Structure of the intermediate state formed on destruction of superconductivity by a current

I. L. Landau

S. I. Vavilov Institute of Physics Problems, USSR Academy of Sciences, Moscow
(Submitted 3 July 1978)
Zh. Eksp. Teor. Fiz. 75, 2295–2308 (December 1978)

It shown that correct allowance for the surface tension at the phase interface yields a configuration in which the shapes of superconducting and normal regions are very different from those used in calculations in all the known models of periodic intermediate-state structures. A numerical calculation of the equilibrium shape of the layers is carried out for various values of the surface tension. A comparison of the calculated and experimental results shows that the structure of the intermediate state is not usually periodic but has a dynamic form proposed by Gorter. Periodic structures can apparently only appear in samples of very large diameter.

PACS numbers: 74.30. – e

There are at present two basically different models of structures of the intermediate state which appear on destruction of superconductivity of a cylindrical sample by a current. One is the periodic (along the cylinder axis) structure of superconducting and normal layers (first proposed[1] by F. London in 1937), shown in Fig. 1(a). The other is a dynamic model of the intermediate state proposed[2,3] by Gorter in 1957. The Gortor structure is in the form of coaxial cylindrical superconducting and normal layers, which appear near the surface of the sample and collapse on its axis [Fig. 1(b)]. In spite of the fact that both models were proposed some time ago, it is not yet possible to say definitely which of these structures occurs in reality. This is basically due to the difficulty of carrying out the appropriate experiments. The nub of the problem is that the destruction of superconductivity by a current produces a region of the intermediate state covered by a normal metal layer so that the structure of the intermediate state can only be deduced from such indirect data as the electrical resistivity, etc.

We carried out a detailed analysis of both models and of the available experimental data. A comparsion of the calculated and experimental results led us to the conclusion that the Gorter structure usually appears in practice. Periodic structures can form only in samples of very large diameter.

Only the London structure can be calculated in full. In fact, the simple condition that the magnetic field in the normal metal has at least the critical value H_c gives[1] the radius of a region of the intermediate state r_{i0} in a sample carrying a current I:

$$r_{i0}=r_0[i-(i^2-1)^{1/2}],\quad i=I/I_c \tag{1}$$

(here, r_0 is the radius of the sample and $I_c=cr_0H_c/2$ is the critical current); in this case, the dependence of the concentration of the normal phase on the distance to the axis of the sample is

$$C_n(r)=r/r_{i0}$$

and the resistance of the sample in the intermediate state is given by

$$R={}^1/_2R_n[1+(i^2-1)^{1/2}/i] \tag{2}$$

(R_n is the resistance in the normal state). In particular, for $I=I_c$, we have $R=0.5R_n$.

The Gorter structure involves continuous formation of new superconducting regions in the bulk of a normal metal and this, in its turn, is associated with supercooling effects, i.e., new superconducting regions appear after some delay relative to the equilibrium situation. The result of this delay is that the radius of the intermediate state in the Gorter structure is always less than the value given by Eq. (1) and the resistance is always greater than that given by Eq. (2). The exact values of these two quantities cannot yet be calculated *a priori*.

The experimental value of the resistance jump at $I=I_c$ is always much greater than the value $0.5R_n$ predicted by the London model; since the Gorter model can, in principle, yield any value of the resistance, one might be tempted to conclude that the latter model is valid. However, two modifications of the London periodic structure model have recently appeared[4,5] and are in satisfactory agreement with the experimental values of the resistance.

It is interesting to note that, although these new mo-

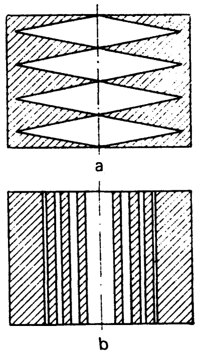

FIG. 1. Schematic representation of the London (a) and Gorter (b) structures.

 0038-5646/78/1201158-08$02.40

dels are based on very different ideas and assumptions, and predict quite different structures, they still give practically the same values of R/R_n for realistic experiments. The considerable discrepancy between the theories of Andreev[4] and Baird and Mukherjee,[5] illustrated in Fig. 10 in Mukherjee's paper,[6] is due to the fact that the curve corresponding to the Andreev theory is plotted using an incorrect value of the parameter, which represents the surface tension at the phase interface. The dependence $\Delta(T)$ at temperatures not too far from the critical value is $\Delta = \Delta_0(1 - T/T_c)^{-1/2}$; for indium, we have $\Delta_0 = 3.3 \times 10^{-5}$ cm (Ref. 7) so that, at the temperatures of the experiment in question, we find that $\Delta = (2.2 - 2.4) \times 10^{-4}$ cm and not 3×10^{-5} cm as assumed by Mukherjee. We shall not consider this point any further because comparisons of the experimental and theoretical results are quite frequently made using the value of Δ_0, which is a factor of tens different from the value of Δ at the temperature of the appropriate experiments.

We shall not carry out a comparative analysis of these new structures because all the periodic models suffer from very serious shortcomings. In all these models, it is assumed that the magnetic field at the phase interface is equal to the critical value. There are no objections to this assumption in the case of the flat parts of the interface but it is absolutely incorrect at the point A (Fig. 2), where the concentration of the superconducting phase vanishes and the curvature of the interface becomes infinite. Since the interface in type I superconductors is characterized by a positive surface tension, the existance of a kink is generally impossible so that, in fact, the interface near the point A should be rounded off. The condition for the magnetic field on the rounded interface is

$$H = H_c(1 - \Delta/2r_{cr}), \tag{3}$$

where $\Delta H_c^2/8\pi$ is the surface tension and r_{cr} is the radius of curvature of the interface at this point. The condition (3) should be satisfied at every point on a stable interface.

We shall now estimate the distortion of the original London structure resulting from allowance for this effect. The radius of curvature at the point A' (Fig. 2) is of the order of

$$r_{cr} \sim ad/2r_{i0}, \quad d = r_{i0} - r_i,$$

where a is the period of the structure; r_{i0} is the radius of an intermediate state region in the original London structure; r_i is the radius of an intermediate state region in the rounded structure. Using this value of r_{cr}, we can easily find the magnetic field at the point A from Eq. (3). On the other hand, the magnetic field at A' is related directly to the total current flowing through the intermediate-state region and this current is governed by the nature of the structure in the region $r < r_i$. For example, in the case of indium at $T = 3$°K, we have $\Delta = 10^{-4}$ cm; if the diameter of the sample is $2r_0 = 0.5$ mm and $a/2r_{i0} = 0.3$, a reduction in the size of the intermediate-state region by 4% (i.e., $d/r_{i0} = 0.04$) gives $H_c - H|_{A'} = 0.1H_c$. Even the roughest estimates indicate that the resultant structure is very far from the original. In fact, allowance for the redistribution of currents in the normal phase increases even further the distortion of the original structure.

Such a qualitative analysis fails to predict even an approximate shape of the equilibrium interface. Therefore, we had to carry out numerical calculations on a computer. All these calculations were carried out on a Hewlett-Packard 21MX computer.

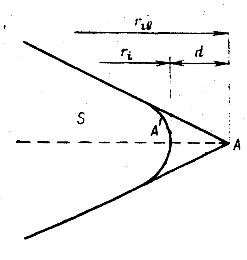

FIG. 2. Schematic diagram of a superconducting layer near the edge of an intermediate-state region.

CALCULATION OF THE SHAPE OF LAYERS IN A PERIODIC STRUCTURE

Our task is to find the shape of an interface such that the magnetic field obeys the condition (3) at each point. We shall use the cylindrical coordinate system (z, r, φ) with the z axis coinciding with the axis of the sample and the $z = 0$ plane passing through the middle of the normal layer. If $z = z_0(r)$ is the equation for the interface, the condition (3) can be rewritten in the form

$$H(r) = H_c\left[1 - \frac{\Delta}{2}\frac{z_0''}{(1 + z_0'^2)^{3/2}}\right] \tag{4}$$

(here, the prime denotes differentiation with respect to r). We then have

$$H(r) = \frac{2}{cr}\int_0^r j(r)\cdot 2\pi r\,dr \tag{5}$$

[here, $j(r)$ is the density of the current flowing into a superconducting layer at a distance r from the axis of the sample]. The value of $j(r)$ can be found by solving the Laplace equation for the electric potential u in the normal layers with suitable boundary conditions (all the calculations are carried out for the case of a local relationship between the current density and ∇u). In view of the axial symmetry, we have $\partial u/\partial \varphi = 0$ and the Laplace equation becomes

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{\partial^2 u}{\partial z^2} = 0. \tag{6}$$

Thus, using Eqs. (4), (5), and (6), we can, in principle, find the equilibrium shape of the interface. However, before considering the results of numerical calculations, we shall try to determine the type of periodic structure which can exist in this case.

We shall first consider the London structure. It is constructed in such a way that the magnetic field is constant along the interface and equal to the critical value. In fact, for the London structure this is only true if the structure period a is much less than the radius of an intermediate-state region r_{i0}. However, we can easily show that, near the axis of the sample, the constancy of the magnetic field along the interface is satisfied for any relationship between a and r_{i0}. In fact, if $r \ll r_{i0}$, the solution of Eq. (6) has the form

$$u \propto x(x^2-1)^{1/2}+\ln[x+(x^2+1)^{1/2}], \quad x=z/r,$$

which gives $j(r) \propto 1/r$. Substituting this dependence j/r in Eq. (5) gives H = const.[1] This macroscopic approach is applicable as long as the distance to the axis of the sample and the thickness of the normal layers are both much greater than the coherence length $\xi(T)$.

For moderately thin samples, we thus have a range of distances to the axis $\xi(T) \ll r \ll r_{i0}$ in which the magnetic field is constant on the interface of a structure of the London type with $C_n \propto r$. Therefore, it is natural to assume that, near the axis of the sample, the shape of the layers should be close to the London shape even in the case of the rounded structure, i.e., near the axis, the dependence of the concentration of the normal phase on the distance to the axis should also have the form $C_n(r)=r/r_{i0}$. The quantity r_{i0} will be called the radius of the original London structure. Its value is given by the difference between the potentials on the sample and may differ considerably from the value calculated from Eq. (1). In this structure, the interface near the axis has zero curvature and the magnetic field is constant and equal to the critical value, i.e., the condition (4) is satisfied.

Equating Eqs. (4) and (5) for H and differentiating with respect to r, we obtain the ordinary differential equation for z_0. In terms of dimensionless variables, $x=r/r_{i0}$ and $y=z/r_{i0}$, this equation now becomes

$$\left.\begin{aligned}&\frac{y_0'''}{y_0''}-3y'y''^{1/3}w^{2/3}-\frac{1-\alpha w}{x\alpha w}+\frac{1}{\alpha w}\frac{j(x)}{j_c}=0,\\ &w=\frac{y_0''}{(1+y_0'^2)^{3/2}} \quad \alpha=\frac{\Delta}{2r_{i0}} \quad j_c=\frac{cH_c}{4\pi r_{i0}}\end{aligned}\right\} \tag{7}$$

[in this equation, a prime denotes differentiation with respect to x and $y=y_0(x)$ is the equation for the interface in terms of new variables].

However, the above equation includes an unknown function $j(x)$. As pointed out earlier, this function can be found by solving the Laplace equation (6). In the present case, this two-dimensional problem is solved for the region in Fig. 3. Line AD represents the middle of a normal layer and, because of symmetry, we have $u|_{AD}$ = const $=u_1$; line AB is the boundary of a superconducting layer and BC joins the middle of the superconducting layer to the surface of the sample so that $u|_{AB}=u|_{BC}$ = const $=u_2$; line CD is the surface of the sample and, on this surface, we have $\partial u/\partial n|_{CD}=0$.

The Laplace equation was solved by the method of networks, in accordance with a program described in Ref. 8. This program was modified somewhat for convience of use in a computer with a relatively small working memory. Calculations could be carried out for 4000–4500 points and the time required to obtain a solution was about 1 h.

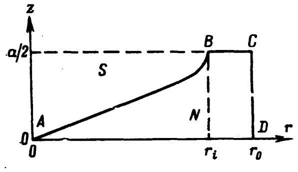

FIG. 3. Schematic representation of the integration domain.

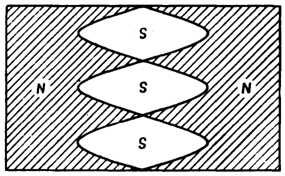

FIG. 4. General appearance of a periodic intermediate-state structure (the normal regions are shown shaded) in the case $I=I_c$, $\Delta/2r_{i0}=3\times10^{-5}$ and $a/2r_{i0}=0.3$.

The equilibrium shape of the interface was found by the method of successive approximations. First, the Laplace equation was solved for some trial interface. The solution was used to find the density of the current flowing into the superconducting region $j(x)$. The dependence $j(x)$ was represented in the form

$$j(x)=j_n/C_n+j_1(y_0')$$

(j_n is the density of the current in the normal metal outside the intermediate-state region) and substituted in Eq. (7), which was then solved subject to the boundary conditions

$$y_0|_{x=0}=0, \quad y_0'|_{x=0}=a/2r_{i0}, \quad y_0'|_{y=a/2r_{i0}}=\infty.$$

The Laplace equation was solved again for the new shape, the new dependence $j(x)$ was found and the process was repeated. After 10–20 such iterations, the solution converged with satisfactory precision. The results of the calculations are presented in Fig. 4 and Table I. Figures 4 and 5, as well as Figs. 6(a) and 7 were plotted by the computer in real time.

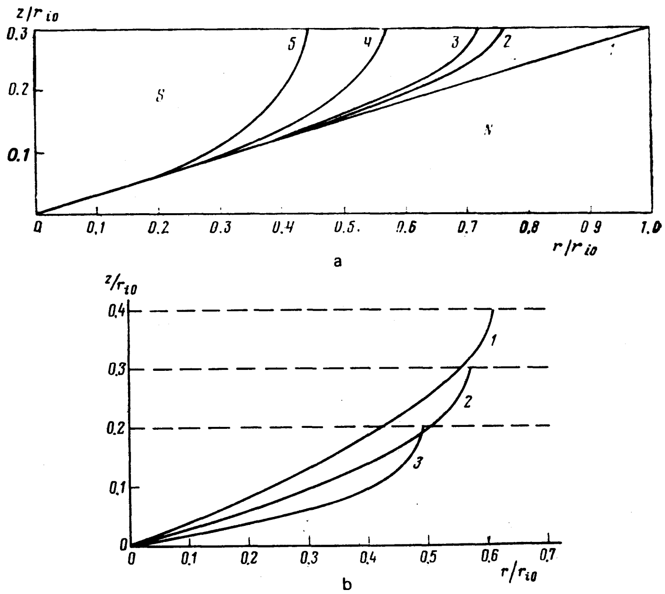

FIG. 5. Equilibrium phase interfaces. a) Results obtained for $a/2r_{i0}=0.3$ and different values of the surface tension: 1) original London structure ($\Delta=0$); 2) $\Delta/2r_{i0}=1.7\times10^{-5}$; 3) $\Delta/2r_{i0}=3\times10^{-5}$; 4) $\Delta/2r_{i0}=10^{-4}$; 5) $\Delta/2r_{i0}=3\times10^{-4}$. b) Results for $\Delta/2r_{i0}=10^{-4}$ and different values of the structure period: 1) $a/2r_{i0}=0.4$, 2) $a/2r_{i0}=0.3$; 3) $a/2r_{i0}=0.2$.

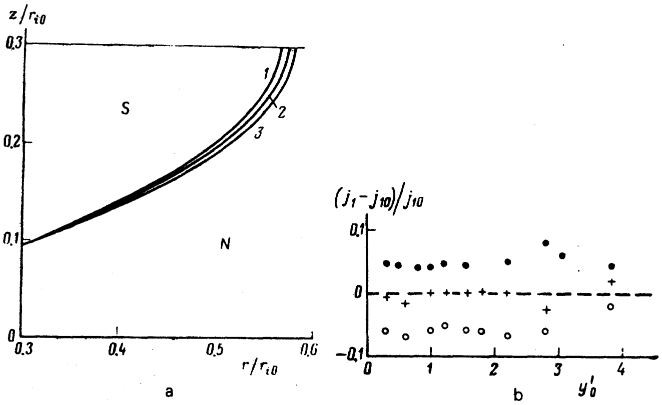

FIG. 6. a) Boundaries of superconducting layers calculated for $\Delta/2r_{i0}=10^{-4}$ and $a/2r_{i0}=0.3$. Curve 1 represents the shape of the boundary when the magnetic field at each point is less than that given by the condition (4), curve 2 corresponds to the equilibrium shape of the boundary, and curve 3 to the case when the magnetic field is always greater than the equilibrium value. b) Dependence of $(j_1-j_{10})/j_{10}$ on y'_0, calculated for the boundaries in Fig. 6a: ○) curve 1; +) curve 2; ●) curve 3.

We shall now consider in some detail the precision of the calculations because this is a subject of very great importance in numerical calculations. Figure 6(a) shows three different interfaces and Fig. 6(b) gives, for the same interfaces, the deviations of j_1 from j_{i0}, showing that the condition (4) is satisfied by H. The difference between the absolute values of the magnetic field at $r=r_{i0}$ for interfaces 1 and 3 is about $0.015H_c$. This figure allows us to estimate the precision of the calculations, which amounts to about 5×10^{-3} in respect of r/r_{i0}. It also follows from Fig. 6(b) that an interface of this kind is stable. In fact, since the displacement of the interface from position 2 to position 3 increases the magnetic field, a force appears which tends to return the interface to its equilibrium position. A similar effect is observed when the interface is displayed toward curve 1.

The shape of superconducting regions obtained as a result of such calculations corresponds to the case when the magnetic field along the phase interface varies in accordance with the condition (4). The absolute value of the magnetic field satisfies the condition (4) when r_{i0} is linked to the difference between the potentials across

TABLE I.

$10^4\Delta/2r_{i0}$	$a/2r_{i0}$	r_{i0}/r_0	r_i/r_{i0}	γ	h_m	R/R_n	F
0.17	0.3	0.68	0.76	0.99	0.94	0.64	0.211
0.3	0.3	0.66	0.73	0.99	0.93	0.68	0.281
1	0.4	0.58	0.6	0.98	0.86	0.75	0.783
1	0.3	0.61	0.57	0.98	0.86	0.75	0.625
1	0.2	0.62	0.49	0.96	0.82	0.77	0.635
3	0.3	0.56	0.45	0.94	0.76	0.82	1.28

Note. The following notation is used above: r_0 is the radius of the sample; r_{i0} is the radius of the London structure; r_i is the radius of the rounded structure; $\gamma=H_0/H_c$, where H_0 is the average magnetic field at the boundary of the intermediate state region; h_m is the minimum magnetic field in the normal state of a metal in units of H_c (see Fig. 8); F is the free energy per unit length of the sample (in units of $H_c^2/8\pi$).

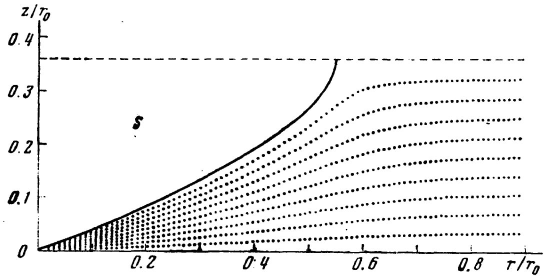

FIG. 7. Lines of constant electric potential calculated for $\Delta/2r_{i0}=10^{-4}$ and $a/2r_{i0}=0.4$.

a sample. As pointed out earlier, Eq. (1) cannot be used to determine r_{i0} in the case of a rounded structure because we then no longer have that relationship between the current and voltage across the sample which is used to derive Eq. (1).

The voltage across the sample is governed by the current density in the normal metal outside the intermediate-state region. The current flowing through this region is $\gamma I_c r_i/r_0$, where I_c is the critical current through the sample and the value γ represents the deviation of the average field on the boundary of the intermediate-state region from H_c and, in general, we have $0<1-\gamma\ll1$. The voltage across the sample can, consequently, be written in the form

$$V=\left(I-\gamma\frac{r_i}{r_0}I_c\right)\frac{r_0^2}{r_0^2-r_i^2}R_n,$$

and the value r_{i0} can be related to the voltage across the sample (see also Ref. 1) by

$$r_{i0}=R_nI_cr_0/2V. \qquad (8)$$

The relationship between r_{i0} and r_i is found as a result of calculations of the shape of the layers and is given in Table I.

The form of the solution generally depends on three parameters, which are the surface tension (governed, in the present case, by the ratio $\Delta/2r_{i0}$), the structure period a, and the proximity of the surface of the sample to the intermediate-state region. However, it is found that, for any current through the sample, the radius of an intermediate-state region r_i is much smaller than r_0 (Table I) and, consequently, the influence of the surface of the sample on the shape of the layers can be ignored. It follows that the solutions obtained are valid for any value of the current through the sample. The resistance of the sample in the $I>I_c$ case can be found by calculating simply the value of r_{i0} in accordance with Eq. (8). Of the two other parameters, the value of $\Delta/2r_{i0}$ is governed by the experimental conditions and the structure period a is, in principle, a free parameter. If such a periodic structure is encountered experimentally, the structure period corresponds to the minimum value of the free energy of the structure. The value of a can be estimated in the $\Delta/2r_{i0}=10^{-4}$ case from the calculations carried out for three different values of $a/2r_{i0}$ (Table I and Fig. 5(a)). The structure with $a/2r_{i0}=0.3$ has a somewhat lower free energy but all the measurable parameters of these structures are practically identical. Therefore, for the other values of the parameter $\Delta/2r_{i0}$, the problem was only solved for the $a/2r_{i0}=0.3$ case.

At first sight, it may seem surprising that the surface tension at the phase interface, which—generally speaking—is small, can alter the structure so considerably. A point to remember is that the surface tension, no matter how small, rounds off a corner of a superconducting layer (Fig. 3). This gives rise to a part of the interface which is parallel to the surface of the sample and the condition of continuity of the tangential component of the electric field on the interface results in a redistribution of the currents in the normal phase in such a way that there is an increase in the current flowing into this region. This is illustrated in Fig. 7, which shows—for one of the calculated cases—the lines of constant electric potential. Thus, it is the need to satisfy simultaneously the condition (4) for the magnetic field and the Laplace equation for the electric field that is responsible for such a considerable difference between the equilibrium and London structures. On the other hand, it is clear that, as the parameter $\Delta/2r_{i0}$ tends to zero, the equilibrium structure tends to the London form although calculations indicate that even when the diameter of the sample is tens of centimeters, the difference between the equilibrium and London structures is still considerable.

DISCUSSION OF RESULTS

It is clear from Figs. 4 and 5 and Table I that the radius of the region occupied by the intermediate state in a rounded structure is considerably less than that given by Eq. (1). This means that in the normal metal with $r>r_i$, there is, as in the Gorter structure, a region where the magnetic field is less than the critical value (Fig. 8). It follows that, in principle, the periodic structure is unstable in the case of nucleation of new superconducting layers in the region where $H<H_c$. On the other hand, it is known that the normal state of type I superconductor can be fairly strongly supercooled. In this case, the important factor is the relationship between the frequency ν of nucleation of new superconducting layers and their lifetime τ in the sample. If $\nu\tau\ll 1$, the structure can be regarded as purely periodic and all the above results apply. However, if $\nu\tau>1$, i.e., when several cylindrical superconducting layers coexist in the sample, the structure is of the Gorter type. We can also have a combined structure if $\nu\tau\lesssim 1$. In the latter case, the central part of the sample has a region occupied by a periodic structure whose parameters are close to those calculated above but new superconducting layers which are from time to time nucleated in the region $r>r_i$ may reduce the resistance of the sample quite considerably.

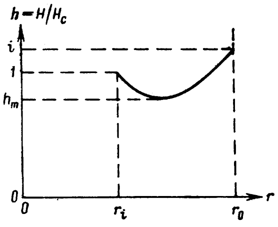

FIG. 8. Schematic distribution of the magnetic field in a normal metal outside the intermediate-state region.

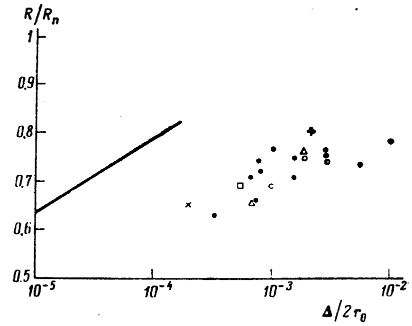

FIG. 9. Dependence of a resistance jump $I=I_c$ on the surface tension. The continuous curve is calculated. The points represent various experimental data: +) Ref. 9; ●) Ref. 10; △) Ref. 11; □) Ref. 12; ×) Ref. 13; ○) Ref. 14.

Figure 9 shows the dependence of R/R_n on $\Delta/2r_0$, calculated for a periodic structure carrying a current $I=I_c$; the points in this figure are the experimental data. It should be mentioned that the approximate linearity of the calculated dependence is purely accidental. Clearly, at high values of $\Delta/2r_0$, the calculated curves should tend asymptotically to unity and, at low values, it should tend to 0.5. We can see from Fig. 9 that the experimental ratios R/R_n are much smaller than the calculated values. This means that the real structure is either of the pure Gorter type or a combination of the Gorter and periodic structures with $\nu\tau\sim 1$. A calculation of the resistance of such a combined structure for various values of $\nu\tau$ is very difficult and, therefore, it was not attempted.

We shall now consider the Gorter structure in somewhat greater detail. In general, this structure should be absolutely unstable in the presence of a longitudinal magnetic field if several cylindrical superconducting layers moving toward the axis are present simultaneously in the sample. In fact, a magnetic field (which may be negligible) should, in this case, be created by the successive motion of superconducting layers. As shown earlier,[13] this process should produce a region in the central part of the sample where the intensity of a longitudinal magnetic field is much higher than the external value. This region is surrounded by a layer of the two-dimensional mixed state in type I superconductors. One should stress that the external longitudinal magnetic field influences only the time taken to establish an equilibrium configuration, i.e., if the Gorter structure appears in a sample on switching on the current, it necessarily results in the penetration of a longitudinal magnetic field into the sample and produces a structure discussed in detail earlier.[13]

It is interesting to note that the processes occurring in a sample have much in common in the Gorter and combined structure cases. Although the structures which exist in the central part of the sample are different in these two cases, outside this region new superconducting layers are nucleated from time to time and move toward the center of the sample.

The experimental data on the jump in the resistance at $I=I_c$ thus indicate that either the Gorter or combined structure is formed in reality. It would be possible to determine which of these two structures does form by, for example, measuring the longitudinal magnetic field inside the sample. The existence of the paramagnetic effect and the tendency of the external longitudinal magnetic field to approach zero would support the Gorter structure. Among the published data, one should mention the work of Hejnowicz and Makiej[15] reporting a discovery of the paramagnetic effect in tin samples for $H_{\parallel}/H_c < 0.006$. Their measurements were carried out using a bismuth magnetic-field sensor, which could be moved in a special aperture at right-angles to the axis of the sample.

Interesting data on the nature of the intermediate-state structures can be obtained by measuring the alternating component of the voltage across a sample whose superconductivity is destroyed by a constant current. Such measurements were recently carried out by Watson and Huebener[14] on indium wires of various diameters. In the case of samples 0.17 mm in diameter, they found an alternating component of the voltage and recorded its spectrum in the frequency range from 20 Hz to 10 kHz. In general, the voltage across the sample could fluctuate also in the presence of a periodic structure moving along the axis. However, in such a case, the amplitude of the alternating voltage $V_\sim$ would be of the order of $aV_=/l$ (here, $V_=$ is the dc voltage across the sample, a is the structure period, and l is the distance between the potential contacts). However, Watson and Huebener[14] discovered that the ratio $V_\sim/V_=$ did not change when l was varied by a factor of 20, which was evidence of the Gorter nature of the structure. One should point out here that the pure Gorter and combined structures should give practically the same results because the rate of contraction of the central region (appearing in the Gorter structure) could be slow compared with the velocity of superconducting regions.

The low value of $V_\sim$ may also be due to the poor quality of the samples (the samples used were prepared by die extrusion). On the other hand, the small value of $V_\sim$ may indicate that the nucleation of new superconducting layers is hindered by some factor or other and that, for a long time, the sample consists only of a system of layers of the two-dimensional mixed state (in the Gorter structure case) or only of a region of the periodic structure (in the combined case). The voltage across the sample is then practically constant or rises slowly for most of the time and drops suddenly at the moment of nucleation of a new superconducting layer. The time dependence of the voltage across a sample in this case is shown schematically in Fig. 10. Here, T_1 can be considerably greater than T_2. These assumptions allow us to ascertain quite easily the nature of the dependence of $V_\sim$ on the frequency f. In fact, if $T_1+T_2>1/\Delta f$ (Δf is the pass band of the measuring amplifier), the frequency dependence of $V_\sim$ can be found simply by spectral expansion of a rectangular pulse of duration T_2 averaging over the various values of T_2. If T_2 has a Gaussian distribu-

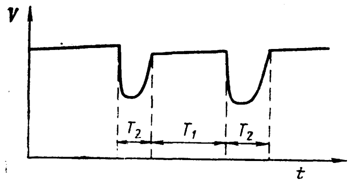

FIG. 10. Schematic representation of the time dependence of the voltage.

tion

$$n(T)=\frac{1}{\sigma(2\pi)^{1/2}}\exp\left[-\frac{1}{2}\left(\frac{T-T_{20}}{\sigma}\right)^2\right] \tag{9}$$

[the quantity $n(T)$ represents the probability that a given pulse is of duration T], we then obtain

$$V_\sim(f)=\frac{A}{\sigma}\int_0^\infty\left|\frac{\sin\omega T}{\omega}\right|\exp\left\{-\frac{1}{2}\left(\frac{T-T_{20}}{\sigma}\right)^2\right\}dT, \tag{10}$$

where $\omega=2\pi f$. Figure 11 gives, on a double logarithmic scale, the experimental results of Watson and Huebener[14] as well as a curve which calculated from Eq. (10) assuming that $T_{20}=3.5$ msec and $\sigma/T_{20}=0.23$ and which represents the dependence of the power (proportional to $V_\sim^2$) on the measurement frequency. It should be noted that variation in the values of T_{20} and A in Eq. (10) results, in the case of double logarithmic coordinates, in a parallel transfer of the calculated curve without change in its shape. The good agreement between the calculated and experimental results confirms the validity of the assumptions made but the value $T_{20}=3.5$ msec seems to be too large for the sample in question.

We shall also consider the experimental evidence on the influence of a transverse magnetic field on the processes occurring during the destruction of superconductivity by a current. As pointed out, such destruction produces an intermediate-state region covered by a normal metal layer of finite thickness. However, the application of a transverse magnetic field makes it possible to displace the intermediate-state region to that surface of the sample where the external magnetic field and the magnetic field created by the current

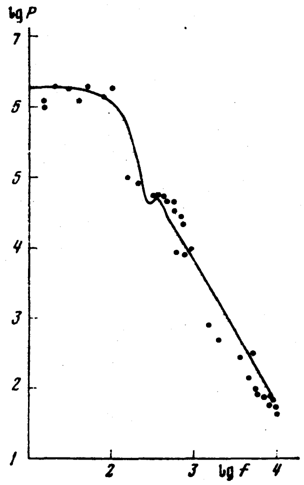

FIG. 11. Dependence of the logarithm of the power (in relative units) on the logarithm of the frequency in hertz. The continuous curve is calculated using Eq. (10) and the points are the results taken from Ref. 14.

have opposite directions. We can easily show that, as long as the intermediate-state region is inside the sample, there are no distortions of the structure and it is displaced as a whole a distance

$$\Delta r=\frac{H_\perp}{H_c}\frac{I_c}{I}\frac{R_n}{R}r_0 \tag{11}$$

($H_\perp$ is the external transverse magnetic field). The intermediate-state region should emerge on the surface of the sample for

$$\Delta r+r_i=r_0. \tag{12}$$

The quantity r_i in the above formula represents the radius of the intermediate-state region in the pure periodic case, whereas, for the Gorter or combined structures, the value of r_i is the maximum radius of the intermediate-state region.

Investigations of the influence of a transverse magnetic field on the destruction of superconductivity by an electric current have been carried out by a number of workers (see, for example, Refs. 15–18). In particular, it has been found that the emergence of the intermediate state on the surface of a sample occurs in transverse fields much higher than those predicted by simple periodic structure models. For example, for $I=I_c$, the external field required is $H_\perp>0.12H_c$ (Ref. 18). Using this value of $H_\perp$ and assuming that $R/R_n=0.7$ (which is a typical ratio), we find from Eqs. (11) and (12) that $r_i/r_0=0.8$, which represents a reasonable value for the maximum radius of a Gorter-type structure.

The fact that a periodic structure appears on the surface of a sample subjected to a transverse magnetic field[16–18] is in no way a conflict with the ideas put forward above because a Gorter-type structure should naturally be converted to periodic under the action of a transverse field. It follows that the emergence of a structure on the surface of a sample should be accompanied by a radical change in this structure. Hence, it follows that the emergence of the intermediate state on the surface of a sample under the influence of a transverse field should be accompanied by considerable hysteresis.[2)] Clearly, the resistance of a sample should also change somewhat at the moment when such a structure emerges on the surface.

Among other experimental data, we may mention here the absence of the intermediate state in a hollow cylindrical sample, investigated by the present author.[19] Use was made[19] of a single-crystal hollow cylinder with an outer diameter of 8 mm and an inner diameter of 4 mm. The current-voltage characteristics showed that the intermediate state did not even appear at $I=I_c$. This apparently strange behavior can be explained naturally on the basis of the above analysis. In fact, for an indium sample of diameter 8 mm, the relevant parameter is $\Delta/2r_0=5\times10^{-5}$ even at the lowest temperatures so that the diameter of the intermediate-state region (Table I) is about 3 mm, i.e., it is considerably less than the radius of the internal cavity of the cylinder.

We can summarize the situation as follows.

1. The values of the resistance of a sample with a periodic structure appearing on destruction of superconductivity by a current, calculated above, represent the upper limit. Other possible intermediate-state structures should be characterized by lower resistances.

2. The experimental values of R/R_n indicate that either a pure Gorter-type structure or a combination of the Gorter and periodic structures appears in reality. This conclusion is in agreement with measurements of the alternating component of the voltage across a sample.[14] The paramagnetic effect in very high external longitudinal magnetic fields[15] also supports the Gorter structure explanation. On the other hand, in samples of much larger diameter, there may be rounded periodic structures.

3. The influence of a transverse magnetic field on the destruction of superconductivity by a current, which is difficult to explain by the existence of a London-type periodic structure, can be accounted for very naturally if we assume the existence of a Gorter-type structure.

4. There are apparently no experimental results which would conflict directly or indirectly with the ideas put forward above.

The author is grateful to Yu. V. Sharvin for numerous fruitful discussions and continuous interest, to A. F. Andreev for discussing the question considered above, and to all the members of the Computing Center of our Institute, whose goodwill has been a great help in completing this investigation.

[1)]From this point of view, it seems to be completely incorrect to state[5] that, in the London structure, the magnetic field at the phase interface is weak in the central part of the sample and vanishes on the axis (see Fig. 1 in Ref. 5). Since there are no explanations of Fig. 1 in Ref. 5, it is not clear what the authors have in mind.

[2)]It should be pointed out that such hysteresis was observed experimentally[17,18] but no special significance was attached to the results and they were not included in the text of the papers.

[1]F. London, Une Conception Nouvelle de la Supraconductibilité, Paris, 1937; F. London, Superfluids, Vol. 1, Wiley, New York, 1950.

[2]C. J. Gorter, Physica (Utrecht) 23, 45 (1957).

[3]C. J. Gorter and M. L. Potters, Physica (Utrecht) 24, 169 (1958).

[4]A. F. Andreev, Zh. Eksp. Teor. Fiz. 54, 1510 (1968) [Sov. Phys. JETP 27, 809 (1968)].

[5]D. C. Baird and B. K. Mukherjee, Phys. Rev. Lett. 21, 996 (1968).

[6]B. K. Mukherjee, J. Low Temp. Phys. 12, 181 (1973).

[7]Yu. V. Sharvin, Zh. Eksp. Teor. Fiz. 38, 298 (1960) [Sov. Phys. JETP 11, 216 (1960)].

[8]V. P. Il'in and S. V. Lebedev, Programma chislennogo resheniya uravneniya Laplasa v dvumernoĭ oblasti (Program for Numerical Solution of the Laplace Equation in Two Dimensions), Novosibirsk, 1972.

[9]N. E. Alekseevskiĭ, Zh. Eksp. Teor. Fiz. 8, 342 (1938).

[10]H. Meissner, Phys. Rev. 109, 668 (1958).

[11]E. Troĭnar, Zh. Eksp. Teor. Fiz. 38, 654 (1960) [Sov. Phys. JETP 11, 470 (1960)].

[12]R. Freud, C. Sulkowski, and B. Makiej, Phys. Lett. A 27, 187 (1968).
[13]I. L. Landau, Zh. Eksp. Teor. Fiz. 64, 557 (1973) [Sov. Phys. JETP 37, 285 (1973)].
[14]H. L. Watson and R. P. Huebener, Phys. Rev. B 10, 4577 (1974).
[15]K. Hejnowicz and B. Makiej, Acta Phys. Pol. A 46, 91 (1974).
[16]A. I. Shal'nikov, Zh. Eksp. Teor. Fiz. 33, 1071 (1957) [Sov. Phys. JETP 6, 827 (1958)].
[17]Yu. V. Sharvin, Proc. Tenth Intern. Conf. on Low-Temperature Physics, Moscow 1966, Vol. 2B, Publ. by VINITI, Moscow, 1967, p. 323.
[18]Yu. V. Sharvin and I. L. Landau, Zh. Eksp. Teor. Fiz. 58, 1943 (1970) [Sov. Phys. JETP 31, 1047 (1970)].
[19]I. L. Landau, Pis'ma Zh. Eksp. Teor. Fiz. 23, 516 (1976) [JETP Lett. 23, 471 (1976)].

Translated by A. Tybulewicz

Optothermodynamic method of diagnostics of the critical point and of the investigation of the equation of state of absorbing liquids

F. V. Bunkin, V. M. Podgaetskiĭ, and M. I. Tribel'skiĭ

P.N. Lebedev Physics Institute, USSR Academy of Sciences
(Submitted 3 July 1978)
Zh. Eksp. Teor. Fiz. 75, 2309–2311 (December 1978)

A new method for the diagnostics of the critical point and for the investigation of the equation of state of an absorbing dielectric liquid is indicated.

PACS numbers: 07.20. – n, 64.30. + t

A method of optothermodynamic action of a laser pulse with a programmed waveform on an absorbing liquid, to bring a liquid is close to the critical point in the focal volume was indicated in Ref. 1. The main shortcomings of this method are the following:

1) the stringent requirements imposed on the waveform of the laser pulse;

2) the short ($\sim r_0/c$, where r_0 is the radius of the focal spot and c is the speed of sound in the liquid) lifetime of the near-critical state;

3) the inapplicability of the method when working with liquids whose critical parameters are unknown.

In the present paper we propose for the investigation of liquids an optothermodynamic method free of these shortcomings. It permits diagnostics of the critical point and investigation of the equation of state of the liquid.

The main difference from Ref. 1 is that now the investigated liquid is placed in a hermetically sealed cell, on the end face of which is incident a homogeneous light beam of intensity $I(t)$ ($I=0$ at $t\leq 0$), so that the problem of the reaction of the liquid to the action of the radiation is one-dimensional (see Fig. 1).[1)] If the laser-pulse duration τ satisfies the condition $c\tau \gg \delta$ (δ is the length of the cell), then the pressure profile established in the liquid on account of the absorption of the laser radiation turns out to be independent of x (the x axis coincides with the propagation direction of the radiation), i.e., it depends only on the time: $p=p(t)$. On the other hand, if $\chi\tau K^2 \ll 1$, where K is the absorption coefficient of the radiation and χ is the thermal diffusivity of the liquid, then the profiles of the temperature T, of the density ρ, of the specific enthalpy w, etc. will depend significantly on x. Thus, on the (p, x) planes, where $X=\rho, T, w\ldots$, the state of the liquid at each instant of time is described by a segment corresponding to a fixed value of $p(t)$ and to a continuum of values of X from a certain interval $X_m(p)\leq X\leq X_M(p)$. During the entire time of action of the radiation pulse the aggregate of the state in which the liquid is situated will occupy on the (p, X) plane a certain two-dimensional region (phase space) bounded by the phase curves $X=X_m(p), X=X_M(p)$ and by the segment $p=p_M$ corresponding to the maximum pressure produced in the liquid at the end of the flash.

In the case $K\delta \gg 1$ the problem has an analytic solution

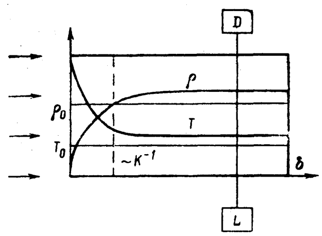

FIG. 1. Schematic profile of the density and temperature of the liquid at a fixed instant of time: L—probing laser, D—detector; the thick line represents the cell.

