Singularities of spin polarization of conduction electrons in rare-earth electrons in the transition from a ferromagnet into a simple helix

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Expressions are obtained for the change (jump) of the magnetic moment of the conduction electrons following a transition from the ferromagnetic into the antiferromagnetic phase. It is shown that the absolute value and the sign of this jump depend substantially on the topology of that Fermi-surface section which is responsible for the formation of the helicoidal magnetic structure. Good qualitative agreement is observed between the relative change of the magnetic moment of the conduction electrons and the relative change of the hyperfine field at the diamagnetic-impurity nuclei in a dysprosium matrix.

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1. In most heavy rare earth metals (REM) the ferromagnetism gives way to antiferromagnetism when the temperature is increased. Antiferromagnetism exists in the temperature interval from the Curie point to the Néel point, and in this interval the REM can have a variety of magnetic structures. For example, in terbium and dysprosium the collinear ordering is replaced by a simple helix, and in erbium the ferromagnetic helix goes over into a static longitudinal spin wave. The specific magnetic properties of REM are due to long-range oscillating exchange interaction between the $4f$ ions via the conduction electrons, which become polarized by the interaction. Information on the spin polarization of the conduction electrons can be obtained by measuring the magnetic fields induced by these electrons at the nuclei of diamagnetic impurities in REM matrices.[1] Measurements of this type were made in the course of investigations[1-5] of the temperature dependence of the hyperfine field at cadmium and tin nuclei implanted in matrices of dysprosium, holmium, and terbium. A jumplike decrease of the hyperfine field at the impurity was observed at the point of transition from a ferromagnet into a simple helix. Since the local magnetization of the matrix is not changed at this point, it can be assumed that the s-f interaction constant also remains unchanged, and we can attempt to ascribe this jump to a change of the spin polarization of the conduction electrons as a result of a change in the topology of the Fermi surface.

We investigate in this paper the singularities of the polarization of the conduction electrons in REM in the transition from the antiferromagnetic state into the ferromagnetic one. We confine ourselves to the case of a helical antiferromagnetic structure, which is observed, for example, in dysprosium and erbium. In this structure the atomic magnetic moments in each of the hexagonal planes are parallel to one another and form ferromagnetic layers whose magnetic moments are perpendicular to the hexagonal axis. The magnetic moment of each successive layer is rotated through a certain angle φ_0. If we introduce for such a structure a wave vector $\mathbf{q}$,

$$|\mathbf{q}|=\varphi_0/d,$$

where d is the distance between the nearest hexagonal planes, then the average magnetic-moment density is given by

$$M_x=M\cos qz,\quad M_y=M\sin qz,\quad M_z=0,\qquad (1)$$

where M is the average modulus of the magnetic moment of the REM ion at the given temperature. Near the point of transition into a collinear ferromagnetic structure with a moment perpendicular to the hexagonal axis, the period of the magnetic structure of the dysprosium spans approximately seven basal planes, while the period of the magnetic structure of terbium spans ten planes (see, e.g., Ref. 6). The angle φ_0 increases with increasing temperature, but near the Néel point the vector $\mathbf{q}$ is much less than the reciprocal-lattice vector.

Dzyaloshinskiĭ[7] has shown that stable helical structures occur when the vector $\mathbf{q}$ is close to an extremal diameter of the Fermi surface. The Fermi surfaces of REM or complicated and their cavities have different dimensions. Since $qd \ll 1$, it is clear that in the "resonant" interaction of the $4f$ ions with the conduction electrons the participating electrons are located either on a small cavity of a complex Fermi surface, or to a section close to its anomalously small diameter. The existence of an anomalously small extremal diameter means that the Fermi surface passes near the critical point $\mathbf{p}_{cr}$ of $\mathbf{p}$-space, in which the electron equal-energy surfaces $\varepsilon(\mathbf{p})=\varepsilon$ change their topology[8]:

$$\varepsilon(\mathbf{p}_{cr})=\varepsilon_{cr},\quad \mathbf{v}(\mathbf{p}_{cr})=\left.\frac{\partial\varepsilon(\mathbf{p})}{\partial\mathbf{p}}\right|_{\mathbf{p}\to\mathbf{p}_{cr}}=0.$$

The equal-energy surfaces located near $\mathbf{p}_{cr}$ can be approximated by surfaces of second order, such as an ellipsoid or a one- or two-cavity hyperboloid. The magnetic-structure vector $\mathbf{q}$ is determined by the extremal diameter $2p_F$ of these surfaces, and its direction is that of the p_z axis: $q\approx 2p_F$ (Fig. 1).

Each of the surfaces shown in Fig. 1 can be either a

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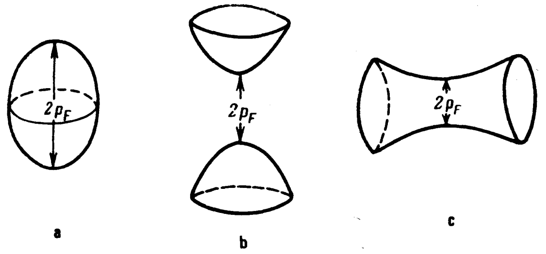

FIG. 1. Various topologies of the Fermi surface near the critical point $\mathbf{p}_{cr}$.

hole or an electron Fermi surface; we consider both variants.

2. The Hamiltonian of the conduction electrons in the field of the magnetic ions is

$$\hat{\mathscr{H}} = \hat{\varepsilon}_0(\mathbf{p}) - \frac{J}{\mu_0 M}(\hat{\boldsymbol{\mu}}_0 \hat{\mathbf{M}}), \tag{2}$$

where $\hat{\varepsilon}_0(\mathbf{p})$ is the operator of the electron energy in the paramagnetic phase, $\hat{\mu}_0 = \mu_0\hat{\sigma} = (e\hbar/2mc)\hat{\sigma}$, $\hat{\sigma}$ are Pauli matrices, and J is the constant of the interaction of the electrons and the magnetic ions. As shown in Ref. 7, $J \sim (T_N \varepsilon_F)^{1/2}$, where T_N is the Nêel temperature and ε_F is the Fermi energy. The interaction constant J will be considered to be the smallest parameter of the problem: $J \ll \varepsilon_0(\mathbf{q}) \ll \varepsilon_F$. The entire calculation is effectively carried out in the form of an expansion in J.

The two component electron eigenfunction $\Psi = \binom{\psi_1}{\psi_2}$ of the operator $\hat{\mathscr{H}}$ satisfies the following system of equations:

$$\hat{\varepsilon}_0(\mathbf{p})\psi_1 - Je^{-iqz}\psi_2 = \varepsilon\psi_1, \tag{3}$$
$$\hat{\varepsilon}_0(\mathbf{p})\psi_2 - Je^{iqz}\psi_1 = \varepsilon\psi_2, \tag{4}$$

where ε are the eigenvalues of the operator (2). We multiply both sides of (3) from the left by e^{iqz} and substitute in this expression ψ_2 from Eq. (4). We obtain

$$e^{iqz}\hat{\varepsilon}_0(\mathbf{p})\psi_1 - \frac{J^2}{\hat{\varepsilon}_0(\mathbf{p}) - \varepsilon} e^{iqz}\psi_1 = \varepsilon e^{iqz}\psi_1$$

or, after multiplying from the right by e^{-iqz}

$$(\hat{\varepsilon}_0(\mathbf{p}) - \varepsilon)(e^{iqz}\hat{\varepsilon}_0(\mathbf{p})e^{-iqz} - \varepsilon)\psi_1 = J^2\psi_1. \tag{5}$$

Since ψ_1 can be represented as a product of a spin function by a spatial function, i.e., $\psi_1 = \psi_1^{(s)}\exp(i\mathbf{p}\cdot\mathbf{r})$ (we have put $\hbar = 1$), we get

$$e^{iqz}\hat{\varepsilon}_0(\mathbf{p})e^{-iqz}\psi_1 = \varepsilon_0(\mathbf{p} - \mathbf{q})\psi_1 \tag{6}$$

and a nonzero solution exists if ε satisfies the following dispersion equation:

$$(\varepsilon_0(\mathbf{p}) - \varepsilon)(\varepsilon_0(\mathbf{p} - \mathbf{q}) - \varepsilon) = J^2. \tag{7}$$

It is convenient to use a more symmetrical form of the dependence on $\mathbf{p}$, by making the change of variable $\mathbf{p} \to \mathbf{p} + \mathbf{q}/2$. According to (7)

$$\varepsilon^{\pm} = \frac{1}{2}\left[\varepsilon_0\left(\mathbf{p} + \frac{\mathbf{q}}{2}\right) + \varepsilon_0\left(\mathbf{p} - \frac{\mathbf{q}}{2}\right)\right] \pm \left\{J^2 + \frac{1}{4}\left[\varepsilon_0\left(\mathbf{p} + \frac{\mathbf{q}}{2}\right) - \varepsilon_0\left(\mathbf{p} - \frac{\mathbf{q}}{2}\right)\right]^2\right\}^{1/2}. \tag{8}$$

At $qd \ll 1$ the accuracy of this expression is determined not so much by the smallness of J as by the following circumstance: if the vector $\mathbf{p}$ were a momentum (and not a quasimomentum), the expression (6), and with it also (7), would be exact. Since the difference between a momentum and a quasimomentum is connected only with the possibility of umklapp, formula (8) is exact if umklapp is neglected. At $qd \ll 1$ this can almost always be done.

The eigenfunctions of the operator (2) are given by

$$\psi^{\pm}(\mathbf{r}) = \frac{e^{i\mathbf{p}\mathbf{r}}}{[[\varepsilon_0(\mathbf{p} + \mathbf{q}/2) - \varepsilon^{\pm}]^2 + J^2]^{1/2}} \binom{[\varepsilon_0(\mathbf{p} + \mathbf{q}/2) - \varepsilon^{\pm}]e^{-iqz}}{J}. \tag{9}$$

In the ferromagnetic phase ($q = 0$) we have $\varepsilon^{\pm} = \varepsilon_0(\mathbf{p}) \pm J$ and hence

$$\psi^{+} = \frac{1}{\sqrt{2}}\left[\binom{0}{1} - \binom{1}{0}\right], \quad \psi^{-} = \frac{1}{\sqrt{2}}\left[\binom{0}{1} + \binom{1}{0}\right],$$

i.e., in the ferromagnetic phase ψ^{+} and ψ^{-} are superpositions of eigenfunctions of the operator $\hat{\sigma}_z$.[2)]

To find the magnetic moment of the conduction electrons we must first calculate the mean values of the operators $\hat{\sigma}_x$, $\hat{\sigma}_y$ and $\hat{\sigma}_z$ in the states (9), and integrate the obtained expressions over the phase-space region inside (or outside) the surfaces $\varepsilon^{\pm}(\mathbf{p}) = \zeta$ where ζ is the chemical potential. Neglecting temperature effects, we can regard the chemical potential as equal to the Fermi energy.

According to (9), we have

$$\langle\hat{\sigma}_x(\mathbf{p})\rangle^{\pm} = \mp\frac{J}{U(\mathbf{p})}\cos qz, \quad \langle\hat{\sigma}_y(\mathbf{p})\rangle^{\pm} = \mp\frac{J}{U(\mathbf{p})}\sin qz,$$
$$\langle\hat{\sigma}_z(\mathbf{p})\rangle^{\pm} = \mp\frac{\varepsilon_0(\mathbf{p} + \mathbf{q}/2) - \varepsilon_0(\mathbf{p} - \mathbf{q}/2)}{2U(\mathbf{p})}, \tag{10}$$

where

$$U(\mathbf{p}) = [{}^1\!/_4(\varepsilon_0(\mathbf{p} + \mathbf{q}/2) - \varepsilon_0(\mathbf{p} - \mathbf{q}/2))^2 + J^2]^{1/2}.$$

For any of the second-degree surfaces considered above, these expressions can be simplified if it is recognized that

$$\varepsilon_0(\mathbf{p} + \mathbf{q}/2) - \varepsilon_0(\mathbf{p} - \mathbf{q}/2) = p_z q/m_\parallel,$$

where $m_\parallel$ is the effective mass. As a result we get

$$\langle\hat{\sigma}_x\rangle^{\pm} = \mp\frac{J}{[(p_z q/2m_\parallel)^2 + J^2]^{1/2}}\cos qz,$$
$$\langle\hat{\sigma}_y\rangle^{\pm} = \mp\frac{J}{[(p_z q/2m_\parallel)^2 + J^2]^{1/2}}\sin qz, \tag{11}$$
$$\langle\hat{\sigma}_z\rangle^{\pm} = \mp\frac{p_z q/2m_\parallel}{[(p_z q/2m_\parallel)^2 + J^2]^{1/2}}.$$

These are the formulas we shall use henceforth. At $q = 0$ we have

$$\langle\hat{\sigma}_x\rangle^{\pm} = \mp 1, \quad \langle\hat{\sigma}_y\rangle = \langle\hat{\sigma}_z\rangle = 0.$$

We proceed now to a concrete form of the Fermi surface near the critical point $\mathbf{p}_{cr}$ (Fig. 1).

3. Let the conduction electrons interacting with the 4f

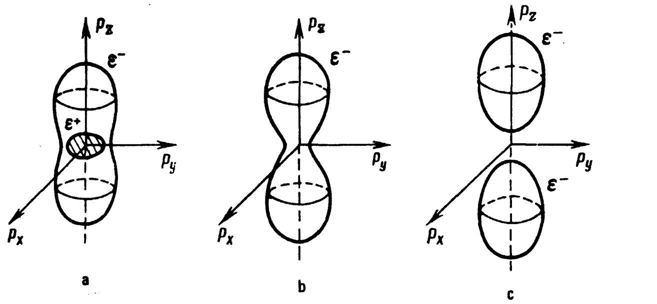

FIG. 2. Topology of the Fermi surface: a—at $\zeta-\varepsilon_{cr}>q^2/8m_\parallel+J$, b—at $q^2/8m_\parallel-J<\zeta-\varepsilon_{cr}<q^2/8m_\parallel+J$, c—at $\zeta-\varepsilon_K<q^2/8m_\parallel-J$. At $\zeta-\varepsilon_{cr}=q^2/8m_\parallel+J$ a new cavity appears, and at $\zeta-\varepsilon_{cr}=q^2/8m_\parallel-J$ the junction breaks.

ions be located inside a small ellipsoid whose equation can be represented as

$$\varepsilon_0(\mathbf{p})=\varepsilon_\kappa+p_z^2/2m_\parallel+p_\perp^2/2m_\perp, \tag{12}$$

where $m_\perp$ is the effective mass. For convenience, we have transferred the origin to the point $\mathbf{p}_{cr}$. The wave vector is $q\approx 2p_F$, where $p_F=(2m_\parallel(\zeta-\varepsilon_{cr}))^{1/2}$. In the antiferromagnetic phase the electrons have, according to (8) and (12), the following dispersion law:

$$\varepsilon^\pm=\varepsilon_{cr}+\frac{p_\perp^2}{2m_\perp}+\frac{p_z^2}{2m_\parallel}+\frac{q^2}{8m_\parallel}\pm\left[\left(\frac{p_zq}{2m_\parallel}\right)^2+J^2\right]^{1/2}. \tag{13}$$

At various relations between the quantities in (13), the Fermi surface $\varepsilon^\pm(\mathbf{p})=\zeta$ has different topologies (Fig. 2).

Inasmuch as in the Dzyaloshinskii theory the critical value of q is known only in order of magnitude ($|q^2/8m_\parallel-|\eta||\sim J$, $\eta=\varepsilon_{cr}-\zeta$), we must consider all three cases shown in Fig. 2.

We start with surface a of Fig. 2, which has, besides the dumbbell, a small ovaloid ($|\eta|>J+q^2/8m_\parallel$). The limits of integration with respect to p_z are obtained from the equation

$$|\eta|-\frac{q^2}{8m_\parallel}-\frac{p_z^2}{2m_\parallel}\mp\left[\left(\frac{p_zq}{2m_\parallel}\right)^2+J^2\right]^{1/2}=0.$$

For the "plus" and "minus" band these limits are respectively

$$p_1=\pm[2m_\parallel|\eta|+q^2/4-(2m_\parallel q^2|\eta|+4m_\parallel^2J^2)^{1/2}]^{1/2},$$
$$p_2=\pm[2m_\parallel|\eta|+q^2/4+(2m_\parallel q^2|\eta|+4m_\parallel^2J^2)^{1/2}]^{1/2}. \tag{14}$$

We see immediately that the z component of the electron magnetic-moment vector vanishes, since it is necessary to integrate an odd function between symmetric limits. The remaining components of this vector can be written in the form

$$\mu_x=\mu\cos qz,\quad \mu_y=\mu\sin qz,$$

where

$$\mu=J\mu_0\frac{m_\perp}{2\pi^2}\left\{\int_0^{p_1}\left[|\eta|-\frac{q^2}{8m_\parallel}-\frac{p_z^2}{2m_\parallel}+\left(\left(\frac{p_zq}{2m_\parallel}\right)^2+J^2\right)^{1/2}\right]\right.$$
$$\times\left[\left(\frac{p_zq}{2m_\parallel}\right)^2+J^2\right]^{-1/2}dp_z-\int_0^{p_2}\left[|\eta|-\frac{q^2}{8m_\parallel}-\frac{p_z^2}{2m_\parallel}\right.$$
$$\left.\left.-\left(\left(\frac{p_zq}{2m_\parallel}\right)^2+J^2\right)^{1/2}\right]\left[\left(\frac{p_zq}{2m_\parallel}\right)^2+J^2\right]^{-1/2}dp_z\right\}. \tag{15}$$

At $q=0$ Eqs. (15) and (14) lead directly to the known expression of the magnetic moment of an electron gas placed in a weak "magnetic field" J/μ_0:

$$\mu_f=J\mu_0m_\perp\pi^{-2}(2m_\parallel|\eta|)^{1/2}. \tag{16}$$

The integrals in (15) can be easily calculated. We, however, do not need the exact expression. Using the fact that $J\ll q^2/8m_\parallel$ and $J\ll|\eta|$, we put $J=0$ in (14) as well as everywhere under the integral sign in (15). In our case this can be done, since the logarithmic singularities cancel each other at the lower limits. From (14) and (15) we have

$$\mu_{af}=\frac{J\mu_0m_\perp}{\pi^2}\left(\frac{(2m_\parallel|\eta|)^{1/2}}{2}+\frac{2m_\parallel|\eta|-q^2/4}{2q}\right.$$
$$\left.\times\ln\left|\frac{q/2+(2m_\parallel|\eta|)^{1/2}}{q/2-(2m_\parallel|\eta|)^{1/2}}\right|\right)+O(J^3). \tag{17}$$

This expression admits of a transition to the limit of the ferromagnetic case: as $q\to 0$ the second term tends to $(2m_\parallel|\eta|)^{1/2}/2$.

We turn now to Fig. 2b. The "plus" band is not populated, and the Fermi surface of interest to us is a dumbbell. The second term of (15) vanishes. To prevent the integral from diverging at the lower limit, we retain J in the denominator of the integrand. The result is

$$\mu_{af}=\frac{J\mu_0m_\perp}{\pi^2}\left[\frac{(2m_\parallel|\eta|)^{1/2}}{4}+\frac{3q}{16}-\frac{2m_\parallel|\eta|}{4q}\right.$$
$$\left.+\frac{2m_\parallel|\eta|-q^2/4}{2q}\ln\frac{2q(q/2+(2m_\parallel|\eta|)^{1/2})}{2m_\parallel J}\right]+O(J^3). \tag{18}$$

The junction breaks at $|\eta|<q^2/8m_\parallel-J$ (Fig. 2c). The corresponding formula for μ_{af} coincides with (17). It is convenient to combine (17) and (18) into one formula by using the fact that

$$|(2m_\parallel|\eta|)^{1/2}-q/2|\approx 2m_\parallel J/q.$$

We obtain

$$\mu_{af}=J\frac{m_\perp\mu_0}{\pi^2}\left(\frac{q}{4}\pm\frac{m_\parallel J}{q}\ln\frac{q^2}{m_\parallel J}\right)+O(J^3). \tag{19}$$

The minus sign in front of the second term pertains to the case c of Fig. 2.

On going from the ferromagnetic into the helical phase, the magnetic moment of the "resonant" conduction electrons decreases by approximately one-half. The jump expressed in terms of the vector $\mathbf{q}$ is

$$\Delta\mu=\mu_f-\mu_{af}\approx Jm_\perp\mu_0q/4\pi^2. \tag{20}$$

We consider now a situation when the critical point $\mathbf{p}_{cr}$ is a maximum point. In place of (12) we have

$$\varepsilon_0(\mathbf{p})=\varepsilon_{cr}-p_z^2/2m_\parallel-p_\perp^2/2m_\perp. \tag{21}$$

The ellipsoid whose diameter along the p_z axis determines the wave vector of the magnetic structure contains not electrons but holes. In the antiferromagnetic phase we obtain the following equation for the hole Fermi surface:

$$\zeta=\varepsilon^{\pm}=\varepsilon_{cr}-\frac{p_z^2}{2m_\parallel}-\frac{p_\perp^2}{2m_\perp}-\frac{q^2}{8m_\parallel}\pm\left[\left(\frac{p_zq}{2m_\parallel}\right)^2+J^2\right]^{1/2}. \tag{22}$$

This surface has the same shape as the electron Fermi surface shown in Fig 2, except that ε^+ is replaced by $\bar{\varepsilon}$ and $\zeta-\varepsilon_{cr}$ by $\varepsilon_{cr}-\zeta$. The volume occupied by the electrons is equal to the volume of the Brillouin zone minus the volume occupied by the holes. Performing the appropriate calculations we verify that regardless of whether the electrons in the paramagnetic phase are outside or inside the ellipsoid, the general formula for the magnetic moment in the antiferromagnetic phase takes the form (19), and expression (20) holds for the jump $\Delta\mu$.

4. Assume that the Fermi surface near the critical point p_{cr} can be approximated by a two-cavity hyperboloid (p_{cr} is the conical point), i.e., the conduction electrons interacting with the $4f$ ions have the following dispersion law:

$$\varepsilon_{cr}-\varepsilon_0(\mathbf{p})=p_z^2/2m_\parallel-p_\perp^2/2m_\perp. \tag{23}$$

In the antiferromagnetic phase we obtain accordingly

$$\varepsilon^{\pm}=\varepsilon_{cr}+\frac{p_\perp^2}{2m_\perp}-\frac{p_z^2}{2m_\parallel}-\frac{q^2}{8m_\parallel}\pm\left[\left(\frac{p_zq}{2m_\parallel}\right)^2+J^2\right]^{1/2}. \tag{24}$$

The structure of the Fermi surface $\varepsilon^{\pm}(\mathbf{p})=\zeta$ is shown in Fig. 3.

In contrast to Sec. 3, the Fermi surface is open. Since the dispersion law (23), and hence also (24), is valid only near the point $\mathbf{p}=\mathbf{p}_{cr}$, the integration with respect to p_z must be restricted to the region $|p_z|\leq p_0$, with $q\lesssim p_0\ll 1/d$. Calculations similar to the foregoing lead to the following formula that is common to the cases $a-c$:

$$\mu_{af}=J\frac{\mu_0 m_\perp}{\pi^2}\left(p_0-\frac{q}{4}\mp\frac{m_\parallel J}{q}\ln\frac{q^2}{m_\parallel J}\right)+O(J^3), \tag{25}$$

whereas the expression for μ in the ferromagnetic phase (in terms of q) is

$$\mu_f=\frac{J\mu_0 m_\perp}{\pi^2}\left(p_0-\frac{q}{2}\right). \tag{26}$$

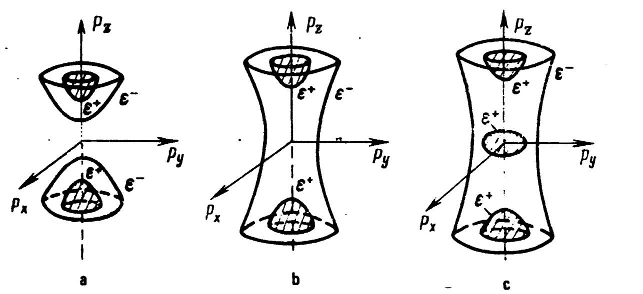

FIG. 3. Topology of Fermi surface: a—at $|\eta|>q^2/8m_\parallel+J$, b—at $q^2/8m_\parallel-J<|\eta|<q^2/8m_\parallel+J$, c—at $|\eta|<q^2/8m_\parallel-J$. The ε^+ band is filled for all possible relations between $|\eta|$, $q^2/8m_\parallel$, and J. At $|\eta|=q^2/8m_\parallel+J$ the junction breaks, and at $|\eta|=q^2/8m_\parallel-J$ a new cavity is produced.

The jump $\Delta\mu$ is negative:

$$\Delta\mu=\mu_f-\mu_{af}\approx-J\mu_0 m_\perp q/4\pi^2. \tag{27}$$

Reversal of the signs of the effective masses does not alter the structure of the Fermi surface $\varepsilon^{\pm}=\zeta$, except that ε^+ is replaced by ε^-. Formulas (25)–(27) remain in force; just as in Sec. 3, the spin polarization of the "resonant" electrons is independent of whether their Fermi surface in the parametric phase is an electron or a hole surface.

5. It remains to consider a Fermi surface of the neck type (see Fig. 1c). Let the conduction-electron dispersion law near the point p_{cr} be

$$\varepsilon_0(\mathbf{p})=\varepsilon_{cr}+\frac{p_z^2}{2m_1}+\frac{p_x^2}{2m_1}-\frac{p_y^2}{2m_2}, \tag{28}$$

where m_1 and m_2 are the effective masses ($m_1, m_2>0$).

In the antiferromagnetic phase we obtain the following spectrum:

$$\varepsilon^{\pm}=\varepsilon_{cr}+\frac{p_z^2}{2m_1}+\frac{p_x^2}{2m_1}-\frac{p_y^2}{2m_2}+\frac{q^2}{8m_1}\pm\left[\left(\frac{p_zq}{2m_1}\right)^2+J^2\right]^{1/2}. \tag{29}$$

The Fermi surface $\varepsilon^{\pm}=\zeta$ and its intersections with the planes $p_y=\text{const}$ are shown in Figs. 4–6. (The junction breaks at $|\eta|=q^2/8m_1+J$, and an arm appears at $|\eta|=q^2/8m_1-J$.)

Just as in Sec. 4, the Fermi surface is open, so that we choose as the limit of integration with respect to p_y the quantity $\tilde{p}_0$, which satisfies the same condition as p_0 in Sec. 4. The expressions used in the calculation of the magnetic moment turn out to be somewhat more complicated than (15), since the surfaces of Figs. 4–6 are not surfaces of revolution. Expanding, as above, the integrands in powers of J, we arrive at the following general expression for μ_{af}:

$$\mu_{af}=J\frac{m_1\mu_0}{\pi^2}\tilde{p}_0+O(J^3). \tag{30}$$

In contrast to (19) and (25), μ_{af} does not contain a correction term of order $J^2\ln J$ and does not depend on q. Since $\mu_f=Jm_1\mu_0\tilde{p}_0/\pi^2$, we have accurate to terms of order J^2

$$\Delta\mu\approx 0. \tag{31}$$

In the case of a hole rather than an electron neck ($m_{1,2}\leq 0$), formulas (30) and (31) remain in force.

6. We have determined the conduction-electron magnetic moment in three cases—at different topologies of the Fermi-surface section responsible for the appearance of long-period magnetic structures (Fig. 1). It turned out that this magnetic moment does not depend

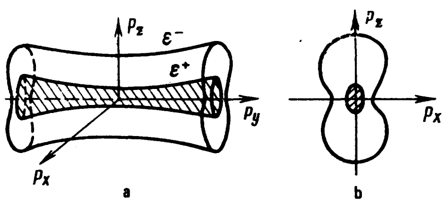

FIG. 4. Fermi surface at $|\eta|>q^2/8m_\parallel+J$ (a) and its intersection by $p_y=\text{const}$ (b).

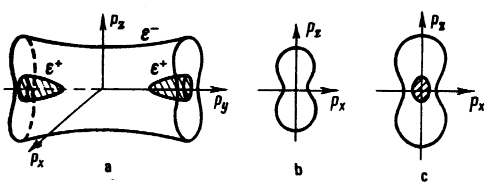

FIG. 5. Fermi surface at $q^2/8m_{\parallel} - J < |\eta| < q^2/8m_{\parallel} + J$ (a) and its sections at $p_y = 0$ (b) and $p_y > [2m_2(q^2/8m_1 - |\eta| - J)]^{1/2}$ (c).

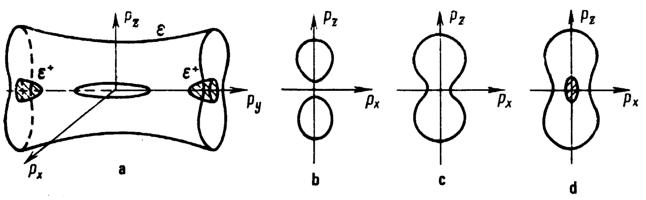

FIG. 6. Fermi surface at $|\eta| < q^2/8m_{\parallel} - J$ (a) and its sections at $p_y = 0$ (b), $[2m_2(q^2/8m_1 - |\eta| - J)]^{1/2} < p_y < [2m_2(q^2/8m_1 - |\eta| + J)]^{1/2}$ (c) and $p_y > [2m_2(q^2/8m_1 - |\eta| + J)]^{1/2}$ (d).

on whether the indicated section is that of an electron or hole Fermi surface. The jump of the magnetic moment (sign and absolute value) depends substantially on the topology of the Fermi surface.

For comparison with experiment we must find the quantity $\Delta\bar{\mu}/\bar{\mu}_f$ where $\bar{\mu}_f$ is the total magnetic moment of all the conduction electrons, including the "nonresonant" ones, for which $2p_F \gg q$, and $\Delta\bar{\mu}$ is the change of the total magnetic moment on going from the ferromagnetic into the antiferromagnetic phase. To calculate the magnetic moment μ' of the electrons belonging to the "large" Fermi surface, we expand the general expression (8), which describes the electron spectrum in the helical phase, in powers of q and neglect terms of order of q^2:

$$\varepsilon^{\pm}(\mathbf{p}) = \varepsilon_0(\mathbf{p}) \pm {}^1/_2|\mathbf{vq}|, \tag{32}$$

where $\mathbf{v} = \partial\varepsilon_0(\mathbf{p})/\partial\mathbf{p}$. An analogous expansion in formulas (10) yields for μ' the expression

$$\mu' = \frac{J\mu_0}{(2\pi)^3}\left\{\int\limits_{\varepsilon<\varepsilon^-} \frac{d^3p}{|\mathbf{vq}|} - \int\limits_{\varepsilon<\varepsilon^+} \frac{d^3p}{|\mathbf{vq}|}\right\}. \tag{33}$$

In the first integral, the integration region is inside the surface $\varepsilon^- = \zeta$, and in the second inside the surface $\varepsilon^+ = \zeta$. According to (32), this means that the integration is between the surfaces $\varepsilon^+ = \zeta$ and $\varepsilon^- = \zeta$, the "distance" between which is $\delta p = |\mathbf{v}_0 \cdot \mathbf{q}|/v_0$, where v_0 is the velocity on the Fermi surface. Therefore

$$\mu' = \frac{2J\mu_0}{(2\pi)^3}\int\limits_{\zeta} \frac{dS}{v_0} = J\nu(\zeta)\mu_0 + O(q^2), \tag{34}$$

where $\nu(\zeta)$ is the density of the electronic states on the Fermi boundary.

Accurate to terms of order q^2, the electrons that do not interact resonantly with the $4f$ ions, have in the antiferromagnetic phase the same magnetic moment as in the ferromagnetic phase. Consequently, at the assumed accuracy we have $\Delta\bar{\mu} = \Delta\mu$, meaning that the jump of the magnetic moment is determined only by the "resonant" electrons.

According to experiment, the discontinuity of the hyperfine field at cadmium and tin nuclei in a dysprosium matrix is $\Delta H = H_f - H_{af} > 0$. Since $\Delta H \sim \Delta\mu$, this means that the Fermi-surface section that determines the vector q in dysprosium is an ellipsoid (or close to it). Next, it follows from experiment[4,5] that the relative discontinuity is $\Delta H/H_f \approx 8\%$. From (19), (20), and (34) we have

$$\frac{\Delta\bar{\mu}}{\bar{\mu}} = \frac{m_\perp q}{4\pi^2\nu(\zeta)} = \frac{m_\perp q}{4m^*\bar{p}_F} \tag{35}$$

where m^* is the effective mass of the "nonresonant" electrons and $\bar{p}_F = (2m^*\zeta)^{1/2}$. Assuming $2\bar{p}_F$ to be of the order of the dimensions of the Brillouin zone for the hcp lattice of dysprosium, and also that $m^* \sim m_\perp$, we get $\Delta\bar{\mu}/\bar{\mu}_f \approx 0.06$. The agreement with experiment can be regarded as satisfactory. A detailed comparison calls for a much more thorough investigation of the Fermi surfaces of f-metals, and in particular for a direct determination of the Fermi-surface section responsible for the formation of the helicoidal structure.

It seems to us that the foregoing analysis demonstrates the feasibility of investigating the role of the conduction electrons in the magnetic properties of metals by studying their electron properties. This makes it possible to separate those electron groups that are sensitive to the magnetic structure of the metal.

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[1)] Hyperfine fields are produced at nuclei not by all the conduction electrons, but only by the s electrons.

[2)] Were we to use a representation in which the matrix $\hat{\sigma}_x$ is diagonal rather than $\hat{\sigma}_z$, then the eigenfunctions obtained by us for the operator (2) at $q = 0$ would go into the states $\binom{0}{1}$ and $\binom{1}{0}$.

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