\left(\frac{evD}{2\pi T}\right)^2\sum_{NN'}L_{NN'}^2(\partial_r-\partial_{r'})\varphi_N(\mathbf{r})\varphi_{N'}^*(\mathbf{r}')|_{r'=r}$$
$$\times\int d\mathbf{r}_0\mathbf{A}_1(\mathbf{r}_0)(\partial_{r_0'}-\partial_{r_0}^*)\varphi_N^*(\mathbf{r}_0)\varphi_{N'}(\mathbf{r}_0')|_{r_0'=r_0}$$
$$\times\int\frac{d\omega}{2\pi}\operatorname{cth}\frac{\omega}{2T}[K_\omega^R(\varepsilon_N)K_\omega^R(\varepsilon_{N'})-K_\omega^A(\varepsilon_N)K_\omega^A(\varepsilon_{N'})], \tag{9}$$

$\varphi_N(\mathbf{r})$ and ε_N are the eigenfunctions and eigenvalues of the operator

$$\partial_r^2=\left(\frac{\partial}{\partial\mathbf{r}}+2ie\mathbf{A}_0\right)^2.$$

In (9) we have

$$L_{NN'}=4\pi T\left[\psi\left(\frac{1}{2}+\frac{D\varepsilon_N}{4\pi T}\right)-\psi\left(\frac{1}{2}+\frac{D\varepsilon_{N'}}{4\pi T}\right)\right]\Big/D(\varepsilon_N-\varepsilon_{N'}). \tag{10}$$

In the Ginzburg–Landau approximation $L_{NN'}=\pi^2/2$.

In the absence of a magnetic field, the wave functions $\varphi_N(\mathbf{r})$ in (9) must be replaced by plane waves, and in the Ginzburg–Landau approximation we obtain Schmid's

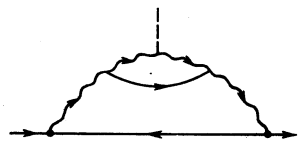

FIG. 1.

results[2] after simple calculations. For a bulky sample we have

$$\chi_\Phi^{(3)}=-\frac{e^2T_{c0}}{6\pi}\xi(T). \tag{11}$$

In the case of a thin film of thickness $d\ll\xi(T)$ we have

$$\chi_\Phi^{(3)}=-\frac{e^2T_{c0}}{3\pi d}\xi^2(T). \tag{12}$$

3. MAGNETIZATION OF BULKY SUPERCONDUCTORS IN A MAGNETIC FIELD

Assume that a stationary magnetic field $\mathbf{A}_0=(0, xH, 0)$ is applied to the sample; we then have in (9)

$$\varphi_N(\mathbf{r})=\frac{1}{2\pi(2^n n!\pi^{1/2}\lambda)^{1/2}}\exp\left\{iq_y y+iq_z z-\frac{(x+x_0)^2}{2\lambda^2}\right\}H_n\left(\frac{x+x_0}{\lambda}\right), \tag{13}$$

$$\varepsilon_N=q_z^2+2eH(2n+1), \tag{14}$$

$N=\{q_y, q_z, n\}$ is the set of Landau quantum numbers, $\lambda=(2eH)^{-1/2}$, $x_0=\lambda^2 q_y$, and $H_n(x)$ are Hermite polynomials.

If we choose the small perturbation $\mathbf{A}_1$ in the form

$$A_{1x}=A_{1z}=0, \quad A_{1y}=a\exp\{ik_z z\}$$

and substitute (13) in (9), we obtain for the current density after some transformations

$$j_\Phi=-2i\left(\frac{evD}{2\pi T}\right)^2\frac{A_{1y}}{(2\pi\lambda^2)^2}\sum_{n=1}^{\infty}n\int_{-\infty}^{+\infty}dq_z\int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}\operatorname{cth}\frac{\omega}{2T}$$

$$\times L^2_{n,q_z;n-1,q_z-k_z}[K_\omega^R(\varepsilon_{n,q_z})K_\omega^R(\varepsilon_{n-1,q_z-k_z})-K_\omega^A(\varepsilon_{n,q_z})K_\omega^A(\varepsilon_{n-1,q_z-k_z})]. \tag{15}$$

To find the magnetization we must now separate the coefficient of $k_z^2A_{1y}$ in (15). Using (2), (4), and (15) we obtain for the magnetization of the system

$$M_\Phi=\frac{4e^2H^2}{\pi}\left(\frac{evD}{2\pi T}\right)^2\sum_{n=1}^{\infty}n\int_{-\infty}^{+\infty}\frac{dq_z}{2\pi}\int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}\operatorname{cth}\frac{\omega}{2T}$$

$$\times\operatorname{Im}K_\omega^R(\varepsilon_{n,q_z})\frac{\partial^2}{\partial q^2}L^2_{n,q_z;n-1,q}K_\omega^R(\varepsilon_{n-1,q})|_{q=q_z}. \tag{16}$$

Prange's limiting case[3] can be easily obtained from (16). In fact, if we put in (16) $T=T_{c0}$ in lieu of $L_{n,qz;n',qz'}$ and $K_\omega^{R(A)}(\varepsilon_{n,qz})$, substitute their values in the Ginzburg–Landau approximation, replace $\operatorname{coth}(\omega/2T)$ under the integral sign by $2T/\omega$, and integrate with respect to ω and q_z, we obtain Prange's result

$$-\frac{\pi M_\Phi}{e^{3/2}H^{1/2}T_{c0}}=\frac{1}{2}\sum_{n=1}^{\infty}\frac{n}{(n^2-1/4)^{1/2}[(n+1/2)^{1/2}+(n-1/2)^{1/2}]^3}\approx 0.18. \tag{17}$$

Experiment[1] has shown that the combination $\pi M/e^{3/2}H^{1/2}T$ with $T=T_{c0}$ is a certain universal function of the ratio H/H^*, where H^* is some characteristic field. Such a universal dependence follows directly from (16). We introduce new symbols

$$\omega/2T=\rho, \quad (D/4\pi T)^{1/2}q_z=\mu, \quad H^*=2\gamma H_{c2}(0), \tag{18}$$

where $H_{c2}(0)=\pi T_{c0}/2\gamma eD$ is the second critical field at $T=0$, and $\gamma=1.78$. Using the symbols (18) we obtain from (16) at $T=T_{c0}$

$$\frac{\pi M_\Phi}{e^{3/2}H^{1/2}T_{c0}}=f\left(\frac{H}{H^*}\right)=\left(\frac{H}{H^*}\right)^{3/2}\sum_{n=1}^{\infty}n\iint_{-\infty}^{+\infty}\frac{d\rho d\mu}{(2\pi)^2}\operatorname{cth}\rho$$

$$\times\operatorname{Im}K_n\left(\rho,\mu;\frac{H}{H^*}\right)\frac{\partial^2}{\partial\mu_0^2}L_n^2\left(\mu,\mu_0;\frac{H}{H^*}\right)K_{n-1}\left(\rho,\mu_0;\frac{H}{H^*}\right)\Big|_{\mu_0=\mu}. \tag{19}$$

It follows from (8), (10), and (18) that

$$K_n\left(\rho,\mu;\frac{H}{H^*}\right)=\left[\psi\left(\frac{1}{2}+\mu^2+\left(n+\frac{1}{2}\right)\frac{H}{H^*}-i\frac{\rho}{2\pi}\right)-\psi\left(\frac{1}{2}\right)\right]^{-1},$$

$$L_n\left(\mu,\mu_0;\frac{H}{H^*}\right)=\left[\psi\left(\frac{1}{2}+\mu^2+\left(n+\frac{1}{2}\right)\frac{H}{H^*}\right)-\psi\left(\frac{1}{2}+\mu_0^2+\left(n-\frac{1}{2}\right)\frac{H}{H^*}\right)\right]\Big/\left(\mu^2-\mu_0^2+\frac{H}{H^*}\right).$$

The function $f(H/H^*)$ can be calculated only numerically.

We consider one more limiting case. We calculate the sample magnetization near the temperature $T\gtrsim T_{c2}(H)$, where

$$T_{c2}(H)=T_{c0}-{}^1/_4\pi eHD. \tag{20}$$

We assume also that the Ginzburg–Landau approximation is valid (the applied magnetic field and the temperature are such that the condition $eHD\ll T_{c2}(H)$, $T-T_{c2}\ll T_{c2}(H)$ is satisfied). Using the Ginzburg–Landau approximation in (16) and retaining in the sum over n only the term $n=1$ (this assumption, strictly speaking is valid in the energy interval $T-T_{c2}(H)\ll eHD$), we obtain after integration the magnetization near $T_{c2}(H)$ in the form

$$M_\Phi=-\frac{e^2HT_{c2}(H)}{2\pi}\left(\frac{\pi D}{2(T-T_{c2}(H))}\right)^{1/2}. \tag{21}$$

4. ROLE OF SURFACE FLUCTUATIONS

The method used by us to calculate the diamagnetic susceptibility is suitable also in the case of an inhomogeneous spatial distribution of the fluctuation electronic pairs. This situation arises, for example, when a stationary magnetic field is applied parallel to the sample surface.

Let the magnetic field be parallel to the y axis (yz is the sample surface plane). We choose the vector potential in the form

$$A_x=A_y=0, \quad A_z=-xH+a\exp\{ik_y y\},$$

$A_{1z}=a\exp\{ik_y y\}$ is an artificially introduced small perturbation. To calculate the response of the system to this perturbation we use formula (9), in which ε_N and $\varphi_N(r)$ stand for the eigenvalues and eigenfunctions of the operator

$$-\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}-\left(\frac{\partial}{\partial z}-2ieHx\right)^2 \tag{22}$$

with boundary conditions on the sample surface. If the surface occupies the half-space $x>0$, a variational procedure for $\varphi_N(r)$ and ε_N yields at the chosen gauge (see, e.g., Ref. 10)

$$\varphi_{q_y,q_z}(\mathbf{r})=\frac{1}{2\pi}\left(\frac{8\alpha}{\pi}\right)^{1/4}\exp\{iq_y y+iq_z z-\alpha x^2\}, \tag{23}$$

where α is the variational parameter,

$$\varepsilon_{q_y,\alpha}=2(1-2/\pi)^{1/2}eH+\varkappa(q_z-q_0)^2+q_y^2,$$
$$\varkappa=2(\pi-2)/(2\pi-3), \quad q_0=[(\pi-2)\pi]^{-1/4}(2eH)^{1/2}, \tag{24}$$
$$\alpha_0=(1-2/\pi)^{1/2}eH.$$

Substituting (23) and (24) in (9) and introducing the surface-magnetization density, we get

$$M_\Phi^s=\int_0^\infty M_\Phi(x)\,dx=2H\left(\frac{evD\varkappa}{2\pi T}\right)^2\int\frac{dq_y dq_z}{(2\pi)^2}(q_z-q_0)^2$$

$$\int\frac{d\omega}{2\pi}\operatorname{cth}\frac{\omega}{2T}\operatorname{Im}K_\omega^R(\varepsilon_{q_y,q_z})\frac{\partial^2}{\partial q^2}L^2_{q_y q_z;q,q_z}K_\omega^R(\varepsilon_{q,q_z})|_{q=q_y}. \quad (25)$$

We introduce a new notation:

$$\frac{\omega}{2T}=\rho,\quad \left(\frac{\varkappa D}{4\pi T}\right)^{1/2}(q_z-q_0)=q_z',\quad \left(\frac{D}{4\pi T}\right)^{1/2}q_y=q_y',$$
$$h=H/4\gamma H_{c3}(0),\quad \tau=T/T_{c0}, \quad (26)$$

H_{c3} is called the third critical field,

$$H_{c3}(0)=\left(1-\frac{2}{\pi}\right)^{-1/2}\frac{\pi T_{c0}}{2\gamma eD}=\left(1-\frac{2}{\pi}\right)^{-1/2}H_{c2}(0)\approx 1.7H_{c2}(0).$$

Substituting (26) in (25) and integrating over the angles of the vector (q_y', q_z') we get

$$\frac{M_\Phi^s}{eT}=\frac{\varkappa^{1/2}}{[\pi(\pi-2)]^{1/2}}\frac{h}{\tau}\int_0^\infty\beta d\beta\int_{-\infty}^\infty\frac{d\rho}{2\pi}\operatorname{cth}\rho\operatorname{Im}K\left(\rho,\beta;\frac{h}{\tau},\tau\right)$$
$$\left(2\frac{\partial}{\partial\beta_0}+\beta_0\frac{\partial^2}{\partial\beta_0^2}\right)L^2(\beta,\beta_0;\frac{h}{\tau})\,K\left(\rho,\beta_0;\frac{h}{\tau},\tau\right)\Big|_{\beta_0=\beta}, \quad (27)$$

where

$$K\left(\rho,\beta;\frac{h}{\tau},\tau\right)=\left[\ln\tau+\psi\left(\frac{1}{2}+\beta+\frac{h}{\tau}-\frac{i\rho}{2\pi}\right)-\psi\left(\frac{1}{2}\right)\right]^{-1},$$
$$L\left(\beta,\beta_0;\frac{h}{\tau}\right)=\left[\psi\left(\frac{1}{2}+\beta+\frac{h}{\tau}\right)-\psi\left(\frac{1}{2}+\beta_0+\frac{h}{\tau}\right)\right]\Big/(\beta-\beta_0). \quad (28)$$
$$\beta=q_y'^2+q_z'^2.$$

It is seen from (27) and (28) that at $T=T_{c0}$ the expression M_f^s/eT is a universal function of the ratio $H/4\gamma H_{c3}(0)$.

We consider now in greater detail the case when the magnetic field and the sample temperature are such that the conditions $eHD\ll T_{c0}$ and $T_{c0}-T\ll T_{c0}$ are satisfied. In this case we can easily find from (27) that at a fixed temperature, in magnetic fields close to $H_{c3}(T)$, we have

$$\frac{M_\Phi^s}{eT}=-\frac{\varkappa^{1/2}H}{6[\pi(\pi-2)]^{1/2}(H-H_{c3}(T))} \quad (29)$$

where $H_{c3}(T)$ is the third critical field at the given temperature:

$$H_{c3}(T)=\frac{4}{[\pi(\pi-2)]^{1/2}}\frac{T_{c0}-T}{eD}. \quad (30)$$

At a fixed value of the magnetic field and at a sample temperature $T\gtrsim T_{c3}(H)$, where

$$T_{c3}(H)=T_{c0}-{}^1/_4[\pi(\pi-2)]^{1/2}eHD, \quad (31)$$

we obtain from (27)

$$M_\Phi^s/eT_{c3}(H)=-\varkappa^{1/2}eHD/24(T-T_{c3}(H)). \quad (32)$$

The contribution of the surface fluctuations to the magnetization of type-II superconductors was experimentally observed in Ref. 7. We shall discuss this effect at the end of the article.

5. DIAMAGNETISM OF THIN FILMS IN A MAGNETIC FIELD

Let the film thickness $d\ll\xi(T)$ and $d\ll(2eH)^{-1/2}$. We assume, as before that the field is parallel to the plane of the film. The plane of the film coincides with the yz plane and occupies the region $|x|<d/2$ in space. In this case the eigenfunctions of the operator (22) are homogeneous in the x coordinate and can be expressed in the form

$$\varphi_{q_y,q_z}(\mathbf{r})=\frac{1}{2\pi d^{1/2}}\exp\{iq_y y+iq_z z\}. \quad (33)$$

The eigenvalues of the operator (22) are given by

$$\varepsilon_{q_y,q_z}=q_y^2+q_z^2+(eHd)^2/3. \quad (34)$$

Using (9), (33), and (34), we can easily show that the fluctuation magnetization of the film is of the form

$$M_\Phi^{(2)}=\frac{e^2HD}{2\pi^2 d}\int_0^\omega\beta d\beta\int_{-\infty}^{+\infty}\frac{d\rho}{2\pi}\operatorname{cth}\rho$$
$$\operatorname{Im}K(\rho,\beta;h_0^2,\tau)\left(2\frac{\partial}{\partial\beta_0}+\beta_0\frac{\partial^2}{\partial\beta_0^2}\right)L^2(\beta,\beta_0;h_0^2)K(\rho,\beta_0;h_0^2,\tau)|_{\beta_0=\beta}, \quad (35)$$

where the functions K and L were defined earlier [see (28)], and

$$h_0=H/H_0^*,\quad H_0^*=(2/ed)(3\pi T/D)^{1/2}.$$

In the Ginzburg-Landau approximation it follows from the presented expressions that the susceptibility of a thin film is

$$\chi_\Phi^{(2)}=-\frac{e^2D}{24d}\left[\frac{T-T_{c0}}{T_{c0}}+\frac{\pi^2}{2}h_0^2\right]^{-1}. \quad (36)$$

At $H=0$ this expression coincides with Schmid's result[2] [see formula (12)].

From (36) we get for the susceptibility at $T=T_{c0}$

$$\chi_\Phi^{(2)}=-\frac{T_{c0}}{\pi d^2H^2}\sim\frac{\lambda^4}{\xi_0 d^2}\chi_A. \quad (37)$$

Here $\xi_0\sim v_F/T_{c0}$, $\lambda\sim(eH)^{-1/2}$. It seems that this dependence is not very difficult to observe in experiment.

6. CONCLUSION

The influence of surface superconducting nucleation centers on the diamagnetic susceptibility of type-II superconductors was experimentally investigated by Gollub *et al.*[7] They studied the temperature dependence of the magnitization of a bulky sample of Pb+4 at% Tl in the surface-superconductivity region. A cylindrical sample several centimeters long and several millimeters in diameter was placed in a magnetic field of 54.5 Oe parallel to the cylinder axis. In the temperature interval $T_{c2}(H)<T<T_{c3}(H)$ the magnetization is subject to hysteresis that is due to the fact that the cylindrical sample is multiply connected in the surface-superconductivity region. A strong change of the magnetization was observed at temperature above $T_{c3}(H)$.

The magnetization formula (32) connected with surface fluctuations is valid strictly speaking for a flat surface, but can be used in estimates for a cylindrical sample. The magnetization per unit volume of the cylindrical sample, recalculated from (32), is of the order of

$$\bar{M}_\Phi\sim e^2HDT_{c3}(H)/R(T-T_{c3}(H)), \quad (38)$$

where R is the radius of the sample. Naturally, as $R\to\infty$ the average magnetization per unit volume, due to surface superconducting fluctuating nucleation centers, tends to zero.

The magnetization connected with volume fluctuations at a given magnetic field H is expressed above $T_{c2}(H)$ by formula (21).

Comparing (21) with (38) we can determine the tem-

perature interval near $T_{c3}(H)$, in which the contribution of the surface fluctuations to the magnetization is more substantial. From (20), (21), (31), and (38) it is seen that this temperature interval should satisfy the condition

$$(T-T_{c3}(H))/T_{c3}(H)<\xi_0 l/R\lambda. \quad (39)$$

Here $\xi_0=v_F/T_{c3}(H)\sim 10^{-4}$ cm. In the experiments of Ref. 7, $R\sim 0.1$ cm and $\lambda=(eH)^{-1/2}\sim 10^{-4}$ cm. If we put $l\sim 10^{-5}$ cm (the order of magnitude of the electron mean free path in the alloy Pb+4 at.% Tl), then the condition (39) is satisfied in the temperature interval $(T-T_{c3})/T_{c3}<10^{-4}$, which agrees with experiment[7] in order of magnitude (the data given in Ref. 7 for the surface-fluctuation diamagnetism are insufficient for a detailed comparison of our results with the experimental data.

In conclusion, the author thanks A. F. Andreev for a discussion of the results.

[1)]In Ref. 9 the author calculated the response of the system to an external perturbation that is a function of only the time coordinate. In the present case A_1 is a function of only the spatial coordinates.

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Anomalous temperature dependence of resistance of doped beryllium

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The resistance of four beryllium samples, differing greatly in respect of the residual mean free path of electrons and magnetic impurity concentration, was measured in the temperature range 2–70°K. All samples exhibited an increase in the resistance as a result of cooling with a minimum in the region between 10 and 17°K. The temperature dependences in a magnetic field were also determined. The observed anomalies were largely due to the influence of magnetic impurities.

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INTRODUCTION

We reported earlier[1] that the temperature dependence of the resistance of beryllium has a minimum and that the position of this minimum and its depth are in agreement with the theoretical estimates of Kozlov and Flerov.[2] The theoretically predicted anomalous temperature dependence[2] is due to the following reasons: at low temperatures in metals when the characteristic wavelength of thermal phonons becomes greater than the electron mean free path governed by the scattering on nonmagnetic impurities and defects, these two processes—i.e., the scattering by phonons and impurities—can no longer be regarded as independent and the phonon contribution to the resistance is no longer described by the usual Bloch–Grüneisen law $\rho\propto T^5$. The characteristic range of temperatures T where this law is replaced with a logarithmic temperature dependence of the resistance is defined by the relationships

$$T_2\ll T\ll T_1;\quad T_1=\frac{\Theta_D\hbar}{\tau\varepsilon_F}\sim c\Theta_D,\quad T_2=\frac{T_1\Theta_D}{\varepsilon_F}.$$

Here, Θ_D is the Debye temperature; $\hbar$ is the Planck constant; τ is the collisional electron lifetime; ε_F is the Fermi energy; c is the concentration of impurities and defects. The effect under consideration is most likely to be exhibited by beryllium: the exceptionally high values of the Debye temperature $\Theta_D=1416$°K (Ref. 3) and of the ratio Θ_D/ε_F (in the case of beryllium this ratio is anomalously large compared with other metals and $\varepsilon_F=0.7$ eV) make this range of temperatures not too narrow and convenient for measurements. For example, if $c\sim 1$–2%, these temperatures are ~10 °K. Moreover, the ratio Θ_D/ε_F determines the absolute value of the effect.

On the other hand, it is known that in the case of metals at low temperatures the scattering by noninteracting magnetic impurities may give rise to a logarithmic temperature dependence of the resistance, first discovered for gold:[4,5] this is known as the Kondo effect,[6] which is responsible for the resistance minimum. The sample investigated by us earlier[1] contained a small amount (~$10^{-2}\%$) of the iron impurity and this could also

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