Determination of the magnetic moment of the He^3 nucleus with an error of $2\times10^{-6}\%$

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The ratio of the free-precession frequencies of helions (He^3 nuclei) and protons was determined for a gaseous mixture of He^3 and hydrogen: $f_{He^3}/f_{H_2} = 0.761\ 786\ 635(4)$. The screening constant of the helion was calculated allowing for the motion of the nucleus relative to the center of mass of the He^3 atom: $\sigma_{He^3} = 59.924(2)\times10^{-6}$. The difference $\sigma_{H_2} - \sigma_{H_2O} = 0.596(13)\times10^{-6}$ at 21°C was determined experimentally and used to find the screening of the nuclei in hydrogen. The published data on the screening of protons in water gave $\sigma_{H_2} = 26.535(19)\times10^{-6}$. These screening constants made it possible to determine the magnetic moment of the helion in units of the magnetic moment of the proton: $\mu_{He^3}/\mu_p = -0.761\ 812\ 070(16)$. Next, a comparison of the magnetic moments of the lightest nuclei and hyperfine splitting of the relevant atoms was used to find the constants of the hyperfine-structure anomalies characterizing the motion of an electron inside a nucleus.

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The magnetic moments of the two lightest nuclei, deuteron and triton, were determined by us earlier in units of the magnetic moment of the proton[1,2] with an error of $\sim10^{-6}\%$. In recent work on the magnetic moment of the proton[3] the error was reduced to the eighth decimal place ($10^{-6}\%$) and the error in the magnetic moment of the neutron[4] was reduced by two orders of magnitude. This should make it possible to identify with a high precision the part of the magnetic moments of the lightest nuclei which is due to the nucleon–nucleon interaction. The present paper reports a determination of the magnetic moment of the He^3 nucleus (helion), which is a mirror-symmetric analog of the triton.

The theory of systems with small numbers of nucleons cannot explain their magnetic moments with the same precision as they can be determined experimentally. Nevertheless, in view of the importance of the few-nucleon problem as a whole both in nuclear physics and in physics of elementary particles, the use of high-precision magnetic moments of the lightest nuclei has become necessary as a consequence of the theoretical work of Coester *et al.*,[5] Pokrovskiĭ,[6] and Sprung.[7] The magnetic moments have been found to be very sensitive to fine features of the nucleon–nucleon potential at short distances where the scattering data are highly ambiguous. Therefore, it has been suggested[6] that the magnetic moments of the deuteron, triton, and helion be used as the experimental parameters for the refinement of the nucleon–nucleon potentials at short internucleon distances. The refinement of the nucleon–nucleon interaction is a pressing problem in connection with the need to check the capabilities of the potential approach in nuclear physics.

On the other hand, determination of the magnetic moments of the light nuclei with an error of $\sim10^{-6}\%$ is essential in an analysis of the hyperfine splitting $\Delta\nu$ of the low-lying S states of the corresponding atoms. The splitting $\Delta\nu$ is one of the most accurately known physical quantities and, therefore, it is used in investigations of sufficiently fine effects. In particular, the splitting $\Delta\nu$ of the ground state of the hydrogen atom is one of the principal quantities used in verifying the validity of quantum electrodynamics.[8] Moreover, the precision achieved in the determination of $\Delta\nu$ for light atoms and the corresponding nuclear magnetic moments makes it possible to consider the motion of an atomic electron inside a nucleus. A comparison of $\Delta\nu$ with the magnetic moments determined by us provides a good basis for checking the models of the interaction of nucleons in the light nuclei. These constants will be compared after presenting the main results of our investigation.

The magnetic moment of the helion can be determined using the following expression:

$$\mu_{He^3}=\mu_p\frac{f_{He}}{f_{H_2}}[1+\sigma_{He}-\sigma_{H_2}], \tag{1}$$

where μ_p is the magnetic moment of the proton, f_{He}/f_{H_2} is the ratio of the free-precession frequencies of the helion and proton in a gaseous mixture of hydrogen and He,[3] and $\sigma_{He}-\sigma_{H_2}$ is the difference between the electron screening of the nuclei in the He^3 atom and in the hydrogen molecule. It should be noted that the magnetic moments of the deuteron and triton can be determined by investigating the isotopic analogs of the hydrogen HD and HT (Refs. 1 and 2), whereas in the present case two experiments have to be carried out. The ratio of the free-precession frequencies of the spins of the helion and nuclei in the hydrogen molecule are determined in the first experiment. In the second, the electron screening of the nuclei in the hydrogen molecule is found and this quantity is of intrinsic interest because there have been many attempts to determine it experimentally and theoretically.[9] In the experimental determination of the screening of the hydrogen nuclei it is sufficient to determine the difference between the precession frequencies of the protons in hydrogen and in water, because the absolute screening of protons in water was recently determined by Phillips *et al.*[10] with an error of $1.4\times10^{-6}\%$ (this error determines largely the current precision with which the magnetic moment of the helion is known). Moreover, to find the magnetic moment of the helion with an error of $10^{-6}\%$, we have to

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calculate the screening of the He^3 nucleus with the same or smaller error.

EXPERIMENTAL INVESTIGATION

The free-precession frequencies of nuclear spins were excited and recorded in a magnetic field of ~15 kG. A pulse spectrometer synthesizer produced a reference network of quartz-oscillator frequencies for three channels (Fig. 1) in accordance with the relationships:

$$f_p{}^0=f_g\,(18+4-2:7),\quad f_{He}{}^0=f_g\,(18-4:3-1:8).\qquad f_d{}^0=f_g\,(2+4:3),\tag{2}$$

where the integers in the parentheses are the coefficients of the multipliers and dividers of the quartz-oscillator frequency $f_q = 2\ 924\ 375$ Hz. The three frequencies in Eq. (2) differ from the free-precession frequencies of the hydrogen, helium, and deuterium nuclei by amounts of the order of a few hundreds hertz. The quartz-oscillator frequency was selected so that the pass band of the three amplifying channels $\Delta\Omega \approx 2$ kHz wide did not coincide with any harmonics of the nearby electronic devices.

The spin-precession frequencies of He^3 and hydrogen differed by 24%, which was sufficient to match one resonance inductor to two channels by a pulse-controlled variable capacitor, as was done by us earlier.[11] On the other hand, this difference was sufficiently small to use a double parallel system of inductors.[12] An orthogonal inductor system could, generally speaking, give rise to a systematic error in the determination of the ratios of the precession frequencies (for example, the ratio of the resonance frequencies of H_2–D_2 obtained by Smaller[13] was underestimated by an amount equal to three errors).[1]

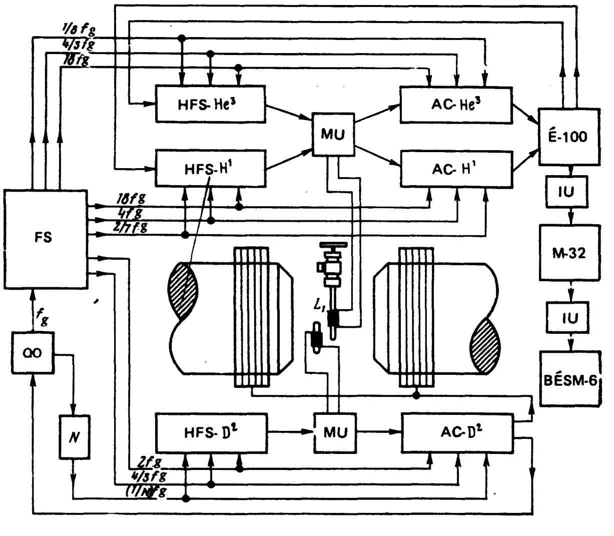

FIG. 1. Block diagram of the apparatus used to determine the magnetic moment of the helion: L_1 is a resonance inductor with an ampoule filled with hydrogen and He^3; MU is a matching unit; AC is an amplifier and converter of free spin precession signals; HFS is a high-frequency pulse shaper; FS is a frequency synthesizer; QO is a quartz oscillator; N is a multistage frequency divider; IU is an interface unit between computers.

Therefore, we used a single resonance inductor L_1 (Fig. 2) and matched it to the proton and helium channels by an antiresonance circuit L_2C_2 tuned to the proton resonance. The input 1 and output 1 were used to apply an excitation pulse and to obtain the proton free-precession signal (Fig. 2), whereas the input 2 and output 2 were employed to excite and extract the He^3 signal. The resonance inductor was tuned to resonance with the proton spin precession by a capacitor C_1 and to resonance with the He^3 spin precession by C_3.

Our sample was a gas-filled glass capillary whose outer diameter was 6 mm and whose wall thickness was 1.5 mm. The upper end of the capillary had a thicker part which was screwed into a brass faucet by a union nut and a Teflon seal. The lower sealed end of the capillary was inserted freely into the resonance inductor L_1 of the signal pickup. Facility for rapid interchange of samples made it possible to ensure carefully the homogeneity of the magnetic field before recording of the spectra; this was done using a strong signal from a standard sample. It should be pointed out that in the earlier investigation of the magnetic moment of He^3 by Williams and Hughes[14] the samples could not be interchanged readily.

The automatic tuning of the frequency f_q and of the magnetic field resonance was ensured by the deuteron resonance signal (Fig. 1) generated in an ampoule with D_2O which contained a mixture of the salt $Gd(NO_3)_3$ to reduce the deuteron relaxation time. The proton and helium spectra were acquired simultaneously by means of an Élektronika-100 computer (E-100 in Fig. 1) which triggered excitation pulses in the two spin systems in turn at intervals of 0.12 sec, and recorded during this interval a total of 528 signal amplitudes. This gave two sets of data on the positions of the proton Ω_{H_2} and helion Ω_{He} signals relative to the reference frequencies given by Eq. (2). The stored data were transferred to a Minsk-32 computer (M-32 in Fig. 1), which printed the spectra for examination, and then—as in the earlier investigations[1,2]—to a BÉSM-6 computer, where the results were analyzed by the conventional least-squares

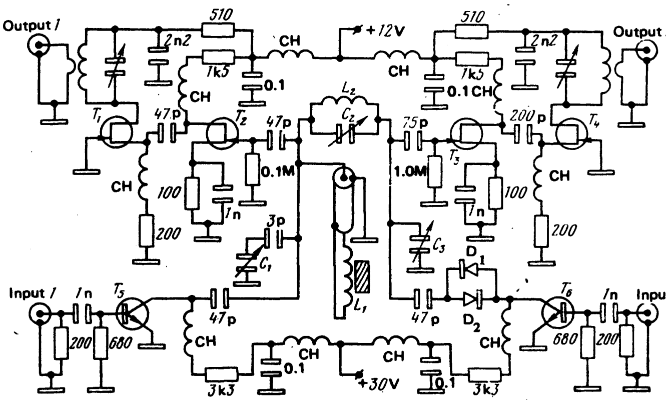

FIG. 2. Matching of a resonance inductor L_1 to the circuits for excitation and recording of free-precession signals of helions and protons; T_1 and T_4 are KP303 vacuum tubes; T_2 and T_3 are KP307 vacuum tubes; T_5 and T_6 are KT315 vacuum tubes; D_1 and D_2 are D311 diodes; CH are chokes.

TABLE I. Experimental data for determination of ratio of precession frequencies of helions and protons in hydrogen.

Ω_{He^3}, Hz	Ω_{H_2}, Hz	Ω^0, Hz	f_{He^3}/f_{H_2}
−413.6	−492.0	−38.8 (2.2)	0.761786670 (35)
−419.7	−498.3	−40.2 (2.4)	0.761786648 (38)
−415.5	−486.8	−44.7 (2.7)	0.761786578 (43)
−414.3	−495.9	−36.5 (2.8)	0.761786705 (44)
−402.8	−472.0	−43.2 (3.6)	0.761786601 (57)
−407.8	−472.5	−48.6 (4.8)	0.761786516 (76)
−402.9	−480.4	−37.0 (5.0)	0.761786699 (79)
−396.6	−464.6	−42.6 (1.9)	0.761786610 (30)
−393.4	−465.3	−38.9 (2.4)	0.761786669 (38)
−395.9	−466.0	−40.9 (2.2)	0.761786637 (35)
−390.1	−457.2	−41.9 (4.8)	0.761786621 (76)

Note. $(f_{He}/f_{H_2})_{av} = 0.761\,786\,637(13)$.

method.

The ratio of the spin-precession frequencies of He^3 and protons in hydrogen were determined for a sample which was a mixture of gases: 7 atm He^3, 52 atm H, and 1 atm O_2. The addition of one atmosphere of oxygen resulted in a slight additional broadening of the hydrogen signal but this oxygen was sufficient to ensure that the spin relaxation time of He^3 was ~1 sec. When the acquisition time was ~1 min, the signal from He^3 had the signal/noise ratio of ~10 and the signal from H_2 had the ratio of ~300. The width of the He^3 signal was ~20 Hz and that of the H_2 signal was ~90 Hz. Although the hydrogen signal was wider, its greater amplitude ensured that the error in the determination of its frequency was approximately the same as the error in the determination of the frequency of the He^3 signal.

Eleven pairs of spectra were recorded and analyzed for a sample with H_2 and He^3. The results are presented in Table I, where the last column gives the ratio of the signal frequencies calculated from

$$\frac{f_{He}}{f_{H_2}}=\frac{f_{He}^0+\Omega_{He}}{f_p^0+\Omega_{H_2}}\approx\frac{f_{He}^0}{f_p^0}+\frac{1}{f_p^0}\left(\Omega_{He}-\frac{f_{He}^0}{f_p^0}\Omega_{H_2}\right). \tag{3}$$

The values of f_{He}/f_{H_2} obtained in our study and in earlier investigations are compared in Fig. 3. It follows from this comparison that either our value or that of Williams and Hughes[14] suffers from a systematic error. In view of this, we shall describe how our error was calculated. An analysis of the i-th set of data by the least-squares method gave the average value of the square of the amplitude of random noise:

$$\Delta_i^2=\sum_{k=N_1}^{N_2}\frac{(y_k-y_{av})^2}{N_2-N_1}, \tag{4}$$

where y_{av} is the average level of the signal found by the least-squares method; $N_1 = 468$ and $N_2 = 528$, i.e., the values of y_k lie in the range of data corresponding to complete disappearance of the free-precession signal. Next, we determined the reduced value of χ^2 from

$$\chi^2=\frac{1}{N_4-N_3-M}\sum_{k=N_3}^{N_4}\frac{[f(t_k)-y_k]^2}{\Delta_i^2}, \tag{5}$$

where $f(t)$ is a function approximating the free-precession signal of the type given by Eq. (2) and taken from Ref. 1; M is the number of the variable parameters; $N_4 - N_3$ is the number of points in the set of data analyzed by the least-squares method.

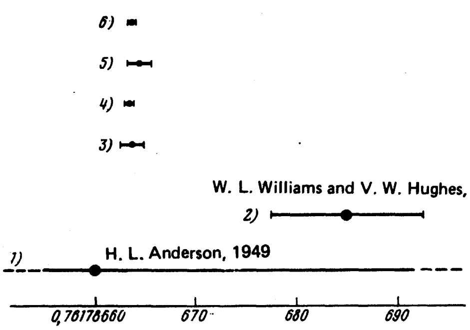

FIG. 3. Comparison of the ratios of the resonance frequencies of the He^3 nuclei and protons in hydrogen: 1) $f_{He^3}/f_{H_2} = 0.761\,786\,6(12) \pm 1.6\cdot10^{-6}$, 2) $f_{He^3}/f_{H_2} = 0.761\,786\,85(7.6) \pm 0.1\cdot10^{-6}$, 3) f_{He^3}/f_{H_2}, 4) $f_{He^3}/f_{H_2} = [f_{He^3}/f_{H^*D}]\,[1-\sigma_{H^*D-H_2}]$, 5) $f_{He^3}/f_{H_2} = [f_{He^3}/f_{HD^*}]\,[f_{HD^*}/f_{H^*D}]\,[1-\sigma_{H^*D-H_2}]$, 6) average $f_{He^3}/f_{H_2} = 0.761\,786\,635(4) \pm 0.0052\cdot10^{-6}$.

The error of the difference

$$\Omega_i=\Omega_{i,\,He}-(f_{He}^0/f_p^0)\,\Omega_{i,\,H_2}$$

was deduced from the relationship

$$\sigma_i=[\delta_{i,He}^2\chi_{i,He}^2+(f_{He}^0/f_p^0)^2\delta_{i,H_2}^2\chi_{i,H_2}^2]^{1/2}, \tag{6}$$

where δ_i is one standard deviation of the variable parameter $\Omega_{i,m}$ deduced by the least-squares procedure. In this way we found the errors σ_i given in parentheses in the third column of Table I. The agreement between the internal and external error for a series of n pairs of spectra was estimated from the inequality

$$\frac{1}{n-1}\sum_{i=1}^{n}\frac{(\Omega_i-\Omega_{av})^2}{\sigma_i^2}<1+\left(\frac{2}{n-1}\right)^{1/2}, \tag{7}$$

where Ω_{av} is the weighted average for $n = 11$ values of Ω_i. The average value of the reduced χ^2 was 1.1 for the set of data on H_2 and 1.19 for He^3. The ratio of the rms scatter of the eleven values of Ω_i to the rms error of the individual pairs was 1.03, i.e., the inequality (7) was satisfied. Therefore, the final result was deduced from

$$\Omega_{av}\pm\sigma_{av}=\sum_{i=1}^{11}\frac{1}{\sigma_i^2}\Omega_i\Big/\sum_{i=1}^{11}\frac{1}{\sigma_i^2}\pm\left[\sum_{i=1}^{11}\frac{1}{\sigma_i^2}\right]^{-1/2}. \tag{8}$$

The stability of the results (Table I) was tested by dropping the first points of the set. The final results are given for $N_4 - N_3 = 300$ and $N_3 = 38$.

This method of analysis of the experimental data could underestimate the error in the case of correlation between the random noise amplitudes. The absence of this correlation was checked by means of the inequality:

$$\sum_{k=N_1}^{N_2}(y_k-y_{av})(y_{k+1}-y_{av})\ll\sum_{k=N_1}^{N_2}(y_k-y_{av})^2. \tag{9}$$

Unfortunately, Williams and Hughes[14] did not describe the method of analysis of their spectra, although they did indicate a number of precautionary measures taken to eliminate the systematic errors. The reason for the discrepancy of the results could be due to the considerable width of the hydrogen signal (Fig. 4). This signal probably overlapped the signal due to an epoxy resin used by Williams and Hughes[14] to locate the in-

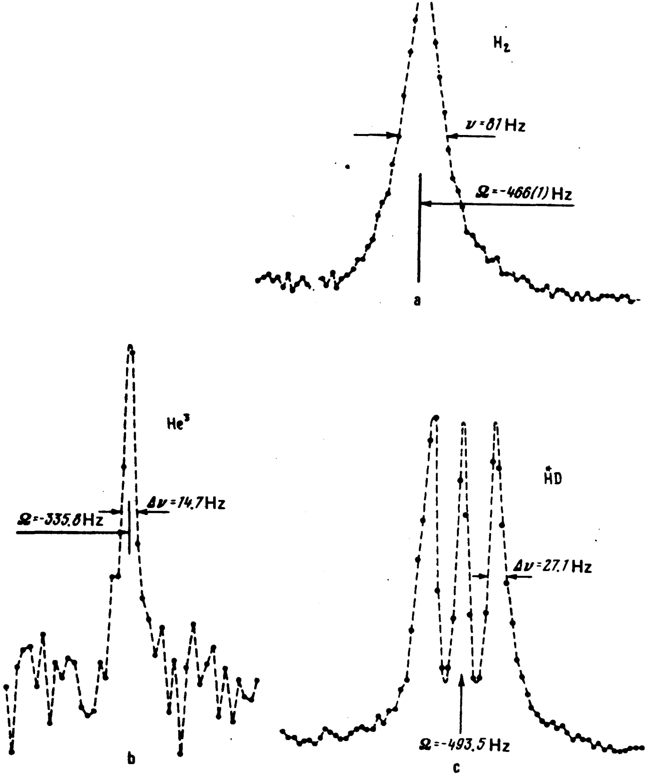

FIG. 4. Fourier transforms of the free-precession signals of: a) hydrogen; b) He^3 nuclei; c) protons of deuterohydrogen H*D. The last two spectra were recorded simultaneously and the ratio of the frequencies for this pair of spectra is given in the first row of Table II.

ductors.

We checked our results for a systematic error by preparing a second sample which was a mixture consisting of 83 atm HD, 6 atm He^3, and 1 atm O$_2$. The proton signal from deuterohydrogen had a triplet structure with components separated by $J = 43.115 \pm 0.012$ Hz (Ref. 1). For our gas mixture the triplet components were ~26 Hz wide and the signal/noise amplitude ratio was ~40. The method described above was applied to 28 pairs of data. Fourier transformation of one pair of data gave the spectrum shown in Fig. 4. The main result was obtained by analyzing 400 numbers from each set, beginning from the 40-th and ending with the 440-th (Table II). The difference between the isotopic screening of protons

$$\sigma(\mathrm{HD})-\sigma(\mathrm{H_2})=(35.9\pm0.2)\cdot10^{-9} \qquad (10)$$

was determined by us earlier in Ref. 1. Application of the correction (10) to the data of Table II made it possible to find the ratio of the free-precession frequencies for the He^3 nuclei and protons in H$_2$ (Fig. 3).

We carried out an additional experiment as a further verification of our data. We determined the ratio of the free-precession frequencies of the He^3 nuclei and deuterons in HD. In this case the resonance conditions were stabilized by reference to the proton signal, which was produced by a control sample with H$_2$O and CuSO$_4$. The investigated sample was a mixture of gases with 71 atm HD, 6 atm He^3, and 1 atm O$_2$. The deuteron and helion channels were matched using the same basic circuit (Fig. 2). The deuteron signal of HD had a doublet structure with components ~7 Hz wide and the signal/noise amplitude ratio was ~6. The results of an analysis of 20 pairs of spectra are given in Table III.

TABLE II. Experimental data for determination of ratio of precession frequencies of helions and protons in deuterohydrogen.

$\Omega^{\mathrm{He^3}}$, Hz	$\Omega^{\mathrm{H^*D}}$, Hz	Ω^0, Hz	$f_{\mathrm{He^3}}/f_{\mathrm{H^*D}}$
−335.8	−536.6	−40.1(8)	0.761786649(13)
−338.5	−536.9	−37.7(2.2)	0.761786687(35)
−338.8	−541.5	−40.8(1.4)	0.761786638(22)
−363.5	−573.6	−40.6(3.0)	0.761786641(46)
−372.5	−580.5	−36.9(1.6)	0.761786700(25)
−359.4	−566.5	−39.3(1.8)	0.761786662(28)
−359.6	−566.4	−39.0(1.6)	0.761786667(25)
−371.7	−579.6	−37.0(1.6)	0.761786699(25)
−399.1	−619.2	−39.8(1.5)	0.761786654(23)
−407.7	−630.6	−39.9(1.4)	0.761786653(23)
−410.4	−632.6	−38.7(1.6)	0.761786671(26)
−404.2	−629.7	−42.2(1.4)	0.761786616(21)
−411.6	−634.1	−38.6(2.3)	0.761786673(36)
−417.9	−646.4	−41.7(2.0)	0.761786625(32)
−414.7	−640.6	−40.5(1.5)	0.761786643(23)
−399.5	−616.3	−37.1(1.6)	0.761786696(26)
−428.8	−657.9	−39.6(1.4)	0.761786657(22)
−449.2	−681.0	−36.7(1.4)	0.761786702(21)
−415.3	−640.1	−39.5(1.5)	0.761786659(24)
−394.0	−613.0	−40.1(1.8)	0.761786649(28)
−430.3	−661.1	−40.5(1.3)	0.761786643(21)
−407.1	−629.7	−39.8(1.4)	0.761786654(23)
−422.9	−652.3	−41.2(1.9)	0.761786632(30)
−347.8	−551.9	−39.8(1.8)	0.761786655(28)
−355.1	−555.0	−34.9(2.1)	0 761786732(33)
−354.2	−559.8	−39.4(1.3)	0.761786661(20)
−351.1	−556 2	−39.7(1.5)	0.761786656(24)
−354.2	−557.5	−37.6(1.2)	0.761786688(20)

Note. $(f_{\mathrm{He}^3}/f_{\mathrm{H^*D}})_{\mathrm{av}} = 0.761\,786\,661\,2(45)$.

The ratio $f_{\mathrm{He}}/f_{\mathrm{HD}^*}$ could be used to find the ratio of the precession frequencies of the He^3 nuclei and of the protons in H$_2$:

$$f_{\mathrm{He}}/f_{\mathrm{H_2}}=(f_{\mathrm{He}}/f_{\mathrm{HD^*}})\,[1-\sigma(\mathrm{H^*D})+\sigma(\mathrm{H_2})]:(f_{\mathrm{H^*D}}/f_{\mathrm{HD^*}}). \qquad (11)$$

The ratio of the proton and deuteron resonance frequencies of HD was taken from our earlier paper[1]:

$$f_{\mathrm{H^*D}}/f_{\mathrm{HD^*}}=6.514399235(16). \qquad (12)$$

Figure 3 shows the results of calculations based on Eq. (11) in which Eqs. (10) and (12) are used. We can see that the results of the three methods for the determination of the ratio of the precession frequencies of helions and protons in hydrogen are in good mutual

TABLE III. Experimental data for determination of ratio of precession frequencies of helions and deuterons in deuterohydrogen.

Ω^{He}, Hz	$\Omega^{\mathrm{HD^*}}$, Hz	Ω^0, Hz	$f_{\mathrm{He^3}}/f_{\mathrm{HD^*}}$
1210.4	61.1	800.4(5.4)	4.96258209(55)
1213.3	60.9	803.9(7.0)	4.96258245(72)
1213.4	61.3	802.5(10.1)	4.96258230(103)
−201.1	−181.2	805.0(2.5)	4.96258256(26)
−190.5	−180.3	811.1(2.4)	4.96258318(25)
−205.9	−182.1	804.7(2.9)	4 96258253(30)
−217.8	−183.8	801.4(3.3)	4.96258219(34)
−231.7	−187.3	804.8(2.5)	4.96258254(25)
−241.4	−187.5	796.0(3.6)	4.96258165(37)
−227.0	−185.8	802.1(2.4)	4.96258226(24)
−226.1	−185.9	803.6(1.9)	4.96258242(20)
−143.5	−170.9	811.4(4.3)	4.96258322(44)
−452.1	−232.1	806.8(2.9)	4.96258275(30)
−418.7	−225.0	805.0(2.6)	4.96258257(26)
−426.7	−226.2	802.9(2.5)	4.96258235(26)
−424.9	−226.5	806.0(2.8)	4.96258266(28)
−409.3	−223.3	805.8(2.7)	4.96258265(28)
−409.6	−222.8	803.2(1.9)	4.96258236(23)
−410.0	−223.2	804.4(3.4)	4.96258250(34)
−379.8	−216.8	803.0(2.2)	4.96258238(19)

Note. $(f_{\mathrm{He}^3}/f_{\mathrm{HD^*}})_{\mathrm{av}} = 4.962\,582\,498(63)$.

agreement. Averaging of these three values gives

$$f_{He}/f_{H_2}=0.761786635(4). \tag{13}$$

The error in the determination of this ratio is 5.2×10^{-9}, which is 53 times less than the difference still remaining between the value reported by Williams and Hughes[14] and the new ratio given by Eq. (13).

The second part of our experimental investigation consisted of a determination of the absolute screening factor of protons in hydrogen. We used the proton screening factor in water obtained at 34.7℃ by Phillips *et al.*[10]:

$$\sigma(H_2O)=25.790(14)\cdot 10^{-6}\ . \tag{14}$$

The factor $\sigma(H_2)$ was found by determining the difference between the screening constants of water and hydrogen at some specific temperature and then making a correction for the temperature dependence of the screening constants.

The difference between the screening constants of water and hydrogen can be found from the difference between the free-precession frequencies of protons in water and hydrogen determined in the same magnetic field, making correction for the difference between the bulk diamagnetic susceptibilities. This frequency difference can be determined either by periodic recording of signals interchanging samples with H_2 and H_2O or by recording signals produced by a rotating coaxial ampoule in which the central capillary is occupied by the H_2 gas and H_2O fills the outer annular gap of the ampoule. Both these methods suffer from shortcomings: in the former case, there may be accidental changes in the magnetic field when the samples are interchanged, whereas in the latter case the field in the central capillary may differ from the average field in the annular gap. Moreover, it is difficult to make a coaxial ampoule with precise dimensions such that the two signals are of comparable amplitude. We used a method which combined the advantages of both these techniques. This was possible because of the two-channel recording of signals of our spectrometer.

Two identical coaxial ampoules were prepared. The outer annular gap in both ampoules was filled with the same standard substance, which was D_2O. The central capillary in one ampoule was filled with twice distilled H_2O and in the other it was filled with gaseous HD under a pressure of 90 atm. The high pressure of HD in the capillary was generated by a chemical reaction using a method described in our earlier paper.[1] Air was evacuated from both central capilliaries so as to remove oxygen, whose presence would have affected the bulk diamagnetic susceptibility.

In this way we determined the difference between the free-precession frequencies of HD and H_2O by periodic recording of the signals when the coaxial ampoules were interchanged. These ampoules were rotated and the simultaneous recording, in the deuteron channel, of the standard signal from D_2O provided the necessary monitoring of changes in the magnetic field.

Table IV gives the results of an analysis of six deuteron–proton spectra. The third column of Table IV lists the frequencies

TABLE IV. Frequencies of spin precession of protons in deuterohydrogen and deuterons in heavy water.

Ω^{D_2O}, Hz	Ω^{HD}, Hz	Ω^0_{HD}, Hz	t, °C	Ω^{D_2O}, Hz	Ω^{HD}, Hz	Ω^0_{HD}, Hz	t, °C
−118.3	−207.9	977.5(3.2)	21	−105.0	−291.4	974.9(2.1)	21
−108.2	−272.0	976.6(2.2)	21	−229.4	520.5	974.0(1.7)	24.6
−106.0	−287.0	977.5(2.1)	21	−229.0	518.0	973.5(1.8)	24.6

Note. $\Omega^0_{HD}(21\,°C)_{av}=976\,43(78)$ Hz.

$$\Omega_{HD}{}^0=\Omega_{HD}+\Omega_{D_2O}(f_p{}^0/f_d{}^0), \tag{15}$$

corresponding to the proton precession frequencies when the frequency of the standard signal of D_2O coincides with $f_d{}^0$. The values of Ω^0_{HD} depend on the temperature of the sample because the screening constant of water changes considerably with temperature.[15] Therefore, corrections are needed to allow for the thermal drift.

We calculated these corrections using the method of Hindman,[15] who determined the change in the screening constant of water at temperatures from −15 to 300℃. Phillips *et al.*[10] give the results of approximation of the Hindman data by a quadratic polynomial. However, Phillips *et al.* included the experimental points obtained by Hindman for supercooled water (from −15℃ to 0℃) and for water at a temperature above 70℃. These points were less reliable than the data of Hindman obtained in the temperature range 0–70℃ (Ref. 15). Phillips *et al.*[10] derived the approximation polynomial with zero at 0℃ although it would be more reasonable to have zero near room temperature: this would minimize the large errors in the determination of the coefficients of the quadratic and cubic terms.

We applied the least-squares method in deriving an approximation polynomial for the screening constant of water[15] in the temperature range 0–70℃. Zero of the polynomial coincided with 21℃. The degree of the polynomial N was selected so that the reduced χ^2 had its minimum for this value of N. The results of our calculations for $N=4$ were as follows

$$\begin{aligned}\sigma(t°\,C)&=\sigma(21°\,C)-1.109\cdot 10^{-8}(t-21)\\&+1.93\cdot 10^{-11}(t-21)^2-8.7\cdot 10^{-14}(t-21)^3.\end{aligned} \tag{16}$$

Substituting the value of $\sigma_{H_2O}(34.7℃)$ from Eq. (14) into Eq. (16), we found that

$$\sigma_{H_2O}(21°\,C)=25.939(14)\cdot 10^{-6}\ . \tag{17}$$

The temperature dependence of the screening constant of gaseous hydrogen was due to a redistribution of the populations of the various rotational-vibrational levels. However, in the temperature range under consideration (20–40℃) it was so slight ($\Delta\sigma_{H_2}<10^{-9}$), that it could be ignored (compare with theoretical estimates[9,16]). Therefore, in averaging the data in Table IV it was sufficient to allow for the temperature correction given by Eq. (16).

An analysis of the deuteron–proton spectra obtained using the second coaxial ampoule (H_2O–D_2O) established that the ratio of the free-precession frequencies

$$\frac{f(H_2O)}{f(D_2O)}=\frac{f_p{}^0}{f_d{}^0}+\frac{1}{f_d{}^0}\left(\Omega_{H_2O}-\frac{f_p{}^0}{f_d{}^0}\Omega_{D_2O}\right) \tag{18}$$

was independent of temperature, which was evidence of the agreement (within the limits of the experimental error) of the coefficients of the temperature dependence of the screening constant deduced from Eq. (16) for H_2O and D_2O. The average value of the ratio in Eq. (18) was 6.514 399 581(29). This value was in good agreement with the ratio obtained earlier for the H_2O–D_2O isotopic mixture.[12] The value of $\Omega^0_{H_2O}$ corresponding to the precession frequency of protons in water in the case of coincidence of the D_2O signal with the frequency $f_d{}^0$ was

$$\Omega^0_{H_2O}=1110.21(28)\ \text{Hz}.$$

This value and the corresponding value for deuterohydrogen (Table IV) were obtained in the same magnetic field. Consequently, the difference between the screening constants of protons in deuterohydrogen and in water is

$$\sigma(H^{\cdot}D)-\sigma(H_2O)=\frac{1}{f_p{}^0}(\Omega^0_{H_2O}-\Omega^0_{HD})+\frac{2\pi}{3}[\chi(HD)-\chi(H_2O)]. \tag{19}$$

Handbooks give two values for the bulk diamagnetic susceptibility of water $\chi(H_2O,\ 20^\circ C)=-0.7205\cdot 10^{-6}$ and $\chi(H_2O, 20^\circ C)=-0.719\cdot 10^{-6}$ (Ref. 17). We shall use the average of these two values $\chi(H_2O, 20^\circ C)=-0.720(2)\cdot 10^{-6}$. In the case of hydrogen the bulk diamagnetic susceptibility is $\chi(H_2,\ 10^\circ C,\ 1\ \text{atm})=-1.78\times 10^{-4}\cdot 10^{-6}$ (Ref. 18). In our case the HD gas pressure was ~90 atm and, consequently, $\chi(HD)=-0.016\cdot 10^{-6}$. Substituting these values into Eq. (19), we found that at 21°C:

$$\sigma(H^{\cdot}D)-\sigma(H_2O)=0.632(13)\cdot 10^{-6}. \tag{20}$$

Using the difference between the screening constants of protons in hydrogen and deuterohydrogen, given by Eq. (10), we obtained the corresponding difference for the screening of protons in H_2 and H_2O:

$$\sigma(H_2)-\sigma(H_2O)=0.596(13)\cdot 10^{-6}. \tag{21}$$

Table V compares the difference between the screening constants given by Eq. (21) with the results of earlier investigations. Reid[9] and Raynes *et al.*[16] found this quantity not by direct measurement but by combination of the chemical shifts known for a number of substances. The new value (21) is in good agreement with the results of earlier investigations and is an order of magnitude more accurate. Further improvement in the precision of $\sigma(H_2)-\sigma(H_2O)$ is at present pointless since the error of the final result—the absolute screening constant of protons in hydrogen $\sigma(H_2)$—is still governed by the error in the determination of the absolute screening constant of water $\sigma(H_2O)$ given by Eq. (14).

TABLE V. Differences between screening constants of protons in hydrogen and water.

$\sigma_{H_2}-\sigma_{H_2O}\cdot 10^{-6}$	Reference	Year	$\sigma_{H_2}-\sigma_{H_2O}\cdot 10^{-6}$	Reference	Year
0.6(3)	[23]	1950	0.61(15)	[16]	1971
0.30(45)	[24]	1951	0.60(15)	[9]	1975
0.60(15)	[25]	1959	0.596(13)	our result	1978

DISCUSSION OF POSSIBLE SYSTEMATIC ERRORS

One source of systematic error could be inconstancy of the time interval between the pulses exciting the free precession signal. This time interval was set by a program controlled by the Élektronika-100 computer and monitored with a ChZ-35A frequency meter. The inconstancy did not exceed 1 μsec, which could give rise to a relative error of the order of 1×10^{-5} on the frequency scale when the time interval was 0.12 sec. It is clear from the tables that the relative random error of the measured frequency was always greater than 3×10^{-4}. Consequently, inconstancy of the time interval could not have a significant influence on the results obtained.

A strict equality of the intervals between the successive activations (switching on) of an amplitude-digital converter of the Élektronika-100 computer was ensured by a special program and checked by control tests. One should bear in mind that the interval between a triggering pulse and the first activation of the converter and the interval from the last activation of the converter to the next triggering pulse were not equal to the intervals between the successive activations of the converter. The positions of the relevant points on the time scale could be found correctly by adjusting the interval between the triggering pulses (measured with a frequency meter) to allow for this inequality. This was done by means of an Élektronika-100 program on the basis of 6 μsec per machine cycle. The correctness of this procedure was checked by control tests. They showed that the error in these calculations did not exceed 2 μsec and, therefore, also did not affect the results.

A systematic error could also arise because of the distortion of the line profile. The χ^2 test was applied to check that the profile was Lorentzian. Moreover, an examination of the stability of the results was made when the position of the initial point (i.e., the first of the experimental points used in the least-squares calculations) was varied. The spectra exhibiting considerable divergences of the results were rejected. The absence of a systematic error in any one group of spectra was checked by the correspondence criterion of the internal and external errors. In all the experiments the discrepancy between the internal and external errors was slight.

The coefficient $2\pi/3$ in Eq. (19) corresponds, as is known, to the case of ideal infinite coaxial cylinders. A theoretical analysis of the influence of departure from the ideal cylindrical shape can be found in the paper of Zimmerman and Foster.[33] The validity of their conclusions has been confirmed experimentally.[33,34] It has been found that when a sample is rotated, departure from the ideal cylindrical shape results in the broadening and, generally, distortion of the signal from the outer (annular) cavity. The signal from the inner cylinder (capillary) is then split. However, the centers of gravity of the signals are not displaced and it is these centers that determine the required frequencies. Moreover, since there was no distortion or splitting in our study, the departure of the cylinders from the ideal shape can be ignored completely. According to Zimmerman and Foster,[33] there may be a line shift due to a mutual inclination of the inner and outer cylinders. They give[33] the following formula for this shift $\Delta\nu$:

$$|\Delta\nu| = |{}^2/_3\pi\Delta\chi\beta^2\nu_0|. \tag{22}$$

Here, $\Delta\chi$ is the difference between the diamagnetic susceptibilities of the substances in the outer and inner cylinders; β is the small angle of inclination of one cylinder relative to the other; ν_0 is the resonance frequency. In our experiments the inner cylinder was specially centered. In estimating β we ignored this centering and estimated the upper limit of β. We designated the diameter of the inner capillary by d_1, the internal diameter of the outer cylinder by d_2, and the height of the inner capillary by h.

The relevant dimensions for our sample were as follows:

$$d_1=3.9\ \text{mm},\quad d_2=4.6\ \text{mm},\quad h=60\ \text{mm}.$$

Therefore, we found that

$$|\beta| < (d_2-d_1)/h = 1.2\cdot 10^{-2}.$$

Hence, we obtained

$$|\Delta\nu/\nu| \leqslant {}^2/_3\pi\Delta\chi\beta^2 < 2\cdot 10^{-10}. \tag{23}$$

Since the relative random error in our investigation was at least 1×10^{-9}, one could ignore the error due to the noncoaxial orientation of the two cylinders.

Similar reasoning applied also to the error associated with the inclination of the ampoules themselves relative to the direction perpendicular to the lines of force of the magnetic field.

ABSOLUTE SCREENING OF NUCLEI IN THE He^3 ATOM AND HYDROGEN MOLECULE

Glick[26] calculated the screening of nuclei in two-electron atoms. However, he ignored the relativistic corrections and those due to the motion of a nucleus relative to the common center of gravity of a system of three particles. The need to allow for these corrections was first pointed out in Ref. 27 without explaining details of the calculation and without giving the final numerical results. In all subsequent studies the Glick result $\sigma_{\text{He}} = 59.935\cdot 10^{-6}$ was used. Since we were interested in obtaining the final result to within ~10^{-6}%, we decided to calculate the relativistic corrections and those allowing for the motion of the nucleus.

This can be done by transforming the relativistic Breit Hamiltonian of a system of three particles to a form convenient for the application of the standard perturbation theory. Grotch and Hegstrom[28] performed this transformation and their results made it possible to calculate the electromagnetic properties of hydrogen-like atoms with an error of the order of $(Z\alpha)^2 m_e^2/m_n^2$ and $\alpha(Z\alpha)^2 m_e/m_n$, where Z and m_n are the charge and mass of the nucleus, m_e is the electron mass, and α is the fine structure constant. Hegstrom[29] generalized directly the results of Ref. 28 to many-particle systems, i.e., he derived—in a form convenient for calculations—the Hamiltonian of such systems in a homogeneous magnetic field, and he included relativistic corrections as well as those for the motion of the nucleus and for the anomalous magnetic moment. Grotch and Hegstrom calculated the g_J factors for the hydrogen, deuterium, and helium atoms.[28,30] The results of their calculations were in excellent agreement with the experimental data. In particular, the ratios of the g_J factors of hydrogen and deuterium agreed to within 1×10^{-11}. This very good agreement between the theory and experiment allowed us to use the method of Grotch and Hegstrom to calculate corrections to the screening of the nuclei in the He^3 atom.

We shall now write down, transforming them to a convenient form, those terms of the Hamiltonian given by Eq. (21) in Hegstrom's paper[29] that contribute to the screening of a nucleus in the He^3 atom:

$$\begin{aligned}
\mathscr{H}_{3,n} &= -\frac{e^2}{2m_n}\left(g\mu_n - \frac{e}{m_n}\right)\mathbf{I}\left\{\frac{1}{r_{1n}^3}[\mathbf{r}_{1n}[\mathbf{H}\mathbf{r}_{1n}]] + \frac{1}{r_{2n}^3}[\mathbf{r}_{2n}[\mathbf{H}\mathbf{r}_{2n}]]\right\} - \frac{e^2}{2m_n}\left(g\mu_n - \frac{e}{m_n}\right)\mathbf{I}\left\{\frac{1}{r_{1n}^3}[\mathbf{r}_{1n}[\mathbf{H}\mathbf{r}_{2n}]]\right. \\
&\quad \left. + \frac{1}{r_{2n}^3}[\mathbf{r}_{2n}[\mathbf{H}\mathbf{r}_{1n}]]\right\} = \mathscr{H}_3^{(1)} + \mathscr{H}_3^{(2)}, \\
\mathscr{H}_{4,n} &= \frac{e^2}{2m_e} g\mu_n \mathbf{I}\left\{\frac{1}{r_{1n}^3}[\mathbf{r}_{1n}[\mathbf{H}\mathbf{r}_{1n}]] + \frac{1}{r_{2n}^3}[\mathbf{r}_{2n}[\mathbf{H}\mathbf{r}_{2n}]]\right\}, \\
\mathscr{H}_{5,n} &= g\mu_n \mathbf{I}\mathbf{H}\frac{(p_1+p_2)^2}{2m_n^2} - \left(g\mu_n - \frac{2e}{m_n}\right)\frac{1}{2m_n^2} \\
&\quad \times\{(\mathbf{I}\mathbf{H})(\mathbf{p}_1+\mathbf{p}_2)^2 - (\mathbf{I}(\mathbf{p}_1+\mathbf{p}_2))((\mathbf{p}_1+\mathbf{p}_2)\mathbf{H})\} = \mathscr{H}_5^{(1)} + \mathscr{H}_5^{(2)},
\end{aligned} \tag{24}$$

where $\mathbf{r}_{in}$ is the radius of the i-th electron measured relative to the nucleus; $\mathbf{p}_i$ is the momentum of the i-th electron in the laboratory coordinate system; I and g are the spin and gyromagnetic ratio of the nucleus in nuclear magnetons; μ_n is the nuclear magneton; H is the external magnetic field.

We shall now consider the term $\mathscr{H}_{4,n}$. Simple transformations yield the following expression for the relevant contribution to the screening constant:

$$\sigma_4 = \frac{2\alpha^2}{3}\left\langle\frac{1}{r_{1n}}\right\rangle,$$

where r_{1n} is expressed in atomic units; σ_4 is the classical Lamb diamagnetic screening. Using the value of $\langle 1/r_{1n}\rangle$, obtained by Pekeris[31] employing a 1078-term wave function and adopting $1/\alpha = 137.035\,987(29)$ given by Olsen and Williams,[32] we find that

$$\sigma_4 = 59.9368\cdot 10^{-6}.$$

(with an error of 3×10^{-11}).

Clearly, the term $\mathscr{H}_3^{(1)}$ can be rewritten as follows:

$$\mathscr{H}_3^{(1)} = -\frac{m_e}{m_n}\frac{g\mu_n - e/m_n}{g\mu_n}\mathscr{H}_{4,n}. \tag{25}$$

Thus, the term relevant to the screening $\sigma_3^{(1)}$ is

$$\sigma_3^{(1)} = -\frac{m_e}{m_n}\frac{g\mu_n - e/m_n}{g\mu_n}\sigma_4 = -12.6\cdot 10^{-3}\cdot 10^{-6}. \tag{26}$$

We can show that if the wave function of the ground state is taken in the Hartree–Fock form, the contribution to the screening $\sigma_3^{(2)}$ due to the term $\mathscr{H}_3^{(2)}$ vanishes.

An estimate of the corresponding error can be obtained using the results of Pekeris,[31] who obtained $\langle \mathbf{r}_{in}\mathbf{r}_{2n}\rangle = -0.0747$ for the exact wave function of the ground state, which should be compared with $\langle r_{in}^2\rangle = 1.193$ (all the results of Pekeris are given in atomic units). Consequently, we may expect $\sigma_3^{(2)}$ to represent about 6% of $\sigma_3^{(1)}$, i.e., $|\sigma_3^{(2)}| < 10^{-9}$.

We shall now consider the term $\mathscr{H}_5^{(2)}$. As above we shall ignore the correlation terms of the $\langle \mathbf{p}_1\mathbf{p}_2\rangle$ type, compared with terms of the $\langle p_i^2\rangle$ type and we shall assume that the problem is spherically symmetric. Then, the corresponding contribution to the screening is

$$\sigma_5^{(2)} = -\frac{g\mu - 2e/m_n}{g\mu_n}\frac{1}{2m_n^2}\frac{4}{3}\langle p_1^2+p_2^2\rangle. \quad (27)$$

Similarly for $\sigma_5^{(1)}$, due to the term $\mathscr{H}_5^{(1)}$, we find that

$$\sigma_5^{(1)} = \frac{1}{2m_n^2}\langle p_1^2+p_2^2\rangle. \quad (28)$$

We shall combine these two contributions and apply the virial theorem, ignoring terms of greater orders of smallness:

$$\sigma_5 = \sigma_5^{(1)}+\sigma_5^{(2)} = -\frac{1}{2m_n}\frac{1}{3}\left(1-\frac{8e}{m_n g\mu_n}\right)\langle p_1^2+p_2^2\rangle$$
$$= \frac{1}{3}\left(1-\frac{16m_p}{gm_n}\right)\frac{m_e^2}{m_n^2}\alpha^2 E, \quad (29)$$

where m_p is the proton mass and E is the energy of the ground state of the He^3 atom (in atomic units). Calculations show that $|\sigma_5| \sim 1\times 10^{-10}$. Consequently, this correction can also be ignored.

We thus find that the final value of the screening constant of a nucleus in the He^3 atom is

$$\sigma_{He} = 59.924(2)\cdot 10^{-6}. \quad (30)$$

The absolute screening of protons in hydrogen is found from Eqs. (17) and (21):

$$\sigma_{H_2} = 26.535(19)\cdot 10^{-6}. \quad (31)$$

Since the precision in the determination of this constant is an order of magnitude higher than before, we can estimate the error in the theoretical calculations which have been carried out many times employing the above very simple procedure.

We shall consider particularly one of the more recent papers published by Reid.[9] He carried out a semiempirical calculation of $\sigma(H_2)$ and $\sigma(D_2) - \sigma(HD^*)$. The adjustable parameters used by Reid[9] were the constants of the spin–rotational interaction of various rotational states H_2 and its isotopic analogs,[19–21] and the isotopic shift was $\sigma(H^*D) - \sigma(H_2)$ (Ref. 22). The difference calculated by Reid was found to be in precise agreement with the results of our experiment reported in Ref. 1. However, this good agreement should be regarded as accidental because Reid ignored the nonadiabatic corrections to the screening, but it should be mentioned that these corrections were included implicitly (although only partly) because of his use of the experimental parameters in the semiempirical calculation.

Reid obtained the following value for the absolute screening constant of hydrogen: $\sigma(H_2) = 26.366(70)$, assuming that the error was equal to five standard deviations obtained in the least-squares method. Even after allowance for this clearly estimated error the theoretical value of Reid did not agree with the new experimental value (31). This discrepancy is evidence of the need for rigorous inclusion of the nonadiabatic corrections. The question of proper inclusion of these corrections is of fundamental importance because they govern the difference between the screening of the nuclei in the HD and HT molecules. A calculation of the difference between the screening of the nuclei in these two molecules has enabled us to determine the magnetic moments μ_d and μ_T with an error of $\sim 10^{-6}\%$. These magnetic moments and also the magnetic moment of the helion, calculated using Eqs. (1), (13), (30), and (31), are given in Table VI together – for the sake of comparison – with the hyperfine splitting constants $\mu_{He^3}/\mu_p = -0.761\,812\,070(16)$.[1)]

COMPARISON OF HYPERFINE SPLITTING CONSTANTS AND MAGNETIC MOMENTS OF LIGHTEST NUCLEI

The hyperfine structure anomaly Δ_A of an atom A is given by

$$\Delta\nu_A = \frac{\Delta\nu_0}{n^3}\left(\frac{m_A}{m_A+m_e}\right)^3(1+\varepsilon)(1-\Delta_A), \quad (32)$$

where $\Delta\nu_A$ is the hyperfine splitting for the S state whose principal quantum number is n; m_A is the mass of the nucleus; m_e is the electron mass; ε represents the relativistic and radiative corrections; $\Delta\nu_0$ is the Fermi nonrelativistic contact hyperfine splitting for a point nucleus of infinite mass:

$$\Delta\nu_0 = \frac{8}{3}Z_A^3\alpha^2 R_\infty c\frac{\mu_A}{\mu_0}\frac{I+{}^1/_2}{I}. \quad (33)$$

Here, Z_A is the charge of the nucleus; R_∞ is the Rydberg constant for the infinite nuclear mass; c is the velocity of light; μ_A/μ_0 is the magnetic moment of the nucleus in Bohr magnetons; I is the nuclear spin.

The hyperfine splittings of the two S states of the same atom with different principal quantum numbers have practically the same anomaly because the wave functions of the s electrons near the nucleus are similar. Therefore, as the first stage, we can ignore all the effects associated with the fact that the nucleus is not point-like, and we can consider the quantity

$$R_{21} = \frac{8\Delta\nu_2}{\Delta\nu_1} - 1. \quad (34)$$

Here, $\Delta\nu_n$ are the values of the hyperfine splitting of the nS states of a hydrogen-like atom.

The relativistic and radiative corrections for the hyperfine splitting of these two levels do not agree. Therefore, R_{21} can be used conveniently to check the limits of validity of quantum electrodynamics. Moreover, a quantity similar to R_{21} can be obtained for the neutral helium atom.

The quantity $\Delta\nu_n$ has been analyzed theoretically for various states of the same atom.[35–37] The agreement between the theory and experiment is good. Consequently, in analyzing Δ_A we can use similar calculations for ε. (A detailed comparison of the theoretical and experimental data for R_{21} can be found in the paper by Prior and Wang.[38])[2] A calculation of Δ_A based on Eq. (32) is subject to a very large error. This error is mainly due to the error in the determination of α. At present, the value of α is known to a precision four orders of magnitude poorer than the precision of $\Delta\nu$. Therefore, it is desirable to introduce a relative anomaly of the hyperfine structure Δ_{AB} which is independent of α:

$$\Delta_{AB}=\frac{\Delta\nu_A\mu_B Z_B^3}{\Delta\nu_B\mu_A Z_A^3}\left(\frac{1+m_e/m_A}{1+m_e/m_B}\right)^3\frac{I_A(I_B+{}^1/_2)}{I_B(I_A+{}^1/_2)}\frac{1+\varepsilon_B}{1+\varepsilon_A}-1, \tag{35}$$

where $\Delta\nu_{AB}$ is the hyperfine splitting of the isotopes A and B; I_{AB}, μ_{AB}, and m_{AB} are the spins, magnetic moments, and masses of the isotopes. Since $\Delta\nu_A$ and $1+m_e/m_A$ are known very accurately for light atoms, the error in the determination of Δ_{AB} is mainly due to the error in the ratio of the magnetic moments.

A theoretical calculation of Δ_{Ap}, which is the anomaly of the hyperfine structure relative to the hydrogen atom, suffers from much poorer precision. A qualitative description of the effects responsible for this anomaly was was given many years ago by Bohr[39] and Low.[40] They reduced to allowance for the motion of an electron inside the nucleus. Calculations of Δ_{Ap} for the nuclei of interest to us have been published in several papers.[41–45] However, these calculations have been carried out mainly over 20 years ago and they have been based on very imperfect wave functions for the ground state of the nucleus. Moreover, at that time the internucleon potential was known only very roughly. In particular, the results depend strongly on the admixture of the forces which vary with the velocity and which have been ignored in these calculations. The present state of the theory of few-nucleon systems[46] makes it possible to calculate much more accurately Δ_{Ap} in the potential approximation. A comparison of these new more reliable theoretical values of Δ_{Ap} with our more accurate experimental values of Δ_{Ap} should make it possible to identify the contribution made to Δ_{Ap} by the meson currents. Estimates of a possible value of this contribution obtained by Drell and Sullivan[45] indicate that it is greater than the experimental error in the determination of Δ_{Ap}.

Williams and Hughes[14] made an attempt to find this contribution. Their result applies to a hypothetical contribution of the meson currents: $\Delta_\mu=(15\pm5)\cdot10^{-6}$. However, the error attributed to this quantity is clearly underestimated. The theoretical value is taken from the papers published in the fifties.[41–43] The error of the total theoretical value Δ_{He^3} is taken to be the error of just one small correction calculated by Greenberg and Foley.[43] Sessler and Foley[41] reported calculations of the main contributions to Δ_{He^3} without giving any estimate of the error. In a similar calculation for the deuteron, Greenberg and Foley[43] gave the probable error of the calculations as 40×10^{-6}. Clearly, in the case of He^3 this error can only increase because the wave function of this nucleus is much more complex than the wave function of the deuteron. It is evident that the "meson current effect" identified by Williams and Hughes[14] disappears within the limits of this error.

TABLE VI. Comparison of hyperfine splittings and magnetic moments of lightest nuclei.

A	$\Delta\nu_A$, Hz	μ_A/μ_p	m_A/m_e	Δ_{Ap}, $\cdot10^{-6}$	Δ_{Ap}, (exp)
^{1}H	1420405751.768(2)	1	1836.15152(70)	–	–
^{2}D	327384352.5222(17)	0.3070121983(8)	3670.4807(15)	170.718(4)	200
^{3}T	1516701470.7919(71)	1.066639908(3)	5496.9181(27)	–5.811(4)	17
^{3}He+	8665649867(10)	–0.761812070(16)	5495.8818(27)	–180.890(18)	–202

Table VI gives the theoretical and experimental values of Δ_{Ap} for the lightest atoms. The first three columns give the experimental quantities used in the calculation of the experimental value of Δ_{Ap}. The hyperfine splittings $\Delta\nu_A$ are taken from Refs. 51–54 and the values of the mass ratio m_A/m_e are reproduced from Refs. 55 and 56. The ratios of the magnetic moments μ_A/μ_p are taken from our earlier papers[1,2] and are compared with the result obtained in the present study.[3] The errors in the theoretical values of Δ_{Ap} for tritium and He^3 have not yet been estimated.[41–45] Clearly, the lower limit of these errors can be taken to be the error in Δ_{dp} given by Greenberg and Foley[43]: $\delta(\Delta_{dp}$, theor.) $\sim40\cdot10^{-6}$.

A number of papers have appeared recently on the subject of those properties of few-nucleon systems which cannot be described by the potential approximation (magnetic and quadrupole moments, form factors, β-decay anomaly).[47–50] Therefore, calculations of Δ_{Ap} (theor.) in the potential approximation and separation from Δ_{Ap}(exp.) of a new constant, the "meson anomaly" of the hyperfine structure $\Delta\mu=\Delta_{Ap}(\text{exp.})-\Delta_{Ap}(\text{theor.})$, is of considerable interest because in this case a low-energy electron provides a very unique probe for the investigation of the structure of the lightest nuclei.

[1] The negative sign of the magnetic moment of the helion was determined earlier in an investigation of the hyperfine structure of the ^3S states of the He^3 atom.[57]

[2] There is an error in the paper of Prior and Wang[38]: instead of the mass of the He^3 nuclei, they used the mass of the He^3 atom in their calculations. The same mistake was made by Rosner and Pipkin[58] in an investigation of $\Delta\nu$ of the neutral helium atom.

[3] The relativistic and radiative corrections for the hydrogen-like atoms and ions depend only on the nuclear charge Z. Therefore, the differences $(\varepsilon_p-\varepsilon_A)$ in Eq. (35) vanish for the deuteron and triton. In the case of (He^3)$^+$, these corrections were calculated using the formulas given in Refs. 14 and 59.

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