Electron heating by a quantizing sound wave. Quantum oscillations of sound absorption and of an electric current

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The propagation, in semiconductors, of a sound wave that quantizes the electron motion in semiconductors and semimetals is investigated. The case of tight binding is considered, when all the electrons are trapped in potential wells produced by the sound, and tunneling can be neglected. The nonequilibrium distribution of the electrons can be described in order of magnitude, by an effective temperature that exceeds noticeably the lattice temperature. This excess is particularly large when the electron scattering by the impurities exceeds the scattering by phonons. Quantization of the electron motion causes the sound absorption coefficient to become an oscillating function of the intensity. The acoustoelectric effect and the influence of the electric field on the sound absorption are investigated.

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1. INTRODUCTION

The availability of high-intensity sound beams in experiments (dozens of watts and hundreds of watts per square cm for volume and surface sound, respectively) raises the question of quantization of electronic states by sound in conductors. We consider here the possible observable effects that can result from such a quantization.

Keldysh was the first to point out that the periodic field of a sound wave should lead to the appearance of forbidden bands in the spectrum of free electrons.[1] For this effect to occur it is necessary that the width ε_g of the forbidden bands be large compared with the electron energy uncertainty:

$$\varepsilon_g \gg \hbar/\tau \qquad (1)$$

(here τ is the electron relaxation time).

In our preceding paper[2] we considered the influence of acoustic quantization of the spectrum on the sound absorption coefficient and on the acoustoelectric current in the case of weak coupling, when the band gap is much less than the width of the allowed band. In the present paper we consider the opposite limiting case.

Small enough band gaps denote localization of the electrons in the potential wells produced by the sound wave. It is then sufficient to consider a system of levels in a single well rather than a system of bands. We call this the tight-binding case. In the next section we analyze the conditions under which tight binding is realized. In Secs. 3 and 4 we solve the kinetic equation for electrons that are scattered by phonons and impurities. It turns out that when the electrons move together with the sound wave they become heated; it is convenient to describe this heating by introducing an effective temperature T_e. If the scattering is by phonons, the heating is small and T_e exceeds the lattice temperature T by not more than two times. If scattering by impurities predominates, then the heating turns out to be quite appreciable, $T_e \sim T\tau_{ph}/\tau_i \gg T$, where τ_{ph} and τ_i are the times of electron relaxation on the phonons and on the impurities. The reason for this heating is that the electrons that move together with the sound are scattered by the impurities inelastically, and this scattering increases the number of high-energy electrons. The heating is stabilized only by phonon emission.

In Sec. 5 we derive a general formula for the sound absorption coefficient, and in Secs. 6 and 7 this formula is analyzed for the quantum and quasiclassical cases. It turns out that the absorption coefficient oscillates as a function of the sound intensity. These oscillations are due to quantization of the electron motion and are reminiscent of Shubnikob–de Haas oscillations, except that they are not washed out when T_e exceeds the distance ε_g between the levels. The amplitude and the period of the oscillations decrease with increasing number of levels in the sound well that is occupied by electrons.

Section 8 is devoted to a study of the effect exerted on the sound absorption coefficient by a longitudinal electric field and to the form of the current-voltage characteristics. At fixed values of the electric field the current-voltage characteristic can exhibit peaks of the same origin as in immobile superlattices.[3]

The last section contains estimates of the parameters at which the effects in question can be observed.

2. THE TIGHT-BINDING APPROXIMATION

The stationary wave function of an electron in the field of a periodic sound wave propagating along the x axis is of the form

$$\psi(\mathbf{r}) = S^{-1/2}\exp(i\mathbf{k}\mathbf{r}_\perp)\varphi_n(\xi), \quad \xi = x - wt. \qquad (2)$$

Here $\mathbf{r}_\perp$ is the radius vector in a plane perpendicular to the x axis; $\mathbf{k}$ is the corresponding two-dimensional wave vector; S is the area of the intersection of the crystal and a plane perpendicular to the x axis; w is the speed of sound; $\varphi_n(\xi)$ is the wave function describing the longitudinal motion of the electron. In a periodic field it is characterized, generally speaking, by the number n of the band and by the longitudinal wave vector.

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We consider tightly bound electrons, i.e., we assume that the characteristic width Δ of the allowed band is much less than the characteristic width ε_g of the forbidden band:

$$\Delta/\varepsilon_g \ll 1. \tag{3}$$

Moreover, we assume that Δ is much less than the electron energy uncertainty $\hbar/\tau$ connected with the scattering:

$$\Delta \ll \hbar/\tau. \tag{4}$$

The last inequality means that before the electron manages to tunnel from one well of the periodic potential into another it will experience a scattering event. It is therefore convenient to choose as the initial states $\varphi_n(\xi)$ of the electrons not its states in a periodic field, but its states in an individual well of a periodic potential. In this case n is the number of the level in the well.

The quantity Δ in inequalities (3) and (4) is determined by the overlap of the wave functions in the neighboring wells; it is easy to calculate Δ for the lowest bands of a sinusoidal potential energy

$$U = U_0 \sin Q\xi. \tag{5}$$

The lower part of each potential well can in this case be regarded as parabolic. In the first order in the parameter (3) the functions $\varphi_n(\xi)$ are then oscillatory. The oscillation levels are equidistant, so that the allowed bands formed by the overlap of the functions $\varphi_n(\xi)$ are also equidistant. Thus, all the lower forbidden bands have the same width, equal to the distance between the oscillator levels:

$$\varepsilon_g = \alpha U_0. \tag{6}$$

Here

$$\alpha = (\hbar^2 Q^2/mU_0)^{1/2}, \tag{7}$$

where m is the effective mass. The width of the nth allowed band ($n = 0, 1, 2, \dots$), calculated by the tight-binding method, is equal to

$$\Delta_n = \frac{4\varepsilon_g}{(2\pi)^{1/2} n!} \left(\frac{2^5}{\alpha}\right)^{n+1/2} e^{-8/\alpha}. \tag{8}$$

We note that expression (8) contains a numerical parameter, by virtue of which the satisfaction of the condition $\alpha \ll 1$ is not obligatory for the calculation of (3), as might be expected. In fact, at $\alpha = 1$ we find from (8) that $\Delta_0/\varepsilon_g \approx 2 \cdot 10^{-3}$.

It is seen from (8) that at low values of n the value of Δ_n increases with increasing n. Obviously, the qualitative deduction that the width of the allowed band increases with increasing number (and perhaps not monotonically at not very large n), and that the width of the forbidden band is bounded, is by itself independent of the actual form of the potential.[1)] In fact, for energies much higher than the amplitude of the potential energy U_0, electrons of almost any energy behave like free ones, Δ_n increases like n^2, and the width of the forbidden bands does not exceed U_0 in order of magnitude. For this reason, the criterion (3) no longer holds for sufficiently high band. This criterion is satisfied, i.e., the electrons that make a substantial contribution to the kinetics are strongly coupled by the sound, if the characteristic energy is much less than the amplitude of the potential energy U_0. For Fermi statistics this condition takes the form

$$\frac{\hbar^2}{m}\frac{N}{Q}\frac{1}{U_0} \ll 1 \tag{9}$$

where N is the electron concentration.

It is important to bear in mind that the potential produced by the sound wave is screened by the electrons. In the case of weak coupling the screening leads only to renormalization of the amplitudes of the potential energy.[4] In the tight-binding case the primarily screened section of the potential well is the region where the electrons are localized. The result is a distortion of the shape of the bottom of the well, and the wave functions and the level positions can differ substantially from the oscillator quantities. At high electron concentration the screening can turn out to be so strong that the depth of the potential well is greatly decreased and the tight-binding condition (3) is violated. The potential produced by the electrons contained in a layer having a thickness of the order of $1/Q$ is $eN/\varkappa Q^2$, where $\varkappa$ is the permittivity. Thus, screening does not violate the tight-binding conditions if

$$e^2 N/\varkappa Q^2 \lesssim U_0. \tag{10}$$

We shall not need the actual form of the wave functions $\varphi_n(\xi)$, and it does not matter therefore whether they are oscillatory or are distorted by the screening. However, the very existence of single-particle wave functions presupposes smallness of the Coulomb energy of the electron interaction compared with their kinetic energy. Under the condition (1), the electrons in each potential well execute two-dimensional motion. The average distance between them is $(Q/N)^{1/2}$. Therefore the condition for the applicability of the single-particle approximation in the case of Fermi statistics can be written in the form

$$\frac{e^2}{\varkappa}\left(\frac{N}{Q}\right)^{1/2} \ll \frac{\hbar^2 N}{mQ}. \tag{11}$$

The conditions (9)–(11) impose limitations on the electron concentration. To estimate the concentration, it is convenient to rewrite the conditions in the form

$$\left(\frac{e^2 Q}{\varkappa}\Big/\frac{\hbar^2 Q^2}{m}\right)^2 \ll \frac{N}{Q^3} \ll U_0\Big/\frac{\hbar^2 Q^2}{m}, \quad U_0\Big/\frac{e^2 Q}{\varkappa}. \tag{12}$$

3. SCATTERING OF ELECTRONS BY PHONONS

In the case of strong coupling and when the conditions (4) are satisfied, equilibrium is independently established in the electron gas in each potential well. We note that equilibrium can be established when the sound-damping length exceeds the electron energy-relaxation length in the well. This condition is usually satisfied. In addition, the damping length can be increased by turning on a longitudinal electric field (see Sec. 8). The electron gas can be described by a distribution function $f_n(\mathbf{k})$ that depends on the number of the level and on the transverse wave vector $\mathbf{k}$. In scattering by acoustic phonons, the kinetic equation for $f_n(\mathbf{k})$ is of the form

$$I_{eph}\{f\}=\sum_{n'}\int\frac{d^3q}{(2\pi)^3}M(q)|\varphi_{nn'}(q_x)|^2\{[(N_q+1)f_{n'}(\mathbf{k}+q_\perp)(1-f_n(\mathbf{k}))$$
$$-N_qf_n(\mathbf{k})(1-f_{n'}(\mathbf{k}+\mathbf{q}_\perp))]\}\delta(\varepsilon_n(\mathbf{k})-\varepsilon_{n'}(\mathbf{k}+\mathbf{q}_\perp)+\hbar\omega_q-\hbar q_xw)$$
$$+[N_qf_{n'}(\mathbf{k}-\mathbf{q}_\perp)(1-f_n(\mathbf{k}))-(N_q+1)f_n(\mathbf{k})(1-f_{n'}(\mathbf{k}-\mathbf{q}_\perp))]$$
$$\times\delta(\varepsilon_n(\mathbf{k})-\varepsilon_{n'}(\mathbf{k}-\mathbf{q}_\perp)-\hbar\omega_q+\hbar q_xw); \qquad (13)$$

here

$$\varepsilon_n(k)=\varepsilon_n+\hbar^2k^2/2m,$$

ε_n is the energy of the nth level of the one-dimensional sound well, $\mathbf{q}_\perp$ is the component of the phonon wave vector $\mathbf{q}$ perpendicular to the x axis, $N_q=[e^{\hbar\omega/T}-1]^{-1}$, ω_q is the phonon frequency, T is the lattice temperature,

$$\varphi_{nn'}(q)=\int\varphi_n(\xi)e^{iq\xi}\varphi_{n'}(\xi)d\xi, \qquad (14)$$

$$M(q)=\pi\Lambda^2q^2/\rho\omega_q \qquad (15)$$

for the deformation interaction (Λ is the constant of the deformation potential and ρ is the crystal density) and

$$M(q)=16\pi^3e^2\beta^2/\varkappa^2\rho\omega_q \qquad (16)$$

for the piezoelectric interaction (β is the piezoelectric modulus). To simplify the notation, we disregard the crystal anisotropy, we assume, for example, that ω_q depends only on the modulus of q, and replace the tensors Λ^2 and β^2 by their values averaged over the directions.

We point out that the δ functions that express the energy conservation law contain the terms $\hbar q_xw$. They stem from the fact that the scattered electrons are not in immobile but in moving potential wells produced by the sound. Such a term can be interpreted, for example, as a Doppler shift of the phonon frequency in a reference frame connected with the sound. There is no momentum conservation law along the x axis, and the characteristic values of q_x are determined by the behavior of the matrix elements (14), i.e., by the localization length $\varphi_n(\xi)$. For the lower oscillator level we have $q_x\sim Q\alpha^{-1/2}$.

We obtain the solution of the kinetic equation in a quasielastic approximation, i.e., in the case when the change of the electron energy $\hbar wq$ in the collision is much less than the characteristic energy interval T_e over which the distribution function changes. The characteristic values are $q\sim\max(k_F, Q\alpha^{-1/2})$, so that the quasi-elasticity conditions can be written in the form

$$\hbar wk_F/T_e\ll1,\quad \hbar wQ/\alpha^{1/2}T_e\ll1. \qquad (17)$$

We note that at $\varepsilon_F\gtrsim\varepsilon_g$, i.e., when more than one level is filled, we have $k_F\gtrsim Q\alpha^{-1/2}$, so that the first of the conditions is the more stringent.

In the elastic approximation, $f_n(k)$ is an arbitrary function of $\varepsilon_n(k)$. To find the form of this function it is necessary to write down Eq. (13) in the first nonvanishing approximation in the quasi-elasticity. Multiplying (13) by $\delta(\varepsilon-\varepsilon_n(k))/4\pi^2$, summing over n, and integrating with respect to k, we obtain after expanding in powers of $\hbar\omega_q$ and $\hbar wq_x$ (in analogy with Ref. 5)

$$\bar{I}_{eph}\{f\}=\sum_n\int\frac{d^2k}{4\pi^2}I_{eph}\{f\}\delta(\varepsilon-\varepsilon_n(\mathbf{k}))$$
$$=\frac{d}{d\varepsilon}\left\{T[A_1(\varepsilon)+A_2(\varepsilon)]\frac{df(\varepsilon)}{d\varepsilon}+A_1(\varepsilon)f(\varepsilon)[1-f(\varepsilon)]\right\}=0. \qquad (18)$$

Here

$$A_l(\varepsilon)=\sum_{nn'}\int\frac{dq}{2\pi}|\varphi_{nn'}(q)|^2M_l\left(\frac{\hbar^2q^2}{2m}\right)\vartheta(\varepsilon-\varepsilon_n)\vartheta(\varepsilon-\varepsilon_{n'}), \qquad (19)$$

$\vartheta(x)=1$ at $x>0$ and $\vartheta(x)=0$ at $x<0$.

For deformation interaction with phonons we have

$$M_1(z)=\frac{\Lambda^2m^3}{2\pi\rho\hbar^5}(2\varepsilon-\varepsilon_n-\varepsilon_{n'}+z), \qquad (20a)$$

$$M_2(z)=\frac{\Lambda^2m^3}{2\pi\rho\hbar^5}\left(\frac{w}{w_{ph}}\right)^2z, \qquad (20b)$$

and for piezoelectric interaction

$$M_1(z)=4\pi\beta^2e^2m^2/\rho\varkappa^2\hbar^3, \qquad (21a)$$

$$M_2(z)=\frac{4\pi\beta^2e^2m^2}{\rho\varkappa^2\hbar^3}\left(\frac{w}{w_{ph}}\right)^2z[(\varepsilon_n-\varepsilon_{n'})^2+2(2\varepsilon-\varepsilon_n-\varepsilon_{n'})z+z^2]^{-1/2}. \qquad (21b)$$

In (20b) and (21b) we distinguish between the sound velocity w and the phonon velocity w_{ph} averaged over all the directions. Putting $w=0$, we obtain an equation valid for a superlattice at rest. Its solution is obviously the Fermi function.

The solution of (18) is

$$f(\varepsilon)=\left\{\exp\left[\int_\mu\frac{d\varepsilon}{\sigma(\varepsilon)}\right]+1\right\}^{-1}, \qquad (22)$$

where

$$g(\varepsilon)=T[1+A_2(\varepsilon)/A_1(\varepsilon)]. \qquad (23)$$

The constant μ is determined by the normalization condition

$$\int_{\varepsilon_0}^{\infty}d\varepsilon\,f(\varepsilon)\sum_n\vartheta(\varepsilon-\varepsilon_n)=\frac{2\pi^2\hbar^2}{mQ}N; \qquad (24)$$

here ε_0 is the energy of the lower level.

It is seen from (22) that $f(\varepsilon)$ is reminiscent of the Fermi function and μ plays the same role as the chemical potential in the equilibrium function. The interval over which it changes is $T_e\sim g(\varepsilon)$, where ε is the characteristic energy. In the case $\mu\gg T_e\sim g(\mu)$, just as the equilibrium function, $f(\varepsilon)$ takes the form of a step, with $\mu\sim\varepsilon_F$, and the disequilibrium manifests itself only in the structure of the step in the region $\varepsilon-\mu\sim T_e$.

Because $A_1\gtrsim A_2>0$, the energy T_e raises the lattice temperature by not more than a factor of two, i.e., the heating is relatively small. In the next section we shall see that in the case of scattering by impurities T_e may turn out to be much larger than T.

4. SCATTERING OF ELECTRONS BY IMPURITIES

The operator of electron-impurity collisions can be expressed in the form

$$I_i\{f\}=\int\frac{d^3q}{(2\pi)^3}\sum_{n'}M_i(q)|\varphi_{nn'}(q_x)|^2$$
$$\times[f_{n'}(\mathbf{k}-\mathbf{q}_\perp)-f_n(\mathbf{k})]\delta(\varepsilon_n(\mathbf{k})-\varepsilon_{n'}(\mathbf{k}-\mathbf{q}_\perp)+\hbar q_xw), \qquad (25)$$

where

$$M_i(q)=2\pi^2\hbar^3N_i\sigma_i/m^2 \qquad (26)$$

for short-range impurities (N_i is the impurity concentration and σ_i is the scattering cross section), and

$$M_i(q)=32\pi^3 N_i Z^2 e^4/\hbar q^4 \varkappa^2 \tag{27}$$

for ionized impurities (Ze is the impurity charge).

We call attention to the fact that the scattering by the impurities is inelastic. This inelasticity, just as in the case of scattering by phonons, stems from the fact that the electrons are not in immobile but in moving potential wells produced by the sound wave. Because of this inelasticity, the integral of the collisions with the impurities does not vanish when an arbitrary function of energy is substituted, as is the case for free electrons in a crystal lattice. It vanishes only when $f=\text{const}$ is substituted. This means that the impurities tend to distribute the electrons uniformly over all the energies.

In fact, let us examine the total change of the electron-gas energy under the influence of collisions with impurities. With the aid of (25) it is easy to obtain the relation

$$\begin{gathered}\frac{d}{dt}2\sum_n\int\frac{d^2k}{4\pi^2}\varepsilon_n(k)f_n(k)=4\pi w\int_0^\infty dq_x\hbar q_x\\ \times\sum_{nn'}\int\frac{d^2k\,d^2k'}{(2\pi)^4}\frac{d^2a}{(2\pi)^2}\delta(\mathbf{k}-\mathbf{k}'-\mathbf{q}_\perp)M_i(q)\\ \times|\varphi_{nn'}(q_x)|^2[f_n(k)-f_{n'}(k')]\delta(\varepsilon_n(k)-\varepsilon_{n'}(k')+\hbar q_x w).\end{gathered}\tag{28}$$

If $f_n(k)=f(\varepsilon_n(k))$ is a decreasing function of the energy, then the integral in the right-hand side is positive, i.e., the total energy of the electron gas increases. In a real crystal this heated can be limited only by the emission of phonons by the electrons. Thus, even if the collisions of the electrons with the impurities in the absence of sound are more frequent than with the phonons, a stationary solution of the kinetic equation for electrons that are strongly coupled by the sound wave can be obtained only by taking simultaneously account the collisions with the impurities and with the phonons.

The scattering by impurities will also be considered here in the quasi-elastic approximation. In this case

$$I_i\{f\}=\sum_n\int\frac{d^2k}{4\pi^2}I_i\{f\}\delta(\varepsilon-\varepsilon_n(k))=\frac{d}{d\varepsilon}A_3(\varepsilon)\frac{df(\varepsilon)}{d\varepsilon}, \tag{29}$$

and in the case of short-range impurities

$$A_3(\varepsilon)=\frac{1}{4}N_i\sigma_i\hbar w^2\sum_{nn'}\vartheta(\varepsilon-\varepsilon_n)\vartheta(\varepsilon-\varepsilon_{n'})\int\frac{dq}{2\pi}q^2|\varphi_{nn'}(q)|^2. \tag{30}$$

For ionized impurities, the contribution made to $A_3(\varepsilon)$ by transitions with conservation of the level number diverges logarithmically at small q_x. This divergence is due to the long-range character of the Coulomb potential and, consequently, to the large contribution of the long-wave components to its Fourier expansion. In the case of free electrons, this divergence, as is well known, is cut off at the Debye screening radius r_D. In this case the situation is more complicated. First, different relations between r_D and the wavelength of the sound are possible. If r_D exceeds the wavelength, then it can be shown that the screening takes place at distances of the order of r_D. In the opposite case the screening is at the wavelength of the sound. Second, in the reference frame concentrated with the sound, the impurities move and the electrons coupled by the sound wave screen the nonstationary potential. The screening can remain static, for example, if $w\tau_M/r_D\ll 1$ (τ_M is the Maxwellian relaxation time), but generally speaking it proceeds dynamically.

In view of the large variety of situation and since the logarithm is only a numerical factor in the function $A_3(\varepsilon)$, we shall denote it by L and will not write out explicitly the parameters under the logarithm sign. We note only that the main contribution to $A_3(\varepsilon)$ is made by transitions with conservation of the level number, since they are the only ones containing a large logarithm. As a result

$$\begin{gathered}A_3(\varepsilon)=B_0\sum_n\frac{\vartheta(\varepsilon-\varepsilon_n)}{(\varepsilon-\varepsilon_n)^{1/2}}\\ B=\frac{(2m)^{1/2}m^3N_iZ^2e^4w^2}{\hbar^6\varkappa^2}L.\end{gathered}\tag{31}$$

The solution of the kinetic equation

$$I_i\{f\}+I_{eph}\{f\}=0, \tag{32}$$

which takes into account scattering by both phonons and impurities, takes the form (22), where

$$g(\varepsilon)=T+[TA_2(\varepsilon)+A_3(\varepsilon)]/A_1(\varepsilon). \tag{33}$$

We note that in the case of scattering by a deformation potential of acoustic phonons and short-range impurities

$$A_3(\varepsilon)/A_2(\varepsilon)=T\tau_{ph}{}^0/\tau_i{}^0, \tag{34}$$

where $\tau_{ph}{}^0$ and $\tau_i{}^0$ are the relaxation times of the free electrons by the deformation potential of the acoustic phonons and the short-range impurities.

If scattering by impurities is more effective than by phonons, then at $\varepsilon<\varepsilon_0$ we have $g(\varepsilon)=T$, and at $\varepsilon>\varepsilon_0$ we have

$$g(\varepsilon)=A_3(\varepsilon)/A_1(\varepsilon). \tag{35}$$

In this case $A_1(\varepsilon)$ remains in the distribution function, meaning that the phonon emission processes must be taken into account. The heating is then much more appreciable than in the case of scattering by phonons only:

$$T_e\sim g(\varepsilon)\gg T. \tag{36}$$

5. ABSORPTION OF SOUND

The energy lost by the sound is transferred to the phonon system, and it is convenient to express it in terms of the phonon-electron collision integral. The change of the density of the sound-energy flux W per unit length is equal to the energy acquired by the phonons per unit time:

$$\frac{dW}{dx}=-\int\frac{d^3q}{(2\pi)^3}\hbar\omega_q I_{phe}\{N_q\}. \tag{37}$$

The phonon-electron collision integral is given by

$$\begin{gathered}I_{phe}\{N_q\}=\frac{Q}{\pi}\sum_{nn'}\int\frac{d^2k}{(2\pi)^2}M(q)|\varphi_{nn'}(q_x)|^2\\ \times\{(N_q+1)f_n(\mathbf{k})[1-f_{n'}(\mathbf{k}-\mathbf{q}_\perp)]\delta(\varepsilon_n(\mathbf{k})-\varepsilon_{n'}(\mathbf{k}-\mathbf{q}_\perp)-\hbar\omega_q+\hbar q_x w)\\ -N_q f_n(\mathbf{k})[1-f_{n'}(\mathbf{k}+\mathbf{q}_\perp)]\delta(\varepsilon_n(\mathbf{k})-\varepsilon_{n'}(\mathbf{k}+\mathbf{q}_\perp)+\hbar\omega_q-\hbar q_x w)\}.\end{gathered}\tag{38}$$

Putting $f_n(\mathbf{k})=f(\varepsilon_n(k))$ we obtain in first-order approximation in the quasielasticity

$$\frac{dW}{dx}=-\frac{Q}{\pi}\int_{\varepsilon_0}^{\infty}d\varepsilon\,A_1(\varepsilon)\left\{T\frac{df(\varepsilon)}{d\varepsilon}+f(\varepsilon)[1-f(\varepsilon)]\right\}. \tag{39}$$

This expression can be simplified by using the electron kinetic equation (32):

$$\frac{dW}{dx}=\frac{Q}{\pi}\int_{\varepsilon_0}^{\infty}d\varepsilon\frac{df}{d\varepsilon}[TA_2(\varepsilon)+A_3(\varepsilon)]. \tag{40}$$

If the electrons are heated by the impurity scattering so strongly that expression (35) can be used for $g(\varepsilon)$ and Boltzmann statistics are valid, the expression for the absorbed energy becomes even simpler:

$$\frac{dW}{dx}=-\frac{Q}{\pi}\int_{\varepsilon_0}^{\infty}d\varepsilon\,f(\varepsilon)A_1(\varepsilon). \tag{41}$$

The sound absorption coefficient Γ is obtained from the relation

$$\Gamma=-\frac{1}{W}\frac{dW}{dx}. \tag{42}$$

6. ULTRAQUANTUM LIMIT

We define as the ultraquantum limit a situation wherein the electrons fill only one lower level. For these electrons the potential produced by the sound wave can be regarded as parabolic, $U\sim -U_0(1-Q^2\xi^2/2)$. The screening can then be neglected. In fact, the electron concentration is limited, when only the lower level is filled, by the condition $N\lesssim \varepsilon_g mQ/\hbar^2$. Their wave functions are localized on the interval $\alpha^{1/2}Q^{-1}$, therefore the ratio of the screening potential to the potential produced by the sound is

$$\left(\frac{e^2\alpha N}{\varkappa Q^2}\right)\Big/\varepsilon_g\lesssim\frac{e^2m}{\varkappa\hbar^2}\left(\frac{Q}{N}\right)^{1/2}\alpha^{1/2}\ll 1 \tag{43}$$

owing to the gas condition (11) and the tight-binding condition. The wave functions of the electrons for this case are well known, and exact analytic expressions can be obtained for the coefficients $A_1(\varepsilon)$ and $A_2(\varepsilon)$. But since the number of cominations of various scattering mechanisms is quite large, we shall not present all the results, and consider only the case which we regard as most likely for piezoelectrics at low temperatures. The principal scattering mechanism is here the scattering by ionized impurities, and the important role in the energy relaxation is played by the piezoelectric interaction with the acoustic phonons. We have

$$A_1=\frac{2^{3/2}\pi^{1/2}\beta^2e^2m^{9/4}Q^{1/2}U_0^{1/4}}{\rho\varkappa^2h^{7/2}}=\text{const},\qquad A_3=B_0\frac{\vartheta(\varepsilon-\varepsilon_0)}{(\varepsilon-\varepsilon_0)^{1/2}}, \tag{44}$$

$$f(\varepsilon)=\left\{\exp\left[\frac{2A_1}{3B_0}[(\varepsilon-\varepsilon_0)^{3/2}-(\mu-\varepsilon_0)^{3/2}]\right]+1\right\}^{-1}. \tag{45}$$

The constant μ is determined from the equation

$$\int_0^{\infty}\frac{x^{-1/2}\,dx}{\exp[x-2A_1(\mu-\varepsilon_0)^{3/2}/3B_0]+1}=\left(\frac{3}{2}\right)^{1/3}\left(\frac{A_1}{B_0}\right)^{2/3}\frac{2\pi^2\hbar^2N}{mQ}. \tag{46}$$

The sound absorption coefficient is given by

$$\Gamma=\frac{QB_0}{\pi W}\left(\frac{mQ}{2\pi^2\hbar^2N}\right)^{1/2} \tag{47}$$

for Fermi statistics and

$$\Gamma=2\pi A_1\hbar^2N/Wm \tag{48}$$

for Boltzmann statistics. Thus, $\Gamma\sim W^{-1}$, i.e., the absorbed energy does not depend on the sound intensity (cf. Ref. 6). We note that for other mechanisms the dependence of Γ on W is generally speaking different. We can write an order-of-magnitude estimate for Γ, applicable to all mechanism, under the condition $T_e\lesssim\mu\sim\varepsilon_g$:

$$\Gamma\sim(\hbar wQ)^2N/\varepsilon_g\tau^0(\varepsilon_g)\alpha W, \tag{49}$$

where $\tau^0(\varepsilon)$ is the relaxation time of the free electrons.

We now turn our attention to the following important fact. The normalization condition (24) in the case of strong degeneracy, when $T_e\ll\varepsilon_g$, can be written in the form

$$(\mu-\varepsilon_0)\sum_n\vartheta(\mu-\varepsilon_n)=2\pi^2\hbar^2N/mQ. \tag{50}$$

With decreasing amplitude of the sound wave, the position of the levels ε_n changes [(see (6)], and as seen from (50), $\partial\mu/\partial U_0$, has discontinuities when U_0 is varied. The position of the first jump is determined by the condition $\mu-\varepsilon_0=\varepsilon_g$ or

$$U_0=4\pi^4\hbar^2N^2/mQ^4. \tag{51}$$

The next jump occurs when U_0 decreases by an amount on the order of (51).

Jumps take place not only in $\partial\mu/\partial U_0$, but also in the functions $A_1(\varepsilon)$, $A_2(\varepsilon)$, and $A_3(\varepsilon)$. Jumps of this kind leads to oscillations of $\Gamma(W)$. The amplitudes of the first oscillations that occur when U_0 is decreased are of the order of $\Gamma(W)$ itself. These oscillations stem from a mechanism similar to those of the de Haas-van Alphen or the Shubnikov-de Haas oscillations. The $\Gamma(W)$ oscillations, however, have an important distinguishing feature. Since one of the causes of the oscillations is the oscillatory dependence of the nonequilibrium distribution function $f(\varepsilon)$ on μ, the amplitude of the $\Gamma(W)$ oscillations does not decrease with rising temperature. We shall prove this statement for the opposite quasiclassical case, when the electrons fill many levels in the well and the structure of the results depends to a somewhat lower degree on the scattering mechanism.

7. QUASICLASSICAL CASE

If the electrons fill many levels in the sound well, i.e., $\max(\mu,T_e)\gg\varepsilon_g$, the principal role is played by levels with large numbers, whose wave functions can be regarded as quasiclassical:

$$\varphi_n(\xi)=\frac{c_n}{p_n^{1/2}}\cos\left(\frac{1}{\hbar}\int_a^{\xi}d\xi\,p_n-\frac{\pi}{4}\right),\qquad p_n=\{2m[\varepsilon_n-U(\xi)]\}^{1/2}. \tag{52}$$

Expanding the cosine in $\varphi_n(\varepsilon)$ in a sum of exponentials, we obtain for $\varphi_{nn'}(q)$ a sum of integrals, each of which is calculated by the saddle-point method. The saddle points ξ_l are defined by a relation that takes the form of the momentum-conservation law:

$$R(\xi)\equiv s_1p_n+s_2p_n'+\hbar q=0 \tag{53}$$

(here s_1 and s_2 take on values ±1). As a result,

$$|\varphi_{nn'}(q)|^2=\frac{\pi}{8}|c_nc_{n'}|^2\sum_{s_1s_2}\sum_l[R'(\xi)p_n(\xi)p_{n'}(\xi)]^{-1}_{\xi=\xi_l}. \tag{54}$$

It is convenient for the subsequent calculations to represent this expression in the form

$$|\varphi_{nn'}(q)|^2=\frac{1}{2\pi\hbar}\frac{d\varepsilon_n}{dn}\frac{d\varepsilon_{n'}}{dn'}\int d\xi\, dp_1\, dp_2\delta(p_1+p_2+\hbar q)$$
$$\times\delta\left(\varepsilon_n-\frac{p_1^2}{2m}-U(\xi)\right)\delta\left(\varepsilon_{n'}-\frac{p_2^2}{2m}-U(\xi)\right). \tag{55}$$

To verify the validity of (55) it suffices to carry out the integration with respect to p_1 and p_2, and then with respect to ξ, after which the expression (54) is obtained. For the normalization constant we have used the relation

$$|c_n|^2=\frac{2m}{\pi\hbar}\frac{d\varepsilon_n}{dn}, \tag{56}$$

which is obtained by comparing the standard expression for this constant[7] with the quasiclassical derivative of the quantization condition.

In the calculation of the functions $A_l(\varepsilon)$ we have used the Poisson summation formula

$$\sum_{n=0}^{\infty}F(n+{}^1\!/_2)=\int_0^{\infty}F(n)dn+2\sum_{s=1}^{\infty}(-1)^s\int_0^{\infty}F(n)\cos 2\pi sn\, dn. \tag{57}$$

The first term in this formula leads to a purely classical result in the form

$$A_l^{\mathrm{cl}}(\varepsilon)=A_{0l}\int d\xi[\varepsilon-U(\xi)]^{\lambda}\vartheta(\varepsilon-U), \tag{58}$$

where for deformation interaction with phonons $\lambda=2$,

$$A_{01}=\frac{2\Lambda^2m^4}{\pi^3\rho\hbar^7},\quad A_{02}=\frac{1}{3}\left(\frac{w}{w_{ph}}\right)^2A_{01}; \tag{59a}$$

for piezoelectric interaction with phonons $\lambda=1$,

$$A_{01}=\frac{8\beta^2e^2m^3}{\pi\rho\varkappa^2\hbar^5},\quad A_{02}=\frac{1}{3}\left(\frac{w}{w_{ph}}\right)^2A_{01}; \tag{59b}$$

and for ionized impurities $\lambda=0$,

$$A_{03}=\frac{B_0}{\hbar}\left(\frac{m}{2}\right)^{1/2}. \tag{59c}$$

For scattering by phonons, we have a Fermi distribution function with a temperature

$$T_e=T\left[1+\frac{1}{3}\left(\frac{w}{w_{ph}}\right)^2\right].$$

The sound absorption coefficient depends, generally speaking, on the form of the potential $U(\xi)$. In the case of scattering by phonons in the case of Boltzmann statistics, however, the integral that contains $U(\xi)$ is eliminated with the aid of the normalization condition (24), and

$$\Gamma_{\mathrm{cl}}(W)=\left(\frac{2\pi\hbar^2}{m}\right)^{3/2}\Gamma(\lambda+1)A_{02}T^{\lambda-1/2}\left[1+\frac{1}{3}\left(\frac{w}{w_{ph}}\right)^2\right]^{\lambda-3/2}\frac{N}{W}. \tag{60}$$

For the deformation integral, this formula differs by a numerical factor from the corresponding result obtained by Kagan.[8] The reason for the difference is that Kagan did not take into account the difference between T_e and T.

To take into account the quantum corrections, we turn to calculation of $A_l(\varepsilon)$ and take into account the sums over s in (19) and (31) when using formula (57). The sums over s, however, are small compared with the classical parts of $A_l(\varepsilon)$ in proportion to some power of the number of the significant levels, inasmuch as in each term of the sum over s the integration with respect to n is carried out with the oscillating function $\cos 2\pi sn$, and the results does not accumulate with increasing number of significant levels. All the double sums over s can therefore be discarded. It is convenient to change from integration with respect to n to integration with respect to ε, after which it remains to calculate the integrals over the momenta and the coordinate, which occur when (55) is substituted in (19) and which contain $\cos[2\pi sn(p^2/2m+U)]$, where

$$n(\varepsilon)=\frac{1}{\pi\hbar}\int dx(2m[\varepsilon-U(x)])^{1/2}\vartheta(\varepsilon-U). \tag{61}$$

In the integration with the coordinate, in view of the large values of $n(p^2/2m+U)$, the cosines oscillate rapidly and the corresponding integrals can be calculated by the saddle-point method. The remaining integrals with respect to the momenta can be calculated with the aid of the asymptotic formula

$$\int_0^{\infty}d\varepsilon\, f(\varepsilon)\cos\left[2\pi sn(\varepsilon)+\frac{\pi}{4}\right]=\frac{f(\varepsilon)\varepsilon_g}{2\pi s}\sin\left[2\pi sn(\varepsilon)+\frac{\pi}{4}\right]\left[1+O\left(\frac{1}{n}\right)\right] \tag{62}$$

where $\varepsilon_g=d\varepsilon/dn$. The final expressions are of the form

$$A_l(\varepsilon)=A_l^{\mathrm{cl}}(\varepsilon)+A_{0l}\sum_{s=1}^{\infty}a_l(\varepsilon,s)\sin[2\pi sn(\varepsilon)+\pi/4], \tag{63}$$

where we have for deformation interaction with phonons

$$a_1(\varepsilon,s)=(-1)^s\frac{\varepsilon\varepsilon_g^{3/2}}{\pi(U_m'')^{1/2}s^{3/2}},\quad a_2(\varepsilon,s)=2a_1(\varepsilon,s); \tag{64a}$$

for piezoelectric interaction with the phonons

$$a_1(\varepsilon,s)=(-1)^s\frac{\varepsilon_g^{3/2}}{\pi(U_m'')^{1/2}s^{3/2}},\quad a_2(\varepsilon,s)=\frac{3}{2}a_1(\varepsilon,s); \tag{64b}$$

and for ionized impurities

$$a_3(\varepsilon,s)=(-1)^s2\hbar(\varepsilon_gms)^{-1/2}. \tag{64c}$$

Here $U_m''=U''(\xi)$ with ξ corresponding to the minimum of $U(\xi)$.

The calculation of the absorption coefficient entails considerable difficulties connected with the calculation of integrals of functions whose form depends on the form of $U(\xi)$. We shall therefore consider hereafter only the limiting case of sufficiently strong degeneracy, when

$$T_e^2\ll\varepsilon_g\mu, \tag{65}$$

and the results can be obtained in closed form. Then, substituting (63) in (40), assuming $\varepsilon=\mu$ in all the smooth functions with the exception of the argument of the cosine, where $n(\varepsilon)$ is expanded in powers of $\varepsilon-\mu$ accurate to terms of first order, we get

$$\Gamma=\Gamma_0(\mu)+\frac{\pi Q}{W}\sum_{s=1}^{\infty}s[A_{02}Ta_2(\mu,s)+A_{03}a_3(\mu,s)]$$
$$\times\frac{\sin[2\pi sn(\mu)+\pi/4]}{\mathrm{sh}^2(2\pi^2sT_e/\varepsilon_g)} \tag{66}$$

$$\Gamma_0(\mu)=\frac{Q}{\pi W}[TA_2^{\mathrm{cl}}(\mu)+A_3^{\mathrm{cl}}(\mu)]. \tag{67}$$

In the calculation of the sum, which is the quantum correction, we have neglected the quantum corrections to $g(\varepsilon)$ in the distribution function. However, $g_{cl}=T+(TA_2^{cl}+A_3^{cl})/A_1^{cl}$ is a smooth function of ε, and we can therefore put $g_{cl}(\varepsilon)=g_{cl}(\mu)\equiv T_e$.

We use now the normalization condition (24) to determine μ. We put

$$g^{-1}(\varepsilon)=\frac{1}{T_e}-\frac{1}{T_e}\sum_{s=1}^{\infty}g_s\sin\left[2\pi sn(\varepsilon)+\frac{\pi}{4}\right], \tag{68}$$

$$g_s(\varepsilon)=[TA_{02}a_2(\varepsilon,s)+A_{03}a_3(\varepsilon,s)-(T_e-T)A_{01}a_1(\varepsilon,s)]/T_eA_1\ (\varepsilon). \tag{69}$$

The sum in (24) is calculated with the aid of the Poisson formula. The integral with respect to ε is calculated in the manner customarily used in the case of a strongly degenerate Fermi gas, using the same procedures as in the derivation of (67). As a result we get

$$\begin{gathered}\frac{2(2m)^{1/2}}{3\pi\hbar}\int dx[\mu-U(x)]^{3/2}\vartheta(\mu-U)\\ -T_e\sum_{s=1}^{\infty}\frac{(-1)^s}{s}\frac{\cos 2\pi sn(\mu)}{\operatorname{sh}(2\pi^2sT_e/\varepsilon_g)}\\ +\frac{n(\mu)\varepsilon_g}{2\pi}\sum_{s=1}^{\infty}\frac{g_s(\mu)}{s}\left[1-\frac{2\pi^2sT_e}{\varepsilon_g}\operatorname{sh}^{-1}\left(\frac{2\pi^2sT_e}{\varepsilon_g}\right)\right]\\ \times\cos\left(2\pi sn(\mu)+\frac{\pi}{4}\right)=\frac{2\pi^2\hbar^2N}{mQ}.\end{gathered} \tag{70}$$

The sums over s in the left-hand side of (70) are small compared with the principal term, and this equation can be solved by iteration:

$$\mu=\mu_0+\Delta\mu, \tag{71}$$

$$\frac{2(2m)^{1/2}}{3\pi\hbar}\int dx[\mu_0-U(x)]^{3/2}\vartheta(\mu_0-U)=\frac{2\pi^2\hbar^2N}{mQ}, \tag{72}$$

$\Delta\mu$ is an oscillating function of U_0. When substituting μ in (66), $\Delta\mu$ need be taken into account only in the first term. The general formula for Γ is too complicated to write out here; we note only two important circumstances.

First, as is obvious from (2),

$$\mu_0=U_0\Phi(\hbar^3N/m^{3/2}U_0^{3/2}),$$

where the form of the smooth function $\Phi(z)$ depends on the form of $U(x)$. As seen from (66), the function (W) executes a single oscillation when μ_0 changes by ε_g, i.e., when U_0 changes by an amount on the order of ε_gU_0/μ_0.

Second, the amplitudes of the oscillations do not decrease with increasing T_e. At $\varepsilon_g\ll T_e$ we have

$$\Delta\mu=-\frac{mQ}{4\pi^3\hbar^2}\frac{d\mu_0}{dN}n(\mu_0)\varepsilon_g\sum_{s=1}^{\infty}\frac{g_s}{s}\cos(2\pi sn(\mu_0)+\pi/4). \tag{73}$$

The sound absorption coefficient is in this case

$$\Gamma=\Gamma_0(\mu_0)+\frac{\partial\Gamma_0(\mu_0)}{\partial\mu_0}\Delta\mu. \tag{74}$$

The amplitude of the oscillations is small compared with the smooth part of Γ in a ratio $1/n(\mu_0)$.

8. INFLUENCE OF ELECTRIC FIELD. CURRENT-VOLTAGE CHARACTERISTIC

We shall consider only not too strong a longitudinal electric field E_x which, while distorting the form of the potential wells, does not violate the strong-coupling conditions. This is the case, at any rate, provided only the upper bound states remain free under the influence of the field, i.e.,

$$eE_x/Q\ll U_0. \tag{75}$$

Under this condition the results of the preceding sections are modified only to the extent that they depend on the shape of the potential well. The only exception is the change of the sound absorption coefficient, due to the work performed by the electric field on the electrons that move together with the sound wave. The absorption coefficient requires an increment

$$\Gamma_E=-eE_xwN/W. \tag{76}$$

This increment does not depend on the states of the electrons in the well or on their statistics. It is the same in the quantum as in the classical case and was first obtained in Ref. 8.

Under the condition (75), the electric field is the total dragged current $j=eNw$ and does not depend on the electric field. At certain values of the electric field, however, peaks are imposed on the total dragged current, from the following sources. The field-induced increments to the electron energy differ in neighboring wells by an amount eE_x/Q. When this difference coincides with ε_g, the positions of the levels in the neighboring wells coincides, and this facilitates the tunneling between them and increases the current by an amount on the order of $eN\Delta/\hbar Q$. The width of this peak, in terms of the field, is of the order of $\hbar Q/e\tau$. Such oscillations in superlattices were predicted by Kazarinov and Suris.[5] It must be noted, however, that a distinct picture of peaks can be observed only if $\max(\mu, T_e)\lesssim\varepsilon_g$. If this condition does not hold, then transitions can occur between different pairs of levels in neighboring wells and the peak patterns become superimposed.

9. POSSIBILITIES OF EXPERIMENTAL OBSERVATION

Most criteria cited in this paper indicate that the amplitude of the periodic potential U_0 produced by the sound wave should be high enough. Without account of the screening, the amplitude U_0 is connected with the amplitude of the elastic displacement in the sound wave u_0 by the relation $U_0=\Lambda Qu_0$ for the deformation interaction or $U_0=4\pi\beta eu_0/\varkappa$ for the piezoelectric interaction.

The most important conditions are the condition (1), to satisfy which it is necessary that the depth of the potential wells exceed the electron energy uncertainty

$$U_0\gg\hbar/\tau, \tag{77}$$

and strong coupling condition $\alpha\lesssim 1$.

We present numerical estimates for three materials: the piezoelectric semiconductor Te, which has one of the largest coupling constants, the piezoelectric semiconductor InSb traditionally used in experiments with sound, and the semiconductor Ge with deformation coupling, in which τ at low temperatures can be large enough. At a lattice temperature equal to several degrees Kelvin, we have $\hbar/\tau\sim 0.1$ K for Ge and ~10 K for

InSb and Te (it is convenient to measure the energy in degrees in these estimates).

The amplitude of the potential is $U_0=2\ \mathrm{K}\cdot(W/W_0)^{1/2}$ in Ge, and $U_0=5\ \mathrm{K}(20\ \mathrm{K})\cdot\ \nu_0 W^{1/2}/\nu W_0^{1/2}$ in InSb and Te, respectively; here W is the intensity and the frequency of the sound wave, $W_0=1\ \mathrm{W/cm^2}$, and $\nu_0=5-10^9$ Hz. Thus, Eq. (77) is satisfied at a sound intensity on the order of 1 W/cm^2 in Ge and on the order of several dozen watts per square centimenter in InSb and Te. At these intensities, the condition $\alpha\lesssim 1$ is satisfied at a sound frequency not exceeding 10^{10} Hz.

In addition, we have stipulated that all the electrons be under tight-binding conditions (9), i.e.,

$$T_e,\ \varepsilon_F<U_0. \tag{78}$$

Finally, the condition (10) meant that the screening of the soundwave field by the conduction electrons must not violate the tight-binding condition. At an electron concentration $N\sim 10^{12}\ \mathrm{cm^{-3}}$ and a sound frequency $\nu\sim 5\cdot 10^9$ Hz the quantity $e^2N/\varkappa Q^2$, which should be less than U_0, is of the order of 1 K. At these sound intensities the condition (10) should be satisfied in Ge at $N\sim 10^{12}$ and in InSb and Te at $N\sim 10^{13}$. At such concentrations we have $\varepsilon_F\sim 0.01$ K in Ge and ~1 K in InSb and Te, i.e., Eq. (78) sets the temperature limit.

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[1] It is seen from (8) that at $n>2^5/\alpha$ the width Δ_n of the allowed band begins to decrease. But expression (8) is likewise no longer valid at these values.

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Plasma-acoustic waves on the surface of a piezoelectric crystal

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A theoretical analysis is made of the interaction between a two-dimensional electron plasma and shear surface piezoacoustic waves. The cases of a plasma layer on the surface of a piezoelectric crystal in vacuum and of an inversion channel in a metal–insulator–semiconductor structure are considered. Renormalization of the velocity of sound and damping of acoustic waves due to their interaction with a plasma are found. A specific damping mechanism of two-dimensional plasmons associated with the emission of acoustic waves is investigated. In all cases considered the characteristic parameters of the wave processes depend strongly on the surface charge density, which should make it possible to control them in experimental studies.

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INTRODUCTION

Electron processes in quasi-two-dimensional systems are attracting considerable attention. Two types of system are being investigated more than others: electrons above the surface of liquid helium (or helium film) and carriers in inversion channels in metal–insulator–semiconductor (MIS) structures. These two types of system are being investigated because they provide experimental means for varying the characteristic parameters in a wide range so that the main parameter, which is the surface charge density, can be varied over four orders of magnitude.

Recent experiments on inversion layers in silicon[1,2] have revealed the presence of two-dimensional plasmons. This is a very important result because the interaction of two-dimensional plasmons with other oscil-