Investigation of distributions of spin waves excited parametrically by parallel pumping of spin-wave instability in ferrites

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The parameters of the distributions of parametrically excited spin waves (PSW's) over the polar and azimuthal angles θ_k and φ_k were determined experimentally. It was found that PSW's form packets with similar wave vectors k and that the angular broadening of the wave vectors is small ($\Delta\theta_k$, $\Delta\varphi_k < 1$), but it depends on the quality of a crystal and pumping rate. The experiments were carried out on single-crystal spheres of yttrium iron garnet at room temperature. The pump frequency was 9370 MHz. The PSW distributions were investigated on the basis of the frequency doubling effect in ferrites in the course of parametric excitation of spin waves.

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INTRODUCTION

Parametric excitation of spin waves by parallel pumping is the technically simplest method for the excitation of large-amplitude spin waves with wave vectors $k \gg \omega_0/c$, where ω_0 is the pump frequency and $c = 3\times10^{10}$ cm/sec is the velocity of light. The theory has been developed furthest for this specific case (see Ref. 1). However, there are certain unexplained circumstances which affect only slightly the integral characteristics of a system of parametrically excited spin waves (PSW's) but are important in practical applications and in the verification of the initial assumptions of the theory. We have in mind here particularly the question of the distribution of PSW's between the wave vectors and especially between the directions of the wave vectors characterized by the polar and azimuthal angles θ_k and φ_k.

Melkov and Grankin[2] showed that the process of doubling of the pump frequency by parametric excitation of spin waves in ferrites can be used to study the PSW distributions. This process occurs in accordance with the scheme shown in Fig. 1: two PSW's of frequency $\omega_k = \omega_0/2$ merge to form a spin wave of frequency ω_0 and two spin waves of frequency ω_0 coalesce to form an electromagnetic wave of frequency $2\omega_0$. The probability of coalescence of two spin waves into one (three-magnon coalescence) depends very strongly on a static magnetic field H_0 applied to a given ferrite and also on the distribution of PSW's over the angles θ_k and φ_k. In particular, the laws of conservation of energy and momentum have the result that the process of three-magnon coalescence exists even in fields $H_0 \leq H_{3m}$ (Ref. 1). The value of H_{3m} is known as the three-magnon coalescence field [see Eq. (12) below]. Therefore, the nature of the PSW distribution can be deduced from the dependence of the power of the radiation at the doubled frequency on the applied static magnetic field. This dependence $P_{2\omega}(H_0)$ or, more exactly, the profile of the radiation line and the position of its maximum on the scale of the static magnetic field can be used to determine the value of θ_k and to estimate the scatter of the wave vectors in respect of their polar angles $\Delta\theta_k$, as well as to find the nature of the distribution over φ_k.

The polar angles θ_k of PSW's are found in Ref. 2 and qualitative estimates of the scatter $\Delta\theta_k$ are obtained; the distribution of PSW's over the azimuthal angles φ_k is not considered at all in Ref. 2 but the results obtained there can be used to judge the nature of this distribution. It is established in Ref. 2 that the intensity of the radiation at the doubled frequency decreases strongly on reduction of the static magnetic field H_0 by just a few oersted below H_{3m}. In fields $H_0 \approx H_{3m}$ the radiation is generated by the coalescence of PSW's with similar values of φ_k, whereas in a field $H_{3m} - H_0 \approx 50$ Oe only the spin waves with azimuthal angles differing by unity, i.e., $\varphi_{k1} - \varphi_{k2} \approx 1$ may coalesce. Therefore, the steep fall of the radiation intensity on reduction of $H_0 < H_{3m}$ shows that PSW's form packets with a small scatter of the waves vectors in respect of φ_k: $\Delta\varphi_k \ll 1$. A rigorous determination of $\Delta\theta_k$ and $\Delta\varphi_k$ requires theoretical calculation of the dependence $P_{2\omega}(H_0)$, which will be done in the first part of the present paper. A comparison of the experimental dependences with the theoretical results, given in the second part, will be used to find the parameters $\Delta\theta_k$ and $\Delta\varphi_k$ of the PSW distribution.

LINE PROFILE AT DOUBLED PUMP FREQUENCY

The Hamiltonian of the interactions responsible for the frequency doubling in parametric excitation of spin waves is

$$\mathcal{H}_{int} = \sum_{k_1k_2k_3} (V_{1,23}^* b_{k_1}^* b_{k_2} b_{k_3} + \text{c.c.})\,\Delta(\mathbf{k}_1 - \mathbf{k}_2 - \mathbf{k}_3) + \sum_{k} [h^*(t) V_k^* b_k b_{-k} + \text{c.c.}]. \quad (1)$$

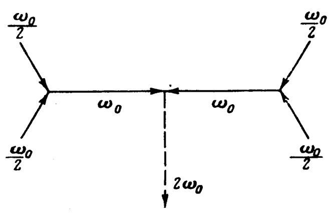

FIG. 1. Doubling of the pump frequency in ferrites in the course of parametric excitation of spin waves. The continuous lines represent spin waves and the dashed line is electromagnetic radiation.

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The notation in Eq. (1) is similar to that adopted in the review in Ref. 1. In particular, b_k represents the amplitude of a spin wave and $h(t)$ is the component of the microwave magnetic field parallel to the magnetization of a crystal M_0 and to a static magnetic field H_0. The first term in the interaction Hamiltonian (1) is responsible for the three-magnon coalescence and the second describes a process which is opposite to the parametric excitation of spin waves by parallel pumping, when two spin waves coalesce forming an electromagnetic wave of doubled frequency. Retaining the most important terms in Eq. (1) in accordance with the frequency-doubling scheme shown in Fig. 1 (only PSW's are taken as the waves with the wave vectors $\mathbf{k}_2$ and $\mathbf{k}_3$; moreover, $\mathbf{k}=\mathbf{k}_1$) and going over from summation to integration, we obtain the following expression for the amplitude of the microwave magnetic field of frequency $2\omega_0$:

$$h(t)=h_{2\omega}e^{-2i\omega_0 t}=e^{-2i\omega_0 t}\int V_{k_1}V^*_{1,23}V^*_{-1,23}n(\mathbf{k}_2)n(\mathbf{k}_3)$$
$$\times\delta(\mathbf{k}_1-\mathbf{k}_2-\mathbf{k}_3)d^3k_1\,d^3k_2\,d^3k_3, \qquad (2)$$

where $n(\mathbf{k})$ is the number of PSW's with the wave vector $\mathbf{k}$ related to the total number of PSW's N deduced from the S theory[1] by the relationship $N=n(\mathbf{k})d^3k$. We shall calculate the integral (2) using the results of Ref. 2 showing that for parallel pumping the angle θ_k differs little from $\pi/2$ and that $\Delta\theta_k$, $\Delta\varphi_k \ll 1$. This makes it possible to use the dispersion law for spin waves and obtain the following expression for PSW's:

$$k^2_{3,2}=k^2_{\pi/2}\left(1+\frac{2\beta_1}{k_{\pi/2}}\theta^2_{3,2}\right), \qquad (3)$$

where $k_{\pi/2}$ is the wave vector of a PSW of frequency $\omega_k=\omega_0/2$ and a polar angle $\theta_k=\pi/2$; $\alpha k^2_{\pi/2}=H_c-H_0$ (Ref. 3); H_c is the static magnetic field in which PSW's with $k\to 0$ are excited; α is the exchange constant;

$$\frac{2\beta_1}{k_{\pi/2}}=\frac{\omega_M}{2}\frac{1-\omega_M/(\omega_0^2+\omega_M^2)^{1/2}}{1/2[(\omega_0^2+\omega_M^2)^{1/2}-\omega_M]-\omega_H}; \qquad (4)$$

$$\theta_{1,2}=\pi/2-\theta_{k_{1,2}}, \quad \theta_{1,2}\ll 1; \qquad (5)$$

$\omega_M=4\pi\gamma M_0$, γ is the gyromagnetic ratio for the electron spin; $\omega_H=\gamma(H_0-N_z M_0)$, N_z is the demagnetization factor for the shape of the sample in the direction of the applied field.

The smallness of $\Delta\theta_k$ and $\Delta\varphi_k$ indicates that PSW's are distributed in a narrow range of the polar and azimuthal angles. It is clear from Eq. (2) that the intensity of the radiation is an integral quantity and, therefore, it cannot be sensitive to fine details of the distribution function. This allows us to use the Gaussian approximation for the distribution function of PSW's in the calculations:

$$\left.\begin{array}{c} n(\theta_k,\varphi_k)d\varphi_k\,d\theta_k=\dfrac{N}{2\pi\Delta\theta_k\Delta\varphi_k}\exp\left[-\dfrac{\theta_k^2}{2\Delta\theta_k^2}-\dfrac{\varphi_k^2}{2\Delta\varphi_k^2}\right]d\varphi_k\,d\theta_k,\\ \Delta\theta_k,\ \Delta\varphi_k\ll 1, \end{array}\right\} \qquad (6)$$

where N is the integral intensity of PSW's.

If PSW's have a polar angle close to $\pi/2$, then spin waves at frequency ω_0 formed as a result of coalescence of two PSW's also have a polar angle close to $\pi/2$: $\theta_k=\pi/2-\theta$, $\theta\ll 1$. Consequently, for these spin waves we may obtain a relationship analogous to Eq. (3):

$$k^2=(2k_{\pi/2})^2\left(1+\frac{\beta}{k_{\pi/2}}\theta^2\right), \qquad (7)$$

$$\frac{\beta}{k_{\pi/2}}=\frac{\omega_M}{8}\frac{1-\omega_M/(4\omega_0^2+\omega_M^2)^{1/2}}{1/2[(\omega_0^2+\omega_M^2)^{1/2}-\omega_M]-\omega_H}. \qquad (8)$$

The integral in Eq. (2) is calculated using also the fact that under real experimental conditions the quantity $2\beta_1/k_{\pi/2}$ [see Eq. (4)] is close to unity:

$$1-2\beta_1/k_{\pi/2}=a_1\ll 1. \qquad (9)$$

In our case, we have $a_1\approx 0.1$.

Using Eqs. (3)-(9), we can obtain the following expression for $H_0<H_{3m}$:

$$h_{2\omega}\propto[P_{2\omega}(H_0)]^{1/2}\propto\exp(-a|\delta H|/2\Delta\varphi_k^2), \qquad (10)$$

whereas for $H_0>H_{3m}$, we find that

$$h_{2\omega}\propto[P_{2\omega}(H_0)]^{1/2}\propto\exp(-2a|\delta H|/b\Delta\theta_k^2), \qquad (11)$$

where

$$\delta H=H_0-H_{3m},\quad a=3/2(H_c-H_{3m}),\quad b=4(2\beta_1/k_{\pi/2}-\beta/k_{\pi/2}),$$

and H_{3m} is the three-magnon coalescence field introduced above:

$$H_{3m}=\frac{\omega_M}{\gamma}\left[\left(\frac{\omega_0^2}{\omega_M^2}+1\right)^{1/2}-\frac{1}{3}\left(\frac{\omega_0^2}{\omega_M^2}+\frac{1}{4}\right)-\frac{1}{2}+N_z\right]. \qquad (12)$$

A comparison of the experimental dependences $P_{2\omega}(H_0)$ with Eqs. (10) and (11) can yield the parameters of the PSW distribution $\Delta\theta_k$ and $\Delta\varphi_k$. The dependence $P_{2\omega}(H_0)$ exhibits resonance with a maximum in the field $H_0=H_{3m}$. If H_0' denotes the static magnetic field in which $P_{2\omega}(H_0)$ decreases by a factor of e relative to the maximum on the left of the resonance ($H_0'<H_{3m}$) and H_0'' denotes a similar field to the right of the resonance ($H_0''>H_{3m}$), the final expressions convenient for the experimental determination of the PSW distribution parameters become

$$\Delta\varphi_k=[a(H_{3m}-H_0')]^{1/2},\quad \Delta\theta_k=\left[\frac{4a}{b}(H_0''-H_{3m})\right]^{1/2}. \qquad (13)$$

EXPERIMENTS

Our experiments were carried out at room temperature using apparatus described in detail in Ref. 2. We shall simply mention here that we employed the usual setup for the observation of parametric instability under parallel pumping conditions[1] except that our resonator had an additional waveguide port for the doubled pump freqeuncy $2\omega_0$. The pump frequency was 9370 MHz, the pulse duration was 200 μsec, and the repetition frequency was 1-50 Hz. The influence of the second harmonic of a magnetron oscillator on the measuring system was suppressed by placing two low-frequency filters in series between the oscillator and resonator. Our samples were single-crystal spheres of yttrium iron garnet (YIG) which were 2.5-3 mm in diameter. The influence of spontaneous oscillations of the magnetization was eliminated by magnetizing the spheres along the [100] difficult axis.

We determined the dependence of the doubled-fre-

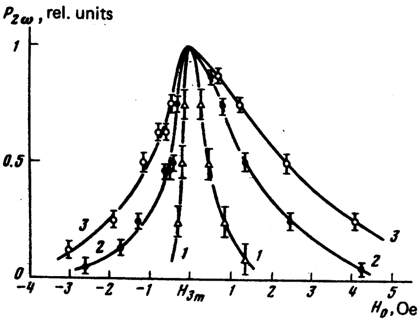

FIG. 2. Dependences of the power $P_{2\omega}$ at the doubled frequency on the static magnetic field H_0 plotted for various values of the excess ζ (dB): 1) 0.2; 2) 1.5; 3) 5. Spherical YIG sample 2.9 mm in diameter, $\mathbf{H}_0 \parallel [100]$. Absolute values of the power at the maxima of curves 1, 2, and 3 are $(1 \pm 0.5) \times 10^{-9}$, and $(2 \pm 1) \times 10^{-8}$ W, respectively.

quency power on the applied static magnetic field $P_{2\omega}(H_0)$ for various values of the excess above the threshold $\zeta = h/h_{\text{th}}$, where h is the amplitude of the magnetic pump field and h_{th} is the threshold value of h corresponding to the appearance of the parametric instability. Typical dependences $P_{2\omega}(H_0)$ are shown in Fig. 2. These dependences, together with Eq. (13), make it possible to calculate the PSW distribution parameters $\Delta\theta_k$ and $\Delta\varphi_k$. The values of $\Delta\theta_k$ and $\Delta\varphi_k$ found in this way are plotted in Figs. 3 and 4 as a function of the excess. The numerical values of the coefficients in the theoretical expressions used in the calculation of the parameters were as follows: $a \approx 0.003$ Oe^{-1}, $b \approx 2.4$, $2\beta/k_{\pi/2} \approx 0.9$, and $a_1 \approx 0.1$.

It is clear from Fig. 3 and 4 that even for very high values of the excess ($\zeta \leqslant 15$dB) the conditions $\Delta\theta_k$, $\Delta\varphi_k \ll 1$, were obeyed, i.e., the main assumption used in the derivation of the theoretical expressions (13) was satisfied. For the minimum excess $\zeta \approx 0.2$ dB at which the frequency-doubling effect was still observed a packet of PSW's was narrowest: $\Delta\theta_k^{\min} \approx 6 \cdot 10^{-2} (\sim 3.5°)$, $\Delta\varphi_k^{\min} \approx 3 \cdot 10^{-2} (\sim 1.7°)$. When the excess was increased, the PSW packet became broader in respect of the polar and azimuthal angles: this broadening was particularly noticeable in the range $\zeta \gtrsim$ 6-9 dB but even at the highest values of the excess reached in our experiments the

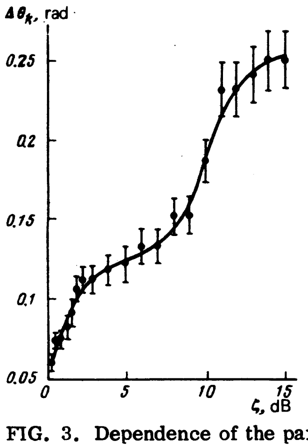

FIG. 3. Dependence of the parameter $\Delta\theta_k$ of the distribution of PSW's over polar angles on the excess ζ for the same sample as in Fig. 2.

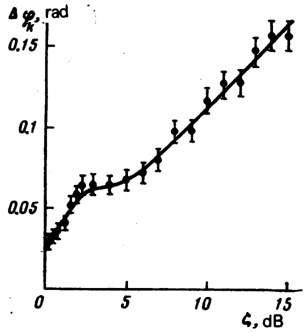

FIG. 4. Dependence of the parameter $\Delta\varphi_k$ of the distribution of PSW's over azimuthal angles on the excess ζ for the same sample as in Fig. 2.

PSW packet remained still relatively narrow ($\Delta\varphi_k < 8°$, $\Delta\theta_k < 14°$).

One should stress that the above values of the PSW distribution parameters $\Delta\theta_k$ and $\Delta\varphi_k$ are reliable only if the experimental dependence $P_{2\omega}(H_0)$ is indeed due to the three-magnon coalescence of PSW's shown in Fig. 1. A detailed proof is given in Ref. 2 that the frequency-doubling process in parallel-pumped ferrites is indeed due to the parametric excitation of spin waves in fields $H_0 \approx H_{3m}$. However, frequency doubling in parametric excitation of spin waves may result not only from the process shown in Fig. 1 but also from many other processes but all of them have a lower probability and, what is more important, they are characterized by a smooth dependence of the intensity of the doubled-frequency radiation on a static magnetic field near $H_0 = H_{3m}$. As shown above, $H_0 = H_{3m}$ is singular only for the three-magnon coalescence process. The doubling effect due to the nonlinear dependence of the longitudinal magnetization m_z on an alternating magnetic field h, $m_z = \chi''(h, H_0)h$, is governed by the nonlinear susceptibility $\chi''(h, H_0)$, which varies only by a few percent in the case of parallel pumping by fields close to H_{3m} (Ref. 1). Thus, this effect also cannot give rise to significant singularities near the field H_{3m}.

It should be noted that the dependences similar to those shown in Figs. 2–4 can be obtained only for very-high-quality single-crystal samples of YIG characterized by a spin-wave resonance line of width $\Delta H_k \lesssim 0.1$ Oe in $H_0 = H_c$ (i.e., these dependences can be obtained for spin waves with the wave vector $k \to 0$). In the case of poorer samples or samples with deliberately introduced impurities the resonance curve $P_{2\omega}(H_0)$ becomes much wider than in Fig. 2 and it has a fine structure, which appears most clearly in the range of small values of the excess $\zeta \leqslant$ 4–5 dB. This fine structure is manifested by one or two additional narrow maxima located above or below the three-magnon coalescence field H_{3m}.

DISCUSSION OF RESULTS

It follows from our investigation that PSW's form a pair or several pairs (see below) of packets of waves with similar values of the wave vectors ($\Delta\theta_k$, $\Delta\varphi_k \ll 1$) and the angular scatter $\Delta\theta_k$ and $\Delta\varphi_k$ increases considerably as the pumping rate rises. The spin-wave Hamiltonian for an ideal YIG sphere magnetized along the [100] axis is symmetric relative to rotation about this axis. A symmetric distribution of PSW's is described in Ref. 1. The PSW distribution found above does not have this symmetry. The asymmetry indicates either that there is a spontaneous violation of the symmetry in the PSW system or that the crystals are characterized by a weak anisotropy which appears during their "firing" and is due to the imperfect treatment of the samples; this anisotropy is sufficient to disturb the symmetry of the distribution. The S approximation (see Ref. 1) is used in Ref. 4 to show that a symmetric PSW distribution in crystals with the same symmetry as ours is unstable, but singular distributions in the form of a finite number of pairs of spin waves with mutually perpendicular spin vectors may be stable. This means that the PSW system does indeed experience spontaneous symmetry breaking. The consequence may be[4] an anisotropy of the attenuation coefficient of spin waves (associated with the hard excitation of PSW's) ensuring an additional stability of the asymmetric PSW distribution. The crystal anisotropy which disturbs the Hamiltonian symmetry enhances the influence of the spontaneous symmetry breaking. Separation of the contribution of these two mechanisms is not possible on the basis of the available experimental data.

The experiments indicate that there is no doubled-frequency radiation in a magnetic field whose intensity corresponds to the coalescence of two spin waves with angles φ_k differing by about $\pi/2$. The emission of such radiation would have indicated the presence of two pairs of PSW packets corresponding to the asymmetric solutions in the form of a quartet of waves, as found in Ref. 4. It is possible that the intensity of this radiation is far too low to be recorded reliably or that only one pair of waves is excited in our experiments.

Scattering of spin waves by inhomogeneities in a crystal and by thermal spin waves can explain why the PSW distribution in respect of φ_k and θ_k is not singular and why the broadening $\Delta\varphi_k$ and $\Delta\theta_k$ is finite, depending strongly on the pumping rate and quality of the crystal. The distribution function of thermal spin waves is closely related to the PSW distribution and may vary considerably with the pumping rate. The probability of scattering of spin waves by inhomogeneities, considered in Ref. 5, may be associated with the line width $\Delta H_{k\,S}$. The formulas of Ref. 5 and the experimental values of $\Delta\theta_k$ allow us to estimate $\Delta H_{k\,S}$. For our crystals, we find that $\Delta H_{kS} \lesssim 10^{-2}\Delta H_k$.

A theory explaining the finite value of $\Delta\varphi_k$ and relating it to some physical characteristics of a crystal should help in developing a new very sensitive method for quality control of ferrite single crystals.

[1]V. E. Zakharov, V. S. L'vov, and S. S. Starobinets, Usp. Fiz. Nauk **114**, 609 (1974) [Sov. Phys. Usp. **17**, 896 (1975)].
[2]G. A. Melkov and V. L. Grankin, Zh. Eksp. Teor. Fiz. **69**, 1415 (1975) [Sov. Phys. JETP **42**, 721 (1975)].
[3]E. Schlömann and R. I. Joseph, J. Appl. Phys. **32**, 1006 (1961).
[4]A. S. Bakaĭ, Zh. Eksp. Teor. Fiz. **74**, 993 (1978) [Sov. Phys. JETP **47**, 522 (1978)].
[5]V. E. Zakharov and V. S. L'vov, Fiz. Tverd. Tela (Leningrad) **14**, 2913 (1972) [Sov. Phys. Solid State **14**, 2513 (1973)].

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Character of submillimeter photoconductivity in *n*-InSb

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A comprehensive investigation was made of the submillimeter photoconductivity of n-InSb in the range of wavelengths $\lambda = 0.6$–8 mm, magnetic fields $H = 0$–30 kOe, electric fields $E = 0.01$–0.5 V/cm, and temperatures $T = 1.3$–30 K. The kinetics of the photoconductivity processes as a function of T, E, and H is investigated. It is shown that impurity photoconductivity does exist for any degree of compensation of extremely purified n-InSb. Particular attention is paid to the hopping photoconductivity realized in strongly compensated n-InSb ($K > 0.8$).

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1. INTRODUCTION

Indium antimonide of n-type (n-InSb) occupies at present a special position among the semiconductors. In its electrophysical properties, it is a unique object for investigations and applications, because it combines isotropy of the conduction band with small effective mass of the free electrons, $m = 0.013 m_e$, and its crystal lattice has an appreciable permittivity, $\varkappa = 16$, and consequently a large Bohr radius $a = 640$ Å of the electron at the hydrogenlike impurity centers. The concentration N_d of the residual donors in extremely purified n-InSb is $\sim 10^{14}$ cm^{-3}, and their compensation is $K = N_a/N_d = 0.3-0.7$. A relatively small change of N_d and K, and also of the external conditions (temperature T, magnetic field H, electric field E, etc.) leads to substantial