Dispersion relation and spin-wave spectroscopy of amorphous ferromagnets

V. A. Ignatchenko and R. S. Iskhakov

L. V. Kirenskiĭ Institute of Physics, Siberian Branch, Academy of Sciences, USSR
(Submitted 21 April 1978)
Zh. Eksp. Teor. Fiz. **75**, 1438–1443 (October 1978)

The effect of spatial fluctuations of the modulus M of the magnetization on the complex modification of the dispersion relation for spin waves, in amorphous ferromagnets, is investigated in the case of strong ferromagnetic exchange, for which only orientational oscillations of $M(r, t)$, described by the Landau–Lifshitz equation, can exist. It is shown that the character of the modification of $\omega(k)$ with fluctuation of M differs essentially from the modifications produced by fluctuations of other magnetic parameters [V. A. Ignatchenko and R. S. Iskhakov, Sov. Phys. JETP **45**, 526 (1977) and **47**, No. 4 (1978)]. Correlation effects that occur in joint fluctuations of several parameters are considered in the illustrative case of simultaneous allowance for fluctuations of α and of M. The possibility is discussed of developing spin-wave spectroscopy of amorphous magnets, i.e. the experimental determination of the basic stochastic characteristics of the fluctuating parameters of a spin system by measurement of the modification of the dispersion relation.

PACS numbers: 75.30.Ds

INTRODUCTION

The phenomenological Hamiltonian that describes the properties of the magnetic system of an ideal crystal has the form

$$\mathcal{H}=\frac{1}{2}\alpha\left(\frac{\partial \mathbf{M}}{\partial x_i}\right)^2-\frac{1}{2}\beta(\mathbf{Ml})^2-\mathbf{MH}-\frac{1}{2}\mathbf{MH}_m, \qquad (1)$$

where α is the exchange constant, β is the anisotropy constant, l is a unit vector along the easy anisotropy axis, H is the external magnetic field, and $\mathbf{H}_m$ is the magnetic dipole field, which is related to the magnetization by Maxwell's equations.

Upon amorphization of a ferromagnet, disorder arises in the material: structural (fluctuations of the magnitude and direction of the interatomic distances, fluctuations of the density of the material) and chemical (fluctuations of the concentrations of the components of an alloy, fluctuations of the concentration of amorphizing additives). This in turn leads to the result that the spin-system parameters α, β, M, and l that occur in (1) became functions of the coordinates. It is clear that the relations between the contributions of the fluctuations of each parameter to the modification of the dispersion relation for spin waves may be different for each specific material; situations are possible in which the effect of one parameter will dominate. It therefore makes sense first to investigate separately the effect of fluctuations of each of the four phenomenological parameters α, β, M, and l on the dispersion law, supposing in each case that the other three parameters are constant.

In our papers,[1,2] such as investigation has been carried out for the parameters α, β, and l. There it was supposed that the fluctuating parameter is a random function of the coordinates, whose basic characteristics—the relative root-mean-square fluctuation γ and the correlation radius r_0—appear in the theory as phenomenological constants describing the amorphous state. It was shown that the character of the modification of the dispersion relation is substantially different for each fluctuating parameter.

The differences in character of the modification of the dispersion law, as it depends on the fluctuating parameter, apparently render practicable the development of spin-wave spectroscopy of amorphous magnets, the problem of which would be to determine the principal fluctuating parameters of the spin system and to measure their basic characteristics. An experimental demonstration that such a formulation of the problem is realistic has been given, in a paper[3] in which the method of spin-wave resonance was first used to observe a characteristic modification of the dispersion law on amorphization of $Co_{1-x}P_x$ films. The character of the modification made it possible to state that the principal fluctuating parameter in amorphous $Co_{1-x}P_x$ films is the exchange parameter α; its root-mean-square fluctuation ($\gamma=\Delta\alpha/\alpha\sim 0.25$) and correlation radius ($r_0\approx 24$ Å) were measured.

The task of the present paper is the further development of the theory that describes the nature of the influence of fluctuating parameters of the spin system on the dispersion law. Section 1 investigates the effect of a fluctuating modulus M of the magnetization on the complex modification of the dispersion relation; Section 2 considers correlation effects that arise in simultaneous fluctuation of several parameters; Sec. 3 discusses the possibility of spin-wave spectroscopy and the necessity for supplementing if with several other methods.

1. DISPERSION RELATION WHEN THE MODULUS OF THE MAGNETIZATION FLUCTUATES

The magnetization (magnetic moment of unit volume) M is defined by averaging of the microstate of the spin system over some volume v_0. Spatial fluctuations of the modulus $M=M(\mathrm{r})$ of the magnetization can be separated into two types according to their origin. The

 0038-5646/78/100726-04$02.40

first type of fluctuations of M can occur in the presence of complete orientation ordering of the spins of the atoms within the averaging volume at $T=0$. This type of fluctuations is characteristic of amorphous materials with strong ferromagnetic exchange and is caused by fluctuation of the number of magnetic atoms in the volume v_0 as a result of structural and chemical disorder. Here a change of M with time (without a change of the density of the material) is impossible; there can occur only orientational oscillations of $\mathbf{M}(\mathbf{r}, t)$, which are described by the Landau-Lifshitz equation,[4] as in an ideal crystal.

The second type of spatial fluctuations of the modulus of the magnetization is a result of averaging of an orientationally disordered basic microstate. This situation is characteristic of spin glasses and of systems similar to them, with weak exchange interaction. Here there is possible also a change of the modulus M with time (because of a change of the orientational microstate within the averaging volume v_0); that is, the appearance of waves of magnetization density $M(\mathbf{r}, t)$, which are not described by the Landau-Lifshitz equation. Such systems were considered in the papers of Halperin and Saslow[5] and of Andreev.[6]

We shall restrict ourselves to consideration of fluctuations of $M(\mathbf{r})$ of the first type, when the situation is described by the Landau-Lifshitz equation. On neglecting, for simplicity, the anisotropy and the magnetic dipole fields, we find that the Hamiltonian (1) corresponds to the following system of equations:

$$\begin{aligned} g^{-1}\dot{M}_x &= \alpha M_z\nabla^2 M_y - (\alpha\nabla^2 M_z + H)M_y, \\ g^{-1}\dot{M}_y &= (\alpha\nabla^2 M_z + H)M_x - \alpha M_z\nabla^2 M_x, \\ M_x^2 + M_y^2 + M_z^2 &= M^2(\mathbf{r}). \end{aligned} \tag{2}$$

Here the third dynamic Landau-Lifshitz equation, for $\dot{M}_z$, has been replaced by an equivalent equation in which the fluctuating parameter $M(\mathbf{r})$ occurs explicitly. We represent this parameter in the form

$$M(\mathbf{r}) = M + \Delta M\rho(\mathbf{r}), \tag{3}$$

where $M = \langle M(\mathbf{r})\rangle$, $\rho(\mathbf{r})$ is a random function with zero mean and unit variance, and ΔM is the root-mean-square fluctuation of $M(\mathbf{r})$.

On linearizing the system (2) for $M_z \approx M(\mathbf{r})$ and $M_x, M_y \ll M$, we obtain for the circular projection $m = M_x + iM_y$ the equation

$$\frac{\omega-\omega_0}{\alpha g M} m + \nabla^2 m + \gamma_m(\rho\nabla^2 m - m\nabla^2\rho) = 0, \tag{4}$$

where $\gamma_m = \Delta M/M$ and $\omega_0 = gH$.

Hence we find, by the method developed in Ref. 1, in the first order of perturbation theory, the complex dispersion relation

$$L(\mathbf{k}) = \gamma_m^2 \int_{-\infty}^{+\infty} \frac{S(\mathbf{k}-\mathbf{k}_1)}{L(\mathbf{k}_1)} (\mathbf{k}\mathbf{k}_1)(\mathbf{k}-2\mathbf{k}_1)(\mathbf{k}_1-2\mathbf{k})\, d\mathbf{k}_1, \tag{5}$$

where $L(\mathbf{k}) = (\omega-\omega_0)/\alpha g M - k^2$, $S(\mathbf{k}) = k_0/\pi^2(k_0^2+k^2)^2$ is the Fourier transform of the correlation function, and $k_0 = r_0^{-1}$.

The imaginary part of the integral in (5) can be calculated exactly by residues; we find for the damping an expression analogous to that obtained earlier[1] for a fluctuating exchange constant:

$$\omega'' = 2\alpha g M \gamma_m^2 \frac{k^5}{k_0(k_0^2+4k^2)}. \tag{6}$$

The real part of (5), after completion of the integration over the azimuthal angle φ_1 and the polar angle θ_1, leads to the following expression for the modified dispersion law:

$$\omega = \omega_0 + \alpha g M k^2[1+\gamma_m^2 J_m(k)], \tag{7}$$

where

$$\begin{aligned} J_m(k) &= -5\frac{k_0^2+5k^2}{k_0^2+4k^2} + \frac{k_0}{\pi k^3}\int_0^\infty \frac{k_1^2+k^2}{k_1^2-k^2} f(x) k_1\, dk_1, \\ f(x) &= \frac{2x}{1-x^2} - \ln\frac{1+x}{1-x}, \quad x = \frac{2kk_1}{k_0^2+k^2+k_1^2}. \end{aligned} \tag{8}$$

On expanding the logarithm as a series, we get for the function $f(x)$ the expression

$$f(x) = \frac{4x^3}{3(1-x^2)}\left[1 + \frac{1}{5}x^2 + \frac{3}{35}x^4 + \ldots\right]. \tag{9}$$

Over the whole range of integration with respect to k_1 and for all finite values of the parameter k, the value of x is less than 1; it reaches unity only for $k \to \infty$ and $k_1 \to \infty$. Therefore the series (9) converges quite rapidly, and we need to keep only its first terms. Thus if we neglect all the terms in square brackets except unity, we obtain for $J_m(k)$ an expression whose graph, as a function of $u = (k/k_0)^2$, is given in Fig. 1, curve 1. If we take account of the next term of the series, we obtain a more complicated expression, whose graph is given in Fig. 1, curve 2. It is seen that within the range $u \lesssim 1$ curves 1 and 2 behave qualitatively in the same way (both have a minimum at $u \approx 0.3$), although quantitatively the difference between them is important, especially when $u > 1$. Allowance for the following term of the series (9) modifies curve 2 only inappreciably in the range $u \leq 1$ that is of interest to us. Within this range, the complicated analytical expression that corresponds to curve 2 can be considerably simplified by expanding the radical that occurs in it:

$$\begin{aligned} J_m = &-3\frac{1+{}^5/_9(13+14u)u}{(1+4u)(1+2u)} + \frac{4u}{15} \\ &\times \frac{1+11u+46u^2+16u^3(7+3u)}{(2+u)(1+4u)(1+2u)^3}. \end{aligned} \tag{10}$$

Here the first term corresponds to the first term of the series (9), the second to a correction introduced by the second term of this series.

The dotted curve in Fig. 1 is from Ref. 1 and describes the modification of ω when it is the exchange constant that fluctuates (the scale for the $J_\alpha(k)$ curve is

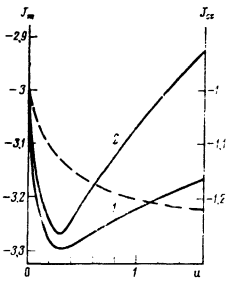

FIG. 1. Variation of the characteristic functions J_m and J_α with the normalized square of the wave number, $u = (k/k_0)^2$. 1, first approximation to $J_m(u)$; 2, second approximation to $J_m(u)$; dotted curve, $J_\alpha(u)$.

shown to the right of the graph). It is evident that the character of the modification of $\omega(k)$ when M fluctuates is substantially different from the modification that corresponds to fluctuation of α. Still more substantial is the difference of the modification caused by $M(r)$ from the modifications produced by fluctuations of $\beta(r)$ and of $1(r)$.[1]

2. ALLOWANCE FOR CORRELATIONS

In Refs. 1 and 2 and in the preceding section, we have discussed the effect of fluctuations of each of the four phenomenological parameters $\alpha, \beta, 1$, and M separately on the dispersion law and damping of spin waves. Here we shall discuss problems that arise when there is simultaneous fluctuation of several parameters; as an example, we choose a medium with a fluctuating exchange constant and magnetization modulus.

The linearized equation of motion for the circular projection of M in this case has the form

$$\frac{\omega-\omega_0}{\alpha g M} m+\nabla^2 m+\gamma_\alpha \nabla(\rho_\alpha \nabla m)+\gamma_m(\rho_m \nabla^2 m-m\nabla^2\rho_m) +\gamma_\alpha\gamma_m[\rho_m\nabla(\rho_\alpha\nabla m)-m\nabla(\rho_\alpha\nabla\rho_m)]=0, \tag{11}$$

where $\gamma_\alpha=\Delta\alpha/\alpha$, $\gamma_m=\Delta M/M$, and ρ_α and ρ_m are stationary random functions of the coordinates with zero means and unit variances.

We obtain the complex dispersion relation in the form

$$L(\mathbf{k})=\int_{-\infty}^{+\infty}\frac{\mathbf{k}\mathbf{k}_1\,d\mathbf{k}_1}{L(\mathbf{k}_1)}\Big\{\gamma_\alpha^2 S_\alpha(\mathbf{k}-\mathbf{k}_1)\mathbf{k}\mathbf{k}_1+\gamma_m^2 S_m(\mathbf{k}-\mathbf{k}_1)(2\mathbf{k}-\mathbf{k}_1)(2\mathbf{k}_1-\mathbf{k}) +\gamma_\alpha\gamma_m\Big[S_{\alpha m}(\mathbf{k}-\mathbf{k}_1)(4\mathbf{k}\mathbf{k}_1-k_1^2-k^2)+L(\mathbf{k}_1)S_{\alpha m}(\mathbf{k}_1)\frac{\mathbf{k}-\mathbf{k}_1}{k_1}\Big]\Big\}, \tag{12}$$

where $S_{\alpha m}(\mathbf{k})$, the Fourier transform of the cross-correlation function, appears on averaging of the products of the Fourier transforms of the random functions ρ_α and ρ_m:

$$\langle\rho_\alpha(\mathbf{k})\rho_m(\mathbf{k}')\rangle=S_{\alpha m}(\mathbf{k})\delta(\mathbf{k}-\mathbf{k}'). \tag{13}$$

In order to calculate this quantity, it is necessary to investigate the origin of the functions ρ_α and ρ_m and to find expressions that relate each of these functions to some initial random functions that describe the structural and chemical disorder in the material. This problem is outside the scope of the present article; we shall restrict ourselves to consideration of the two simplest limiting cases.

If the functions ρ_α and ρ_m are uncorrelated, then the dispersion relation (12) contains the sum of the previously calculated complex modifications of the frequency with respective weighting factors γ_α^2 and γ_m^2. The opposite case of complete correlation can be expressed as a linear relation between the fluctuations of α and of M; then $S_{\alpha m}=S_\alpha=S_m$, although the values of γ_α and γ_m may remain different. In the case of such correlation, we get for ω'' the expression

$$\omega''=2(\gamma_\alpha+\gamma_m)^2\alpha g M\frac{k^5}{k_0(k_0^2+4k^2)}. \tag{14}$$

The real part of (12), after completion of the integration over the azimuthal angle φ_1 and the polar angle θ_1,

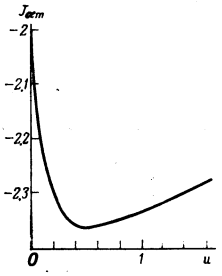

FIG. 2. Characteristic function $J_{\alpha m}(u)$ of the cross-correlation of the fluctuations of α and of M.

takes the form

$$\omega=\omega_0+\alpha g M k^2[1+\gamma_\alpha^2 J_\alpha(k)+\gamma_m^2 J_m(k)+\gamma_\alpha\gamma_m J_{\alpha m}(k)], \tag{15}$$

where

$$J_{\alpha m}=1+4J_\alpha+\frac{k_0}{2\pi k^3}\int_0^\infty\frac{k_1^2+k^2}{k_1^2-k^2}f(x)k_1\,dk_1, \qquad J_\alpha=-(k_0^2+5k^2)/(k_0^2+4k^2). \tag{16}$$

On calculating the integral over k_1 in the same approximations as in (8), we get for $J_{\alpha m}(k)$ a complicated expression whose graph is given in Fig. 2. It is evident that the function $J_{\alpha m}(k)$, with respect to the character of its behavior, is close to an average between the functions $J_\alpha(k)$ and $J_m(k)$. For $k\to 0$, the dispersion relation (15) takes the form

$$\omega\approx\omega_0+\alpha g M k^2(1-\gamma_\alpha^2-3\gamma_m^2-2\gamma_\alpha\gamma_m). \tag{17}$$

3. SPIN-WAVE SPECTROSCOPY OF AMORPHOUS MAGNETS

The investigations presented in Refs. 1 and 2 and in the present paper show that fluctuation of each of the spin-system parameters considered,[1)] α, M, β, and 1, leads to a modification of the dispersion law that is characteristic of that parameter. It is therefore possible, in principle, to develop a spin-wave spectroscopy (SWS) of amorphous magnets. For materials in which the correlation radius of the fluctuations of the parameter under investigation is sufficiently large, one can use the improved method of spin-wave resonance at frequencies ~9 GHz;[3] the study of shorter correlation radii requires the development of higher-frequency methods.

The difficulties that arise in the interpretation of a dispersion law in which superposition of the characteristic functions $J_j(k)$ of several fluctuating parameters has occurred can be overcome by investigation of the damping ω'' of spin waves caused by inhomogeneities and of a number of other characteristics. Thus according to the character of the damping, the parameters under consideration separate into two pairs: a) isotropic inhomogeneities (α, M), characteristic of which are an increase of the damping with increase of k and a change of the law from k^5 to k^3 at the point $k_0/2$; b) anisotropic inhomogeneities $(\beta, 1)$, which lead to a maximum of the damping at the point $k_0/2$. Fluctuations of M lead to an actual decrease of the saturation magnetization, which can be measured by independent methods. Fluctuations of β and 1 shift the frequency of uniform ferromagnetic resonance (FMR) in a sphere, while fluctuations of α and M do not; in bodies of other shapes, because of the effect of the demagnetizing fields, the FMR frequency will be shifted also by fluctuations of M.

Fluctuations of 1 lead to a change of the macroscopic ground state: a static stochastic magnetic structure appears, which can be investigated by electron-optical methods. Use of such supplementary methods can appreciably increase the informational content of the SWS method.

The greatest difficulty of the theory of SWS lies in allowance for the magnetic dipole fields. This problem has so far been solved in analytical form only for fluctuations of α;[2] for fluctuations of β and 1, the shift of the uniform FMR frequency has been calculated.

Expressions obtained without allowance for the magnetic dipole fields are useful only for qualitative interpretation of experiment.

[1)]As has already been mentioned,[7] fluctuations of β and 1 describe not only inhomogeneity of the crystallographic anisotropy, but also any inhomogeneities whose effect can be approximately described by a term $\beta(\mathbf{M})^2$ in the phenomenological Hamiltonian (internal elastic stresses, inclusions, etc.).

[1]V. A. Ignatchenko and R. S. Iskhakov, Zh. Eksp. Teor. Fiz. **72**, 1005 (1977) [Sov. Phys. JETP **45**, 526 (1977)].
[2]V. A. Ignatchenko and R. S. Iskhakov, Zh. Eksp. Teor. Fiz. **74**, 1386 (1978) [Sov. Phys. JETP **47**, 725 (1978)].
[3]V. A. Ignatchenko, R. S. Iskhakov, L. A. Chekanova, and N. S. Chistyakov, Zh. Eksp. Teor. Fiz. **75**, 653 (1978) [Sov. Phys. JETP **48**, 328 (1978)].
[4]L. D. Landau and E. M. Lifshitz, Phys. Z. Sowjetunion **8**, 153 (1935) (reprinted in L. D. Landau, Collected Works, Pergamon, 1965, No. 18 and in D. ter Haar, Men of Physics: L. D. Landau, Vol. 1, Pergamon, 1965, p. 178).
[5]B. I. Halperin and W. M. Saslow, Phys. Rev. B **16**, 2154 (1977).
[6]A. F. Andreev, Zh. Eksp. Teor. Fiz. **74**, 786 (1978) [Sov. Phys. JETP **47**, 411 (1978)].
[7]V. A. Ignatchenko, Zh. Eksp. Teor. Fiz. **54**, 303 (1968) [Sov. Phys. JETP **27**, 162 (1968)].

Translated by W. F. Brown, Jr.

State density in a one-dimensional disordered system in the two-band approximation

S. A. Gredeskul and L. A. Pastur

Physicotechnical Institute of Low Temperatures, Ukrainian Academy of Sciences

(Submitted 25 April 1978)
Zh. Eksp. Teor. Fiz. **75**, 1444–1457 (October 1978)

An exact solution is obtained of the problem of the density of one-electron states for two models of a disordered semiconductor that is described by a system consisting of two first-order equations and corresponding to the two-band approximation. In the first model, where the disorder is produced by a random impurity potential of a definite type, the state density has no singularities when the gap collapses. In the second model, where the fluctuation parameter is the gap width, the state density can have a singularity at the center of the forbidden band, when the fluctuations of the gap are large enough. Asymptotic formulas are obtained for the state densities in characteristic sections of the spectrum and at the most interesting values of the parameters.

PACS numbers: 71.20. + c

INTRODUCTION

The interest presently shown in the spectra of one-dimensional disordered systems, with allowance for the band structure, is natural. Up to now, the equivalent-mass approximation was used most treatments of the electron spectrum or of the equivalent concept of the structure of the unperturbed (ordered) crystal for other types of excitations (see, e.g., Refs. 1–5 and the bibliographies therein). Yet situations exist when allowance for the periodicity of the initial ordered system, meaning also the band structure of the bare spectrum, is important. This problem arises, in particular, when an attempt is made to explain some observed singularities of physical quantities (for example, the low-temperature behavior of the magnetic susceptibility[6]) in quasi-one-dimensional compounds.[7-9] Since full allowance for the band structure entails great difficulties, it is natural to turn to the simplest case of two broad resolved bands with a narrow gap between them. In this case (see Ref. 10) the spectrum and the states of the quasiparticles in the vicinity of the gap in the presence of a random potential are described by a system of two first-order equations of the Dirac type.

This paper consists of two parts. In the first (Secs. 1 and 2) we derive a number of relations that hold for an arbitrary random potential, and calculate the state density within the framework of the indicated equations for a model in which the potential is a sequence of rec-

0038-5646/78/100729-07$02.40