Some features of optical nutation in a gas

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A study is made of the oscillations of the amplitude of an electric field near the leading edge of a rectangular pulse due to reemission of photons by resonant atoms of molecules in a gas (nutation effect). Optical nutation due to a pair of pulses is also considered. Allowance is made for the level degeneracy associated with various projections of the total momentum and for the thermal motion of atoms and irreversible relaxation. It is shown that a change of the input-pulse polarization from linear to circular makes nutational oscillations dependent on the nature of the atomic resonance transition. In the absence of irreversible relaxation the change from linear to circular polarization is accompanied by an increase in the nutation period and slower decay of the amplitude of nutational oscillations if the resonance transition is of the $J \to J$ type. The reverse is true of $J \rightleftarrows J+1$ atomic transitions. These relationships can be used in experimental determinations of the type of atomic transition. For high values of the momentum J the nutation period and the nature of decay of the nutational oscillation amplitude are independent of J but are governed by the reduced dipole moment of the resonance transition. A comparison of the theoretical and experimental nutational oscillation curves makes it possible to determine this dipole moment. When the nutation period is comparable with the irreversible relaxation time, the nutation amplitude decay is basically exponential and this makes it possible to calculate the relaxation time of a resonant medium.

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The optical nutation consists in oscillations of the amplitude of an electric field near the leading edge of a rectangular light pulse passing through a resonant medium. This effect is analogous to the familiar nutation in NMR when an ensemble of nuclear spins subjected to a static magnetic field interacts with a weak alternating magnetic field of the resonance frequency.[1] Optical nutation is due to the fact that the majority of resonant molecules is transferred to an excited state in the leading edge of a high-power light pulse. These molecules then emit induced radiation under the action of the same pulse and drop to a lower energy state. This reemission process is repeated so that the amplitude of the resultant field is modulated at a frequency Ω_n, which is known as the nutation frequency. This effect can be observed if the nutation period $2\pi/\Omega_n$ is long compared with the leading edge of an input pulse but short compared with the irreversible relaxation time, so that the interaction with a medium remains coherent.

Optical nutation was predicted by Tang and Statz[2] and then observed experimentally by Hocker and Tang.[3] However, in the latter case,[3] the leading edge of the light pulse was comparable with the nutation period and the pulse itself was far from rectangular and it was absorbed strongly in the investigated medium. More recently, Alimpiev and Karlov[4-6] suggested a method for bypassing these difficulties: they illuminated a resonant medium with two superimposed light pulses of the same carrier frequency but of different durations and intensities. The first pulse was very short but of high intensity and it acted as a characteristic perturbation after which optical nutation was observed against the background of the longer second pulse of lower intensity. The experimental investigation of nutation[4-6] made if possible to identify certain molecular transitions and to determine the corresponding dipole moments. Another original method for investigating nutation was suggested by Brewer and Shoemaker.[7]

We shall allow more consistently for the resonance level degeneracy and thermal motion of atoms, and we shall investigate fully the influence of irreversible relaxation on the nutation effect. Our treatment applies to any degree of level degeneracy and to an arbitrary inhomogeneous broadening of a resonance transition. This makes it possible to study nutation in the case of linear and circular polarization of light pulses under on-resonance and slightly off-resonance conditions. The approach reveals new features of optical nutation, which extend the possibilities of using this effect in investigations of resonant media alongside with self-induced transparency[8,9] and with photon echo.[10,11]

1. PRINCIPAL EQUATIONS

We shall consider a gas composed of identical atoms (or molecules) such that one of the atomic transition frequencies ω_0 is close to the carrier frequency ω of a light pulse traveling along the Z axis and having the shape of a step:

$$\mathbf{E}=\begin{cases} \mathbf{l}ee^{i(kz-\omega t)}+\text{c.c.}, & 0\leqslant t-z/c, \\ 0, & t-z/c<0. \end{cases} \tag{1}$$

Here, $\mathbf{E}$ is the electric field intensity; $\mathbf{l}$ is a unit polarization vector of the light wave; e is a slowly varying amplitude of the wave; $\omega=kc$ and $|\omega-\omega_0|\ll\omega_0$. Here, $\mathbf{l}=\mathbf{l}_x$ for linear polarization and $\mathbf{l}=2^{-1/2}(\mathbf{l}_x+i\mathbf{l}_y)$ for right-handed circular polarization; $\mathbf{l}_x$ and $\mathbf{l}_y$ are unit vectors along the indicated Cartesian axes.

We shall use $\rho_{mm'}$ and $\rho_{\mu\mu'}$ for the density matrices representing the state of an atom at the lower and upper levels whose energies are ε_1 and ε_2 and total momenta are J_1 and J_2, respectively ($\varepsilon_2-\varepsilon_1=\hbar\omega_0$). Let $\rho_{\mu m}$ be the density matrix describing transitions between these two levels, whose degeneracy is due to the projections μ

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and m of the total momenta. Propagation of an ultrashort pulse (1) in such a resonant medium can be described by the d'Alembert equation

$$\left(\frac{\partial^2}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{E}=\frac{4\pi}{c^2}\frac{\partial^2}{\partial t^2}\int \mathrm{Sp}\,\mathbf{P}\,dv \tag{2}$$

and by the quantum-mechanical equation for the density matrix ρ

$$\left(\frac{\partial}{\partial t}+v\frac{\partial}{\partial z}\right)\rho=\frac{i}{\hbar}[\rho(H-\mathbf{Ed})-(H-\mathbf{Ed})\rho]-\hat{\Gamma}\rho. \tag{3}$$

Here, H is the Hamiltonian of an atom in a system based on its center of inertia; $\mathbf{p}=\rho\mathbf{d}$ is the operator of the polarization of the medium, $\mathbf{d}$ is the operator of the dipole moment of the resonance transition; the last term on the right-hand side of Eq. (3) allows for irreversible relaxation in accordance with the formulas

$$(\hat{\Gamma}\rho)_{mm'}=\Gamma_1\rho_{mm'},\quad (\hat{\Gamma}\rho)_{\mu\mu'}=\Gamma_2\rho_{\mu\mu'},\quad (\hat{\Gamma}\rho)_{\mu m}={}^1/_2(\Gamma_1+\Gamma_2)\rho_{\mu m},$$

where $1/\Gamma_1$ and $1/\Gamma_2$ are the relaxation times of an excited atom at the lower and upper levels, respectively. The values of Γ_1 and Γ_2 are governed by collisions and radiative decay.

The density matrix $\rho=\rho(z,t)$ applies to the atoms moving along the Z axis at a velocity v. Before the passage of a light pulse $t-z/c\leqslant 0$ the elements of the density matrix satisfy the relationships

$$\rho_{mm'}(z,t)=\frac{n_1 f(v)}{2J_1+1}\delta_{mm'},\quad \rho_{\mu\mu'}(z,t)=\frac{n_2 f(v)}{2J_2+1}\delta_{\mu\mu'},\quad \rho_{\mu m}(z,t)=0 \tag{4}$$

for $t-z/c\leqslant 0$, $0\leqslant z$. Here, the point $z=0$ corresponds to the boundary of the medium; n_1 and n_2 are the densities of atoms at the lower and upper levels in the absence of an external field; $f(v)$ is the Maxwellian distribution function

$$f(v)=\frac{1}{\pi^{1/2}u}\exp\left(-\frac{v^2}{u^2}\right), \tag{5}$$

where u is the average thermal velocity of atoms. Equation (4) is derived on the assumption that the atoms are distributed homogeneously in space and that the Zeeman sublevels have all the same populations before the passage of a light pulse.

The polarization of an ultrashort pulse (1) does not change during its propagation in the medium. This allows us to obtain the equation for the pulse amplitude by equating the factors in front of the exponential function $e^{i(kz-\omega t)}$ on both sides of Eq. (2). It is convenient to use

$$\rho_{\mu m}d^{\alpha}_{m\mu'}l_\alpha^*=p_{\mu\mu'}e^{i(kz-\omega t)},$$

where $p_{\mu\mu'}$ is a slow function of time, and the repeated matrix and vector indices always imply summation.

It follows from Eqs. (2) and (3) that the equations for slow functions have the following form in the resonance approximation

$$\left(\frac{\partial}{\partial t}+c\frac{\partial}{\partial z}\right)e=i2\pi\omega\int p_{\mu\mu}\,dv, \tag{6}$$

$$\left(\frac{\partial}{\partial t}+i(kv-\Delta\omega)+\frac{\Gamma_1+\Gamma_2}{2}\right)p_{\mu\mu'}+\frac{i}{\hbar}|d(J_2,J_1)|^2e(N_{2\mu\mu'}-N_{1\mu\mu'})=0, \tag{7}$$

$$\left(\frac{\partial}{\partial t}+\Gamma_1\right)N_{1\mu\mu'}-\frac{i}{\hbar}(e^*\Pi_{\mu\mu''}p_{\mu''\mu'}-ep^+_{\mu\mu''}\Pi_{\mu''\mu'})=0, \tag{8}$$

$$\left(\frac{\partial}{\partial t}+\Gamma_2\right)N_{2\mu\mu'}+\frac{i}{\hbar}(e^*p_{\mu\mu''}\Pi_{\mu''\mu'}-ep^+_{\mu\mu''}\Pi_{\mu''\mu'})=0, \tag{9}$$

where

$$N_{1\mu\mu'}=d^{\alpha}_{\mu m}\rho_{mm'}d^{\beta}_{m'\mu'}l_\beta^*l_\alpha/|d(J_2,J_1)|^2,$$

$$N_{2\mu\mu'}=\rho_{\mu\mu''}d^{\alpha}_{\mu''m}d^{\beta}_{m\mu'}l_\beta^*l_\alpha/|d(J_2,J_1)|^2,$$

$$\Pi_{\mu\mu'}=d^{\alpha}_{\mu m}d^{\beta}_{m\mu'}l_\beta^*l_\alpha/|d(J_2,J_1)|^2,\quad \Delta\omega=\omega-\omega_0.$$

Here, $d(J_2,J_1)$ is the reduced dipole moment of a $J_2\to J_1$ atomic transition, related to the probability γ of spontaneous emission of a photon $\hbar\omega_0$ from an isolated atom by the expression

$$\gamma=4|d(J_2,J_1)|^2/3(2J_2+1)\hbar\lambda^3,\quad \lambda=c/\omega.$$

The term kv on the left-hand side of Eq. (7) allows for the Doppler frequency change during the motion of an atom.

The initial and boundary conditions for Eqs. (6)–(9) are, in accordance with Eqs. (4) and (5),

$$e(z,t)=p_{\mu\mu'}(z,t)=0,\quad N_{1\mu\mu'}(z,t)=\frac{n_1 f(v)}{2J_1+1}\Pi_{\mu\mu'},\quad N_{2\mu\mu'}=\frac{n_2 f(v)}{2J_2+1}\Pi_{\mu\mu'}\ \text{ for } t\leqslant\frac{z}{c}; \tag{10}$$

$$e(0,t)=a(t)\ \text{ for } 0\leqslant t, \tag{11}$$

where $a(t)$ is the profile of a step-like pulse entering the medium.

In the case of linear polarization the quantization axis can be taken parallel to 1 and for the circular polarization along the direction of propagation of the wave. Then, the principal matrix $\Pi_{\mu\mu'}$ becomes diagonal

$$\Pi_{\mu\mu'}=\Pi_\mu\delta_{\mu\mu'} \tag{12}$$

with the following diagonal elements[12,13]:

a) for the linear polarization

$$\Pi_\mu=\frac{\mu^2}{J(J+1)(2J+1)}\ \text{ for } J_2=J\to J_1=J, \tag{13}$$

$$\Pi_\mu=\frac{(J+1)^2-\mu^2}{(J+1)(2J+1)(2J+3)}\ \text{ for } J_2=J\to J_1=J+1 \text{ and } J_2=J+1\to J_1=J; \tag{14}$$

b) for the right-handed circular polarization

$$\Pi_\mu=\frac{(J+\mu)(J-\mu+1)}{2J(J+1)(2J+1)}\ \text{ for } J_2=J\to J_1=J, \tag{15}$$

$$\Pi_\mu=\frac{(J-\mu+1)(J-\mu+2)}{2(J+1)(2J+1)(2J+3)}\ \text{ for } J_2=J\to J_1=J+1, \tag{16}$$

$$\Pi_\mu=\frac{(J+\mu)(J+\mu+1)}{2(J+1)(2J+1)(2J+3)}\ \text{ for } J_2=J+1\to J_1=J. \tag{17}$$

If a left-handed polarized circular wave is used, the sign in front of μ on the right-hand side of Eqs. (15)–(17) should be reversed.

When the matrix $\Pi_{\mu\mu'}$ has the diagonal form (12), the diagonal elements of the required matrices

$$p_{\mu\mu}=p_\mu,\quad N_{1\mu\mu}=N_{1\mu},\quad N_{2\mu\mu}=N_{2\mu}$$

form an independent closed system of equations. For simplicity, we shall solve it in the constant-field ap-

proximation ignoring the reaction of the medium on the transmitted light pulse. This is possible if

$$2\pi^{1/2}L|N|T_0|d(J_2,J_1)|^2/\hbar\chi\ll 1,$$

where $N=n_1/(2J_1+1)-n_2/(2J_2+1)$ is the initial density of the change in the population of the Zeeman sublevels; L is the length of gaseous medium; $T_0=1/ku$ is the time of reversible Doppler relaxation.

In the constant-field approximation the amplitude in Eqs. (7)–(9) can be replaced with the profile of the pulse entering the medium $e=a(t-z/c)$ and the equations become

$$\left(\frac{\partial}{\partial t}+\frac{\Gamma_1+\Gamma_2}{2}+i(kv-\Delta\omega)\right)p_\mu+\frac{i}{\hbar}|d(J_2,J_1)|^2a(N_{2\mu}-N_{1\mu})=0, \quad (18)$$

$$\left(\frac{\partial}{\partial t}+\Gamma_1\right)N_{1\mu}-\frac{i}{\hbar}\Pi_\mu(a^*p_\mu-ap_\mu^*)=0, \quad (19)$$

$$\left(\frac{\partial}{\partial t}+\Gamma_2\right)N_{2\mu}+\frac{i}{\hbar}\Pi_\mu(a^*p_\mu-ap_\mu^*)=0. \quad (20)$$

Having determined the function p_μ from Eqs. (18)–(20) using Eqs. (10) and (11), we can easily apply Eq. (6) to find the amplitude $e_n=e_n(t-z/c)$ of the intensity of an electric field due to the reemission from resonant molecules:

$$e_n=2i\pi\chi^{-1}L\int\sum_{\mu=-J_2}^{J_2}p_\mu dv. \quad (21)$$

This quantity is taken at the exit from the medium. For $0\leq t-z/c$ it describes optical nutation with allowance for the level degeneracy, thermal motion of atoms, and irreversible relaxation. The amplitude e of the total electric field is $e=a+e_n$.

The intensity I of an electromagnetic pulse at the exit from the medium is

$$I=I_0(1+2e_n'/e_0),$$

where $I_0=ce_0^2/2\pi$ is the time-average intensity of the light pulse at the entry to the medium for linear or circular polarizations, and the prime denotes the real part of the amplitude (21).

2. CASE OF A SINGLE RECTANGULAR PULSE

Let us assume that a light pulse (1) with a constant real amplitude e_0 enters a resonant medium. If we ignore the irreversible relaxation, Eqs. (18)–(20) are easily integrated and the real part of the amplitude (21) for $0\leq t-z/c$ is

$$e_n'=-E_0\int_{-\infty}^{\infty}\exp(-\eta^2)\sum_{\mu=-J_2}^{J_2}\frac{\Pi_\mu}{\Omega_\mu T_0}\sin\Omega_\mu\left(t-\frac{z}{c}\right)d\eta, \quad (22)$$

where the parameter Π_μ is defined in Eqs. (13)–(17) for the linear and circular polarization cases, and the other quantities are

$$E_0=2\pi^{1/2}LNT_0|d(J_2,J_1)|^2e_0/\hbar\chi,$$
$$\Omega_\mu^2=(\eta/T_0-\Delta\omega)^2+(2J_2+1)\Pi_\mu\Omega^2,$$
$$\Omega=2|d(J_2,J_1)|e_0/\hbar(2J_2+1)^{1/2}=(3\gamma\chi^2/\hbar)^{1/2}e_0.$$

In the case of linear polarization and the atomic $J\to J$ transition with large values of the momentum $J\gg 1$ it is possible to sum the series in Eq. (22) for $\Delta\omega=0$ and $\Omega T_0\gg 1$, expressing it in terms of a Bessel function of order $\frac{3}{2}$:

$$e_n'=-\frac{E_0}{4\pi^{1/2}}\frac{t-z/c}{T_0}\left(\frac{2\pi}{\Omega(t-z/c)}\right)^{1/2}J_{1/2}\left(\Omega\left(t-\frac{z}{c}\right)\right), \quad \Omega\left(t-\frac{z}{c}\right)\ll J. \quad (23)$$

Numerical calculations show that the formula (23) applies also to $J\rightleftarrows J+1$ atomic transitions excited by a light pulse with circular polarization. The period of nutations described by Eq. (23) can be used to determine γ because other quantities can be found experimentally. The value of γ is independent of J for $J\gg 1$, since the reduced dipole moment $d(J,J)$ is proportional to $J^{1/2}$ (Ref. 14). However, the factor E_0 increases on increase in J because of enhancement in the change in the population of the active levels $2JN\propto(n_1-n_2)$. The decay of nutation (23) is solely due to the resonant level degeneracy.

An analysis of a large number of graphs of the amplitude (22) reveals the following features. If the intensity of an input pulse (1) is constant and the $J\to J$ atomic transition is characterized by large momenta $J\gg 1$, the nutation period in the case of linear polarization is approximately $2^{1/2}$ times shorter than in the circular polarization case. The amplitude of nutation in the linear polarization case is less and it decays more rapidly than in the circular case. For $J\rightleftarrows J+1$ ($J\gg 1$) transitions the nature of nutation in the linear and circular polarization cases is diametrically opposite to the nature in the case of $J\to J$ transitions (curves 1 and 2 in Fig. 1). This behavior is due to the different probabilities of the emission (absorption) of a photon with linear and circular polarizations as a result of a transition of an atom from one Zeeman sublevel to another $\varepsilon_2 J_2\mu\to\varepsilon_1 J_1 m$,

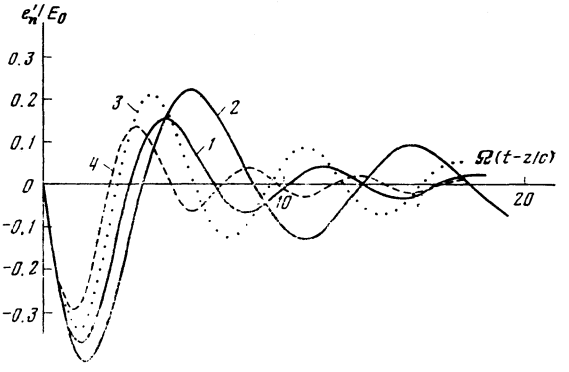

FIG. 1. Optical nutation against background of rectangular pulses in the case of large values of the momentum $J\gg 1$. Curves 1 and 2 correspond to the case when the intensities of the pulses entering a medium are the same for linear and circular polarizations. In the case of a $J\to J$ atomic transition the first curve corresponds to the linear polarization case and the second to the circular case. For a $J\rightleftarrows J+1$ transition these curves are interchanged so that the linear and circular polarizations correspond to curves 2 and 1. If the projections of the electric field intensity of the incident pulses with the linear and circular polarizations are the same, nutation is described by curves 1, 2, 3, and 4. For a $J\to J$ transition, curves 1 and 3 correspond to the linear and circular polarizations. In the case of a $J\rightleftarrows J+1$ transition, the linear and circular polarizations correspond to curves 2 and 4. It is assumed that $T_0=1/\Omega$ and $\Delta\omega=0$.

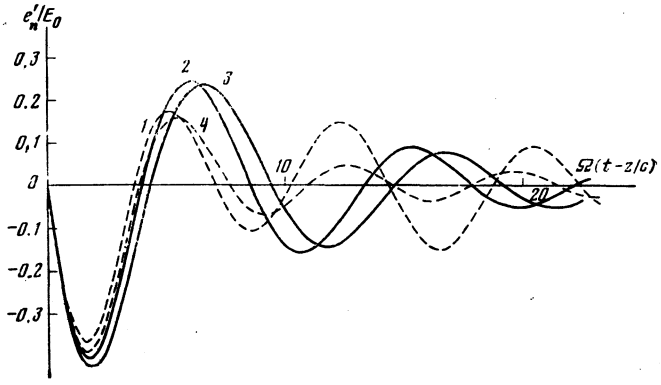

FIG. 2. Optical nutation due to rectangular pulses with identical intensities at the entry to a medium in the case of small momenta. The linear and circular polarizations correspond to curves 1 and 2 for a $2\to 2$ transition and curves 3 and 4 for $2\rightleftarrows 3$ transitions. It is assumed that $T_0=1/\Omega$ and $\Delta\omega=0$.

which is reflected in the term $(2J_2+1)\Pi_\mu\Omega^2$ occurring in Ω_μ^2.

The nature of nutation in the case of small values of J is illustrated in Fig. 2, which gives the results for $J=2$. We can see that for each of the $J_2=2\to J_1=2, J_2=2\to J_1=3$ and $J_2=3\to J_1=2$ transitions the rates of decay of the nutation amplitudes in the linear and circular polarization cases become approximately equal but the periods are still different. The relationships obtained can be used to distinguish experimentally the $J\to J$ and $J\rightleftarrows J+1$ transitions by altering the polarization of a light pulse (1) but keeping its intensity (at the entry to the medium) constant.

An interesting feature of nutation appears when the projections of the electric field intensities of the incident pulses with linear and circular polarizations are the same but the intensities differ by a factor of 2. Then, in the $J\to J$ ($J\ll 1$) atomic transition case the change from the linear to circular polarization reduces slightly the nutation period and results in slower decay of the amplitude. However, in the $J\rightleftarrows J+1$ ($J\gg 1$) case the change from the linear to circular polarization is accompanied by a considerable (by a factor of about 2) reduction in the nutation period (curves 1, 2, 3, and 4 in Fig. 1). This difference between the periods is retained also for small values of J. Once again such behavior can be used in experimental identification of atomic transitions.

For each of the $J\to J$ and $J\rightleftarrows J+1$ atomic transitions and a specific polarization of a light pulse the period and behavior of the nutation amplitude is independent of J in the range $J\gg 1$. The nutation period is governed entirely by the value of $d(J_2, J_1)$ and the amplitude of the incident pulse, and its order of magnitude is $2\pi/\Omega$. Consequently, a comparison of the theoretical curve representing Eq. (22) with an experimental result can be used to find the dipole moment of an atomic transition. For arbitrary values of the momentum J the decay of nutational oscillations is stronger for smaller values of the parameter ΩT_0. In the $\Omega T_0\lesssim 1$ case the decay is due to the degeneracy and the Doppler scatter of the resonant levels.

Figures 1 and 2 give the results obtained for the on-resonance case when $\Delta\omega=0$. However, these results still apply for a slight detuning from resonance. According to Eq. (22), a small detuning $|\Delta\omega|\ll 1/T_0$ is unimportant. If $|\Delta\omega|\leqslant 1/T_0$ and $1/T_0<\Omega$, the spectral composition of that part of a light pulse (1) which corresponds to the optical nutation duration is so wide that it excites the majority of molecules inside a Doppler profile. Therefore, detuning within the range $0\leqslant|\Delta\omega|\leqslant 1/T_0<\Omega$ does not interfere with experimental identification of atomic transitions and determination of γ in accordance with the method described above. However, if the detuning is large $|\Delta\omega|\gg 1/T_0$, only a small proportion of the resonant atoms is excited and the nutation amplitude decreases strongly, compared with the exact on-resonance case.

If the relaxation terms are allowed for in Eqs. (18)–(20), the function (22) has to be multiplied by the factor $\exp[-(\Gamma_1+\Gamma_2)(t-z/c)/2]$, and the frequency Ω_μ has to be replaced with

$$\bar{\Omega}_\mu=2^{-1/2}[\Omega_\mu^2-\Gamma_{12}^2+((\Omega_\mu^2-\Gamma_{12}^2)^2+4\Gamma_{12}^2(\eta/T_0)^2)^{1/2}]^{1/2},$$

where $\Gamma_{12}=(\Gamma_1-\Gamma_2)/2$. The new terms which appear in the amplitude are small if $\Gamma_{12}^2/\Omega_\mu^2\ll 1$.

When both active levels are excited, the parameters Γ_1 and Γ_2 differ only slightly. In particular, for molecular transitions the relaxation time of each of the excited states is usually the same $1/\Gamma_1=1/\Gamma_2=T_r$ (Ref. 4). In this case the influence of irreversible relaxation reduces to multiplication of the amplitude (22) by the factor $\exp[-(t-z/c)/T_r]$. A study of the amplitude of nutation for various moments of atomic transitions and gas temperatures shows that in the absence of irreversible relaxation $1/\Omega\ll T_r$ the oscillation decay is due to the level degeneracy and inhomogeneous broadening of the resonance transition. However, in the range $1/\Omega\gtrsim T_r$ the amplitude of nutational oscillations decays basically in accordance with the exponential law $\exp[-(t-z/c)/T_r]$ because of irreversible relaxation. This is in agreement with the conclusions reached in Refs. 4–6. Variation of the amplitude of the incident pulse makes it possible to satisfy easily the inequalities $1/\Omega\ll T_r$ and $1/\Omega\gtrsim T_r$.

3. EXCITATION OF A MEDIUM BY A PULSE PAIR

We shall assume that a resonant medium is subjected to a pulse of type (1), which is a superposition of two rectangular pulses with the same carrier frequency $\omega=\omega_0$ but different amplitudes. The combined amplitude (11) of a light pulse entering the medium is

$$a(t)=\begin{cases} e_1e^{i\Phi}, & 0\leqslant t\leqslant T_1,\\ e_0, & T_1\leqslant t,\end{cases} \tag{24}$$

where e_1 and e_0 are the constant real amplitudes of the first and second pulses; T_1 is the duration of the first pulse; Φ is a possible constant phase shift of these pulses. The first pulse is very short $T_1<1/\Omega$, but its intensity is high: $e_1\gg e_0$; it excites resonant atoms which then reemit photons during the second longer pulse. Thus, the first pulse acts as an initial perturbation, like the leading edge of a single rectangular pulse

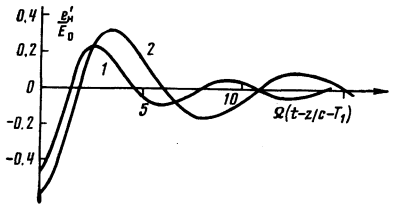

FIG. 3. Optical nutation for a pulse pair. For a $J \to J$ ($J \gg 1$) transition, curves 1 and 2 correspond to the linear and circular polarizations. Conversely, for a $J \rightleftarrows J+1$ ($J \gg 1$) transition, the linear and circular polarizations correspond to curves 2 and 1. It is assumed that $T_0 = 1/\Omega$, $\Omega T_1 = 0.1$, $\Delta\omega = 0$, $e_1 = 25e_0$, $\Phi = 2\pi n$, when $n = 0, \pm 1, \pm 2, \ldots$.

discussed above. Since it is difficult to generate a single extended pulse (1) with a very steep leading edge, a pulse pair described by Eq. (24) is preferable in the experimental sense.[4-6]

In the approximation of constancy of the field (24), Eqs. (18)–(20) are easily integrated in each of the ranges $0 \leqslant t - z/c \leqslant T_1$ and $T_1 \leqslant t - z/c$ subject to the appropriate initial and boundary conditions, and also subject to Eq. (21). Consequently, the real part of the nutation amplitude in the range $T_1 \leqslant t - z/c$ becomes

$$\left.\begin{aligned} e_n' = -E_0 \int_{-\infty}^{\infty} e^{-\eta^2} \sum_{\mu=-J_2}^{J_2} \frac{\Pi_\mu}{\Omega T} \left[a_\mu \sin \Omega_\mu \left(t - \frac{z}{c} - T_1 \right) \right. \\ \left. + b_\mu \cos \Omega_\mu \left(t - \frac{z}{c} - T_1 \right) \right] d\eta; \\ a_\mu = 1 - \frac{e_1}{e_0} \frac{\Lambda_\mu^2}{\Omega_{1\mu}^2} (1 - \cos \Omega_{1\mu} T_1), \quad b_\mu = \frac{e_1}{e_0} \frac{\Omega_\mu}{\Omega_{1\mu}} \cos \Phi \sin \Omega_{1\mu} T_1, \\ \Lambda_\mu^2 = (\eta/T_0)^2 \cos \Phi + (e_1/e_0)(2J_2+1)\Pi_\mu \Omega^2, \\ \Omega_{1\mu}^2 = (\eta/T_0)^2 + (e_1/e_0)^2 (2J_2+1)\Pi_\mu \Omega^2. \end{aligned}\right\} \quad (25)$$

The constant phase shift Φ, the duration of the perturbation T_1, and the ratio e_1/e_0 affect the amplitude nutational oscillations and shift the graph of the function (25) as a whole, but the nutation period is independent of Φ, T_1, and e_1/e_0. It is important to note that all the relationships governing nutational oscillations are found above for a single rectangular pulse and remain in force for a pulse pair of Eq. (24) characterized by $\Omega T_1 < 1$. This is demonstrated clearly by the nutational oscillation curves in Fig. 3, which gives the results applicable to the case of identical intensities of the incident pulses with linear and circular polarizations.

If a strong amplitude perturbation in the form $e(t - z/c)$ $e_1/e^{i\Phi}$ for $0 \leqslant t - z/c \leqslant T_1$ is experienced by a monochromatic wave $\mathbf{E} = \mathbf{l} 2e_0 \cos(\omega t - kz)$, then nutation in the range $T_1 \leqslant t - z/c$ is still described by Eq. (25) but now the coefficients a_μ and b_μ are fairly complex. The nutation amplitude depends not only on the phase Φ, perturbation duration T_1, and ratio e_1/e_0, but also on some fixed values of the quantities $p_\mu(0)$, $N_{1\mu}(0)$, and $N_{2\mu}(0)$ taken at the moment $t = z/c$ when the perturbation is activated. However, the nutation period is independent of these quantities and the above relationships still govern the behavior of nutational oscillations.

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