the decay of the excited state. The potential V is attractive, so that its mean value is negative. Therefore, the energy of the slow electron is decreased and that of the fast electron is increased.

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Theory of stimulated coherent emission from atoms in spatially separate optical fields

E. V. Baklanov, B. Ya. Dubetskiĭ, and V. M. Semibalamut

Institute of Heat Physics, Siberian Branch of the Academy of Sciences of the USSR, Novosibirsk
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The problem of the interaction of a gas of atoms with spatially separate waves is considered. It is shown that the action of the field of two separate standing waves in a gas generates coherent radiation at distances which are multiples of the separation between the fields. The profile of the coherent emission line has a narrow resonance whose width is equal to the reciprocal of the transit times of the atoms between the field. An analysis is made of the influence of the quantization of the atomic motion (recoil effect) on the line profile. It is shown that in a wide range of the parameters the interaction of atoms with a standing wave can be regarded as a sudden perturbation. This makes it possible to solve the problem without iteration in respect of the field intensity. The coherent emission line profile is considered in the presence of high-power heterodyne laser radiation in the reception region. It is shown that it is identical with the profile of an absorption line of a weak plane probe wave localized in the region of formation of coherent radiation. The recoil effect splits the profile into a generally infinite series of components and under certain conditions the component at the line center is retained. This distinguishes fundamentally the resonance considered here from other nonlinear optical resonances. The results are given of numerical calculations of the line profile in the case when the spatially separate fields have Gaussian profiles. The optimal (in respect of the field intensity and pressure) conditions for the observation of the coherent emission effect are found. An estimate is obtained of this effect for a transition in CH_4 giving rise to emission at $\lambda = 3.39\mu$, for which the effect has been observed experimentally.

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§1. INTRODUCTION

If the lifetime of an atomic oscillator is sufficiently long, the dipole moment induced by an external field can be transported over long distances. In principle, this makes it possible to observe the emission from an atom in a region outside the range of action of the exciting field.

In the case of an ensemble of atoms with velocities exhibiting a scatter we can find the macroscopic polarization by averaging the dipole moment of an atom over the velocities. At considerable distances from an exciting optical field the phase of the dipole moment, considered as a function of velocity, changes (in the optical frequency range) by a very large value and, consequently, the macroscopic polarization vanishes.

However, in the case of a nonlinear interaction between atoms and optical radiation the polarization may be transported in a system with spatially separate optical fields.[1] This results in generation of coherent radiation in a region where there is no exciting field.[2] The present paper deals with the theory of such co-

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herent emission in separate fields (CESF).

The CESF effect was recently observed experimentally. Coherent radiation due to the $\lambda = 3.39\ \mu$ transition in methane was generated in spatially separate fields.[3] The temporal analog of this effect (coherent emission under the action of standing-wave pulses separated in time) was observed[4] as a result of the $\lambda = 10.6\ \mu$ transition in SF_6. New characteristics of the CESF effect are of interest in applications but this requires a more detailed theoretical analysis.

We shall consider the case when coherent radiation is formed by two separate standing waves. In this case the power of stimulated coherent radiation (CESF power), considered as a function of frequency detuning, has a narrow resonance of width of the order of the reciprocal transit time of an atom between the two fields. The polarization transport is known to give rise to narrow resonances in a system of three separate standing-wave beams.[1,5,6] These resonances are analogous to the Ramsey resonance in the rf range.[7] Such a resonance has been observed in a system with three separate fields at the $\lambda = 0.5882\ \mu$ wavelength of neon.[8]

In the theory of resonances in separate fields[5,6] the internal degrees of freedom of an atom are considered quantum-mechanically and the motion of an atom as a whole is analyzed classically. In the present paper the resonance of the CESF line profile will be considered allowing for the quantization of the motion of an atom along the direction of propagation of the wave.[1)] It is pointed out in Ref. 1 that the width of resonances in separate fields may reach 100 Hz. The quantization of the atomic motion (recoil effect) distorts a resonance in a frequency range 10^3–10^5 Hz. It follows that allowance for this quantization is essential.

We shall consider the CESF effect in spatially separate fields. In § 2 we shall discuss the physical mechanism of CESF and in § 3 we shall estimate the magnitude of the effect. A qualitative analysis of the influence of the recoil effect on the resonance line profile in separate fields is given in § 4. Evolution of the density matrix of a gas of atoms interacting with separate fields is considered in § 5 allowing for the recoil effect. The relationships obtained are sufficient to analyze any resonance in separate fields. The theory of the CESF effect is presented in §§ 6 and 7. The results of a numerical analysis and a comparison with the experimental data are given in § 9. The authors hope to consider later the CESF effect for pulses separated in time and to generalize the results of Refs. 5 and 6 by allowing for the quantization of the atomic motion.

§2. QUALITATIVE ANALYSIS OF THE MECHANISM OF POLARIZATION TRANSPORT

The appearance of macroscopic polarization in standing waves separated by large distances can be interpreted geometrically as follows (Fig. 1). For simplicity, we shall consider a beam of atoms emerging from one point of the first light beam at right-

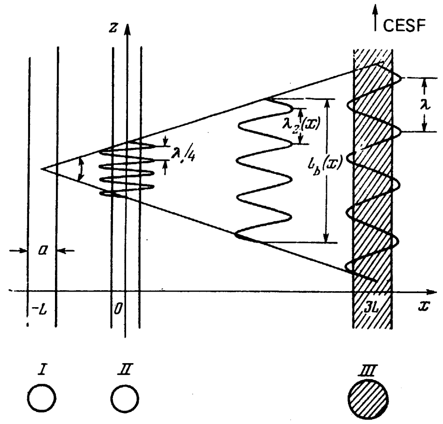

FIG. 1. Appearance of CESF at a distance of $3L$.

angles to this beam. The interaction of atoms with the standing-wave field gives rise to a resonance whose width is of the order of the reciprocal of the transit time $\tau_a = a/u$, where u is the velocity of the atoms in the atomic beam. This means that the scatter of atoms in respect of the projections of the velocity along the direction of the light propagation (z axis) is $v \sim \lambda/\tau_a$, where λ is the wavelength and the angular divergence of a beam of atoms with an induced dipole moment is $\theta \sim v/u \sim \lambda/a$. Hence, it follows that in the $x<0$ case the beam polarization is a smooth function of z with a characteristic size $\lambda L/a$, where L is the distance between the light beams. After nonlinear interaction with the standing-wave field of the second light beam the polarization acquires spatial harmonics whose period is $\lambda_m(0) = \lambda/(2m)$, where $m = 1, 2, 3, \ldots$. During the subsequent motion of the atomic beam the harmonic period increases geometrically in a similar manner:

$$\lambda_m(x) = \lambda_m(0)\,(x+L)/L. \tag{1}$$

The polarization frequency is equal to the frequency of the two separate light waves ω and the wave vector $k_m(x) = 2\pi/\lambda_m(x)$ is not generally equal to ω/c.

This means that the phase-matching condition is not satisfied and there is no stimulated emission. A polarization wave with $\lambda_m(x_m) = \lambda$ is localized only at the points $x_m = (3m-1)L$, and this gives rise to coherent emission. We shall estimate the transverse size of the region where the polarization is localized. It is governed by the condition that the polarization wavelength should be close to λ: $(\lambda_m^{-1}(x) - \lambda^{-1}) l_b(x) \lesssim 1$, where $l_b(x) = (x+L)\theta$ is the size of the region (along the z axis) occupied by the atomic beam at a distance x. Hence, we obtain $m \sim 1(x - x_m) \lesssim a$.

The transfer of the population difference between the fields gives rise to polarization harmonics of period $\lambda/(2m-1)$ and these are localized at distances $x = 2mL$. Thus, coherent emission appears in the field of strong standing waves at all distances which are multiples of L.

§3. ESTIMATE OF THE CESF POWER

Let us assume that in region I (Fig. 1) the induced polarization is of the order of $P_1 \sim \chi E_0$, where E_0 is the field amplitude in region I, χ is the susceptibility of a gas of atoms which is of the order of $\alpha\lambda$, and α is the absorption coefficient per unit length. The fraction of the total number of atoms polarized in region I which reach region III from region I is a/L because these atoms intersect light beam II. We shall assume that the field in region II is strong so that the polarization harmonics are of the order of P_1. Thus, polarization in region III is

$$P\sim(a/L)P_1\sim(a/L)\alpha\lambda E_0.$$

This polarization in region III gives rise to coherent emission of radiation whose field amplitude is $E\sim(l/\lambda)P$, where l is the size of region III along the z axis. Since the transverse size of region III is of the order of a, the emitted radiation power $W_+\sim cE^2a^2$ is described by

$$W_+\sim(a/L)^2(\alpha l)^2W_0, \tag{2}$$

where $W_0\sim cE_0^2a^2$ is the field power in region I.

Since this effect is nonlinear in relation to the field of the two standing waves, the characteristic power W_0 is found from the condition of saturation of the transition involved:

$$(d_{21}E_0\tau_0/\hbar)^2\sim d_{21}^2W_0/(c\hbar^2v_0^2)\sim1,$$

where d_{21} is the dipole matrix element of the transition between energy levels, $\tau_0=a/v_0$ is the transit time of atoms across the field, and v_0 is the thermal velocity. If we assume that $d_{21}\sim0.1D$ and $v_0\sim5\times10^4$ cm/sec, we find that $W_0\sim10^{-4}$–10^{-5} W.

For typical values of $\alpha\sim10^{-4}$ cm^{-1}, $l\sim10$ cm, $a\sim1$ cm, and $L\sim10$ cm, the radiation power W_+ is of the order of 10^{-12}–10^{-13} W. Direct recording of such weak signals is hardly possible but if in the region where signal reception takes place there is a field of a heterodyne laser of high power W' and this field is frequency detuned from the field W_0, then the detected power exhibits beats of amplitude $W\sim(W+W')^{1/2}$, which can be estimated from Eq. (2):

$$W\sim(a/L)\alpha l(W_0W')^{1/2}. \tag{3}$$

For the same parameters as before the power of the beat signal is $W\sim10^{-7}$–10^{-8} W (for $W'\sim10^{-2}$ W), which is one or two orders of magnitude higher than the sensitivity limits of the existing detectors.

It should also be pointed out that since the quantity in Eq. (3) is linear in the response of the system to the separate fields, we can analyze more fully the effect in the cases encountered experimentally. The profile of the beat signal is identical with the profile of a resonance line exhibited by the absorption of a weak probe wave in the region where the CESF signal is formed.

We shall confine our attention to the case of two-level atoms and we shall ignore the magnetic degeneracy of the sublevels. In the case of linearly polarized waves, which are in resonance with transitions between degenerate levels, the problem reduces to the interaction between the field and an ensemble of uncoupled two-level systems. Thus, the results obtained can be used to calculate the CESF effect for any specified transition. The authors hope to apply later their results to CESF in methane at the $\lambda=3.39\ \mu$ transition.

§4. CHARACTERISTICS OF THE RECOIL EFFECT IN A SYSTEM OF SEPARATE FIELDS

The recoil effect splits a nonlinear power resonance (Lamb dip) into two components (recoil doublet) which are separated on the frequency scale by twice the recoil frequency $\Delta=\hbar k^2/2M$ (Ref. 9). The effect has been observed experimentally[10,11] for the $\lambda=3.39\ \mu$ transition in methane. The appearance of a recoil doublet can be interpreted as a consequence of the splitting of the emission and absorption line profiles because of the recoil (a change in the momentum) of an atom in an elementary event of interaction with an external field. When an atom interacts with fields separated by large distances, the resonance splitting is due to a different mechanism, which we shall now consider.

A resonance whose width is the reciprocal of the transit time between separate fields appears as follows.[7] The dipole moment acquired by the atom in the first field oscillates freely in the space between the fields at the frequency of an atomic transition ω_{21} and during the transit time $T=L/u$ it acquires a phase $\omega_{21}T$. During this time the field of the second wave acquires a phase ωT, i.e., the "atom–separate fields" system is phase matched for $\Omega\lesssim T^{-1}$, where $\Omega=\omega-\omega_{21}$ is the detuning of the field frequency relative to the atomic transition frequency; this gives rise to an additional absorption resonance associated with the transport of polarization between the fields.

The need to allow for the recoil effect occurs when Δ becomes comparable with the resonance width: $\Delta\sim T^{-1}$. When the fundamental condition of the theory $a/L\ll1$ is satisfied, the recoil frequency is $\Delta\ll\tau_a^{-1}$. Since τ_a^{-1} governs the transit width of the profile of a line due to a single atom, the splitting of the absorption and emission lines can be ignored.[2)] However, an elementary event of interaction between an atom and the field alters the longitudinal velocity of the atom v by an amount $\pm\hbar k/M$, which gives rise to an additional Doppler-induced change in the phase of the dipole moment of an atom by an amount $\pm\Delta T$, i.e., allowance for the recoil has the effect that there is no phase mismatch in the system for $\Omega\sim\pm\Delta$. These values of detuning correspond to resonance maxima in separate fields and they can be resolved if $\Delta\gtrsim T^{-1}$.

We shall not use perturbation theory and we shall solve the problem for fields of arbitrary intensity. We shall consider briefly the solution method. It has been pointed out[5] that if $a/L\ll1$, then we can assume that the frequency detuning is $\Omega=0$ in the space occupied by the field and we can ignore the relaxation of atomic levels in this region. It is known (see, for example, Ref. 12) that under these conditions the problem of the interaction of an atom with a standing wave reduces to the problem of the motion of a quasiparticle in a potential $U(z)\propto\cos(kz)$, where z is the direction of propagation of the wave. In a system in which an atom is at rest the problem of the interaction with separate standing waves reduces to a set of perturbations of the motion of quasiparticles of given momentum p and these perturbations can be described by $U(z,t)=\varphi(t)\cos kz$, their duration [size $\varphi(t)$] being τ_a. If the above conditions are satisfied, such perturbations can be regarded as sudden. The usual condition is then $\omega_{mn}\tau_a\ll 1$, where m and n are quantum numbers whose values in the absence of perturbation are such that U_{mn} differs from zero. In our case the corresponding inequality $kv\tau_a\ll 1$ ($v=p/M$) is insufficient because the characteristic velocities of the particles interacting nonlinearly with a wave (the size of a Bennett dip) are $\sim(k\tau_a)^{-1}$. However, we shall show (see the Appendix) that when a perturbation mixes states p and p' with similar momenta $(p-p')\sim\hbar k\ll p \sim M/(k\tau_a)$, a much weaker condition is sufficient to simplify the problem: $(\partial^2\varepsilon_p/\partial p^2)(p-p')^2\tau_a/\hbar\ll 1$ ($\varepsilon_p=p^2/2M$ is the energy of the state p) or $\Delta\tau_a\ll 1$. We have to satisfy the equality $U_{mn}=U(m-n)$, which applies to the Hamiltonian describing the perturbation of the motion of free particles.

We shall assume the following relationship between the relevant parameters:

$$\Delta\sim T^{-1}\sim\Gamma\ll\tau_a^{-1}\ll ku\theta_0, \tag{4}$$

where Γ is the homogeneous width of an atomic emission line and θ_0 is the characteristic angle of elastic collisions between atoms [the last inequality in Eq. (4) is discussed in Footnote 5].

§5. PRINCIPAL RELATIONSHIPS

We shall consider a gas of atoms in the field of a standing wave:

$$\left.\begin{aligned} &E(\mathbf{r},t)=e^{-i\omega t}E(\mathbf{r})+\text{c.c.},\\ &E(\mathbf{r})=2E\varphi(x-L,y)\cos(kz+\alpha),\end{aligned}\right\} \tag{5}$$

where ω, E, k, and α are—respectively—the frequency, amplitude, wave vector, and phase of the wave; the z axis is directed along the direction of propagation of the wave; the function $\varphi(x,y)$ describes the dependence of the wave amplitude on the transverse coordinate; the vector $(L,0,0)$ gives the coordinates of the wave center. The Hamiltonian of a two-level atom experiencing the field (5) is[3]

$$H=\mathscr{H}_0+V,$$

$$V=\begin{pmatrix}0 & \exp(-i\Omega t)U\\ \exp(i\Omega t)U & 0\end{pmatrix},$$

$$U=-d_{21}E(\mathbf{r})\qquad \mathscr{H}_0=-\frac{1}{2M}\frac{\partial^2}{\partial\mathbf{r}^2},$$

where the upper state is

$$|2\rangle=\begin{pmatrix}1\\0\end{pmatrix},$$

whereas the lower state is

$$|1\rangle=\begin{pmatrix}0\\1\end{pmatrix},$$

d_{21} is the matrix element of the dipole moment of the 2–1 transition in an atom.[4] The equation for the density matrix of the atomic gas is

$$i\dot\rho(t)=-\frac{i}{2}\{\gamma,\rho(t)\}+[H,\rho(t)], \tag{6}$$

where

$$\{\gamma,\rho(t)\}=\gamma\rho(t)+\rho(t)\gamma,$$

$$\gamma=\frac{1}{2}\begin{pmatrix}\gamma_2 & 0\\ 0 & \gamma_1\end{pmatrix}$$

γ_i is the relaxation of the i-th level.[5] The solution (6) is sought in the form $\rho(t)=U(t)\rho U^+(t)$, where $U(t)=\exp(-i(\Omega/2)\sigma_3 t)$ and σ_3 is the Pauli matrix. We then obtain

$$\frac{i}{2}\{\gamma,\rho\}+\frac{\Omega}{2}[\sigma_3,\rho]=[\mathscr{H}_0+U\sigma_1,\rho],$$

where

$$\sigma_1=\begin{pmatrix}0&1\\1&0\end{pmatrix},\quad \sigma_3=\begin{pmatrix}1&0\\0&-1\end{pmatrix}.$$

In the absence of a field the density matrix is

$$\rho^0_{\mathbf{p},\mathbf{p}'}=\frac{1}{V}\left(\frac{2\pi}{M}\right)^3\delta_{\mathbf{p},\mathbf{p}'}W_M(\mathbf{s})\frac{1}{2}(1-\sigma_3),$$

where $\mathbf{p}=p\mathbf{n}_z+\mathbf{q}$ is the momentum of an atom; $\mathbf{q}$ is the transverse component of the momentum; $\mathbf{n}_z$ is a unit vector along the z axis; $\mathbf{s}=\mathbf{p}/M$ is the velocity of the atom; $W_M(\mathbf{s})=(\pi^{1/2}v_0)^{-3}\exp(-\mathbf{s}^2/v_0^2)$ is the Maxwellian distribution function of the atomic velocities; V is the normalization volume. Going over to the Wigner representation in respect of the transverse coordinates

$$\rho(\mathbf{r}_\perp,\mathbf{u})=\left(\frac{MV^{1/2}}{2\pi}\right)^2\sum_{\varkappa}\rho_{\mathbf{q}-\varkappa/2,\mathbf{q}+\varkappa/2}e^{-i\varkappa\mathbf{r}_\perp},$$

we obtain the following expression ignoring the recoil in the direction transverse to the wave:

$$i\left(\mathbf{u}\frac{\partial\rho}{\partial\mathbf{r}_\perp}+\frac{1}{2}\{\gamma,\rho\}\right)=[H_0+U\sigma_1-(\Omega/2)\sigma_3,\rho], \tag{7}$$

where $H_0=-(1/2M)\partial^2/\partial z^2$, and $\mathbf{u}=\mathbf{q}/M$ is the transverse velocity of the atom. We shall now go over to a coordinate system in which the atom is at rest: $\mathbf{r}_\perp=\mathbf{r}_{\perp0}+\mathbf{u}\tau$, where $\mathbf{r}_{\perp0}$ is defined by the condition $\mathbf{r}_{\perp0}\cdot\mathbf{u}=0$. We shall seek the solution of Eq. (7) in the form

$$\rho_{p,p'}(\mathbf{r}_\perp,\mathbf{u})=(2\pi/MV^{1/2})W_M(v)W_M(\mathbf{u})\langle p|e^{-iH_0\tau}\rho(\tau)e^{iH_0\tau}|p'\rangle,$$

where

$$v=p/M,\quad W_M(v)=(\pi^{1/2}v_0)^{-1}\exp(-v^2/v_0^2),$$
$$W_M(\mathbf{u})=(1/2\pi)W_M(u),$$
$$W_M(u)=(2u/v_0^2)\exp(-u^2/v_0^2),$$

where $u=|\mathbf{u}|$. Then, for atoms moving close to the x axis (the angle φ between $\mathbf{u}$ and the x axis is of the order of $a/L\ll 1$), we obtain

$$\left.\begin{aligned} &i\left(\frac{\partial\rho}{\partial\tau}+\frac{1}{2}\{\gamma,\rho\}\right)=\left[\tilde U\sigma_1-\left(\frac{1}{2}\Omega\right)\sigma_3,\rho\right],\\ &\rho(-\infty)=\tfrac{1}{2}(1-\sigma_3),\end{aligned}\right\} \tag{8}$$

where

$$\tilde U=2G\varphi(\tau-T)e^{iH_0\tau}\cos(kz+\alpha)e^{-iH_0\tau},$$
$$\varphi(\tau-T)=\varphi(x(\tau)-L,y(\tau)),\quad G=-d_{21}E.$$

In the absence of the wave field the solution of Eq. (8) is

$$\left.\begin{aligned} &\rho(\tau)=S(\tau-\tau')\rho(\tau')S^+(\tau-\tau'),\\ &S(\tau)=\exp(-\tfrac{1}{2}(\gamma-i\Omega\sigma_3)\tau);\end{aligned}\right\} \tag{9}$$

here, $\tau>\tau'$.

We shall be interested in the relationship between the density matrix before the interaction with the field $\rho(T-0)$ and after this interaction $\rho(T+0)$. Allowing for the conditions $\gamma_i \tau_a \ll 1$ and $\Omega\tau_a \ll 1$, we obtain

$$\rho(T+0)=e^{iH_0T}\Lambda_G e^{-iH_0T}\rho(T-0)\,e^{iH_0T}\Lambda_G^+ e^{-iH_0T}, \tag{10}$$

where the "perturbation matrix" of the system is

$$\left.\begin{aligned}&\Lambda_G=T\exp\left[-iG\sigma_1\int_{-\infty}^{\infty}d\tau\,U(\tau)\right],\\&U(\tau)=2\varphi(\tau)\,e^{iH_0\tau}\cos(kz+\alpha)\,e^{-iH_0\tau}.\end{aligned}\right\} \tag{11}$$

It is clear from Eq. (A.3) that the commutator $[U(\tau), U(\tau')]$ contains a small parameter of the problem

$$\Delta\tau_a \ll 1. \tag{12}$$

Therefore, the T-ordering symbol in Eq. (11) can be omitted. We then find that Λ_G is given by

$$\frac{d\Lambda_G}{dG}=-i\sigma_1\int_{-\infty}^{\infty}d\tau\,U(\tau)\Lambda_G. \tag{13}$$

We shall seek the solution of Eq. (13) in the form

$$\langle p|\Lambda_G|p'\rangle=\sum_n \Lambda_{Gn}\delta_{p,u'+nk}\quad (n=0,\pm1,\dots).$$

Then, in the ξ representation

$$\Lambda_G(\xi)=\sum_n \Lambda_{Gn}e^{in\xi}$$

subject to the condition (12), we finally obtain

$$\left.\begin{aligned}&\Lambda(\xi)=\exp[-i\sigma_1\lambda(\xi)],\quad \lambda(\xi)=\psi\cos(\xi+\alpha),\\&\psi=2G\tau_a\Phi(kv\tau_a),\quad \Phi(kv\tau_a)=\int_{-\infty}^{\infty}\frac{d\tau}{\tau_a}\cos(kv\tau)\varphi(\tau),\end{aligned}\right\} \tag{14}$$

where $v=p/M$ is the atomic velocity along the z axis.[6)]

It should be noted that $\Lambda(\xi)$ considered as a function of v has a characteristic size of the same order as the size of a Bennett dip $(k\tau_a)^{-1}$ and, therefore, it changes little over a distance $\sim k/M$. Hence, it follows that the produce of the two operators of the Eq. (14) type $\Lambda=\Lambda_1\Lambda_2$ considered in the ξ representation is

$$\Lambda(\xi)=\Lambda_1(\xi)\Lambda_2(\xi). \tag{15}$$

§6. CALCULATION OF THE POLARIZATION OF THE MEDIUM

We shall consider a gas of atoms in the field of two spatially separate standing waves:

$$E(\mathbf{r},t)=e^{-i\omega t}E_0 f(\mathbf{r})+\text{c.c.}, \tag{16}$$

where

$$f(\mathbf{r})=\begin{cases}2\sum\limits_{j=0,-1}\varphi(x-jL,y)\cos(kz+\alpha_j), & 0<z<l,\\ 0,\quad z<0, & z>l.\end{cases} \tag{16a}$$

Coherent emission in separate fields appears because of the transport of the macroscopic polarization to distances which are multiples of L. The polarization induced by the field (16) is of the form $P(\mathbf{r},t)=\exp(-i\omega t)P(\mathbf{r})+\text{c.c.}$ We shall be interested in the first Fourier harmonic of the polarization

$$P_+(\mathbf{r}_\perp)=\int_0^{2\pi/k}\frac{k\,dz}{2\pi}e^{-ikz}P(\mathbf{r})=N\int d^2u\,W_M(\mathbf{u})P_+(\mathbf{r}_\perp,\mathbf{u})$$

(N is the density of atoms), where the polarization density in the $\mathbf{u}$ space is

$$P_+(\mathbf{r}_\perp,\mathbf{u})=d_{12}\int_{-\infty}^{\infty}dv\,W_M(v)\,\mathrm{Sp}(\sigma_-\langle p|e^{-iH_0\tau}\rho(\tau)\,e^{iH_0\tau}|p-k\rangle),$$

$$\sigma_-=\begin{pmatrix}0&0\\1&0\end{pmatrix}.$$

If $\tau\gg\tau_a$, we find that the application of Eqs. (9) and (10) gives

$$\left.\begin{aligned}&P_+(\mathbf{r}_\perp,\mathbf{u})=-\frac{d_{12}}{2}\int_{-\infty}^{\infty}dv\,W_M(v)\\&\times\exp\{-(\Gamma-i\Omega)\tau-i(kv-\Delta)\tau\}\,\mathrm{Sp}(\sigma_-\langle p|\Lambda_0\tilde{M}_-\Lambda_0^+|p-k\rangle),\\&\text{where}\\&\tilde{M}_-=\exp(-iH_0T)M_-\exp(iH_0T),\quad M_-=S(T)\Lambda_{-1}\sigma_3\Lambda_{-1}^+S^+(T),\end{aligned}\right\} \tag{17}$$

Λ_j is the "matrix of the perturbation" by the j-th field, described by Eq. (14). The operator $\tilde{M}$ is given by

$$\left\langle q+\frac{\varkappa}{2}\right|\tilde{M}_-\left|q-\frac{\varkappa}{2}\right\rangle=\int_0^{2\pi}\frac{d\xi}{2\pi}\sum_n M_-(\xi)\,e^{-in(\xi+kvT)}\delta_{\varkappa,nk}, \tag{18}$$

where $v=q/M$. Hence,

$$P_+(\mathbf{r}_\perp,\mathbf{u})=-\frac{d_{12}}{2}\int_{-\infty}^{\infty}dv\,W_M(v)\,\exp(-(\Gamma-i\Omega)\tau-i(kv-\Delta)\tau)$$

$$\times\int_0^{2\pi}\frac{d\xi_1}{2\pi}\int_0^{2\pi}\frac{d\xi_2}{2\pi}\int_0^{2\pi}\frac{d\xi_3}{2\pi}\sum_{n_1,n_2}\exp[-i(n_1\xi_1+n_2(\xi_2+(kv-(2n_1+n_2)\Delta)T)$$

$$-(n_1+n_2-1)\xi_3)]\,\mathrm{Sp}(\sigma_-\Lambda_0(\xi_1)M_-(\xi_2)\Lambda_0^+(\xi_3)). \tag{19}$$

After the transition $v\to v-(2n_1+n_2)\Delta/k$, we can ignore the changes in $W_M(v)$ and Λ_j in a distance $\sim\Delta/k$. Then, the summation and integrations in Eq. (18) can be carried out in an elementary manner. The result is

$$P_+(\mathbf{r}_\perp,\mathbf{u})=-\frac{i}{2}d_{12}e^{-(\Gamma-i\Omega)\tau}\int_{-\infty}^{\infty}dv W_M(v)\int_0^{2\pi}\frac{d\xi}{2\pi}e^{-i(\xi+kv\tau)}\,.$$

$$\times\Big\{e^{-\Gamma T}[\cos(\Omega T)\cos(2\psi_0\cos(\Delta\tau)\cos(\xi+\alpha_0))$$

$$+i\sin(\Omega T)\cos(2\psi_0\sin(\Delta\tau)\sin(\xi+\alpha_0))]\sin(2\psi_{-1}\cos(\xi-kvT+\alpha_{-1}))$$

$$+\frac{1}{2}[(e^{-\gamma_2T}+e^{-\gamma_1T})\sin(2\psi_0\cos(\Delta T)\cos(\xi+\alpha_0))$$

$$-(e^{-\gamma_2T}-e^{-\gamma_1T})\sin(2\psi_0\sin(\Delta\tau)\sin(\xi+\alpha_0))]\cos(2\psi_{-1}\cos(\xi-kvT+\alpha_{-1}))\Big\}. \tag{20}$$

The integrand in Eq. (20) is a rapidly oscillating function of v if $a/L\ll1$. Averaging over fast oscillations, we find that the polarization differs from zero only in the regions

$$|x-mL|\leqslant a. \tag{21}$$

The odd values of m correspond to the first term in the braces of Eq. (20) and the even terms correspond to the second term. Consequently, going over to the integration variable $kv\tau_a$ in the region $|x-(2n-1)L|\lesssim a$, we obtain the following expression for the polarization density:

$$P_+^{2n-1}(\mathbf{r}_\perp,\mathbf{u})=\frac{id_{12}u}{2\pi^{1/2}kav_0}\exp[i(2n\alpha_0-(2n-1)\alpha_{-1})]$$

$$\times\exp[-(\Gamma-i\Omega)\tau-\Gamma T]\int_{-\infty}^{\infty}dv\,\exp(-iv(x-(2n-1)L)/a)$$

$$\times J_{2n-1}(2\psi_{-1})\,(\cos(\Omega T)J_{2n}(2\psi_0\cos(\Delta\tau))+i(-1)^n\sin(\Omega T)J_{2n}(2\psi_0\sin(\Delta\tau))), \tag{22}$$

whereas in the region $|x-2nL|\lesssim a$, this density is

$$P_+^{2n}(\mathbf{r}_\perp,\mathbf{u})=-\frac{id_{12}u}{4\pi^{1/2}kav_0}\exp[i((2n+1)\alpha_0-2n\alpha_{-1})-(\Gamma-i\Omega)\tau]$$
$$\times\int_{-\infty}^{\infty}dv\exp[-iv(x-2nL)/a]J_{2n}(2\psi_{-1})((e^{-\gamma_2T}+e^{-\gamma_1T})J_{2n+1}(2\psi_0\cos(\Delta\tau))$$
$$+i(-1)^n(e^{-\gamma_2T}-e^{-\gamma_1T})J_{2n+1}(2\psi_0\sin(\Delta\tau))),\qquad(23)$$

where $J_m(x)$ is a Bessel function of order m.

§7. PROFILE OF A CESF LINE

The Maxwell equation for the field in a medium is

$$\frac{1}{c^2}\frac{\partial^2E}{\partial t^2}-\Delta E=-\frac{4\pi}{c^2}\frac{\partial^2P}{\partial t^2}.\qquad(24)$$

Let us assume that the polarization induced in the medium is

$$P(\mathbf{r},t)=(P_+(\mathbf{r})e^{ikz}+P_-(\mathbf{r})e^{-ikz})e^{-i\omega t}+\text{c.c.},$$

where

$$P_\pm(\mathbf{r})=\begin{cases}P_\pm(\mathbf{r}_\perp), & 0<z<l,\\ 0, & z<0,\quad z>l,\end{cases}$$

and l is the length of the cell with the gas along the z axis.

We shall seek the solution of Eq. (24) in the form

$$E(\mathbf{r},t)=(E_+(\mathbf{r})e^{ikz}+E_-(\mathbf{r})e^{-ikz})e^{-i\omega t}+\text{c.c.},$$

where $E_\pm(\mathbf{r})$ are slowly varying functions. We then obtain[7]

$$\frac{\partial E_\pm}{\partial z}=\pm2\pi ikP_\pm(\mathbf{r}).\qquad(25)$$

The solution of Eq. (25) for $z>l$ gives the amplitude of the coherent radiation emitted in such a gas:

$$E_+(\mathbf{r}_\perp)=2\pi iklP_+(\mathbf{r}_\perp).$$

We shall now give the expression for the radiation energy flux $|E_+(\mathbf{r}_\perp)|^2$ in the case when the function $\varphi(\mathbf{r}_\perp)$ is

$$\varphi(\mathbf{r}_\perp)=\begin{cases}1, & |x|<a,\quad|y|<a,\\ 0, & \text{in other cases}\end{cases}$$

[in this case we can ignore the contribution of the particles moving in the shades regions in Fig. 2; then, $\Phi_p(v)=2\sin(kv\tau_a)/(kv\tau_a)$ is independent of φ], and if we ignore the recoil effect, then

$$S_+^m(\mathbf{r}_\perp)=S_0\left(\frac{a\alpha l}{LG\tau_0}\right)^2s_1^m(x)s_2^m(y),\qquad(26)$$

$$s_1^m(x)=\frac{1}{\pi^4(m+1)^2}\left|\int_0^\infty du\,u^2e^{-u^2-m(\Gamma-i\Omega)T}\right.\times\begin{Bmatrix}e^{-\Gamma T}\cos(\Omega T), & m=2n+1,\\ {}^1/_2(e^{-\gamma_1T}+e^{-\gamma_2T}), & m=2n.\end{Bmatrix}\qquad(26a)$$

$$\left.\times\int_0^\infty dv\cos(v(x-mL)/a)J_m(8(G\tau_0/u)\sin v/v)J_{m+1}(8(G\tau_0/u)\sin v/v)\right|^2.$$

$$s_2^m(y)=\begin{cases}1, & |y|<a,\\ [(2m+1-|y|/a)/(2m)]^2, & a<|y|<(2m+1)a,\\ 0, & (2m+1)a<|y|,\end{cases}\qquad(26b)$$

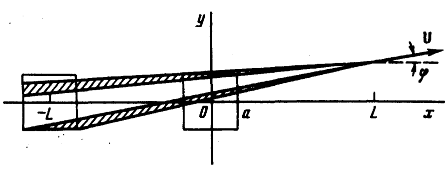

FIG. 2. Derivation of Eq. (26).

where $T=T_0/u$; $T_0=L/v_0$ is the transit time of an atom between the fields; $\tau_0=a/v_0$ is the transit time across a light beam. Figure 3 shows a graph of the function $s_1^m(x)/s_1^m(mL)$, calculated from Eq. (26a) for $m=1$, 2, and 3 for $8G\tau_0=1$, $\Omega=0$, and $\gamma_1=\gamma_2=\Gamma=0$. The total radiation power is

$$W_+^m=W_0(\alpha la/L)^2w^m,$$

where for the same values of the parameters we have $w^1=3.58\times10^{-4}$, $w^2=7.75\times10^{-5}$, and $w^3=1.11\times10^{-5}$; $W_0=4a^2S_0$ is the radiation power in the separate wave fields.

Let us now assume that in the region of reception of the emitted radiation there is a heterodyne laser field

$$E'(\mathbf{r},t)=E'\exp(ikz-i\omega't)+\text{c.c.}\qquad(27)$$

Then, the power flux of the combined emitted and heterodyne fields can be represented by $S_\Sigma=S_++S'+S$, where S_+ is the power of the radiation emitted by the gas as calculated above, $S'=c|E'|^2/(2\pi)$ is the power flux of the field (27), and S is the interference term which is of interest to us and which describes beats between the field (27) and the CESF field. The last term is

$$S(\mathbf{r}_\perp)=\frac{c}{\pi}\operatorname{Re}E_+(\mathbf{r}_\perp)E'^*e^{i(\omega'-\omega)t}.\qquad(28)$$

Substituting in Eq. (28) the expression for the field $E_+(\mathbf{r}_\perp)$ and carrying out elementary integration[8)] with respect to x and v, we find that the "average" power flux

$$\bar S=\int\frac{d^2r_\perp}{a^2}S(\mathbf{r}_\perp)$$

is given by

$$\bar S^m=(S_0S')^{1/2}(\alpha l)(G\tau_0)^{-1}\frac{a}{L}s^m(\Omega)\cos((\omega'-\omega)t+\varphi^m(\Omega)+\varphi^m),\qquad(29)$$

$$s^{2n-1}(\Omega)\exp(i\varphi^{2n-1}(\Omega))=\frac{L}{a}\int_0^\infty du\frac{u}{v_0}W_M(u)e^{-2n\Gamma T+(2n-1)i\Omega T}$$
$$\times\int_{-\infty}^\infty\frac{dy}{a}\int_0^{2\pi}\frac{d\varphi}{2\pi}[\cos(\Omega T)J_{2n}(4G\tau_a\Phi_0(0)\cos(\Delta\tau))$$
$$+i(-1)^n\sin(\Omega T)J_{2n}(4G\tau_a\Phi_0(0)\sin(\Delta\tau))]J_{2n-1}(4G\tau_a\Phi_{-1}(0)),\qquad(29a)$$

$$s^{2n}(\Omega)\exp(i\varphi^{2n}(\Omega))=\frac{L}{2a}\int_0 du\frac{u}{v_0}W_M(u)e^{-2n(\Gamma-i\Omega)T}$$
$$\times\int_{-\infty}^\infty\frac{dy}{a}\int_0^{2\pi}\frac{d\varphi}{2\pi}[(e^{-\gamma_2T}+e^{-\gamma_1T})J_{2n+1}(4G\tau_a\Phi_0(0)\cos(\Delta\tau))$$
$$+i(-1)^n(e^{-\gamma_2T}-e^{-\gamma_1T})J_{2n+1}(4G\tau_a\Phi_0(0)\sin(\Delta\tau))]J_{2n}(4G\tau_a\Phi_{-1}(0)),\qquad(29b)$$

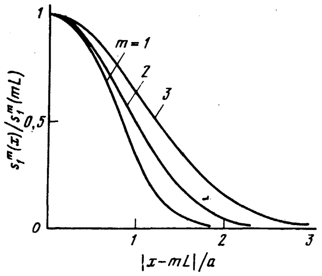

FIG. 3. Distribution of the CESF power density in transverse directions.

where $\tau = mT$; $\alpha = 4\pi^{3/2}N|d_{21}|^2/v_0$ is the unsaturated absorption coefficient of the gas; $S_0 = c|E_0|^2/2\pi$ and $S' = c|E'|^2/2\pi$ are the power densities of the field (16) and of the heterodyne laser field; $\tau_0 = a/v_0$ is the duration of interaction between the original field and an atom moving at a thermal velocity. We have allowed here for the fact that the size of the domain of integration with respect to φ is $\sim a/L \ll 1$. In the limit $\Delta \to 0$, Eq. (29) becomes

$$s^m(\Omega)\exp(i\varphi^m(\Omega)) = \int_0^\infty du \frac{u}{v_0} W_M(u) e^{-m(\Gamma - i\Omega)T} F_m(G\tau_0)$$

$$\times \begin{cases} e^{-\Gamma T}\cos \Omega T, & m=2n+1, \\ \frac{1}{2}(e^{-\gamma_1 T}+e^{-\gamma_2 T}), & m=2n, \end{cases} \tag{30}$$

where

$$F_m(x) = \frac{L}{a}\int_{-\infty}^{\infty}\frac{dy}{a}\int_0^{2\pi}\frac{d\varphi}{2\pi} J_{m+1}(4x\Phi_0(0))J_m(4x\Phi_{-1}(0)).$$

If $\Delta \gtrsim T^{-1}$, Eq. (29) can be represented conveniently in the form

$$s^m(\Omega)\exp(i\varphi^m(\Omega)) = \sum_{\nu=-\infty}^{\infty} s_\nu^m(\Omega-\Omega_\nu^m)\exp(i\varphi_\nu^m(\Omega-\Omega_\nu^m)). \tag{31}$$

Here,

$$s_\nu^{2n-1}(\Omega)\exp(i\varphi_\nu^{2n-1}(\Omega)) = \int_0^\infty du \frac{u}{v_0} W_M(u)$$

$$\times \exp[-2n\Gamma T + i\Omega(2n-1+(-1)^{\nu+n})T] F_{2n-1,\nu}(G\tau_0), \tag{32}$$

where

$$F_{2n-1,\nu}(x) = \frac{L}{a}\int_{-\infty}^{\infty}\frac{dy}{a}\int_0^{2\pi}\frac{d\varphi}{2\pi} J_{2n-1}(4x\Phi_{-1}(0))J_{n-\nu}(2x\Phi_0(0))J_{n+\nu}(2x\Phi_0(0)),$$

$$\Omega_\nu^{2n-1} = (2\nu(2n-1)/(2n-1+(-1)^{\nu+n}))\Delta,$$

$$s_\nu^{2n}(\Omega)\exp(i\varphi_\nu^{2n}(\Omega)) = \int_0^\infty du \frac{u}{v_0} W_M(u) F_{2n,\nu}(G\tau_0)$$

$$\times \exp[-2n(\Gamma - i\Omega)T]\cdot \begin{cases} e^{-\gamma_1 T}, & \nu+n=2m, \\ e^{-\gamma_2 T}, & \nu+n=2m+1. \end{cases} \tag{33}$$

Here,

$$F_{2n,\nu}(x) = \frac{L}{a}\int_{-\infty}^{\infty}\frac{dy}{a}\int_0^{2\pi}\frac{d\varphi}{2\pi} J_{2n}(4x\Phi_{-1}(0))$$

$$\times J_{n-\nu}(2x\Phi_0(0))J_{n+\nu+1}(2x\Phi_0(0)), \quad \Omega_\nu^{2n} = (2\nu+1)\Delta.$$

We have used above the identity

$$J_m^\nu(t) = \int_0^{2\pi}\frac{d\xi}{2\pi} J_m(t\cos\xi)e^{i\nu\xi} = J_{(m+\nu)/2}\left(\frac{t}{2}\right)J_{(m-\nu)/2}\left(\frac{t}{2}\right),$$

which follows from the integral representation of the Bessel function.

For a Gaussian profile

$$\varphi(\mathbf{r}_\perp) = \exp(-\mathbf{r}_\perp^2/a^2) \tag{34}$$

subject to the conditions $|x - mL| \lesssim a$ and $\varphi \ll 1$, we obtain

$$\Phi_p(0) = \pi^{1/2}\exp[-((y-(m-p)L\varphi)/a)^2]$$

and $F_m(x)$, $F_{m,\nu}(x)$ are given by

$$F_m(x) = \frac{2}{\pi}\left(\int_0^\infty dy\, J_m(4\pi^{1/2}xe^{-y^2})\right)\left(\int_0^\infty dy\, J_{m+1}(4\pi^{1/2}xe^{-y^2})\right) \tag{35}$$

$$F_{2n-1,\nu}(x) = \frac{2}{\pi}\left(\int_0^\infty dy\, J_{2n-1}(4\pi^{1/2}xe^{-y^2})\right)\left(\int_0^\infty dy\, J_{n-\nu}(2\pi^{1/2}xe^{-y^2})J_{n+\nu}(2\pi^{1/2}xe^{-y^2})\right), \tag{35a}$$

$$F_{2n,\nu}(x) = \frac{2}{\pi}\left(\int_0^\infty dy\, J_{2n}(4\pi^{1/2}xe^{-y^2})\right)\left(\int_0^\infty dy\, J_{n-\nu}(2\pi^{1/2}xe^{-y^2})J_{n+\nu+1}(2\pi^{1/2}xe^{-y^2})\right). \tag{35b}$$

It should be noted that the quantity (29) appears also in the problem of calculation of a correction to the power absorbed by atoms from the field of a weak probe wave. Let us assume that at a distance $x = mL$ there is a traveling plane wave

$$\mathscr{E}(\mathbf{r}, t) = \mathscr{E}e^{-i\omega t + ikz + i\eta} + \text{c.c.},$$

which is postulated to be weak ($g\tau_0 \ll 1$, where $g = d_{21}\mathscr{E}$). The correction to the absorbed power $\beta = \int d^2r_\perp \langle \mathscr{E}(\mathbf{r}, t)P(\mathbf{r}, t)\rangle_{z,t}$, associated with the transport of polarization to the region $|x - mL| \lesssim a$, is given by the following expression derived from Eq. (22):

$$\beta = (-1)^{m+1}\frac{c\mathscr{E}^2}{2\pi}\frac{\alpha}{g\tau_0}\frac{a^3}{L}s^m(\Omega)\cos(\varphi^m(\Omega)+(m+1)\alpha_0 - m\alpha_{-1} - \eta).$$

If at a distance $x = mL$ there is a weak plane standing wave

$$\mathscr{E}(\mathbf{r}, t) = \mathscr{E}e^{-i\omega t + i\xi}(e^{i(kz+\eta)} + e^{-i(kz+\eta)}) + \text{c.c.},$$

then the quantity β is given by

$$\beta = (-1)^{m+1}\frac{c\mathscr{E}^2}{\pi}\frac{\alpha}{g\tau_0}\frac{a^3}{L}s^m(\Omega)\cos(\varphi^m(\Omega) - \xi)\cos((m+1)\alpha_0 - m\alpha_{-1} - \eta).$$

§8. DISCUSSION

We shall consider only the case of separate standing waves. We shall note simply the qualitative features of the CESF effect in fields of other geometry. The action of two separate unidirectional traveling-wave fields produces coherent radiation only in the vicinity of the points $x = L$ and $-2L$ (Fig. 1) and it is essentially a spatial analog of the photon echo in a gas with a Doppler-broadening transition.[14] In this case a resonance in the profile of the CESF line has the Doppler width $\omega_D = kv_0$. If one of the fields is a standing wave and the other a traveling wave,[9)] coherent radiation is emitted along the z axis at $x = L$ and $-2L$ and, against the background of a Doppler-broadened resonance, there is a Lamb dip of width which is the reciprocal of the interaction time τ_0 of an atom with the field. The radiation is emitted in the opposite direction only at $x = L$ and, against the background of resonances of width of the order of ω_D and τ_0^{-1}, there is a resonance of width which is the reciprocal of the transit time between the fields $T_0 = L/v_0$. It should be noted that T_0^{-1} can be regarded as the homogeneous width of a resonance in separate fields.[7] Thus, the change to a standing wave removes the inhomogeneous broadening of resonance in separate fields, in complete analogy with other optical resonances.[15-17]

It should be stressed that the theory of the CESF effect in the form presented here applies only to standing waves because the Hamiltonian of the interaction between a two-level atom and a traveling spatially confined wave cannot be diagonalized. The exception to this rule is only the case of a traveling wave with a constant distribution of the amplitude across a light

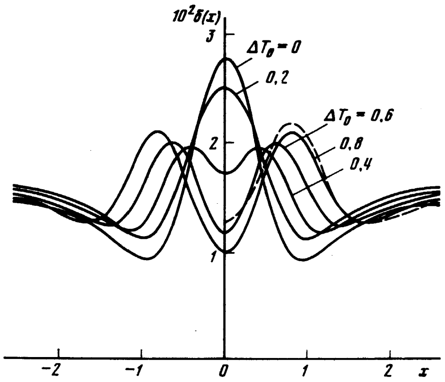

FIG. 4. Splitting of the CESF line profile in the vicinity of $x=L$ due to the recoil effect. The continuous curves represent $s^1(\Omega T_0)$ and the dashed curve is $s^{1,1}(\Omega T_0-0.8)$.

beam.[18]

The formulas (29) and (30) give the profile of a CESF line as a whole. The formulas (31)–(33) describe the splitting of the profile (because of the recoil effect) into a number of components separated from one another by a frequency interval of $\sim\Delta$. We shall note the following qualitative features of the splitting of the CESF line.

1. The resonance in the CESF line profile in the vicinity of a point $x=(2n-1)L$ is, as a whole, symmetric relative to the change in the sign of Ω. This is due to the fact that the process of polarization transport between the waves is responsible for the CESF effect over odd distances. Asymmetry of the resonance appears in the next, in respect of $\gamma_i\tau_a$, orders because of the difference between the relaxations of atomic levels during the time of interaction with the field.

2. Among the components of the resonance of the radiation in the vicinity of a point $x=(2n-1)L$ (for $n\neq1$) there is a component which differs from zero and is located at the line center ($\Omega=0$). This is a basic feature which distinguishes the resonance considered here from other nonlinear optical resonances.

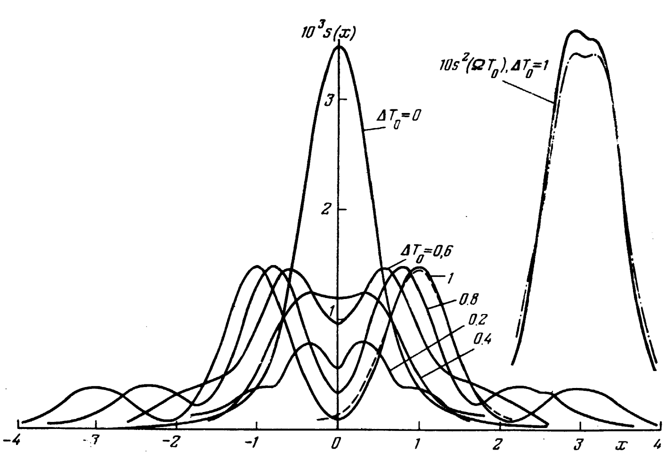

FIG. 5. Same as Fig. 4 but for the CESF effect in the vicinity of $x=2L$. The continuous curves represent $s^2(\Omega T_0)$, the dashed curve is $s^{2,1}(\Omega T_0-1)$, and the chain curve is $10s^{2,3}(\Omega T_0-3)$.

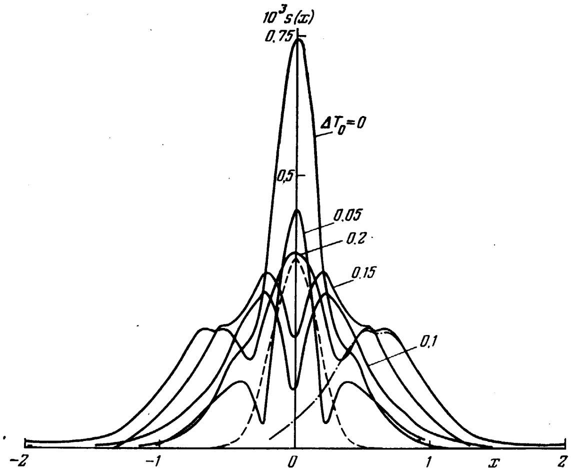

FIG. 6. Same as Fig. 4 but for the CESF effect in the vicinity of $x=3L$. The continuous curves represent $s^3(\Omega T_0)$, the dashed curve is $s^{3,0}(\Omega T_0)$, and the chain curve is $s^{3,3}(\Omega T_0-0.6)$.

It is associated with the difference between the mechanisms of resonance splitting in separate fields and of the Lamb dip, mentioned in § 4. It is difficult to describe directly the process of a resonance at the line center because it is predicted only in the seventh order of perturbation theory for CESF in the vicinity of $x=3L$.

Further analysis can be made and the results of calculations will be quoted for the CESF line profile with $m=1$, 2, and 3. If $\Delta T_0\gg1$, the radiation in the vicinity of a point $x=mL$ depends resonantly on the detuning in the intervals

$$|\Omega-(2l+1)\Delta|\leqslant T_0^{-1}, \quad m=1,2,$$
$$|\Omega-3l\Delta|\leqslant T_0^{-1}, \quad m=3,$$

and then from Eqs. (31)–(33) we obtain

$$\bar{S}^{m\nu}(\Omega)=(S_0S')^{1/2}(\alpha l)(G\tau_0)^{-1}\frac{a}{L}s^{m,\nu}(\Omega-\nu\Delta)\times\cos((\omega'-\omega)t+\varphi^{m,\nu}(\Omega-\nu\Delta)+\varphi^{m}),$$

where $\bar{S}^{m\nu}(\Omega)$ is the beat signal in the radiation in the vicinity of $x=mL$ or detuning in the interval $|\Omega-\nu\Delta|\lesssim T^{-1}$,

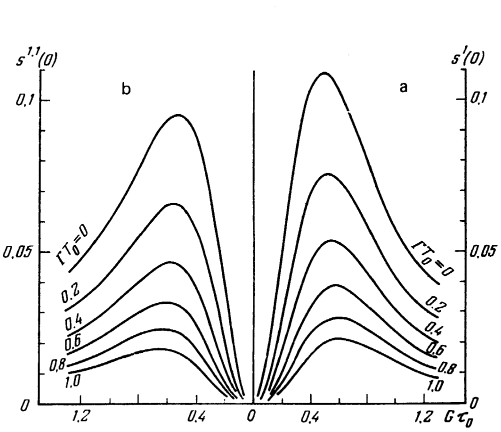

FIG. 7. Field dependences of the beat signal amplitude in the vicinity of $x=L$: a) without allowance for the recoil effect at the line center; b) the component at $\Omega=\Delta$.

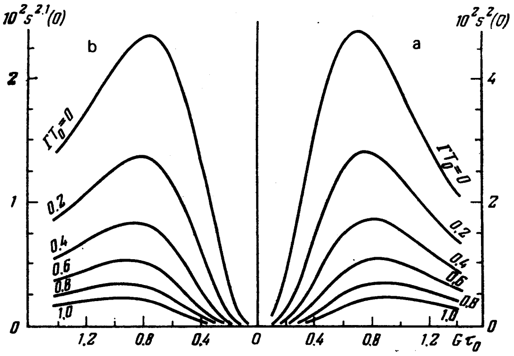

FIG. 8. Same as Fig. 7, but for the CESF effect in the vicinity of $x=2L$.

$$s^{1,2l+1}(\Omega)\exp(i\varphi^{1,2l+1}(\Omega))=s_0{}^1+\overset{1}{s}_{2l+1}(\Omega)\exp(i\overset{1}{\varphi}_{2l+1}(\Omega)), \tag{36a}$$

$$s^{2,2l+1}(\Omega)\exp(i\varphi^{2,2l+1}(\Omega))=s_{2l+1}(\Omega)\exp(i\varphi_{2l+1}(\Omega)), \tag{36b}$$

$$s^{3,6n}(\Omega)\exp(i\varphi^{3,6n}(\Omega))=s_{4n}{}^3(\Omega)\exp(i\varphi_{4n}{}^3(\Omega)), \tag{36c}$$

$$s^{3,3(2n+1)}(\Omega)\exp(i\varphi^{3,3(2n+1)}(\Omega))$$

$$=\overset{3}{s}_{2n+1}(\Omega)\exp(i\overset{3}{\varphi}_{2n+1}(\Omega))+s_{2(2n+1)}(\Omega)\exp(i\varphi_{2(2n+1)}(\Omega)). \tag{36d}$$

§ 9. RESULTS OF THE CALCULATIONS. COMPARISON WITH THE EXPERIMENTAL DATA

We carried out numerical calculations for the CESF effect in a field of a Gaussian profile (34) using the formulas (29)–(33), (35), and (36).

Figures 4–6 show the CESF line profile (29) for the radiation emitted in the vicinity of the points $x=L$, $2L$, and $3L$, respectively in the case when $G\tau_0=(4\pi^{1/2})^{-1}$, $\gamma_1=\gamma_2=0$ and the parameter ΔT_0 has various values. The same figures include, for comparison, graphs of the components (36) into which these resonances are split. It should be noted that the resonance components shown in the figures are sufficient to deduce the overall CESF line profile to within $\lesssim 10\%$ for any value of the parameter ΔT_0.

In an analysis of the field dependence of resonances we shall consider only the following two limiting cases: a) $\Delta=0$, b) $\Delta \gg T_0^{-1}$ and—for simplicity—we shall assume that $\gamma_1=\gamma_2=\Gamma$. In case b) we shall discuss only the strongest components of a resonance.

Figures 7–9 give graphs of the dependences of

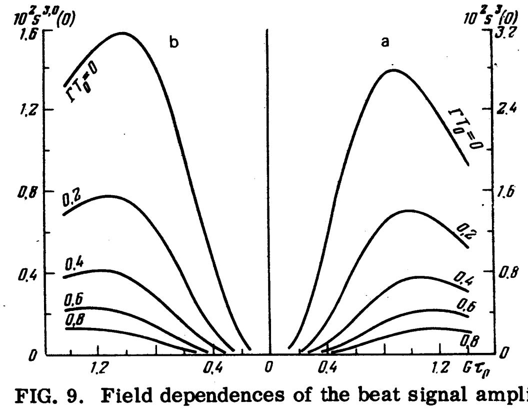

FIG. 9. Field dependences of the beat signal amplitude in the vicinity of $x=3L$ at the line center.

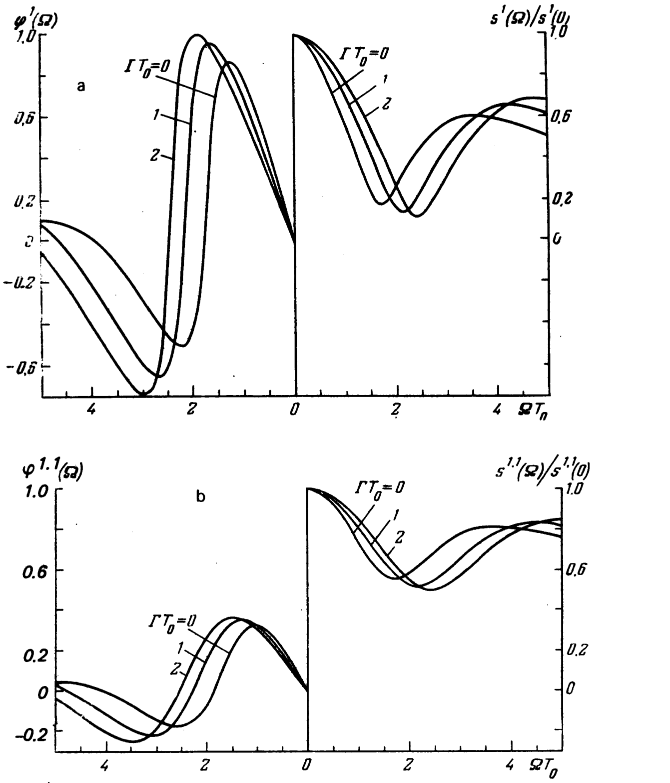

FIG. 10. Dependences of the beat signal amplitude and phase in the vicinity of $x=L$ on the detuning Ω for the optimal value of the parameter $G\tau_0$: a) $s^1(\Omega)$, $\varphi^1(\Omega)$; b) $s^{1,1}(\Omega)$, $\varphi^{1,1}(\Omega)$.

$s^{\cdots}(0)$ on the parameter $G\tau_0$ for the values $m=1$, 2, and 3, respectively. We can see that in a certain optimal field the beat signal amplitude is maximal. An analysis of the dependences of the beat signal amplitude and phase on Ω was made for the optimal field intensity. Figure 10a shows the dependences $s^1(\Omega)$ and ${}^1(\Omega)$ for various values of the parameter ΓT_0. Figure 10b gives the corresponding graphs of $s^{1,1}(\Omega)$ and $\varphi^{1,1}(\Omega)$. Figures 11 and 12 give the dependences $s^{\cdots}(\Omega)$ for various values of the parameter ΓT_0 in the two cases of $m=2$ and 3, respectively.[10]

Figure 13 shows the dependences of the resonance width, deduced from the equation $s^{\cdots}(\Gamma_R)=(s^{\cdots}(0)+s^{\cdots}(\infty))/2$, and of $s^{\cdots}(0)$ on the parameter ΓT_0. Figure 14 shows the dependences of $G_{opt}\tau_0$ on ΓT_0.

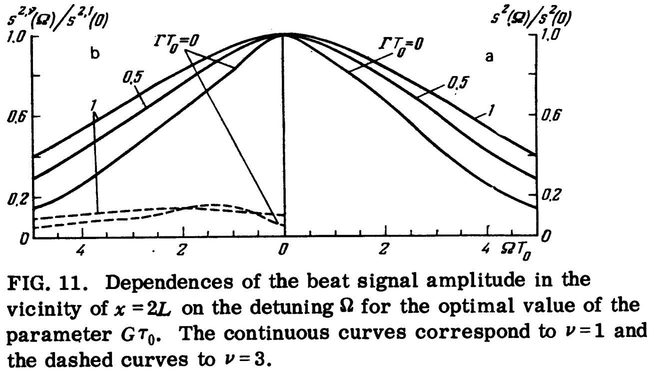

FIG. 11. Dependences of the beat signal amplitude in the vicinity of $x=2L$ on the detuning Ω for the optimal value of the parameter $G\tau_0$. The continuous curves correspond to $\nu=1$ and the dashed curves to $\nu=3$.

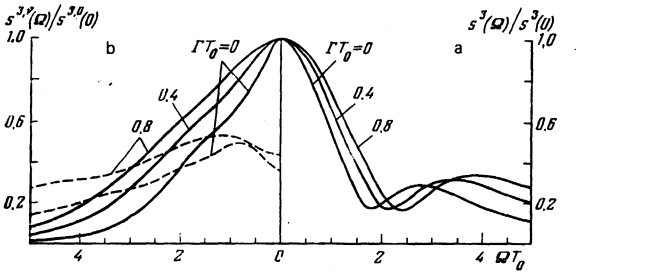

FIG. 12. Same as in Fig. 11, but in the vicinity of $x=3L$. The continuous curves correspond to $\nu=0$ and the dashed curves to $\nu=3$.

Since the unsaturated absorption coefficient α and the homogeneous line width Γ are linear functions of pressure (we shall ignore the radiative widths of the energy levels), the beat signal amplitude is maximal at some specific pressure. The values of the parameters ΓT_0, $\Gamma_R T_0$, $s^{\cdots}(0)$, $G_{opt}\tau_0$ corresponding to this pressure are listed in Tables I and II.

In the absence of recoil under conditions optimal in respect of the pressure and field the beat signal power in the vicinity of $x=L$ at the line center ($\Omega=0$) is (see Tables I and II)

$$W^1=\bar{S}^1a^2=0.0303\alpha l\frac{a^2}{L}\left(\frac{W_0}{(G\tau_0)^2}S'\right)^{1/2}, \tag{37a}$$

where $W_0=(\pi/2)S_0a^2$ is the power of a standing wave with a Gaussian profile.

In case b) the radiation power depends resonantly on the frequency at the line center only at the point $x=3L$. The beat signal power (Table II) is then

$$W^{3,0}=a^2\bar{S}^{3,0}=0.41\cdot10^{-2}\alpha l\frac{a^2}{L}\left(\frac{W_0}{(G\tau_0)^2}S'\right)^{1/2}. \tag{37b}$$

We shall now estimate the CESF effect for methane applying Eqs. (37a) and (37b) approximately under the conditions realized in the experiments described in Ref. 3: $\partial\alpha/\partial p=0.2$ cm$^{-1}\cdot$Torr^{-1}, $\partial\Gamma/\partial p=15$ MHz/Torr, $\Delta\approx1$ kHz, $a=0.5$ cm, $L=3.5$ cm, $l=115$ cm, $T=300$°K, $S'=10^{-3}$ W/cm^2, and $W_0=10^{-4}$ W. In these experiments

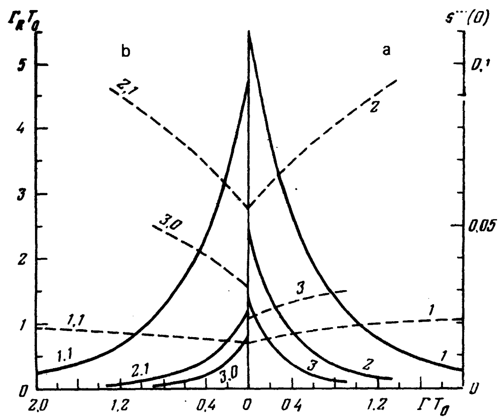

FIG. 13. Dependences of the amplitude (continuous curves) and width (dashed curves) of resonance on the parameter ΓT_0. The numbers alongside the curves give the values of m in Fig. 13a and the values of m and ν in Fig. 13b.

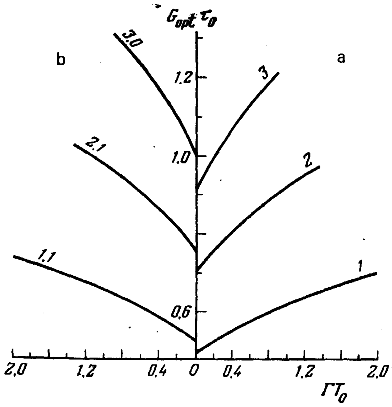

FIG. 14. Dependences of the optimal amplitude on the parameter ΓT_0. The numbers alongside the curves give the values of m in Fig. 14a and the values of m and ν in Fig. 14b.

the recoil doublet was not resolved. The optimal pressure for the observation of the effect was $p=1.03\times10^{-4}$ Torr and the resonance width was $\Gamma_R=2.2$ kHz. Assuming that the power of separate waves used in Ref. 3 correspond to the conditions optimal in respect of the field, we found that the CESF power given by Eq. (37a) was $W^1=2.8\times10^{-9}$ W, which was of the same order of magnitude as the experimentally observed signal. The splitting of resonances in methane by $\Delta>\Gamma_R$ because of recoil corresponded to a distance between the standing waves $L>17$ cm, so that the optimal pressure was $p\lesssim1.1\times10^{-5}$ Torr. Hence, for the same values of the parameters the component of the radiation at the line center obtained from Eq. (37b) was $W^{3,0}\lesssim0.85\times10^{-11}$ W. In the above estimates we ignored the influence of the magnetic hyperfine structure of the $\lambda=3.39$ μ transition and of the spatial degeneracy of the levels. The influence of these factors on the CESF effect in methane will be allowed by us elsewhere. Here, we shall simply note that inclusion of these factors does not alter qualitatively the CESF line profile nor does it alter significantly the signal amplitudes.

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APPENDIX

The commutator of perturbations separated in time

$$U(\tau)=\varphi(\tau)\exp(iH_0\tau)U\exp(-iH_0\tau), \tag{A.1}$$

where H_0 is the Hamiltonian in the absence of perturbation, is

$$\langle m|[U(\tau),U(\tau')]|n\rangle=2i\varphi(\tau)\varphi(\tau')\sum_l U_{ml}U_{ln}$$
$$\times\exp(i(\varepsilon_m-\varepsilon_n)(\tau+\tau')/2)\sin((\varepsilon_n+\varepsilon_m-2\varepsilon_l)(\tau'-\tau)/2), \tag{A.2}$$

TABLE I.

	m		
	1	2	3
ΓT_0	0.61	0.42	0.32
$\Gamma_R T_0$	0.87	3.49	1.27
$s^m(0)$	0.038	0.017	0.0095
$G_{opt}\tau_0$	0.58	0.81	1.03

TABLE II.

	m, ν		
	1. 1	2. 1	3. 0
ΓT_0	0.65	0.43	0.33
$\Gamma_R T_0$	0.79	3.5	1.96
$s^{m,\nu}$	0.031	0.0078	0.0052
$G_{opt}\tau_0$	0.61	0.87	1.15

where $|n\rangle$ and ε_n are the eigenstate and eigenvalue H_0. In the case when the perturbation (A.1) mixes states of similar energies, we find—retaining only the terms linear in $(i-k)$ in the expansion $\varepsilon_i = \varepsilon_k + (\partial\varepsilon_k/\partial k)(i-k) + \frac{1}{2}(\partial^2\varepsilon_k/\partial k^2)(i-k)^2$—that instead of Eq. (A.2) we now have

$$\langle m|[U(\tau),U(\tau')]|n\rangle = i\varphi(\tau)\varphi(\tau')\exp(-i(\varepsilon_n-\varepsilon_m)(\tau+\tau')/2)$$
$$\times\sum_l \sin\left(\frac{\partial\varepsilon_n}{\partial n}(m+n-2l)\left(\frac{\tau'-\tau}{2}\right)\right)(U_{ml}U_{ln}-U_{m,m-l+n}U_{m-l+n,n}).$$

Hence, we can see that for $U_{ml}=U(m-l)$ the expansion in Eq. (A.2) begins with the term $\sim(\partial^2\varepsilon_n/\partial n^2)(\Delta n)^2(\tau'-\tau)^2$, where Δn is the size of $U(n)$. The exact value of (A.2) in the case of perturbation of the motion of a free particle is

$$\langle p|[U(\tau),U(\tau')]|p'\rangle = 2i\varphi(\tau)\varphi(\tau')\exp[i(\varepsilon_p-\varepsilon_{p'})(\tau+\tau')/2]\frac{1}{R}$$
$$\times\int\frac{dq}{2\pi}U_{\varkappa-q}U_{\varkappa+q}\cos(q\tilde{v}(\tau-\tau'))\sin\left(\frac{(\varkappa^2-q^2)}{2M}(\tau-\tau')\right), \quad \text{(A.3)}$$

where U_q is the Fourier transform of the potential $U(z)$, $\varkappa=(p-p')/2$, $\tilde{v}=(p+p')/(2M)$, and R is the normalization length.

[1)]The motion of an atom at right-angles to a wave is considered classically. This is permissible if the frequency $\hbar/Ma^2$ (a is the transverse size of the wave and M is the mass of an atom) is much less than the characteristic frequency intervals of the resonances. The small parameter is now λ/a, where λ is the wavelength. Thus, we can speak of an atom with definite transverse coordinates and velocities.

[2)]We are effectively assuming that the Lamb dip in the absorption of one beam does not resolve the recoil doublet.

[3)]Here and later, we shall assume that $\hbar=1$.

[4)]The phases of the states $|2\rangle$ and $|1\rangle$ are selected in such a way that U is real.

[5)]The form of Eq. (6) presupposes that the homogeneous line width is $\Gamma=(\gamma_1+\gamma_2)/2$. It is shown in Ref. 13 that this is justified in the case when the characteristic angle of elastic scattering θ_0 of an atom is much greater than the ratio of the width of a Bennett dip to the transverse atomic velocity or, in our case, that $\theta_0 \gg \lambda/a$. Collisional line shift is assumed to be included in the definition of the transition frequency ω_{21}.

[6)]We are assuming here that $\varphi(\tau)$ is an even function and also that its characteristic size is $\sim\tau_a$. The index G of Λ in Eq. (14) will be omitted later.

[7)]In Eq. (25) we are ignoring diffraction, which is permissible for $l \ll a^2/\lambda$.

[8)]It should be noted that this is possible only in the case when $E'(r,t)$ is a plane wave.

[9)]To be specific, we shall assume that a traveling wave is located at $x=-L$ and it propagates along the z axis.

[10)]It should be noted that the profile of the line representing the phase of the beat signal $\varphi(\Omega)$ exhibits a dispersion resonance only at $m=1$. In fact, if R.p. 500 B, coherent radiation is analogous to a photon echo (see §8) and, therefore, it appears only in the vicinity of $x=L$, i.e., R.p. 500 C applies only when $m=1$. An analysis shows that the complex quantity R.p. 500 D follows, on increase of Ω, a helical path relative to the value $s^m(\infty)$. Hence, it follows that $\varphi^m(\infty)$ vanishes for radiation in the vicinity of $x=L$ and in other cases it rises without limit.

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