Nonlinear effects in threshold absorption of sound in superconductors

B. I. Ivlev and N. B. Kopnin

L. D. Landau Institute of Theoretical Physics, Academy of Sciences of the USSR, Moscow
(Submitted 8 June 1978)
Zh. Eksp. Teor. Fiz. **75**, 1828–1836 (November 1978)

The absorption coefficient of longitudinal sound γ^s calculated in the linear approximation has a discontinuity at $\omega = 2\Delta(T)$. If allowance is made for the finite intensity of the acoustic wave, the coefficient γ^s becomes a continuous function of Δ but there is a jump in the temperature dependence of Δ. This again gives rise to a discontinuity of the absorption coefficient. The position of the discontinuity on the temperature scale shifts by an amount proportional to the intensity of sound and may exhibit hysteresis.

PACS numbers: 74.30. – e

1. INTRODUCTION

It is well known[1] that the absorption coefficient of longitudinal sound in superconductors exhibits a discontinuity

$$\frac{\delta\gamma^s}{\gamma^n}=\frac{\pi}{2}\operatorname{th}\frac{\Delta}{2T} \tag{1}$$

at the acoustic frequency $\omega=2\Delta(T)$. The abrupt increase in the absorption coefficient γ^s at the threshold frequency is due to the appearance of a new absorption mechanism, which is the direct creation of a pair of excitations by a phonon. The classical result (1) is obtained in the linear approximation in respect of the acoustic field and is valid at low sound intensities. Recent experiments (see, for example, Ref. 2) are evidence of increasing interest in the nonlinear effects in the threshold absorption, i.e., in the influence of the sound intensity on the absorption singularities near $\omega=2\Delta(T)$. In view of this it would be desirable to consider the problem theoretically.

We shall show that if allowance is made for the finite intensity of the acoustic wave, the absorption coefficient γ^s becomes a continuous function of the order parameter Δ and a jump changes to a narrow transition region of width proportional to the square of the intensity. However, although the discontinuity disappears in γ^s considered as a function of Δ, it is still present in the temperature dependence of the order parameter at some fixed frequency (or in the dependence of Δ on ω at a fixed temperature). The jump $\Delta(T)$ occurs in the region of $\Delta(T)\approx\omega/2$ and it is proportional to the intensity of sound. It is associated with the strong dependence of the distribution function of excitations on the sound intensity near the absorption threshold where phonons of energy 2Δ create excitations.

The jump $\Delta(T)$ is superimposed on the dependence γ^s and it again gives rise to a discontinuity of $\gamma^s(T)$ considered as a function of temperature. In the first approximation, the magnitude of this jump is given by Eq. (1) and its position on the temperature axis shifts relative to T_0 [$\Delta(T_0)=\omega/2$ in the absence of sound] by an amount proportional to the sound intensity and it may exhibit hysteresis.

For simplicity, we shall consider a pure superconductor so that $\tau\Delta\gg1$, where τ is the electron transit time between collisions with impurities. This restriction is not important because the real parameter of the expansion is not $(\tau\Delta)^{-1}$ but $(\tau v_F k)^{-1}$, where $k=\omega/s$ is the wave vector of sound. For $\omega\sim\Delta$, this parameter is of the order of $(\Delta\tau v_F/s)^{-1}$, which is much less than unity right up to very long transit times.

2. ABSORPTION COEFFICIENT NEAR THE THRESHOLD

The absorption of sound is governed by the imaginary part of the self-energy of phonons which results from the interaction with electrons. In the case of longitudinal sound, this interaction is described by a correction χ to the energy (chemical potential) of electrons, which is related to the change in the charge density in the lattice. The Hamiltonian of the interaction is

$$H_{int}=\int\psi^+\psi\chi\,d\mathbf{r}=g\int\psi^+\psi\varphi\,d\mathbf{r}, \tag{2}$$

where ψ and φ are the electron and phonon field operators. We can go over to the phonon field in Eq. (2) employing the relationship

$$-{}^1/_3 v_F p_F \operatorname{div}\mathbf{u}+\chi=0$$

(where $\mathbf{u}$ denotes the displacements of ions), which follows from the electrical neutrality condition in the principal approximation in respect of s/v_F.

We can show that the self-energy of phonons is given by (the upper index R represents the retarded function)

$$\Sigma_\omega^{(ph)R}(\mathbf{k})=-(\chi_\mathbf{k})^{-1}g^2\int\frac{d\varepsilon}{4\pi i}\frac{d\mathbf{p}}{(2\pi)^3}\operatorname{Sp}\left\{\hat{G}_{\varepsilon+\omega/2,\varepsilon-\omega/2}\left(\mathbf{p}+\frac{\mathbf{k}}{2},\mathbf{p}-\frac{\mathbf{k}}{2}\right)\right\}. \tag{3}$$

Here, $\hat{G}$ is the correction to the total electron Green function

$$\hat{G}=\begin{pmatrix} G & F \\ -F^+ & \bar{G}\end{pmatrix},$$

which is due to the interaction of χ with phonons. In the approximation which is linear in respect of this interaction, Eq. (3) represents the usual loop of the polarization operator.

The Green function G can be found from the Éliash-

 0038-5646/78/110921-05$02.40

berg system of equations[3] for the functions integrated with respect to $\xi_p = p^2/2m - E_F$:

$$\hat{g}_{\varepsilon,\varepsilon'}(\mathbf{p}_F,\mathbf{k}) \equiv \begin{pmatrix} g & f \\ -f^+ & \bar{g} \end{pmatrix} = \int \frac{d\xi_p}{\pi i} \hat{G}_{\varepsilon,\varepsilon'}\left(\mathbf{p}+\frac{\mathbf{k}}{2}, \mathbf{p}-\frac{\mathbf{k}}{2}\right).$$

The matrix form of these equations is

$$v_F \mathbf{k}' \hat{g}_{\varepsilon_+,\varepsilon_-} - \varepsilon_+ \tau_z \hat{g}_{\varepsilon_+,\varepsilon_-} + \hat{g}_{\varepsilon_+,\varepsilon_-} \tau_z \varepsilon_- + \{\hat{h}\hat{g} - \hat{g}\hat{h}\}_{\omega',k'} = \hat{I}^{ph}_{\varepsilon_+,\varepsilon_-} + \hat{I}^{imp}_{\varepsilon_+,\varepsilon_-}, \quad (4)$$

where τ_z is the Pauli matrix; $\varepsilon_\pm = \varepsilon \pm \omega'/2$; $I^{imp}_{\varepsilon_+,\varepsilon_-}$ is the integral representing collisions with impurities and $I^{ph}_{\varepsilon_+,\varepsilon_-}$ is the integral representing collisions with phonons, where

$$\hat{I}^{ph}_{\varepsilon_+,\varepsilon_-} = \{\hat{\Sigma}^R \hat{g} - \hat{g}\Sigma^A + \hat{\Sigma}\hat{g}^A - \hat{g}^R\hat{\Sigma}\}_{\omega',k'}. \quad (5)$$

Here, the matrix $\hat{\Sigma}$ is the electron self-energy part; the braces $\{\ldots\}\omega', k'$ denote contraction over ω and $\mathbf{k}$. The matrix $\hat{h}$ is given by

$$\hat{h} = \begin{pmatrix} \chi & -\Delta \\ \Delta & \chi \end{pmatrix}$$

(the order parameter is assumed to be real).

The function $g_{\varepsilon_+,\varepsilon_-}$, which should satisfy the normalization condition of Larkin and Ovchinnikov,[3a] can be represented in the form

$$\hat{g}_{\varepsilon_+,\varepsilon_-} = \hat{g}^R_{\varepsilon_+,\varepsilon_-} f_{\varepsilon_-} - f_{\varepsilon_+} \hat{g}^A_{\varepsilon_+,\varepsilon_-} + \hat{g}^{(a)}_{\varepsilon_+,\varepsilon_-}, \quad (6)$$

where f_ε is related to the excitation distribution function. In equilibrium, we have $f_\varepsilon = 1 - 2n_\varepsilon$, where n_ε is the Fermi function. The Green functions $g_{\varepsilon+\omega/2,\varepsilon-\omega/2}$ are governed by the elements of Eq. (4) which are nondiagonal in respect of the energy ($\omega' = \omega$). Since $v_F k = v_F\omega/s \gg \tau_{ph}^{-1}$, τ^{-1}, where τ_{ph} is the phonon relaxatime, we can find $\hat{g}^{(a)}_{\varepsilon+\omega/2,\varepsilon-\omega/2}$ ignoring the phonon relaxation. The expression for this function is obtained from Eq. (4) or, from[4]

$$\hat{g}^{(a)}_{\varepsilon+\omega/2,\varepsilon-\omega/2}(\mathbf{p},\mathbf{k}) = -\int \frac{d\xi_p}{\pi i} \hat{G}^R_{\varepsilon+\omega/2}\left(\mathbf{p}+\frac{\mathbf{k}}{2}\right)\hat{h}_\omega(\mathbf{k}) \times \hat{G}^A_{\varepsilon-\omega/2}\left(\mathbf{p}-\frac{\mathbf{k}}{2}\right)(f_{\varepsilon+\omega/2} - f_{\varepsilon-\omega/2}). \quad (7)$$

In the expression (7) the functions $G^{R(A)}$ are taken in the zeroth order in χ. This is done because the difference between $G^{R(A)}$ and the unperturbed functions is governed by the ratio χ/ε (where $\varepsilon \sim \Delta$), whereas the nonlinear effects of interest to us are associated with the departure of the "distribution function" f_ε from equilibrium, which occurs much earlier and which is due to the slowness of the electron–phonon relaxation. For the same reason, in the case of the regular functions $\hat{g}^{R(A)}_{\varepsilon_+,\varepsilon_-}$ in Eq. (6) we need to consider only the principal approximation in χ:

$$\hat{g}^{R(A)}_{\varepsilon_+,\varepsilon_-} = \hat{g}^{R(A)}_{\varepsilon} \cdot 2\pi\delta(\omega') - \int \frac{d\xi_p}{\pi i} \hat{G}^{R(A)}_{\varepsilon_+}\left(\mathbf{p}+\frac{\mathbf{k}'}{2}\right)\hat{h}_{\omega'}(\mathbf{k}')\hat{G}^{(A)}_{\varepsilon_-}\left(\mathbf{p}-\frac{\mathbf{k}'}{2}\right). \quad (8)$$

The formulas (7) and (8) are deduced directly from the Feynman graphs.

The function f_ε is governed by the terms in Eq. (4) which are diagonal in respect of energy (and $\mathbf{k}$), i.e., by the terms with $\omega' = k' = 0$. After averaging over the directions of the vector $\mathbf{v}_F$, we find from Eq. (5) that[3]

$$\int \frac{dO_p}{4\pi} \mathrm{Sp}\{\hat{h}\hat{g} - \hat{g}\hat{h}\}_{\omega'=k'=0} = \int \frac{dO_p}{4\pi} \mathrm{Sp}\{\hat{I}^{ph}_\varepsilon(\mathbf{p}, k'=0)\}. \quad (9)$$

We must bear in mind that the order parameter Δ should be found self-consistently using the equation

$$\frac{\Delta_{\omega'}(k')}{|\lambda|} = \nu(0) \int \frac{d\varepsilon}{4} \frac{dO_p}{4\pi} f_{\varepsilon_+,\varepsilon_-}(\mathbf{p},k'). \quad (10)$$

Equations (3) and (6)–(10) allow us to consider the nonlinear effects of interest to us near the threshold. The transport equations (4) and (9) have been used earlier[5] to study the nonlinear absorption of sound in superconductors without allowance for the threshold effects.

We shall discuss a spatially homogeneous superconductor. Moreoever, for simplicity, we shall assume that $v_F k \approx v_F 2\Delta/s \gg T$, i.e., we shall ignore a very narrow range of temperatures near T_c where $(1 - T/T_c) \lesssim 10^{-6}$.

The frequency corrections ($\omega' \neq 0$) to the order parameter in Eq. (10) are small and, therefore, we can then assume in Eqs. (7) and (8) that $h_\omega = \chi_\omega$. Taking the imaginary part of $\Sigma^{(ph)R}_\omega(k)$, we find that the absorption coefficient is given by

$$\frac{\gamma'}{\gamma^n} = -\frac{1}{\omega}\int_{-\infty}^{\infty} \frac{d\varepsilon}{2} \theta(\varepsilon_+^2 - \Delta^2)\theta(\varepsilon_-^2 - \Delta^2) \frac{\varepsilon_+\varepsilon_- - \Delta^2}{\xi^R_{\varepsilon_+}\xi^R_{\varepsilon_-}}(f_{\varepsilon_-} - f_{\varepsilon_+}). \quad (11)$$

Here, $\varepsilon_\pm = \varepsilon \pm \omega/2$ and $\xi^{R(A)}_\varepsilon$ are the values of the function $(\varepsilon^2 - \Delta^2)^{1/2}$, which is analytic in the plane of the complex variable ϵ with cuts from $-\infty$ to $-\Delta$ and from Δ to ∞; these functions are taken along the upper (lower) edge of the cut where the condition $\varepsilon > +\Delta$ leads to the value $\xi^{R(A)}_\varepsilon = \pm(\varepsilon^2 - \Delta^2)^{1/2} + i\delta$ of this function. Equation (11) is a generalization of the familiar expression for γ^S to the nonlinear case.

The distribution function obtained from Eq. (9) is (compare with Refs. 3 and 5)

$$\varepsilon_0 \left\{ (f_\varepsilon - f_{\varepsilon+\omega}) \frac{\varepsilon(\varepsilon+\omega) - \Delta^2}{\xi^R_\varepsilon \xi^R_{\varepsilon+\omega}} \theta[(\varepsilon+\omega)^2 - \Delta^2] - (f_{\varepsilon-\omega} - f_\varepsilon) \frac{(\varepsilon-\omega)\varepsilon - \Delta^2}{\xi^R_{\varepsilon-\omega}\xi^R_\varepsilon} \theta[(\varepsilon-\omega)^2 - \Delta^2] \right\} = J^{ph}_\varepsilon. \quad (12)$$

Here, the "intensity" of sound is

$$\varepsilon_0 = \frac{\pi^2 N_0 k |\mathbf{u}_k|^2}{6m_e} = \frac{\pi^2 Z}{6} \frac{\hbar^2 w}{m_e M_{at} \omega s}.$$

In the above expression, N_0 is the electron number density; Z and M_{at} are the atomic valence and mass; $w = \rho|\omega u|^2$ is the acoustic energy density in a sample. According to Eliashberg,[3] the collision integral is

$$J^{ph}_\varepsilon = -\frac{\varepsilon}{\xi^R_\varepsilon}\theta(\varepsilon^2 - \Delta^2)\frac{\nu(0) g^2 \pi}{2(sp_F)^2}\int_{-\infty}^{\infty} \frac{\varepsilon'}{\xi^R_{\varepsilon'}}(\varepsilon - \varepsilon')^2 \times \mathrm{sign}(\varepsilon - \varepsilon') d\varepsilon' \left(1 - \frac{\Delta^2}{\varepsilon\varepsilon'}\right)\left[\mathrm{cth}\left(\frac{\varepsilon' - \varepsilon}{2T}\right)(f_\varepsilon - f_{\varepsilon'}) - f_\varepsilon f_{\varepsilon'} + 1\right]\theta(\varepsilon'^2 - \Delta^2).$$

We are interested in the absorption coefficient of sound in the direct vicinity of the threshold, i.e., in the case when

$$\omega/2 = \Delta(1+\alpha),$$

where $\alpha \ll 1$. The intensity of sound will be assumed to be small: $\varepsilon_0 \tau_{ph} \lesssim \Delta/T \ll 1$. We shall show later that the near-threshold nonlinear effects are governed by the parameter $p \propto \varepsilon_0 \tau_{ph} \alpha^{-1/2}$, which can be greater or

smaller than unity. We shall consider only temperatures close to the critical value. For this reason we can reduce the collision integral to the τ approximation[6]:

$$J_\varepsilon^{ph} = -\frac{\varepsilon}{\xi_\varepsilon^R}\theta(\varepsilon^2-\Delta^2)\frac{\tilde f_\varepsilon}{\tau_{ph}},$$
$$\tau_{ph}^{-1} = \frac{2\nu(0)g^2\pi}{(sp_F)^2}\int_0^\infty \frac{\varepsilon^2\,d\varepsilon}{\operatorname{sh}(\varepsilon/T)} = \frac{7\zeta(3)\pi\nu(0)g^2T^3}{(sp_F)^2}, \tag{13}$$

where $\tilde f_\varepsilon$ is the nonequilibrium correction to the distribution function:

$$f_\varepsilon = f_\varepsilon^{(0)} + \tilde f_\varepsilon, \quad f_\varepsilon^{(0)} = \operatorname{th}(\varepsilon/2T).$$

Equation (13) is valid for $\varepsilon\sim\Delta \ll T$. It should be noted that the nonequilibrium correction f_ε is not necessarily small compared with $f_\varepsilon^{(0)}$ if $\varepsilon\sim\Delta$. For example, in the threshold region, where the zero values of the radicands in the denominators of the left-hand sides of Eq. (12) approach one another, the correction $\tilde f_\varepsilon$ can be of the same order of magnitude as the equilibrium function $\tanh(\varepsilon/2T)$ in the $\varepsilon\sim\Delta$ case. This circumstance results in strong nonlinearities near the threshold.

The frequency integral in Eq. (11) can be split into three ranges: $-\infty<\varepsilon<-(\Delta+\omega/2)$, $\Delta+\omega/2<\varepsilon<\infty$, and $-(\omega/2-\Delta)<\varepsilon<\omega/2-\Delta)$. The last range exists only for $\omega/2>\Delta$ (i.e., for $\alpha>0$) and is responsible for the jump in γ^s considered in the linear approximation. The contribution of the first two integration ranges at zero intensity is a continuous function of Δ, and when the nonlinearity is allowed for, a correction of the order of $\varepsilon_0\tau_{ph}\ll 1$ has to be added. If Eqs. (12) and (13) are used to find the correction to the equilibrium distribution function, the following expression is obtained for the subthreshold range $\omega=2\Delta-0$ (i.e., for $\alpha\to-0$):

$$\frac{\gamma^s}{\gamma^n} = 1-\frac{\pi\Delta}{4T} - \varepsilon_0\tau_{ph}\frac{\Delta}{T}C, \tag{14}$$

where $C=1.67$.

We shall now consider the integration range $-(\omega/2-\Delta)<\varepsilon<\omega/2-\Delta$, where the nonlinear effects are particularly strong. By analogy with the linear case, we shall denote the contribution of this range by $\delta\gamma^s$. Using Eqs. (12) and (13), we find that when $-\Delta\alpha<\varepsilon<\Delta\alpha$, $\alpha=(\omega/2-\Delta)/\Delta>0$, then

$$f_{\varepsilon+\omega/2}-f_{\varepsilon-\omega/2} = \frac{2\operatorname{th}(\Delta/2T)}{1+2\varepsilon_0\tau_{ph}\Delta\{[(\varepsilon+\omega/2)^2-\Delta^2]^{-1/2}+[(\omega/2-\varepsilon)^2-\Delta^2]^{-1/2}\}}. \tag{15}$$

After substitution in Eq. (11), we find that the absorption jump is

$$\frac{\delta\gamma^s}{\gamma^n} = \frac{\pi}{2}\operatorname{th}\left(\frac{\Delta}{2T}\right)\Phi(p),$$
$$\Phi(p) = \frac{2}{\pi}\int_0 \frac{dx}{(1-x^2)^{1/2}+p[(1+x)^{1/2}+(1-x)^{1/2}]} \tag{16}$$

where $p=\varepsilon_0\tau_{ph}(2/\alpha)^{1/2}$. In the immediate vicinity of the threshold, where $\alpha\to 0$ and $p\gg 1$, we find that

$$\Phi(p)\propto p^{-1}=\alpha^{1/2}/2^{1/2}\varepsilon_0\tau_{ph},$$

i.e., $\delta\gamma^s$ vanishes at $\alpha=0$. An increase in α at a given intensity reduces the parameter p and for $p\ll 1$ we have

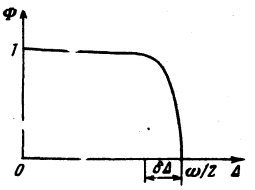

FIG. 1.

$$\Phi(p) = 1-2^{1/2}p\pi^{-1}\ln p^{-1}, \tag{17}$$

where $\delta\gamma^s$ is given by Eq. (1) in the zeroth approximation with respect to the intensity. The dependence of $\Phi(p)$ on Δ is shown schematically in Fig. 1. We can see that if allowance is made for the nonlinearity, the absorption coefficient becomes a continuous function of Δ and the jump is smeared out into a transition region of width $\delta\Delta\sim\Delta(\varepsilon_0\tau_{ph})^2$.

We have considered the behavior of the absorption coefficient as a function of the order parameter. However, in a real situation the order parameter at a given intensity is a function of temperature and should be found from Eq. (10), which we shall do in the next section.

3. THRESHOLD EFFECTS AND ORDER PARAMETER

The nonequilibrium nature of the excitations created by an acoustic wave alters the order parameter. This effect, known as superconductivity stimulation, has been investigated earlier[6-8] without allowance for the threshold phenomena. Near the threshold the deviations of the distribution function f_ε from equilibrium are large even at low intensities, and, therefore, we can expect the occurrence of singularities of the order parameter. The steady-state part of the order parameter is found from Eq. (6), in which the anomalous function is $\hat g^{(a)}=0$ at $\omega'=0$, and separating the equilibrium part in f_ε, we obtain

$$\nu(0)\left[\frac{T-T_c}{T_c}\Delta+\frac{7\zeta(3)}{8\pi^2T^2}\Delta^3\right]$$
$$-\nu(0)\int_0^\infty \frac{\Delta}{\xi_\varepsilon^R}\tilde f_\varepsilon\theta(\varepsilon^2-\Delta^2)\,d\varepsilon. \tag{18}$$

The nonequilibrium function $\tilde f_\varepsilon$ differs from zero, when one of the functions $\theta[(\varepsilon+\omega)^2-\Delta^2]$ or $\theta[\varepsilon-\omega)^2-\Delta^2]$ differs from zero. Therefore, the process of integration in Eq. (18) can be divided to two ranges: $\Delta+2\Delta\alpha<\varepsilon<+\infty$ and $\Delta<\varepsilon<\Delta+2\Delta\alpha$. The contribution of the first range is a smooth function of Δ and we can calculate the integral assuming that $\alpha=0$. With the aid of Eq. (12), we find that the contribution of this range to Eq. (18) is

$$-\nu(0)a(\Delta^2/T)\varepsilon_0\tau_{ph},$$

where

$$a=\pi-\pi/\sqrt{3}.$$

Integration over the range $\Delta<\varepsilon<\Delta+2\Delta\alpha$ gives rise to a singularity in Eq. (18). Consequently, we find that $\tilde f_\varepsilon$ is described by

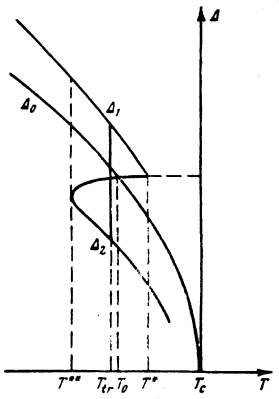

FIG. 2.

$$f_\varepsilon = -2\varepsilon_0\tau_{ph}\frac{\Delta}{[(\omega-\varepsilon)^2-\Delta^2]^{1/2}}(f_\varepsilon - f_{\varepsilon-\omega}).$$

The difference $f_\varepsilon - f_{\varepsilon-\omega}$ can be found from Eq. (15) by a suitable frequency shift. Integration yields the following contribution to Eq. (18):

$$\nu(0)\pi\Delta^2 T^{-1}\varepsilon_0\tau_{ph}\Phi(p)\theta(\alpha).$$

The function $\Phi(p)$ is defined above.

The final equation for Δ becomes

$$\nu(0)\left\{\frac{T-T_c}{T_c}\Delta + \frac{7\zeta(3)}{8\pi^2 T_c^2}\Delta^3 - \frac{\Delta^2}{T_c}\varepsilon_0\tau_{ph}\left[a - \pi\Phi(p)\theta\left(\frac{\omega}{2}-\Delta\right)\right]\right\} = 0. \tag{19}$$

Figure 2 shows schematically the temperature dependence of Δ. It has a beak-shaped singularity at $\Delta(T) = \omega/2$ and the width of this singularity along the temperature axis is of the order of $\Delta\varepsilon_0\tau_{ph}$. The temperature T^* at which $\Delta(T^*) = \omega/2$ is

$$T^* = T_0 + {}^1/_2\omega a\varepsilon_0\tau_{ph},$$

where T_0 is found from the condition $\Delta(T_0) = \omega/2$ when the intensity is zero. Differentiating Eq. (19) with respect to Δ, we find that T^{**} corresponds to

$$p \sim ((\omega/T)\varepsilon_0\tau_{ph})^{1/2} \ll 1, \quad \alpha = (T/\omega)^{1/2}(\varepsilon_0\tau_{ph})^{1/2} \ll 1.$$

For these values of p we have $\Phi(p) \approx 1$ and

$$T^{**} = T_0 + {}^1/_2\omega(a-\pi)\varepsilon_0\tau_{ph}.$$

In the interval between T^{**} and T^* we can have generally three different values of Δ at any given temperature. Only the following regions of $\Delta(T)$ are stable and at any given temperature one falling region corresponds to a stable state and the other to a metastable state. The limit of metastability can be found by the Langevin method of random forces (see, for example, Refs. 9 and 10). Therefore, the right-hand side of Eq. (10) should be supplemented by the random force $f(\mathbf{r}, t)$, about which we shall assume that

$$\langle f(\mathbf{r}, t) f(\mathbf{r}', t')\rangle = A\delta(\mathbf{r}-\mathbf{r}')\delta(t-t'),$$

where the averaging is carried out over the random distribution. The correlation function A is selected so that in the absence of the acoustic wave the probability of this state is equal to $\exp(-V\mathscr{F}_{GL}/T)$, where $\mathscr{F}_{GL}$ is the density of the free Ginzburg–Landau energy and V

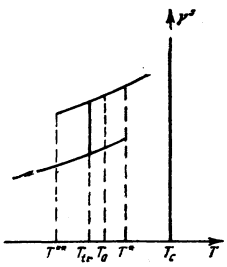

FIG. 3.

is the volume of the superconductor.

Applying the standard procedure (see, for example, Ref. 11), we shall now go over from Eq. (10) to a random force in the Fokker–Planck equation for the probability of a given state $W(\Delta, t)$. The stationary solution of this equation is

$$W(\Delta) = W(\Delta_0)\exp\left\{-\frac{V}{T}\int_{\Delta_0}^{\Delta} F(\Delta)\,d\Delta\right\},$$

where $F(\Delta)$ denotes the lefthand side of Eq. (19). In the absence of the acoustic wave the integral of $F(\Delta)$ reduces to $\mathscr{F}_{GL}$ and we return to the usual expression for $W(\Delta)$. The temperature T_{tr} which sets the limit of metastability of the upper and lower branches $\Delta_1(T)$ and $\Delta_2(T)$ (see Fig. 2) is found from the condition

$$\int_{\Delta_1(T_{tr})}^{\Delta_2(T_{tr})} F(\Delta)\,d\Delta = 0.$$

The application of Eq. (19) gives

$$T_{tr} = T_0 + {}^1/_2\omega(a-\pi/2)\varepsilon_0\tau_{ph} = T_0 - {}^1/_{12}\omega(2\cdot 3^{1/2}-3)\pi\varepsilon_0\tau_{ph}.$$

We can thus see that the dependence $\Delta(T)$ has a singularity near $\Delta(T) = \omega/2$ where, at a certain temperature, there is an abrupt change from the upper branch in Fig. 2 to the lower one. This transition should rise to hysteresis. The jump is not very large, of the order of $T_c\varepsilon_0\tau_{ph}$, but the absorption coefficient changes discontinuously by a finite amount because $p \ll 1$ everywhere on the lower branch. The discontinuity of γ^s is given by Eq. (16) where the function $\Phi(p)$ should be taken from Eq. (17) for the parameter p in the range $p \ll 1$, corresponding to Δ on the lower branch of Fig. 2 at a given temperature. Thus, if an abrupt transition occurs at T_{tr}, then

$$p = \left(\frac{7\zeta(3)\omega\varepsilon_0\tau_{ph}}{2\pi^3 T_c}\right)^{1/2}.$$

Figure 3 shows schematically the dependence $\gamma^s(T)$ at a fixed acoustic frequency ω.

The results obtained are valid if $\varepsilon_0\tau_{ph} \ll 1$. At higher intensities corresponding to $\varepsilon_0\tau_{ph} \sim 1$, the nonlinear contribution to the subthreshold absorption coefficient [Eq. (14)] and to the equation for Δ [Eq. (19)] can be found by solving the infinite system of algebraic equations (12), which is generally difficult. However, it is qualitatively clear that at high intensities the abrupt transition in Fig. 2 takes place from a state Δ_1 to a state Δ_2 corresponding to the parameter $p \gtrsim 1$. The absorption coefficient is reduced as a result of this

transition.

We shall conclude by estimating the intensity of sound at which these phenomena can be expected. If $\omega = 10^{10}$ sec^{-1} and $\tau_{ph} = 10^{-9}$ sec, we find that

$$w = \left(\frac{e_0\tau_{ph}}{\hbar}\right)\frac{6}{\pi^3 Z}\frac{m_e M_{at}\omega s}{\hbar\tau_{ph}} \sim \left(\frac{e_0\tau_{ph}}{\hbar}\right)\frac{6A}{\pi^3 Z}\ \text{erg/cm}^2 .$$

Here, A is the atomic weight. The appearance of the Planck constant in the above expressions is due to the selection of the units employed in the present paper.

[1]V. L. Pokrovskiĭ, Zh. Eksp. Teor. Fiz. **40**, 143 (1961) [Sov. Phys. JETP **13**, 100 (1961)]; I. A. Privorotskiĭ, Zh. Eksp. Teor. Fiz. **43**, 1331 (1962) [Sov. Phys. JETP **16**, 945 (1963)].
[2]K. R. Lyall, D. J. Meredith, and E. R. Dobbs, J. Phys. F **6**, 807 (1976).
[3]G. M. Éliashberg, Zh. Eksp. Teor. Fiz. **61**, 1254 (1971) [Sov. Phys. JETP **34**, 668 (1972)].
[4]L. P. Gor'kov and G. M. Éliashberg, Zh. Eksp. Teor. Fiz. **54**, 612 (1968) [Sov. Phys. JETP **27**, 328 (1968)].
[5]I. É. Bulyzhenkov and B. I. Ivlev, Zh. Eksp. Teor. Fiz. **70**, 1405 (1976) [Sov. Phys. JETP **43**, 731 (1976)].
[6]B. I. Ivlev, S. G. Lisitsyn, and G. M. Eliashberg, J. Low Temp. Phys. **10**, 449 (1973).
[7]G. M. Éliashberg, Pis'ma Zh. Eksp. Teor. Fiz. **11**, 186 (1970) [JETP Lett. **11**, 114 (1970)].
[8]B. I. Ivlev and G. M. Éliashberg, Pis'ma Zh. Eksp. Teor. Fiz. **13**, 464 (1971) [JETP Lett. **13**, 333 (1971)].
[9]A. Schmid, Phys. Rev. **180**, 527 (1969).
[10]L. P. Gor'kov and N. B. Kopnin, Zh. Eksp. Teor. Fiz. **59**, 234 (1970) [Sov. Phys. JETP **32**, 128 (1971)].
[11]S. Chandrasekhar, "Stochastic problems in physics and astronomy," Rev. Mod. Phys. **15**, 1 (1943) (Russ. Transl., IIL, M., 1947).

Translated by A. Tybulewicz

Relaxation of nuclear magnetization under many-pulse NMR experimental conditions

L. N. Erofeev, B. A. Shumm, and G. B. Manelis

Division of the Institute of Chemical Physics, Academy of Sciences of the USSR

(Submitted 9 June 1978)
Zh. Eksp. Teor. Fiz. **75**, 1837–1846 (November 1978)

An investigation was made of the behavior of nuclear magnetization subjected to a pulse sequence $90°_y$–τ–$(\varphi_x - 2\tau)^N$ under off-resonance conditions. Establishment of quasiequilibrium regimes in the spin system was investigated. The effects of many-spin resonance absorption of the energy of an external agency were detected. The concept of an effective field ω_e, of magnitude and direction governed by the parameters of the exciting pulses and detuning Δ, was introduced. The measured resonance values of φ_x and $\Delta\tau$ were in good agreement with those calculated in the effective field framework. The experimental results were compared with the conclusions of a thermodynamic theory of narrowing of NMR lines of a solid, given in the following paper in the present issue.

PACS numbers: 76.60.Es

Investigations of NMR in a rotating coordinate system in the presence of a continuously acting pulsed hf magnetic field are used widely to study solids. Experiments of this kind have frequently improved the sensitivity of the NMR method[1] and have given information on relatively slow molecular motion in matter,[2] as well as NMR spectra of solids with much enhanced resolution.[3,4]

A pulse variant of such experiments, called the spin-locking method, has been proposed recently.[5] This method is of great practical importance because it can be used to measure the spin–lattice relaxation time $T_{1\rho}$ in a rotating coordinate system much faster and more conveniently than by the traditional method with a continuously acting hf field.[2,6] Moreover, pulse spin locking is the simplest many-pulse experiment which can be used as a satisfactory model in theoretical analyses of the behavior of nuclear magnetization under the action of pulse sequences. It is pointed out in Refs. 7 and 8 that some of the phenomena observed under pulse spin-locking conditions cannot be explained by the theory of the average Hamiltonian[9] usually employed in dealing with such experiments. A different approach has been developed[8] for the spin dynamics of many-pulse NMR experiments: it is based on the determination of quasiequilibrium states of the spin system and allowance for many-spin processes of the absorption of energy of an external agency. This approach gives a satisfactory agreement with the results of experimental investigations[5,7] employing pulse spin locking in the specific case when the exact resonance conditions are satisfied. In the present paper, which is a continuation of Ref. 7, we shall consider the processes of establishing quasiequilibrium states and relaxation of a spin system with the dipole coupling in the case of pulse spin locking in the more general off-resonance case.

1. EXPERIMENTAL METHOD

Our measurements were carried out using a many-pulse NMR spectrometer[10] tuned to the resonance fre-

0038-5646/78/110925-06$02.40