inhomogeneous broadening of the absorption line is important, $T_2^* < t_p$, we can carry out a similar analysis representing the polarization of the medium as factorized in respect of the frequency:

$$P_2(\Delta\omega)=\chi(\Delta\omega)P_2(0), \quad \chi(\Delta\omega)=\frac{1}{1+(\Delta\omega t_p)^2}.$$

In all the above expressions one then has to replace the detuning parameter $\Delta\omega t_p$ with its effective value $(\Delta\omega t_p)_{eff}=\langle \Delta\omega\chi\rangle/\langle \chi\rangle$, where the averaging is carried out over the profile of an inhomogeneously broadened line. The effective detuning parameter is proportional to $\Delta\omega T_2^*$ and the coefficient of proportionality depends on the actual line profile and on $\Delta\omega$. Detuning of the frequency of light with a narrow spectrum from the center of an inhomogeneously broadened absorption line has the greatest influence on the diffraction instability for $|\Delta\omega T_2^*|\sim 1$. The asymmetric dependence of the growth rate of transverse perturbations on the sign of detuning [see Eq. (9)] is in agreement with the experimental results on the influence of detuning of the light frequency on the passage of coherent pulses through a resonantly absorbing medium[11] and on the transverse structure of the transmitted radiation.[6,7]

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Light generation by a moving active medium

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The influence of motion of the active medium between the mirrors of an open resonator on the interaction of the generated radiation with its ensuing periodic structure of the inverted population is investigated theoretically. It is shown that the distributed feedback of the opposing light waves decreases when the active medium moves; this leads to establishment of single-frequency stationary generation at velocities exceeding the calculated critical value (which agrees well with experiment).

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An electromagnetic analysis of a resonator filled with a moving medium[1] is carried out here for the purpose of studying the progagation and generation of light in a dispersive active medium that moves uniformly along the optical axis of an open resonator. The problem is both of independent interest and serves to reveal the role of the spatial-periodic structure which is produced in the active medium because of the inhomogeneous saturation of the inverted population,[2,3] since motion of the medium is one method of eliminating structure effects.[4]

It will be shown that the distributed feedback that is self-induced in the active medium decreases rapidly with increasing velocity of the active medium, both because of the smoothing of the periodic structure of the inverted population, and because it lags in phase the generating standing light wave. As a result, at sufficiently high velocities exceeding a certain critical value, stable stationary generation is produced in the medium and has been observed in a number of experiments.[4,5] The model considered here does not take into account the modulation that can occur in the generated radiation when the dielectric boundaries move parallel to the resonator mirrors[4–6] and does not occur, for example, when the end faces of the active element are cut at the

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Brewster angle,[5,6] or when they are excited only resonantly when the boundary is made self-transparent.[4] At velocities lower than critical, near the state of stationary generation, the space–time modulation of the radiation, which corresponds to excitation of longitudinal resonator modes, increases and can lead to homogeneous pulsations of the generated radiation.

1. FORMULATION OF PROBLEM

We consider an open resonator of length 2λ, made up of ideal flat mirrors, and assume that the active medium filling the resonator moves homogeneously with constant velocity V in a positive direction of the resonator z axis. The initial equations for the propagation and generation of the light are most easily written out in an inertial reference frame that moves together with the active medium. We denote by z_c and t_c respectively the spatial and temporal coordinates of this reference frame. The independent variables z and t of the immobile system of the resonator ($|z|\leq l, t>0$) are expressed in terms of z_c and t_c by means of approximate ($V\ll c$) Lorentz transformations

$$z=z_c+Vt_c, \quad t=t_c+Vz_c/c^2. \tag{1}$$

We introduce the mutually perpendicular components of the electric and magnetic fields, E and H in the resonator system and E_c and H_c in the coordinate system of the active medium. Under the transformation (1), they go over into each other

$$E=E_c+VH_c/c, \quad H=H_c+VE_c/c. \tag{2}$$

In the reference frame connected with the medium, E_c and H_c satisfy the one-dimensional wave equations

$$\begin{gathered}\frac{\partial^2}{\partial z_c^2}E_c=\frac{\varepsilon}{c^2}\frac{\partial^2}{\partial t^2}E_c,\\ \frac{\partial^2}{\partial z_c^2}H_c=\frac{\varepsilon}{c^2}\frac{\partial^2}{\partial t^2}H_c+\frac{1}{c}\frac{\partial\varepsilon}{\partial z_c}\frac{\partial}{\partial t_c}E_c,\end{gathered} \tag{3}$$

with permittivity $\varepsilon(z_c, t_c)$ given near the working transition of the active medium by

$$\varepsilon(z_c,t_c)=\varepsilon_0+\frac{\varepsilon_0}{\omega_0}[(i\alpha_1+\alpha_2)n_p(z_c,t_c)-i\alpha_r], \tag{4}$$

where ε_0 is the contribution of all the other transitions, ω_0 is the central frequency of the working transition, α_r is the loss coefficient in the medium,

$$\alpha_1(\omega-\omega_0)=\frac{\alpha_0}{1+T_2^2(\omega-\omega_0)^2}, \quad \alpha_2(\omega-\omega_0)=\frac{\alpha_0(\omega-\omega_0)T_2}{1+T_2^2(\omega-\omega_0)^2}. \tag{5}$$

Here $\alpha_0=T_2\omega_0|d|^2/\hbar\varepsilon_0$, d is the matrix element of the dipole moment of the transition, T_2 is the spin–spin relaxation time (T_2^{-1} is the half-width of the spectral line of the working transition), and ω is the frequency of the acting electromagnetic field.

The change of the population difference of the levels of the working transition $n_p(z_c, t_c)$ is determined by the average value of the field intensity $|E_c|^2$ during the period of oscillation in accordance with the kinetic equation

$$\frac{\partial}{\partial t_c}n_p=\gamma_1(n^0-n_p)-2\alpha_1 n_p|E_c|^2, \tag{6}$$

in which $\gamma_1=T_1^{-1}$, T_1 is the time of the spin-lattice relaxation of the inversion, and n^0 is the value of the inverted population produced by the pump in the absence of the field. In a field E_c (and correspondingly in H_c, E, and H) we assume that a normalization factor has been introduced, such that $|E_c|^2$ has the meaning of the quantity usually called the photon density in the resonator.

Equation (6) and the first equation of (3) constitute under ordinary conditions a closed system for the field and for the inverted population. However, the boundary conditions on the resonator mirrors

$$E(z=\pm l, \ t)=0 \tag{7}$$

in the immobile reference frame connect, in accordance with the transformation (2), the electric and magnetic components of the field that acts in the moving medium. Taking this connection into account, we can immediately derive from (2) and (3), in terms of z_c and t_c, a single equation for E

$$\frac{\partial^2}{\partial z_c^2}E=\frac{\varepsilon}{c^2}\frac{\partial^2}{\partial t_c^2}E+\frac{V}{c^2}\frac{\partial\varepsilon}{\partial z_c}\frac{\partial}{\partial t_c}E. \tag{8}$$

In the immobile reference frame the field $E(z, t)$ will be represented in the form of a sum of opposing waves:

$$\begin{aligned}E(z,t)=&R\exp\{i[\omega_0 t-\beta_0 z+\delta_R]\}\\&+S\exp\{i[\omega_0 t+\beta_0 z+\delta_S]\},\end{aligned} \tag{9}$$

with amplitudes $R(z, t)$ and $S(z, t)$ and with phase shifts $\delta_R(z, t)$ and $\delta_S(z, t)$, which are assumed to be slowly varying both over the wavelength $\lambda=2\pi/\beta_0$ and during the period $T=2\pi/\omega_0$ of one oscillation of the light. Substituting (9) in terms of the variables z_c and t_c in (2) and (8) we find that, accurate to quantities of first order in V/v ($v=\omega_0/\beta_0=c/\varepsilon_0^{1/2}$ is the speed of light in the medium) the light intensity which varies slowly in time (but not in coordinate) is a standing wave in which the nodes and antinodes move in the reference frame of the medium with velocity V relative to the active atoms

$$\begin{aligned}|E_c|^2=&R^2(1-2V/v)+S^2(1+2V/v)\\&+2RS\cos[2\beta_0(z_c+Vt_c)+\delta_S-\delta_R].\end{aligned} \tag{10}$$

This standing wave duplicates the spatial-periodic structure of the inverted population, which we seek in the form of the expansion

$$\begin{aligned}&n_p(z_c,t_c)=n(z_c,t_c)\\&+n_1(z_c,t_c)\cos[2\beta_0(z_c+Vt_c)+\delta(z_c,t_c)]+\dots,\end{aligned} \tag{11}$$

confining ourselves in weak fields to the first spatial harmonics. Here n (the average value of the inverted population) and n_1 and δ (the depth and phase shift of its spatial modulation) are also slowly varying functions of the time and of the spatial coordinates.

2. COUPLED WAVE GENERATION EQUATIONS

The presence of a periodic structure in the active medium causes the opposing waves (9) to become coupled via Bragg reflections from this structure.[2,3] The Bragg condition for normal reflection from the periodic structure is satisfied automatically, since the structure is produced by the same field that propagates in it. It is therefore possible to use here the coupled-wave approximation,[7] wherein the resultant field can be represented as before in the form (9), but each of the opposing waves is already a superposition of a direct and Bragg-reflected waves.

We express (9) in terms of the variables z_c and t_c and substitute it together with (10) and (11) in (6) and (8). Neglecting the higher orders of the Bragg reflections, we obtain from (8) four quasilinear partial differential equations of first order for the slowly varying amplitudes and phases of the opposing waves; a similar restriction to the lowest spatial harmonics in (6) yields three ordinary differential equations for the components of the inverted population. We write down directly the resultant equations in the immobile reference frame connected with the resonator mirrors; for this purpose, in accordance with transformation (1) we replace the derivatives in accordance with the relations

$$\frac{\partial}{\partial z_c}=\frac{\partial}{\partial z}+\frac{V}{c^2}\frac{\partial}{\partial t},\quad \frac{\partial}{\partial t_c}=\frac{\partial}{\partial t}+V\frac{\partial}{\partial z}$$

and obtain ultimately a closed system of seven partial differential equations of first order

$$\begin{aligned}
&\left(\frac{\partial}{\partial t}+v\frac{\partial}{\partial z}\right)R=\frac{n}{2}(\hat{\alpha}_1\cos\varphi_R+\hat{\alpha}_2\sin\varphi_R)R\\
&\qquad+\frac{n_1}{4}(\hat{\alpha}_1\cos\Phi_S-\hat{\alpha}_2\sin\Phi_S)S-\frac{\alpha_p}{2}R,\\
&\left(\frac{\partial}{\partial t}-v\frac{\partial}{\partial z}\right)S=\frac{n}{2}(\hat{\alpha}_1\cos\varphi_S+\hat{\alpha}_2\sin\varphi_S)S\\
&\qquad+\frac{n_1}{4}(\hat{\alpha}_1\cos\Phi_R+\hat{\alpha}_2\sin\Phi_R)R-\frac{\alpha_p}{2}S,\\
&\left(\frac{\partial}{\partial t}+v\frac{\partial}{\partial z}\right)\delta_R=\frac{n}{2R}(\hat{\alpha}_1\sin\varphi_R-\hat{\alpha}_2\cos\varphi_R)R\\
&\qquad-\frac{n_1}{4R}(\hat{\alpha}_1\sin\Phi_S+\hat{\alpha}_2\cos\Phi_S)S+\beta_0V\chi,\\
&\left(\frac{\partial}{\partial t}-v\frac{\partial}{\partial z}\right)\delta_S=\frac{n}{2S}(\hat{\alpha}_1\sin\varphi_S-\hat{\alpha}_2\cos\varphi_S)S\\
&\qquad+\frac{n_1}{4S}(\hat{\alpha}_1\sin\Phi_R-\hat{\alpha}_2\cos\Phi_R)R-\beta_0V\chi,\\
&\left(\frac{\partial}{\partial t}+V\frac{\partial}{\partial z}\right)n=\gamma_1(n^0-n)-2n\hat{\alpha}_1(R^2+S^2)-2n_1\hat{\alpha}_1RS\cos\Phi,\\
&\left(\frac{\partial}{\partial t}+V\frac{\partial}{\partial z}\right)n_1=-\gamma_1n_1-2n_1\hat{\alpha}_1(R^2+S^2)-4n\alpha_1RS\cos\Phi,\\
&\left(\frac{\partial}{\partial t}+V\frac{\partial}{\partial z}\right)\delta=4\frac{n}{n_1}\hat{\alpha}_1RS\sin\Phi-2\beta_0V,
\end{aligned}\tag{12}$$

with boundary conditions

$$R(\pm l,t)=S(\pm l,t),\quad \delta_R(\pm l,t)=\delta_S(\pm l,t),$$

which follow from (7).

In the derivation of the system (12) we took into account the fact that the factors α_1 and α_2 in (4) have, generally speaking, the meaning of integral operators $\hat{\alpha}_1$ and $\hat{\alpha}_2$ whose eigenvalues are given for a monochromatic field by expressions (5). For the sake of brevity the operator symbols $\hat{\alpha}_1$ and $\hat{\alpha}_2$ include also the variables on which these operators do not act. They are included in the notation

$$\varphi_R=\delta_R-(\delta_R),\quad \varphi_S=\delta_S-(\delta_S),$$
$$\Phi_R=\delta_R+(\delta-\delta_S),\quad \Phi_S=(\delta_R+\delta)-\delta_S,\quad \Phi=\delta_R+(\delta)-\delta_S$$

and are enclosed there in the parentheses. The factors $1\pm\chi V/v$ of the derivatives in the first four equations of (12) have been omitted here inasmuch as the velocities in question (on the order of several dozen centimeters per second) they differ little from unity and cause all the frequencies of the system to change by a factor $1\pm\chi V/v$. This differs substantially from the corresponding changes that occur when the active medium moves perpendicular to the optical axis,[1] but nevertheless the effects of the displacement of the structure of the field and of the inverted population, which correspond to the terms $\sim\beta_0V$, turn out to predominate to a considerable degree.

The quantity $\chi=(1-1/\varepsilon_0)$ is the so called Fresnel coefficient of the "ether" drag, as can be easily verified by calculating the phase velocity of the constant-amplitude wave without allowance for the activity of the medium:

$$v_\phi=\frac{\omega_0+\partial\delta_R/\partial t}{\beta_0-\partial\delta_R/\partial z}\approx v+\left(\frac{\partial}{\partial t}\delta_R+v\frac{\partial}{\partial z}\delta_R\right)\Big/\beta_0=v+V\chi,$$

which, as is well known, explains the result of the Fizeau experiment for a wave traveling, for example, in the same direction as the medium.

The right-hand sides of the first four equations in (12) determine the changes of the amplitudes and phases of the opposing waves on account of the variance of the gains and of the refractive indices in the presence of an opposing wave in the periodic structure of the inverted population. The last terms of the equations for the amplitudes correspond to the radiation losses, which are uniformly distributed over the length of the resonator. The last three equations of (12) describe the changes of the components of the inverted population (11), which occur when the active medium moves in the presence of a field in the resonator. At $V=0$ the system (12) is a generalization of the previously obtained system[3] to include the case of dispersion of the active medium.

3. STATIONARY GENERATION

The time-independent solutions of (12), which satisfy the boundary conditions, determine a stationary regime of generation of opposing waves of light by the moving medium. For a generation frequency that coincides with the frequency of the working transition, the stationary values of the variables are respectively

$$\begin{aligned}
&R_0=S_0=\left(\frac{\gamma_1\eta}{2\alpha_0}\right)^{1/2},\quad \delta_{R0}=\delta_{S0}=\text{const}+\frac{1}{4v}(4\beta_0V\chi-\alpha_0\nu_0)z,\\
&n_0=\frac{\alpha_r}{\alpha_0}\left[1+\frac{\eta(1+2\eta)}{\delta_V^2+(1+\eta)(1+2\eta)}\right],\\
&n_{10}=-2\eta\frac{\alpha_r}{\alpha_0}\frac{[\delta_V^2+(1+2\eta)^2]^{1/2}}{\delta_V^2+(1+\eta)(1+2\eta)},\\
&\delta_0=\Phi_0=-\operatorname{arctg}\frac{\delta_V}{1+2\eta},
\end{aligned}\tag{13}$$

where

$$\nu_0=n_{10}\sin\Phi_0=\frac{\alpha_r}{\alpha_0}\frac{2\eta\delta_V}{\delta_V^2+(1+\eta)(1+2\eta)},\quad \delta_V=\frac{2\beta_0V}{\gamma_1}$$

and the dimensionless parameter η which characterizes the stationary intensity is determined from the equation

$$4\eta^3+(10-2n^0\alpha_0/\alpha_r)\eta^2+\left(2\delta_V^2+6-3n^0\frac{\alpha_0}{\alpha_r}\right)\eta-(\delta_V+1)(\alpha_0n^0/\alpha_r-1)=0,$$

from which it is seen that the threshold of the stationary generation ($\eta>0$) is usually given by the condition $\alpha_0n^0>\alpha_r$ corresponding to an excess of the gain over the losses.

The stationary values depend on the velocity of the active medium via the parameter δ_V, which is proportional to the ratio of the inverted-population relaxation time T_1 to the time interval within which the atoms of the active medium are displaced by an amount equal to the wavelength of the light in the medium.

In the immobile medium we have

$$\eta=\frac{1}{4}\left[\frac{\alpha_0}{\alpha_r}n^0-4+\left[\left(\frac{\alpha_0}{\alpha_r}n^0\right)^2+8\right]^{1/2}\right]$$

and $\eta\approx\frac{1}{3}(\alpha_0 n^0/\alpha_r-1)$ near the generation threshold, whereas at sufficiently large velocities of the medium ($\delta_V\gg 1$) we have $\eta\approx\frac{1}{2}(\alpha_0 n^0/\alpha_r-1)$, i.e., at the same pump power one obtains in the moving medium single-frequency radiation intensities larger by a factor 1.5, as observed by direct measurements.[4] This is due to the smoothing of the structure of the inverted population ($n_{10}\sim\delta_V^{-1}$) when the active medium moves and to its phase lag relative to the standing light wave that produces the structure.

If the phase difference between the field and the structure in the immobile medium is $\Phi_0=0$, i.e., if the antinodes of the light wave correspond to minima of the gain, a situation highly unfavorable for effective amplification,[3,8] then at sufficiently high velocities of the medium we have $\Phi_0\to-\pi/2$, i.e., the structure lags in phase in such a way that the nodes and antinodes of the field are under practically the same amplification conditions. The Bragg-reflection coefficient, which determines the connection between the opposing waves, is in the stationary regime

$$\rho_0=n_{10}\cos\Phi_0=-\frac{\alpha_r}{\alpha_0}\frac{2\eta(1+2\eta)}{\delta_V^2+(1+\eta)(1+2\eta)}$$

and at high velocities $\rho_0\sim\delta_V^{-2}$, i.e., the coupling between the opposing is decreased just as rapidly as the result of the change of the phase relations as a result of the smoothing of the structure.

4. MODULATION OF RADIATION

We seek a solution of (12) near the stationary state (13). To this end we represent all the variables as sums (13) and small deviations

$$R=R_0+\Delta R,\quad S=S_0+\Delta S,\quad \delta_R=\delta_{R0}+\delta_{R1},$$
$$\delta_S=\delta_{S0}+\delta_{S1}\quad n=n_0+\Delta n,\quad n_1=n_{10}+\Delta n_1,\quad \delta=\delta_0+\delta_1. \tag{14}$$

Substituting (14) in (12) we obtain, in the approximation linear in the deviations, a system of equations which describes two types of oscillations at substantially different frequencies. One of them characterizes the homogeneous oscillations of the sum of the amplitudes of the opposing waves and of the components of the inverted population, while the other is determined by the time of travel of the light in the resonator and exceeds significantly (usually by three–four orders of magnitude) the former. This circumstance enables us to resort, in the analysis of each type of oscillation, to perfectly justifiable approximations. Thus, when high-frequency oscillations are considered it turns out that the changes of the components of the inverted population at these frequencies are negligibly small and the excitation of these oscillations can be treated assuming constant values of the parameters of the periodic structure.

We separate the spatially inhomogeneous and homogeneous parts of the amplitude deviations:

$$\Delta R=\overline{\Delta R}(t)+\Delta R_1(z,t),\quad \Delta S=\overline{\Delta S}(t)+\Delta S_1(z,t),$$

and obtain for the variables

$$\Delta_m=\Delta R_1-\Delta S_1,\quad \Delta_r=\Delta R_1+\Delta S_1,$$
$$\delta_m=\delta_{R1}-\delta_{S1},\quad \delta_r=\delta_{R1}+\delta_{S1}$$

in the indicated approximation a system of partial differential equations of first order

$$\frac{\partial\Delta_m}{\partial t}+v\frac{\partial\Delta_r}{\partial z}=\frac{1}{2}(\varkappa_1\hat{\alpha}_1-\alpha_r)\Delta_m-\frac{\nu_0}{4}\hat{\alpha}_2\Delta_r+\frac{\varkappa_1}{2}R_0\hat{\alpha}_2\delta_m-\frac{\nu_0}{4}R_0(\alpha_0-\hat{\alpha}_1)\delta_r$$
$$\frac{\partial\Delta_r}{\partial t}+v\frac{\partial\Delta_m}{\partial z}=\frac{\nu_0}{4}\hat{\alpha}_2\Delta_m+\frac{1}{2}(\varkappa_2\hat{\alpha}_1-\alpha_r)\Delta_r-\frac{\nu_0}{4}R_0(\alpha_0+\hat{\alpha}_1)\delta_m+\frac{\varkappa_2}{2}R_0\hat{\alpha}_2\delta_r, \tag{15}$$
$$\frac{\partial\delta_m}{\partial t}+v\frac{\partial\delta_r}{\partial z}=-\frac{\varkappa_1}{2R_0}\hat{\alpha}_2\Delta_m+\frac{\nu_0}{4R_0}(\alpha_0-\hat{\alpha}_1)\Delta_r+\frac{1}{2}(\varkappa_1\hat{\alpha}_1-\alpha_r)\delta_m-\frac{\nu_0}{4}\hat{\alpha}_2\delta_r,$$
$$\frac{\partial\delta_r}{\partial t}+v\frac{\partial\delta_m}{\partial z}=\frac{\nu_0}{4R_0}(\alpha_0+\hat{\alpha}_1)\Delta_m-\frac{\varkappa_2}{2R_0}\hat{\alpha}_2\Delta_r+\frac{\nu_0}{4}\hat{\alpha}_2\delta_m+\frac{1}{2}(\varkappa_2\hat{\alpha}_1-\alpha_r)\delta_r$$

with boundary conditions

$$\Delta_m(\pm l,t)=0,\quad \delta_m(\pm l,t)=0.$$

Here $\varkappa_1=n_0-\frac{1}{2}\rho_0$, $\varkappa_2=n_0+\frac{1}{2}\rho_0$.

a) *Immobile medium.* We consider first the excitation of the radiation modulation at $V=0$. If we neglect the small increment to the real part of the permittivity ($\alpha_2\approx 0$), then the system (15) breaks up into two independent systems, each of which can be reduced to a single second-order equation of the form

$$\frac{\partial^2\delta_m}{\partial t^2}-2a_1\frac{\partial\delta_m}{\partial t}+(a_1^2-a_2^2)\delta_m=v^2\frac{\partial^2\delta_m}{\partial z^2},\quad \delta_m(\pm l,t)=0.$$

A similar equation holds for $\Delta_m(z,t)$. Its general solution is

$$\delta_m=\sum_{\Omega}C_\Omega\exp\{[a_1(\Omega)\pm i\Omega]t\}\Delta(z), \tag{16}$$

where the functions

$$\Delta(z)=\begin{cases}\cos kz, & k=(m-1/2)\pi/l,\\ \sin kz, & k=m\pi/l,\end{cases}$$

with $m=1,2,\ldots$ correspond to solutions with different spatial symmetry, and

$$a_1(\Omega)=a_2(\Omega)[\eta-(1+\eta)T_2^2\Omega^2],\quad a_2(\Omega)=1/2\alpha_r[(1+\eta)(1+T_2^2\Omega^2)]^{-1},$$

where

$$\Omega\approx(k^2v^2-[a_2(kv)]^2)^{1/2}.$$

The constants C_Ω are determined by the arbitrary values of the deviations. At $\eta>T_2^2\Omega^2(1-T_2^2\Omega^2)^{-1}$, which is readily satisfied for media with a broad ($T_2\ll\Omega^{-1}$) luminescence line at insignificant excess over the generation threshold, we have $\alpha_1(\Omega)>0$ and both the phase and the amplitude space-time modulations of the generated radiation increase near the stationary state. The condition $\alpha_1(\Omega)>0$ determines the spectrum of the radiation. The phase and amplitude modulations can be represented in the form of expansions in the modes of the "hot" resonator, and it turns out naturally in this case that the criterion for the growth of the modulation coincides with the threshold of excitation of the neighboring axial modes.[8] The relation between the values of the phase and amplitude modulation is determined in the linear approximation by the initial conditions. Usually radiation detectors based on the photoeffect do not register a pure phase modulation, and predominance of the amplitude modulation corresponds to the case of mode locking[9] and generation of short light pulses.

b) *Moving medium*. Without touching upon the analytic determination of the steady-state regimes in the immobile medium, let us examine the changes produced by the motion of the active medium in the conditions of the radiation-modulation radiation excitation. We turn to the system (15). For the variable δ_m (or Δ_m) it has a solution satisfying the boundary condition in the form

$$\delta_m = \sum_\Omega [C_\Omega \exp\{[a(\Omega_1) \pm i\Omega_1]t\} + D_\Omega \exp\{[a(\Omega_2) \pm i\Omega_2]t\}]\Delta(z), \qquad (17)$$

where

$$a(\Omega) = \frac{1}{2}\frac{\alpha_r}{1+T_2^2\Omega^2}\left[\frac{\eta(1+2\eta)}{\delta_v^2+(1+\eta)(1+2\eta)} - T_2^2\Omega^2\right],$$

$$\Omega_{1,2} = [k^2v^2 + {}^1/_4\alpha_0^2\text{v}_0^2 - {}^1/_{16}\alpha_1^2(kv)n_{10}^2 \pm kv({}^1/_{16}\alpha_0^2\text{v}_0^2 + {}^1/_4\alpha_0\text{v}_0\alpha_2(kv)n_0 - {}^1/_{16}\alpha_2^2(kv)\rho_0^2)^{1/2}]^{1/2}.$$

At velocities close to zero, the radicand becomes complex and contributes also to the growth rate, thus causing small changes in the growth rate and in the frequency in solution (16) for an immobile medium; these changes are due to allowance for the dispersion of the real part of the dielectric constant. At not too low velocities, the modulation frequencies are always real, and the values of the growth rate $a(\Omega)$ become negative for active-medium velocities $V > V_{cr}$

$$V_{cr} = \frac{\gamma_1 v}{2\omega_0}\left(\frac{\eta(1+2\eta)}{T_2^2\Omega^2} - (1+\eta)(1+2\eta)\right)^{1/2}. \qquad (18)$$

The modulation frequencies are determined principally by the time of travel of the light through the resonator, and at typical resonator dimensions they exceed by several orders of magnitude not only the relaxation constant of the inverted population, but also, as will be shown below, the frequency of the natural oscillations of the inverted-population components in the absence of a field.

c) *Homogeneous pulsations*. For homogeneous deviations of the sum of the amplitudes $\overline{\Delta}_r = \overline{\Delta S} + \overline{\Delta R}$ of the inverted population $\overline{\Delta n}$ and of the depth of its modulation $\overline{\Delta n}_1$ we can obtain, after substituting (14) in (12) and averaging over the resonator length, a system of ordinary differential equations

$$\frac{d\overline{\Delta}_r}{dt} = \frac{1}{2}\alpha_r\left(\frac{\hat{\alpha}_1}{\alpha_0} - 1\right)\overline{\Delta}_{1r} + \alpha_0 R_0\overline{\Delta n} + \frac{1}{2}\alpha_0 R_0\overline{\Delta n_1} - \frac{1}{4}\alpha_0\rho_0 R_0\langle\delta_m^2\rangle,$$

$$\frac{d\overline{\Delta n}}{dt} = -4\alpha_r R_0\overline{\Delta}_r - (\gamma_1 + 4\alpha_0 R_0^2)\overline{\Delta n} - 2\alpha_0 R_0^2\overline{\Delta n_1} + \alpha_0 R_0^2 n_{10}\langle\delta_m^2\rangle$$
$$-\frac{1}{2}\alpha_0\rho_0\langle\Delta_m^2 + \Delta_r^2\rangle, \qquad (19)$$

$$\frac{d\overline{\Delta n_1}}{dt} = -4\alpha_0 R_0(n_{10} + n_0\cos\Phi_0)\Delta_r - 4\alpha_0 R_0^2\cos\Phi_0\overline{\Delta n}$$
$$-(\gamma_1 + 4\alpha_0 R_0^2)\overline{\Delta n_1} + 2\alpha_0 R_0^2 n_0\langle\delta_m^2\rangle - 2\alpha_0 n_{10}\langle\Delta_r^2 + \Delta_m^2\rangle.$$

The eigensolutions of (19) take the form $\exp\{-\frac{1}{2}(\gamma_1 + \alpha_r T_2^2\omega_p^2)t \pm i\omega_p t\}$ and describe pulsations, weakly damped in the linear approximation, of the field and of the inverted population at a frequency

$$\omega_p = \left(\gamma_1\alpha_r\eta\left[2 + \frac{(\delta_v^2+(1+2\eta)^2)^{1/2}}{\delta_v^2+(1+\eta)(1+2\eta)}\right]\right)^{1/2}. \qquad (20)$$

The inhomogeneous terms of (12) take the form e^{2at}, or more accurately

$$\sum_\Omega C_\Omega^2 e^{2a(\Omega)t},$$

and represent those components of the squares of the space-dependent deviations of the phase and amplitudes of the field which do not vanish after spatial averaging over the length of the resonator ($\langle A\rangle = (2l)^{-1}\int A dz$), and vary slowly compared with $e^{i\Omega t}$. At $a(\Omega) > 0$ they act in (19) as a driving force that takes the system out of the equilibrium state and initiates a radiation spike.

If all the $a(\Omega) < 0$, then the stationary generation regime turns out to be stable both to homogeneous deviations and to the excitation of space-time amplitude and phase modulation of the radiation. In a moving medium this is possible if the condition (18) is satisfied. For the parameters of a laser using yttrium aluminum garnet activated with neodymium ions,[4] $T_1 = 2.3\times 10^{-4}$ sec, $\omega_0 = 2\times 10^{15}$ sec^{-1}, $\varepsilon_0^2 = 1.83$, $2l = 72$ cm, $T_2 = 2\times 10^{-12}$ sec, $(\Delta\nu = (T_2\pi)^{-1} = 156$ GHz) and $\eta = \frac{1}{4}$ we obtain in accordance with (18) a critical velocity $V_{cr} = 7.5$ cm/sec, in good agreement with the values observed by Danielmeyer and Nilsen[4] ($V_{cr} = 10$ cm/sec). Tursunov's measurements were made at a velocity $V = 55$ cm/sec. Calculation by formula (18) at pumps 1.4 times larger than threshold ($\eta \approx 0.2$) in the case of the crystal $CaWO_4$ ($T_1 = 0.13\times 10^{-3}$ sec, $(T_2\pi)^{-1} = 5.3$ cm^{-1}, $2l = 60$ cm, $\omega_0 = 2\times 10^{15}$ sec^{-1}) yields $V_{cr} \approx 10$ cm/sec, and for ruby the values of the critical velocity are lower by one order of magnitude. Similar observations of stationary generation were also made by others[5,6] at velocities from 35 to 80 cm/sec.

Our analysis leads also to some conclusions with respect to the role of the spatially inhomogeneous burnout of the inverted population in the dynamics of light generation in solid-state active media. As shown above, the induced periodic structure of the active medium and the distributed coupling of the opposing waves can lead to pulsations of the radiation only via excitation of side modes, i.e., it is a secondary cause of pulsations and cannot influence directly on the stability of single-mode generation. This is confirmed, in particular, by reports of a low level of the fluctuations of the radiation of single-mode generators with stabilization of the resonator elements,[10] and also by observation of spikeless generation in an active medium with a narrow luminescence line,[11] when only one longitudinal mode lands in the gain contour. We note here that although the permittivity approximation is valid only for broad spectral lines, nonetheless a formal treatment of the increment in the solution (16) for a line with a half-width $T_2^{-1} < \Omega$ shows that in this case, too, a stable stationary generation regime should be observed. On the other hand, if several longitudinal modes are excited, the inhomogeneous burnout of the inverted population can lead to development of radiation pulsations, in agreement with the large accumulated experimental material,[12] and is confirmed by theoretical calculations.

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Quasiclassical calculation of transition probabilities

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A simple general formula is derived for the probability of an allowed transition between quasiclassical states.

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It is well known that the calculation of a transition probability between quasiclassical states reduces to the calculation, by the method of steepest descents, of matrix elements of the perturbation potential. If the transition is classically forbidden, the saddle point lies in the complex plane and the result depends on details of the analytic behavior of the potential in which the quasiclassical motion occurs (Ref. 1, Sec. 51). If the transition is forbidden, simple general formulas can be derived; the present note deals with this case.

We shall calculate the square of the absolute value of the matrix element

$$\varphi_{nn'}(q)=\int \varphi_n(x)e^{iqx}\varphi_{n'}(x)\,dx \tag{1}$$

with the quasiclassical functions

$$\varphi_n(x)=\frac{c_n}{\sqrt{p_n}}\cos\left(\frac{1}{\hbar}\int_a^x p_n dx-\frac{\pi}{4}\right),\quad p_n=\{2m[\varepsilon_n-U(x)]\}^{1/2}, \tag{2}$$

where $U(x)$ is the potential in which the quasiclassical motion of a particle with mass m occurs, ε_n is an energy eigenvalue, a is the position of one of the classical turning points, and c_n is a normalization constant. We write the integral (1) in the form of a sum

$$\varphi_{nn'}(q)=\frac{c_nc_{n'}}{4}\sum_{s_1,s_2=\pm1}\int \exp\left[if(x)-i\frac{\pi}{4}(s_1+s_2)\right]\frac{dx}{(p_np_{n'})^{1/2}}, \tag{3}$$

where

$$f(x)=\frac{1}{\hbar}\int_a^x (s_1p_n+s_2p_{n'})\,dx+qx. \tag{4}$$

In the sense of the quasiclassical approximation, $f(x)$ is a rapidly varying function. Therefore in calculating the integrals in Eq. (3) we can use a steepest descent (or stationary phase) method. The position of the saddle points is given by the equation

$$\hbar f'(x_l)=s_1p_n(x_l)+s_2p_{n'}(x_l)+\hbar q=0,$$

which expresses the law of conservation of momentum. For allowed transitions all of the x_1 are real. The result of the calculations is

$$|\varphi_{nn'}(q)|^2=\pi\frac{|c_nc_{n'}|^2}{8}\sum_{s_1,s_2=\pm1}\sum_l [|f''(x_l)|p_n(x_l)p_{n'}(x_l)]^{-1}. \tag{5}$$

Differentiating the quasiclassical quantization condition with respect to n and comparing the result with the well known expression for the normalization constant (Ref. 1, Sec. 48), we obtain the relation

$$|c_n|^2=\frac{2m}{\pi\hbar}\frac{d\varepsilon_n}{dn}. \tag{6}$$

Equations (5) and (6) give the required result. This expression can be written in a more intuitive, and in some cases much more convenient form: