where

$$\Psi_i^\alpha=(\partial u_{ki}/\partial k_\alpha)_{k=0}.$$

The second equation in the system (25) is associated with the first. We shall now expand Eq. (19) up to k^2 and sum again over i; next, we shall multiply the obtained equation by u_0^* and use the second equation in the system (25). Then, simple transformations carried out on the assumption that $\rho_s=c^2\langle\chi\rangle$ give the following expression for ρ_s:

$$\rho_s=\frac{2}{9N}\sum_{ij}J_{ij}(\mathbf{S}_{0i}\mathbf{S}_{0j})(\mathbf{R}_i-\mathbf{R}_j)^2-\frac{1}{3N|\mathbf{u}_0|^2}\sum_{ij\alpha}\Psi_i^{\alpha*}\hat{A}_{ij}(\Psi_i^\alpha-\Psi_j^\alpha). \quad (26)$$

Bearing in mind that in the case of a positive definite matrix any form obeys $(\varphi^*\hat{U}\varphi)>0$, we find from Eq. (26) that the second sum in that equation is positive and, therefore, the expression

$$\rho_{s0}=\frac{2}{9N}\sum_{ij}J_{ij}(\mathbf{S}_{0i}\mathbf{S}_{0j})(\mathbf{R}_i-\mathbf{R}_j)^2 \quad (27)$$

represents an upper limit of ρ_s. This upper limit is identical with that obtained by Halperin and Saslow.[2]

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Properties of superfluid ^{3}He-A near the transition into the A_1 phase

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Some characteristic singularities of the behavior of superfluid ^{3}He-A in the presence of a strong magnetic field are observed in the immediate vicinity of the transition into the A_1 phase. It is shown that in narrow gaps of width $L\gtrsim\xi_D$ the texture of the anisotropy vector **l** should change abruptly when the temperature of the $A\to A_1$ transition is approached, leading to a considerable variation of the spectrum of the NMR frequencies. Coupled oscillations of the density and of the longitudinal magnetization near the $A\to A_1$ transition are investigated. These oscillations merge in the A_1 phase with the magnetosonic mode typical of triplet pairing in the state with a single spin projection.

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1. INTRODUCTION

It is presently customary to identify the superfluid A phase of liquid ^{3}He with the Anderson-Brinkman-Morel state (see, e.g., Ref. 1). For this anisotropic state, the nine-component complex order parameter that describes the coherent phase with triplet spin pairing in the p wave has a relatively simple multiplicative structure

$$A_{\mu i}=\Delta d_\mu u_i, \quad (1)$$

where $\Delta=\Delta(T)$ specifies the amplitude of the order parameter, and the complex vector $\mathbf{u}=(\mathbf{u}_1+i\mathbf{u}_2)\sqrt{2}$ describes an orbital state whose projection of the relative angular momentum of the Cooper pairs along the axis $\mathbf{l}=\mathbf{u}_1\times\mathbf{u}_2$ ($\mathbf{u}_1$ and $\mathbf{u}_2$ are real orthogonal unit vectors) is equal to +1. As for the spin part of the order parameter, in the absence of a magnetic field it is described by a real (unit) vector **d**, along which the projection of the summary spin of the pair is equal to zero.

It must be recognized, however, that in the presence of a sufficiently strong magnetic field, and close enough to the temperature of the transition to the normal phase, the pairing amplitudes of the quasiparticles in states with summary-spin projection $S_z=\pm1$ are not equal ($\Delta_\uparrow\neq\Delta_\downarrow$) (Ref. 2). It is then no longer possible to describe the spin part of the order parameter by a single real vector, and the vector **d** in (1) must be represented by a linear superposition

$$\mathbf{d}=c_\uparrow\mathbf{d}_\uparrow+c_\downarrow\mathbf{d}_\downarrow, \quad (2)$$

where the complex vectors $\mathbf{d}_\uparrow=(\mathbf{d}_1+i\mathbf{d}_2)/\sqrt{2}$ and $\mathbf{d}_\downarrow=\mathbf{d}_\uparrow^*$ describe states of pairs whose summary-spin projection on the direction $\mathbf{s}=\mathbf{d}_1\times\mathbf{d}_2$ is equal to +1 or − 1,

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respectively ($\mathbf{d}_1$ and $\mathbf{d}_2$ are real orthogonal unit vectors). In Eq. (2), the coefficients are

$$c_{\uparrow,\downarrow}=\Delta_{\uparrow,\downarrow}/(\Delta_\uparrow^2+\Delta_\downarrow^2)^{1/2}, \tag{3}$$

and it must be borne in mind here that in (1) we have $\Delta(T)=(\Delta_\uparrow^2+\Delta_\downarrow^2)^{1/2}/\sqrt{2}$. It will be convenient in what follows to express the spin vector (2) in the form

$$\mathbf{d}=\alpha_+\mathbf{d}_1+i\alpha_-\mathbf{d}_2, \tag{4}$$

where $\alpha_\pm=(\Delta_\uparrow\pm\Delta_\downarrow)/2\Delta$.

It is easy to verify that at equilibrium the orthogonal pair of vectors $(\mathbf{d}_1, \mathbf{d}_2)$ lies in a plane perpendicular to the direction of the external magnetic field. On the other hand, the dipole-dipole part of the free energy is given by

$$\mathscr{F}_D=-{}^1/_2\chi_s\Omega_A^2\{\alpha_+^2(\mathbf{d}_1\mathbf{l})^2+\alpha_-^2(\mathbf{d}_2\mathbf{l})^2\}, \tag{5}$$

where χ_s is the spin susceptibility of the normal phase of liquid ^{3}He and Ω_A is the characteristic dipole frequency proportional to Δ. If the orientation of the orbital vector $\mathbf{l}$ is specified by polar and azimuthal angles (v, χ), then, directing the polar axis along $\mathbf{H}$, we find that

$$\mathscr{F}_D=-{}^1/_2\chi_s\Omega_A^2\{\alpha_+^2\cos^2\chi+\alpha_-^2\sin^2\chi\}\sin^2\vartheta, \tag{6}$$

and since $\alpha_+^2\geqslant\alpha_-^2$ we have at equilibrium $\chi=0$ ($\mathbf{d}_1=y, \mathbf{d}_2=-x$). The dipole-dipole forces tend at the same time to press the vector $\mathbf{l}$ towards the $(\mathbf{d}_1, \mathbf{d}_2)$ plane, but near the vessel wall $\mathbf{l}$ prefers to assume an orientation perpendicular to the wall, and in general the angle has an equilibrium value $v\neq0$ as a result of the competition between the two aforementioned tendencies. We shall begin therefore with the expression

$$\mathscr{F}_D=-{}^1/_2\chi_s\alpha_+^2\Omega_A^2\sin^2\vartheta=-{}^1/_4\chi_s(1+\beta)\Omega_A^2\sin^2\vartheta, \tag{7}$$

where the parameter $\beta(T)=\Delta_\uparrow\Delta_\downarrow/\Delta^2$ characterizes the splitting of the spin states in the magnetic field ($0\leqslant\beta\leqslant1$). In a weak field and far from T_c the splitting can be neglected ($\Delta_\uparrow=\Delta_\downarrow$) and $\beta=1$. On the other hand, as the temperature of the transition to the A_1 phase is approached, β decreases and vanishes at the transition point itself. In this region, the parameter $\beta(T)$ varies strongly with temperature, and this can lead to a number of curious effects that are readily observable in experiment.

2. TEXTURE OF ^{3}He-*A* IN NARROW GAPS NEAR THE TRANSITION TO THE A_1 PHASE

As one of the illustrations, we consider the superfluid *A* phase of liquid ^{3}He located in a gap between two parallel plates, in the presence of a strong magnetic field directed perpendicular to the planes of the plates. The described situation was considered theoretically many times (see, e.g., Ref. 3) and was investigated experimentally.[4] These investigations were made however at $\beta=1$, when the equilibrium texture along $\mathbf{l}$ does not vary with temperature if the magnetic field intensity is specified and the gap width L is fixed. As will be shown below, in the immediate vicinity of the transition to the A_1 phase (where $\beta=0$) the picture changes substantially with temperature, and this should manifest itself primarily in an unusual behavior of the NMR signal.

To determine the equilibrium orientation of the orbital vector $\mathbf{l}$, we must add to the dipole part of the free energy (7) the inhomogeneity energy, given near T_c by

$$\mathscr{F}_{\mathrm{grad}}={}^1/_2\chi_s c_\parallel^2\{3|\nabla\mathbf{u}|^2+|[\nabla\times\mathbf{u}]|^2+3|\mathbf{u}\times\nabla d_\mu|^2+|[\mathbf{u}\times\nabla]d_\mu|^2 +2\mathrm{Re}[3(\nabla\mathbf{u}^*)(\mathbf{u}d_\mu^*\nabla d_\mu)-[\nabla\mathbf{u}\cdot][\mathbf{u}d_\mu^*\times\nabla d_\mu]]\}, \tag{8}$$

where $c_\parallel$ is the velocity of the spin wave propagating along $\mathbf{l}$ ($c_\parallel$ is proportional to Δ), and the interference term, which contains the vector $d_\mu^*\nabla d_\mu\sim\alpha_+\alpha_-\nabla\psi$, is the result of the splitting of the spin states of the Cooper pairs (the angle ψ specifies the orientation of the unit vectors $\mathbf{d}_1$ and $\mathbf{d}_2$ in a plane with a normal $\mathbf{s}=\mathbf{d}_1\times\mathbf{d}_2$). Since $\nabla\psi=0$ in our case, the interference term drops out. It must be borne in mine, however, that in more complicated situations (when $\nabla\psi\neq0$) the interference term in (8) plays an important role. An example of this situation will be considered in the next section (see also Refs. 5 and 6).

It is easy to verify that for the one-dimensional problem considered below (the z axis is oriented along $\mathbf{H}$) the inhomogeneity energy takes the following simple form:

$$\mathscr{F}_{\mathrm{grad}}={}^1/_2\chi_s c_\parallel^2(1+2\cos^2\vartheta)\times(d\vartheta/dz)^2. \tag{9}$$

To determine the equilibrium value of the angle $\vartheta=\vartheta(z)$ we must combine expression (9) with (7) and minimize the functional

$$\int_{-L/2}^{L/2}\left\{(1+2\cos^2\vartheta)\left(\frac{d\vartheta}{dz}\right)^2-(1+\beta)\xi_D^{-2}\sin^2\vartheta\right\}dz, \tag{10}$$

where the temperature-independent dipole length is $\xi_D=c_\parallel/\Omega_A$. The first integral of this variational problem is of the form

$$(1+2\cos^2\vartheta)(d\vartheta/dz)^2+(1+\beta)\xi_D^{-2}\sin^2\vartheta=(1+\beta)\xi_D^{-2}\sin^2\vartheta_0, \tag{11}$$

with ϑ_0 equal to the maximum deviation of the equilibrium value of the angle ϑ from zero (attained at the center of the gap between two parallel walls of the vessel). It is easy to verify that the equation for the angle ϑ_0 is of the form (see Ref. 3)

$$\mathrm{K}(\sin\vartheta_0)[\mathrm{K}(\sin\vartheta_0)+2\mathrm{E}(\sin\vartheta_0)]={}^1/_4(1+\beta)(L/\xi_D)^2, \tag{12}$$

where $K(x)$ and $E(x)$ are complete elliptic integrals of the first and second kind, respectively.

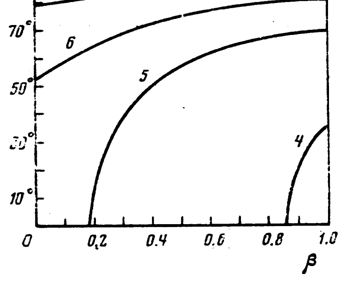

FIG. 1. Plots of the equilibrium angle ϑ_0 against the parameter $\beta(T)$ for different gap widths L. The numbers on the curves are the corresponding values of the ratio L/ξ_D.

Numerical solution of (12) for different values of the ratio L/ξ_D leads to the plot of ϑ_0 against the parameter β shown in the figure. Far from the transition into the A_1 phase (i.e., in the region where $\beta=1$), after the critical gap width $L_c=\sqrt{\frac{3}{2}\pi}\,\xi_D$ is reached, the angle ϑ_0 begins to increase from zero to the value $\pi/2$ characteristic of open geometry ($L \gg \xi_D$). At $L<L_c$ a decrease of β to zero does not change the homogeneous texture with $\vartheta\equiv 0$, for in this case the dipole energy is incapable of counteracting the orienting influence of the vessel walls. At $L>L_c$, however, the picture changes substantially. As seen from an examination of the figure, the angle ϑ_0 has a strong temperature dependence at $L/\xi_D=4$. As the transition to the A_1 phase is approached, ϑ_0 decreases rapidly and vanishes at $\beta\approx 0.85$; this corresponds to a transition into a state that is homogeneous along $\mathbf{l}$, since the effectiveness of the dipole-dipole forces decreases with decreasing β. A similar situation is observed also at $L/\xi_D=5$. At $L/\xi_D=6$ the homogeneous state is no longer reached, but the angle ϑ_0 still has a noticeable dependence on β. With further increase of the gap width L, the temperature dependence of ϑ_0 weakens and at $L/\xi_D=10$ the angle ϑ_0 is close to $\pi/2$ independently of β.

The described behavior of the texture of the vector $\mathbf{l}$ in narrow gaps in the immediate vicinity of the $A\to A_1$ transition can be investigated by observing the singularities of the NMR signal. An analysis of the tensor $\hat{\Omega}^2$ with components

$$(\hat{\Omega}^2)_{\mu\nu}=\mathrm{Re}\{|\mathbf{dl}|^2\delta_{\mu\nu}-(\mathbf{d}^*\mathbf{l})d_\mu l_\nu-[\mathbf{d}\times\mathbf{l}]_\mu[\mathbf{d}^*\times\mathbf{l}]_\nu\}, \tag{13}$$

which enters in NMR theory for a superfluid Fermi liquid,[7] shows that at $L<L_c$ ($\vartheta_0=0$, $\mathbf{l}\equiv\mathbf{z}$) we have

$$(\hat{\Omega}^2)_{xx}=-{}^1/_2(1+\beta)\Omega_A^2,\quad (\hat{\Omega}^2)_{yy}=-{}^1/_2(1-\beta)\Omega_A^2,\quad (\hat{\Omega}^2)_{zz}=0. \tag{14}$$

This leads to a transverse resonance with

$$\omega_{tr}^2\approx\omega_L^2-\Omega_A^2-{}^1/_2(1-\beta^2)\Omega_A^4/\omega_L^2,\quad \Omega_A\ll\omega_L. \tag{15}$$

Far from the transition to the A_1 phase we have $\beta=1$ and we arrive at the negative frequency shift predicted in Ref. 8 and observed in experiment.[4] It follows from (15) that near the $A\to A_1$ transition there should be observed an additional negative shift of the resonance frequency. At the same time, since $(\hat{\Omega}^2)_{xx}(\hat{\Omega}^2)_{yy}\neq 0$ at $\beta<1$, a new transverse low-frequency resonance appears at

$$\tilde{\omega}_{tr}\approx\left[\frac{1}{2}(1-\beta^2)\right]^{1/2}\frac{\Omega_A^2}{\omega_L^2}\omega_L.$$

Unfortunately, its intensity is limited by the smallness of $(\Omega_A/\omega_L)^4$ (see Ref. 7).

An even more pronounced manifestation of the splitting of the spin states near the transition into the A_1 phase should be expected in the case $L\gtrsim L_c$, when the texture of the vector $\mathbf{l}$ changes rapidly with decreasing β (see the figure). In this region, the spectrum of the NMR signal should have an unusually strong temperature dependence, which would be easy to investigate in experiment.

In the limit of open geometry with $L\gg L_c$, the effect of the vessel walls is negligible and a state with $\vartheta=\pi/2$ ($\mathbf{l}=\mathbf{y}$) sets in over practically the entire volume. In this case we have[9,10] $\omega_{tr}^2=\omega_L^2+\frac{1}{2}(1+\beta)\Omega_A^2$, and the frequency of the longitudinal resonance is $\omega_l=\beta^{1/2}\Omega_A$; these anomalous temperature dependences of the resonant frequencies in the vicinity of the $A\to A_1$ transition have already been observed in experiment.[9] However, as shown above, the behavior of ^{3}He-A in narrow gaps ($L\gtrsim L_c$) should exhibit a number of new singularities, due to the temperature dependence of the texture in the immediate vicinity of the transition into the A_1 phase.

3. COUPLED OSCILLATIONS OF THE DENSITY AND OF THE MAGNETIZATION IN THE VICINITY OF THE TRANSITION INTO THE A_1 PHASE

We turn now to an investigation of the collective oscillations of the density and magnetization of superfluid ^{3}He-A in the immediate vicinity of the transition into the A_1 phase. In this region, an important role is played by the interference term in the inhomogeneity energy [see (8)], which describes the effects of the entanglement of the spin and orbital degrees of freedom in the presence of a strong magnetic field.

To study the influence of the foregoing effects on the character of the collective oscillations of the magnetization (spin waves) we can start from the generalized Leggett equation, which takes the form of a hydrodynamic equation for the fluctuations of the magnetization $\mathbf{m}(\mathbf{r},t)$ (we consider below a situation wherein the motion of the normal component is blocked):

$$\frac{\partial m_\mu}{\partial t}+\gamma\nabla\mathbf{J}_\mu=\gamma[\mathbf{m}\times\mathbf{H}_0]_\mu+\chi_s\Omega_A^2\,\mathrm{Re}\{\gamma(\mathbf{d}^*\mathbf{l})[\mathbf{d}\times\mathbf{l}]_\mu\}. \tag{16}$$

Here γ is the gyromagnetic ratio for the ^{3}He nuclei, $\mathbf{H}_0$ is the external static magnetic field, and $\mathbf{J}_\mu$ is the density of the superfluid flux of the μ-th component of the spin, with

$$J_{\mu i}=\chi_s c_\parallel^2 a_{ij}\{-\mathrm{Re}[\mathbf{d}^*\times(\nabla_j\mathbf{d})]_\mu+2\alpha_+\alpha_-[\mathbf{d}_1\times\mathbf{d}_2]_\mu\nabla_j\Phi\}. \tag{17}$$

Equation (17) contains the anisotropy tensor $a_{ij}=2\delta_{ij}-l_il_j$, and Φ is the phase shift of the orbital part of the order parameter and describes the rotation of the pair of unit vectors $(\mathbf{u}_1,\mathbf{u}_2)$ around the direction $\mathbf{l}$ (it is assumed below that $\mathbf{l}$ is constant). The presence of a term with $\nabla\Phi$ in the equation for the spin flux is a reflection of the aforementioned mixing of the spin and orbital degrees of freedom, which increases with splitting of the spin states; at $\Delta_\uparrow\neq\Delta_\downarrow$ the mass transport is connected with a magnetization flux.

Substituting (17) in (16) we get for the magnetization

$$\begin{aligned}\frac{\partial\mathbf{m}}{\partial t}&=\gamma[\mathbf{m}\times\mathbf{H}_0]+\chi_s\Omega_A^2\,\mathrm{Re}\{\gamma(\mathbf{d}^*\mathbf{l})[\mathbf{d}\times\mathbf{l}]\}\\&+\gamma\chi_s c_\parallel^2 a_{ij}\{\mathrm{Re}\,\nabla_i[\mathbf{d}^*\times(\nabla_j\mathbf{d})]-2\alpha_+\alpha_-\nabla_i([\mathbf{d}_1\times\mathbf{d}_2]\nabla_j\Phi)\}.\end{aligned} \tag{18}$$

We must now use the dynamic equation for the order parameter $A_{\mu i}(\mathbf{r},t)$. Since we are considering a situation in which the magnetization oscillations must be accompanied be density oscillations $\delta n(\mathbf{r},t)$, the adiabatic Hamiltonian must include the term $(\delta n)^2/2\varkappa$, where $\varkappa$ is the compressibility of liquid ^{3}He. As a result we arrive at the following Leggett-Josephson equation:

$$\frac{\partial \mathbf{d}}{\partial t}+\mathbf{d}\left(\mathbf{u}^*\frac{\partial \mathbf{u}}{\partial t}\right)=-\frac{1}{\gamma\chi_s}[\mathbf{d}\times\mathbf{m}]-\frac{2i}{\varkappa}\mathbf{d}\delta n, \tag{19}$$

which reduces to two independent relations

$$\mathbf{u}^*\frac{\partial\mathbf{u}}{\partial t}=-\frac{2i}{\varkappa}\delta n, \tag{20a}$$

$$\frac{\partial\mathbf{d}}{\partial t}=-\frac{1}{\gamma\chi_s}[\mathbf{d}\times\mathbf{m}], \tag{20b}$$

whose structure is insensitive to the splitting of the spin states of the Cooper pairs. Differentiating (18) with respect to time and taking relations (20) into account, we verify that in the approximation linear in $\mathbf{m}$ and δn

$$\frac{\partial^2 m_\mu}{\partial t^2}-\gamma\left[\frac{\partial\mathbf{m}}{\partial t}\times\mathbf{H}_0\right]_\mu+(\hat\Omega^2)_{\mu\nu}m_\nu-(\hat c^2)_{\mu\nu}D^2m_\nu-\alpha_+\alpha_-\gamma c_4^2[\mathbf{d}_1\mathbf{d}_2]_\mu D^2 n=0, \tag{21}$$

where

$$(\hat c^2)_{\mu\nu}=\mathrm{Re}(\delta_{\mu\nu}-d_\mu^* d_\nu)c_\parallel^2,\quad D^2=a_{ij}\nabla_i\nabla_j=2\nabla^2-(\mathbf{l}\nabla)^2,$$

and the speed of fourth sound is $c_4=2(\chi_s/\varkappa)^{1/2}c_\parallel$.

On the other hand, using the continuity equation $\partial n/\partial t=-\nabla\mathbf{j}$ and noting that the density of the superfluid particle flux is

$$j_i=2\chi_s c_\parallel^2 a_{ij}(\nabla_j\Phi+\mathrm{Im}\,d_\mu^*\nabla_j d_\mu), \tag{22}$$

we easily obtain the equation

$$\partial^2 n/\partial t^2-c_4^2D^2n-4\alpha_+\alpha_-c_\parallel^2[\mathbf{d}_1\mathbf{d}_2]D^2\mathbf{m}/\gamma=0, \tag{23}$$

which forms together with (21) a closed system that describes the coupled oscillations of the density and the magnetization.

As seen from an examination of (21) and (23), in the immediate vicinity of the transition to the A_1 phase (where $\alpha_+\alpha_-\neq 0$) the density fluctuation δn is "hooked" to the fluctuation of the longitudinal magnetization m_z. Changing to Fourier components and turning on a weak alternating field with amplitude $h_z(\mathbf{q}\omega)$, we arrive at the following system of linear inhomogeneous equations for $m_z(\mathbf{q}\omega)$ and $\delta n(\mathbf{q}\omega)$:

$$(\omega_s^2(\mathbf{q})-\omega^2)m_z(\mathbf{q}\omega)+\alpha_+\alpha_-\gamma\omega_4^2(\mathbf{q})\delta n(\mathbf{q}\omega)=\gamma^2\chi_s\omega_s^2(\mathbf{q})h_z(\mathbf{q}\omega),$$

$$(\omega_4^2(\mathbf{q})-\omega^2)\delta n(\mathbf{q}\omega)+4\alpha_+\alpha_-\left(\frac{c_\parallel}{c_4}\right)^2\frac{\omega_4^2(\mathbf{q})m_z(\mathbf{q}\omega)}{\gamma}=\alpha_+\alpha_-\gamma\varkappa\omega_4^2(\mathbf{q})h_z(\mathbf{q}\omega), \tag{24}$$

where the bare spectra of the longitudinal spin waves and of the fourth sound are given by

$$\omega_s^2(\mathbf{q})=\beta\Omega_A^2+c_\parallel^2q^2a(\mathbf{q}/q),\quad \omega_4^2(\mathbf{q})=c_4^2q^2a(\mathbf{q}/q), \tag{25}$$

the anisotropy coefficient being $a(\mathbf{e})=2-(\mathbf{e}\cdot\mathbf{l})^2$.

By solving the system (24) we readily find that the longitudinal dynamic spin susceptibility is

$$\chi_{zz}(\mathbf{q}\omega)=\chi_s\frac{\beta(\Omega_A^2+\beta c_\parallel^2q^2a)\omega_4^2-\omega_s^2\omega^2}{(\omega^2-\omega_+^2)(\omega^2-\omega_-^2)}, \tag{26}$$

where the eigenfrequencies $\omega_\pm(\mathbf{q})$ of the coupled magnetosonic oscillations are given by

$$\omega_\pm^2(\mathbf{q})=\tfrac12\{\omega_s^2+\omega_4^2\pm[(\omega_s^2+\omega_4^2)^2-4\beta(\Omega_A^2+\beta c_\parallel^2q^2a)\omega_4^2]^{1/2}\}. \tag{27}$$

It is easy to verify that the upper branch with frequency $\omega_+(\mathbf{q})$ is transformed in the A_1 phase itself ($\beta=0$) into a zero-gap magnetosonic mode of the fourth-sound type[11] with modified velocity

$$\tilde c_4=\left(1+\frac{1+F_0^a}{1+F_0^s}\right)^{1/2}c_4. \tag{28}$$

where F_0^s and F_0^a are respectively the zeroth harmonic of the non-exchange (symmetrical) and exchange (antisymmetrical) parts of the Fermi-liquid Landau parameters.

It is easy to verify that excitation of the longitudinal magnetization is accompanied by density fluctuations, with

$$\delta n(\mathbf{q}\omega)=\frac{2(1-\beta^2)^{1/2}c_\parallel^2q^2a\omega^2}{\omega_s^2\omega^2-\beta(\Omega_A^2+\beta c_\parallel^2q^2a)\omega_4^2}\,\frac{m_z(\mathbf{q}\omega)}{\gamma}. \tag{29}$$

It can be shown similarly (by perturbing the density with an external source) that near the transition to the A_1 phase the dynamic susceptibility is

$$\varkappa(\mathbf{q}\omega)=\varkappa\frac{\beta(\Omega_A^2+\beta c_\parallel^2q^2a)\omega_4^2-\omega_4^2\omega^2}{(\omega^2-\omega_+^2)(\omega^2-\omega_-^2)}, \tag{30}$$

and excitation of density oscillations is accompanied by oscillation of the longitudinal magnetization, with amplitude

$$m_z(\mathbf{q}\omega)=\frac{(1-\beta^2)^{1/2}\omega^2}{\omega^2-\beta(\Omega_A^2+\beta c_\parallel^2q^2a)}\,\frac{\gamma}{2}\,\delta n(\mathbf{q}\omega). \tag{31}$$

In conclusion, a few words on transverse spin-wave modes that do not mix with density oscillations. Turning to (21), we easily verify that under conditions of splitting of the spin states there exist two transverse modes. One has a gap equal to the frequency of the transverse NMR

$$\omega_{tr}=(\omega_L^2+{}^1\!/_2(1+\beta)\Omega_A^2)^{1/2},\quad \omega_L\gg\Omega_A, \tag{32}$$

and is characterized by a dispersion ($(q\xi_D)^2\ll 1$)

$$\omega_+^2(\mathbf{q})=\omega_{tr}^2+c_+^2q^2a, \tag{33}$$

where

$$c_+^2=[1-{}^1\!/_4\,(1-\beta^2)(\Omega_A/\omega_L)^2]c_\parallel^2. \tag{34}$$

The second zero-gap spin-wave mode is due to the splitting effect, and for it we have $\omega_-^2(\mathbf{q})=c_-^2q^2a$, and a corresponding propagation velocity

$$c_-=1/2(1-\beta^2)^{1/2}(\Omega_A/\omega_L)c_\parallel\,.$$

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