Kinetic properties of an exciton ferromagnet

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Kinetic properties (absorption of ultrasound, spinlattice relaxation, and absorption of light) are considered for an exciton ferromagnet whose ferromagnetism is due to the simultaneous presence in it of charge- and spin-density waves, with different concentrations of electrons and of holes. The results obtained agree qualitatively with experimentally observed features of these kinetic properties in band ferromagnets, such as Ni, Sc, In, and $ZrZn_2$. The possibility of ferromagnetic ordering is considered in the case when the Fermi surfaces of the electrons and of the holes have different shapes, and also in the presence of a nonmagnetic electron or hole reservoir of arbitrary capacity.

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1. INTRODUCTION

The exciton-ferromagnetism model was first proposed by Volkov and Kopaev[1] and was investigated in detail by Volkov *et al.*[2,3] These papers treated systems that are unstable with respect to electron-hole pairing in singlet or triplet states,[4-6] with formation of charge-density waves (CDW) or spin-density waves (SDW) respectively. It was shown that when there is a different concentration of electrons and of holes, simultaneous existence of CDW and SDW is possible in such systems, with order parameters Δ_s and Δ_t respectively. Then the spin degeneracy of the electron and hole bands is removed, and a nonvanishing magnetic moment appears in the system.

Such a model of ferromagnetism can apparently be applied in order to describe the properties of the weak ferromagnets $ZrZn_2$ and Sc_3In. Since these compounds are formed from nonmagnetic elements, the ferromagnetic properties observed in them are caused by collective effects of nonlocalized electrons.

A criterion for band ferromagnetism in metals was proposed by Stoner[7] and consists in the following. The appearance of ferromagnetic properties in a metal indicates a splitting of the conduction band into two spin subbands. Such a splitting will occur when the advantage in exchange energy of the electrons with ordering of the spins is greater than the disadvantage in the kinetic energy. For the single-band model of a metal, this is possible only when the coupling constants $e^2/\hbar v_F$ are larger than or of the order of unity (e is the electron charge, $\hbar$ is Planck's constant, v_F is the Fermi velocity). Therefore it is quite difficult to construct a systematic theory of ferromagnetism in the Stoner model. On the other hand, the presence of ferromagnetic properties is observed also in metals with smaller coupling constants.

In the exciton-ferromagnetism model, magnetic ordering occurs with an arbitrarily small coupling constant. In this model, actually, the Stoner criterion is weakened because, along with the conduction band, the valence band is also taken into account. Since the spin splitting occurs in both bands, the kinetic-energy disadvantage resulting from it is exactly compensated.

It may be anticipated that the exciton-ferromagnetism model is quite general and can describe qualitatively the properties of systems with $e^2/\hbar v_F \sim 1$. Then the temperature of the structural transition (necessary in this model) may prove to be above the melting point, i.e., unobservable. Therefore the features of the kinetic properties of exciton ferromagnets that are obtained below are apparently general for ferromagnets with nonlocalized electrons, even when the coupling constants are large.[1)] Unfortunately, in this case a systematic calculation of the correlation effects that are responsible for the features considered is difficult.

The present paper treats certain kinetic properties (absorption of ultrasound, nuclear spin-lattice relaxation, and absorption of light) of an exciton ferromagnet: that is, of an exciton phase with two order parameters Δ_s and Δ_t (the properties of an exciton phase with a single parameter Δ_s or Δ_t were considered earlier[10]). It is shown that the absorption coefficient of ultrasound (US) reaches a maximum somewhat below the Curie temperature T_c. The temperature variation of the rate of nuclear spin-lattice relaxation (NSR) has a similar form, with a maximum whose position, in all cases considered, coincides with T_c. These results agree qualitatively with experimental data for Sc_3In (Ref. 11) and $ZrZn_2$,[12] and also for Ni,[8,9] whose ferromagnetism is also caused by collectivized electrons.[13] Study of the frequency variation of the absorption coefficient of light at $T = 0$ shows the presence of two characteristic thresholds, due to the presence of two order parameters in the system.

It should be mentioned that all the calculations were made for an isotropic model of a semimetal, with a given difference δn of the electron and hole concentrations. The Fermi surfaces of the electrons and holes were assumed to be congruent in the limit $\delta n = 0$. Therefore one must expect only qualitative agreement of the results of the present calculation with experimental data.

More realistic is the case in which a certain part of the electron and hole Fermi surfaces is almost congruent. Electron-hole pairing occurs only on this part. The remaining part of the Fermi surfaces plays the role of an electron reservoir. In the conclusion of the paper,

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the possibility of ferromagnetic ordering in such a case is considered, and a phase diagram is constructed for an exciton ferromagnet in the presence of an electron reservoir of arbitrary capacity.

2. ABSORPTION OF ULTRASOUND

We shall consider absorption of US by an exciton ferromagnet in the example of a semimetal in which the extrema of the electron and hole bands in momentum space coincide[1,2]; we shall restrict ourselves to the case of high-frequency ultrasound, with frequency $\omega\tau \gg 1$, where τ is the relaxation time of the quasiparticles. In this case it may be supposed that the interaction of the sound wave with the electronic system occurs as absorption and radiation of phonons by the quasiparticle excitations. The Hamiltonian of such interaction has the form

$$H_{ep} = \sum_{\mathbf{k},\mathbf{k}',\sigma,i\neq j} (g_0 a_{i\mathbf{k}\sigma} a_{i\mathbf{k}'\sigma} + g_1 a^+_{i\mathbf{k}\sigma} a_{j\mathbf{k}'\sigma})(b_{\mathbf{q}} + b^+_{-\mathbf{q}}), \tag{1}$$

where $a_{i\mathbf{k}\sigma}$ is the annihilation operator of an electron in the i-th band (i, $j = 1, 2$) with momentum $\mathbf{k}$ and spin σ, g_0 and g_1 are matrix elements of the electron-phonon interaction, and $b_{\mathbf{q}}$ is the phonon operator. In the expression for H_{ep}, the law of conservation of momentum, $\mathbf{k}' = \mathbf{k} - \mathbf{q}$, is satisfied.

In order to find the absorption coefficient of US, it is convenient to transform, in the Hamiltonian (1), from the electron operators a_i to the operators α_i of the quasiparticles that describe the elementary excitations in the exciton phase. It was shown in Ref. 2 that the equations for the Green's functions of an exciton ferromagnet contain no anomalous functions nondiagonal with respect to the spin. This means that the wave function of a quasiparticle in the reorganized phase can be represented as a superposition of Bloch waves with the same spin indices from the different bands; that is, the canonical transformation to the new quasiparticle operators will have the form

$$\begin{aligned} \alpha_{1\mathbf{k}\sigma} &= u_{k\sigma} a_{1\mathbf{k}\sigma} - v_{k\sigma} a_{2\mathbf{k}\sigma}, \\ \alpha_{2\mathbf{k}\sigma} &= u_{k\sigma} a_{2\mathbf{k}\sigma} + v_{k\sigma} a_{1\mathbf{k}\sigma}, \end{aligned} \tag{2}$$

where the coefficients $u_{k\sigma}$ and $v_{k\sigma}$ are determined by the formulas[2]

$$u_{k\sigma}^2,\ v_{k\sigma}^2 = {}^1/_2[1 \pm \xi_k/\varepsilon_\sigma(\mathbf{k})]. \tag{3}$$

Here $\varepsilon_\sigma(\mathbf{k})$ is the spectrum of quasiparticles with spin σ:

$$\begin{aligned} \varepsilon_\sigma(\mathbf{k}) &= (\xi_k^2 + \Delta_\sigma^2)^{1/2}, \\ \xi_k &= (\mathbf{k}^2 - \mathbf{k}_F^2)/2m, \quad \Delta_{\pm\sigma} = \Delta_{\pm} = \Delta_s \pm \Delta_t. \end{aligned} \tag{4}$$

After the transformation (2), the Hamiltonian of the electron-phonon interaction takes the form (we omit terms that describe the creation of new quasiparticles, since we suppose that $\omega \ll |\Delta_\sigma|$)

$$H_{ep} = \sum_{\mathbf{k},\mathbf{k}',\sigma,i} [g_0 n_\sigma(\mathbf{k},\mathbf{k}') \mp g_1 m_\sigma(\mathbf{k},\mathbf{k}')] \alpha_{i\mathbf{k}\sigma} \alpha_{i\mathbf{k}'\sigma} (b_{\mathbf{q}} + b^+_{-\mathbf{q}}), \tag{5}$$

where the upper sign pertains to the term with $i = 1$, the lower with $i = 2$, and n_σ and m_σ are coherence factors for scattering of quasiparticles by phonons and are equal to

$$n_\sigma^2,\ m_\sigma^2 = {}^1/_2[1 + (\Delta_\sigma \pm \xi_k \xi_{k'})/\varepsilon_\sigma(\mathbf{k})\varepsilon_\sigma(\mathbf{k}')]. \tag{6}$$

Now, as in Ref. 10, one can obtain from the Hamiltonian (5) an expression for the absorption coefficient α_f of US. We give the final expression:

$$\begin{aligned} \frac{\alpha_f}{\alpha_n} = \frac{1}{2\omega} \sum_\sigma \int_{-\infty}^{\infty} \Bigg\{ & \frac{\varepsilon(\varepsilon+\omega) + \Delta_\sigma^2}{(\varepsilon^2 - \Delta_\sigma^2)^{1/2}[(\varepsilon+\omega)^2 - \Delta_\sigma^2]^{1/2}} \\ & - \operatorname{sign}(\varepsilon) \frac{2g_0 g_1}{g_0^2 + g_1^2} \frac{(2\varepsilon+\omega)\Delta_\sigma}{(\varepsilon^2 - \Delta_\sigma^2)^{1/2}[(\varepsilon+\omega)^2 - \Delta_\sigma^2]^{1/2}} \Bigg\} \\ & \times [f(\varepsilon-\mu) - f(\varepsilon+\omega-\mu)]\theta(|\varepsilon| - \Delta_\sigma)\, d\varepsilon, \end{aligned} \tag{7}$$

where α_n is the absorption coefficient of US in the semimetallic phase, $f(\varepsilon)$ is the Fermi distribution function of the quasiparticles, μ is the chemical potential, which is nonzero because of the doping, and the θ function takes account of the presence of a dielectric gap in the density of states of the quasiparticles:

$$\theta(x) = \begin{cases} 1, & x > 0, \\ 0, & x < 0. \end{cases} \tag{8}$$

We note that in the absence of doping, the second term in (7) disappears. Furthermore, when $\Delta_s = 0$ or $\Delta_t = 0$ the expression (7) agrees with the corresponding expression of Ref. 10.

In order to calculate the integral (7), it is necessary to know the temperature variation of the order parameters Δ_s and Δ_t and of the chemical potential μ. In Ref. 3, integral equations were obtained for Δ_s and Δ_t at finite temperature. But to explain the qualitative $\alpha_f(T)$ relation, one may assume that the temperature variation of Δ_s and Δ_t has a simple square-root form:

$$\Delta_s = \Delta_{s0}(1 - T/T_s)^{1/2}, \quad \Delta_t = \Delta_{t0}(1 - T/T_c)^{1/2}, \tag{9}$$

where T_s and T_c are the temperatures of the structural and magnetic transitions, respectively. This is a sufficiently good approximation for temperatures close to the critical and for low temperatures at small concentrations of the excess carriers.[3]

From the condition of a prescribed excess concentration of carrier,[3] one can by means of (9) find the $\mu(T)$ relation. By substituting it in (7), one can determine the qualitative form of the $\alpha_f(T)$ relation. The results of numerical calculations are shown in Fig. 1. The calculations were made for the case $g_1 = 0$, $\Delta_{t0}/\Delta_{s0} = T_c/T_s = 0.8$, and $\delta n/\Delta_{s0} = N/4N(0) = 0.69$, where N is the concentration of excess carriers and $N(0)$ is the density of states at the Fermi level. It is seen from the figure that the curve has a maximum below the Curie point (the temperature T_c, at which there first appears, against the CDW background, a nonvanishing parameter Δ_t accompanied by appearance of SDW). When g_1 is

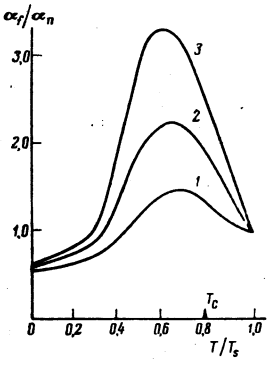

FIG. 1. Absorption of ultrasound. Curves 1, 2, and 3 correspond to frequencies ω/T_s respectively equal to 10^{-1}, 10^{-3}, and 5.10^{-4}.

taken into account, the value of the maximum decreases. Such a temperature variation (Fig. 1) of the absorption coefficient of US has been observed experimentally in Ni.[8] We note that an attempt to explain an $\alpha_f(T)$ relation like Fig. 1 was made by Kim.[14]

3. NUCLEAR SPIN RELAXATION

Similarly to the preceding (cf. Ref. 10), one can consider the temperature dependence of the rate of nuclear-spin relaxation (NSR) in an exciton ferromagnet. The Hamiltonian of interaction of a nuclear spin with the electrons has a form analogous to (1), in which electrons are scattered with turnabout of the spin, and the phonon operators are replaced by creation and annihilation operators of the spin of the nucleus. On taking account of this after the transformation (2), we get instead of the coherence factors n_σ and m_σ, respectively

$$l^2_{\sigma,-\sigma},\ p^2_{\sigma,-\sigma}=\frac{1}{2}\left[1+\frac{\Delta_\sigma\Delta_{-\sigma}\pm\xi_{\mathbf{k}}\xi_{\mathbf{k}'}}{\varepsilon_\sigma(\mathbf{k})\varepsilon_{-\sigma}(\mathbf{k}')}\right]. \tag{10}$$

Hence we get for the rate of NSR

$$\left(\frac{t_f}{t_n}\right)^{-1}=\frac{1}{\omega}\int_{-\infty}^{\infty}\left\{\frac{\varepsilon(\varepsilon+\omega)+\Delta_+\Delta_-}{(\varepsilon^2-\Delta_-^2)^{1/2}[(\varepsilon+\omega)^2-\Delta_+^2]^{1/2}}\right.$$
$$\left.-\operatorname{sign}(\varepsilon)\frac{2g_0g_1}{g_0^2+g_1^2}\frac{(\varepsilon+\omega)\Delta_-+\varepsilon\Delta_+}{(\varepsilon^2-\Delta_-^2)^{1/2}[(\varepsilon+\omega)^2-\Delta_+^2]^{1/2}}\right\} \tag{11}$$
$$\times[f(\varepsilon-\mu)-f(\varepsilon+\omega-\mu)]\theta(|\varepsilon|-\Delta_++\omega)\,d\varepsilon,$$

where t_f and t_n are the spin-lattice relaxation times in the exciton and semimetallic phases, respectively. By the frequency ω in (11) one must understand the energy of spin splitting in a magnetic field.[2)] The results of numerical calculations of (11), in the same approximation (9) as for US, are shown in Fig. 2 (for the case $T_c/T_s=0.8$) and Fig. 3 (for the case $T_c/T_s=0.25$). In both cases, the maximum of the NSR rate is reached at the Curie temperature.

We note also that the results of Fig. 2 relate to the case of strong ferromagnetism, when at $T=0$ the Fermi level is located in one of the spin subbands, i.e., $|\Delta_-|<\mu<|\Delta_+|$. Therefore upon decrease of the temperature the relaxation rate decreases exponentially to zero, since at $T=0$ there are carriers only with a single spin direction.

But in the case of weak ferromagnetism ($\mu>|\Delta_+|$, $|\Delta_-|$ at $T=0$), the NSR rate remains finite even at $T=0$; at low temperatures there even occurs an increase of the relaxation rate (Fig. 3). This is apparently due to an increase of the density of states with energies of order μ on lowering of the temperature (the case is being

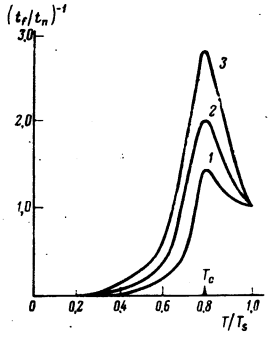

FIG. 2. Rate of nuclear spin-lattice relaxation: strong ferromagnetism. Curves 1, 2, and 3 correspond to the same values of ω/T_s as in Fig. 1.

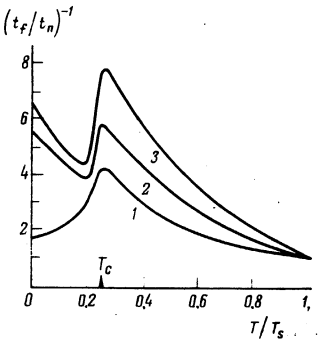

FIG. 3. Rate of nuclear spin-lattice relaxation: weak ferromagnetism. Curves 1, 2, and 3 correspond to the same values of ω/T_s as in Fig. 1.

considered in which the value of μ is comparable with $|\Delta_-|$, $|\Delta_+|$). At large frequencies (Curve 1, Fig. 3) this effect of course disappears.

Experimentally, variations of the NSR rate with temperature similar to Figs. 2 and 3 have been observed for weak ferromagnets Sc_3In (Ref. 11) and $ZrZn_2$ (Ref. 12) and also for Ni.[9]

Here it must be mentioned that in Ref. 9 a very abrupt increase of the NSR rate, by an order of magnitude, was detected on approach to T_c from the low-temperature side:

$$t_f^{-1}\sim(1-T/T_c)^{-n}, \tag{12}$$

where $n\approx0.7$. Such behavior of the NSR rate can be explained within the framework of the model of ferromagnetism under consideration, by supposing that the temperatures of the structural and magnetic transitions are close together.

In fact, it is evident from (11) that in the ferromagnetic phase ω may tend to zero. Then the integral (11) remains convergent (at the point T_c and above, the integral diverges logarithmically at $\omega=0$). We consider the temperature range $1-T/T_c\ll1$. Furthermore, we shall suppose that the following inequalities are satisfied:

$$\Delta_t(T)\ll\Delta_s(T)\ll T_c. \tag{13}$$

The right-hand inequality reflects the closeness of the structural and magnetic transition points; the left-hand corresponds to the fact that the magnetic transition point T_c lies below the structural transition point T_s. In this case, one finds from (11) that

$$\left(\frac{t_f}{t_n}\right)^{-1}\approx\frac{\Delta_s(T_c)}{T_c}\ln\frac{2\Delta_s(T_c)}{\Delta_t(T)}. \tag{14}$$

On expressing Δ_s and Δ_t according to formulas (9) and using the fact that (12), with $n=0.7$, holds over the temperature range $1-T/T_c\sim10^{-2}$ to 10^{-3} (Ref. 9), we find from (14) that

$$1-T_c/T_s\sim10^{-2}-10^{-3}.$$

This means that in Ni, near the ferromagnetic transition, there should also be observed a structural phase transition, with appearance of CDW.

4. ABSORPTION OF LIGHT

In this section we shall consider absorption of light by an exciton ferromagnet at temperature $T=0$. The

Hamiltonian of the interaction of the electrons with an electromagnetic field, in the linear approximation, looks as usual:

$$H_{eA}=\sum_{i,k,\sigma}\frac{e}{2m_i}\mathbf{A}_q(\mathbf{k}+\mathbf{k}')a^+_{ik\sigma}a_{ik'\sigma}+\text{H.c.} \tag{15}$$

where i is the band index, $\mathbf{k}'=\mathbf{k}+\mathbf{q}$, and the vector potential $\mathbf{A}(\mathbf{r})$ of the external field has the form

$$\mathbf{A}(\mathbf{r})=\mathbf{A}_q e^{i(\mathbf{qr}-\omega t)}+\text{c.c.} \tag{16}$$

After the transformation (2), the Hamiltonian (16) takes the form

$$H_{eA}=\frac{e}{2m}\sum_{\mathbf{k},\sigma}\mathbf{A}_q(\mathbf{k}+\mathbf{k}')p_\sigma(\mathbf{k},\mathbf{k}')(\alpha^+_{1k\sigma}\alpha_{2k'\sigma}+\alpha^+_{2k\sigma}\alpha_{1k'\sigma})+\text{c.c.} \tag{17}$$

Here only those terms have been kept that are connected with generation of new quasiparticles. The contribution of scattering processes of quasiparticles already there because of the doping will have order of smallness μ/ω; they may therefore be neglected when we are considering cases of weak doping. The coherence factor p_σ has the form

$$p_\sigma^2(\mathbf{k},\mathbf{k}')=\frac{1}{2}\left[1+\frac{\Delta_\sigma^2-\xi_{\mathbf{k}}\xi_{\mathbf{k}'}}{\varepsilon_\sigma(\mathbf{k})\varepsilon_\sigma(\mathbf{k}')}\right]. \tag{18}$$

Calculations similar to the previous ones give the following expression for the absorption coefficient σ_f of the light:[3)]

$$\frac{\sigma_f}{\sigma_n}=\frac{1}{2\omega}\sum_\sigma\int_{\mu_\sigma}^{\omega-\Delta_\sigma}\frac{[\varepsilon(\omega-\varepsilon)+\Delta_\sigma^2]d\varepsilon}{(\varepsilon^2-\Delta_\sigma^2)^{1/2}[(\omega-\varepsilon)^2-\Delta_\sigma^2]^{1/2}}, \tag{19}$$

where σ_n is the conductivity at frequency ω in the semimetallic phase, and where

$$\mu_\sigma=\begin{cases}\mu, & \mu>|\Delta_\sigma|,\\ \Delta_\sigma, & \mu<|\Delta_\sigma|.\end{cases} \tag{20}$$

The frequency dependence of the ratio σ_f/σ_n of absorption coefficients of light is shown in Fig. 4. Curve 1 relates to the case of strong ferromagnetism, that is to the case $|\Delta_-|<\mu<|\Delta_+|$. The absorption begins with the threshold frequency $\omega=\Delta_-+\mu_1$. At frequency $\omega=2\Delta_+$, where the second spin subband begins to take part in the absorption, there is an abrupt jump, which occurs, as in the case of an undoped exciton dielectric,[10] because of a singularity in the density of states. Curve 2 relates to the case of weak ferromagnetism, in which the chemical potential lies above the extrema of both spin subbands. It is seen from the figure that in this case the absorption also begins with frequency $\Delta_-+\mu_2$, while at frequency $\Delta_++\mu_2$ there occurs a break due to the contribution of the second subband. Here a jump, as in Curve 1, does not occur, since finite states with high density are occupied in consequence of the doping. In this case of weak absorption ($v_f k\ll T_s$) the break on the curve is absent.

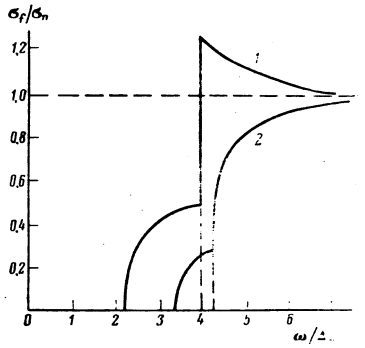

FIG. 4. Absorption of light: 1, strong ferromagnetism; 2, weak ferromagnetism. $\Delta_+/\Delta_-=2$, $\mu_1/\Delta_-=1.25$, $\mu_2/\Delta_-=2.25$.

5. EXCITON FERROMAGNETISM IN THE CASE OF NONCONGRUENT FERMI SURFACES OF ELECTRONS AND HOLES

As was mentioned earlier, the treatment of exciton ferromagnetism in Refs. 1–3 was carried out under the condition of a prescribed difference of concentrations of electrons and holes (δn), and also for congruent Fermi surfaces. But if there are in the system, besides sections of the Fermi surface that have singular spectral properties, also sections that play the role of a reservoir of electrons and holes, then the question of the possibility of band ferromagnetism has not been investigated for arbitrary reservoir capacity. Below, we shall consider a model in which the Fermi surfaces of the electrons and of the holes coincide everywhere except in isolated solid-angle regions. In the angular region $\Delta\Omega_1$, the electron and hole spectra are connected by the relation

$$\varepsilon_1(\mathbf{p})=-\varepsilon_2(\mathbf{p})+u, \tag{21}$$

and in the angular region $\Delta\Omega_2$, by the relation

$$\varepsilon_1(\mathbf{p})=-\varepsilon_2(\mathbf{p})-u. \tag{22}$$

In the remaining angular region, we take, as usual, $\varepsilon_1(\mathbf{p})=-\varepsilon_2(\mathbf{p})$. We shall investigate the system by a method proposed in Ref. 2. We shall not write the Hamiltonian in explicit form, and we shall retain the notation of Ref. 2 for the coupling constants. We shall at first suppose that, despite the presence of noncoincident sections of the Fermi surfaces in the region $\Delta\Omega_1$ and $\Delta\Omega_2$, a gap occurs in the spectrum of electrons and holes over the whole Fermi surface and is isotropic. This approach is evidently correct for a not too large noncoincidence of the Fermi surfaces (that is, for sufficiently small u). The system of self-consistency equations has the following form:

$$\begin{aligned}&\Delta_+\ln\gamma+\Delta_-\ln\delta=\Delta_+[\ln|\Delta_+|+\varphi(|\Delta_+|)],\\ &\Delta_-\ln\gamma+\Delta_+\ln\delta=\Delta_-[\ln|\Delta_-|+\varphi(|\Delta_-|)].\end{aligned} \tag{23}$$

Here Δ_+ and Δ_- are order parameters, $\gamma=(\Delta_{s0}\Delta_{t0})^{1/2}$, $\delta=(\Delta_{s0}/\Delta_{t0})^{1/2}$, Δ_{s0} and Δ_{t0} are quantities defined in Ref. 2, and

$$\begin{aligned}\varphi(x)=(1-2v)\,\mathrm{Arch}\left(\frac{\mu}{|x|}\right)\theta\left(\frac{\mu}{|x|}-1\right)+v\,\mathrm{Arch}\left(\frac{u+\mu}{|x|}\right)\theta\left(\frac{u+\mu}{|x|}-1\right)\\ +v\,\mathrm{Arch}\left(\left|\frac{u-\mu}{x}\right|\right)\theta\left(\left|\frac{u-\mu}{x}\right|-1\right).\end{aligned} \tag{24}$$

In the expression (24), $v=|\Delta\Omega_1|/4\pi=|\Delta\Omega_2|/4\pi$ (for simplicity we suppose that the solid-angle regions $|\Delta\Omega_1|$ and $|\Delta\Omega_2|$ coincide in magnitude), and μ is the shift of the chemical potential with respect to the middle of the forbidden band of the exciton dielectric. It is known that for $v=0$ and fixed μ, the system (23) has no magnetic solutions. We shall first investigate the case $\mu=0$, computationally the simplest. The system of equations (23) can be reduced, by a substitution suggested by Kopaev and Tugushev,[15] to a single equation, which was solved numerically for various values of u

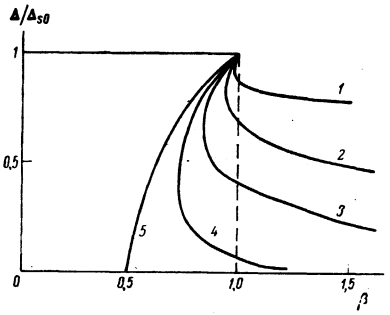

FIG. 5. Phase diagram for a nonmagnetic exciton dielectric in the presence of a reservoir of arbitrary capacity and of noncoincident sections of the Fermi surface. Curves 1, 2, 3, 4, and 5 are shown for values of the parameter v 0.1, 0.2, 0.3, 0.4, and 0.5 respectively.

and v. The results of the calculation for nonmagnetic solutions are shown in Fig. 5. The magnetic solutions that formally exist for the system (23) are nonphysical, in the same way as was true in the case of noncoincident Fermi surfaces in the absence of doping. For the nonmagnetic solutions, the curves 1, 2, 3, 4, 5 correspond to values of v respectively equal to 0.1, 0.2, 0.3, 0.4, 0.5. Similar numerical calculations for fixed $\mu \neq 0$, which we carried out over a broad range of μ, u, and v, show that all the magnetic solutions that occur are nonphysical. Thus in the presence of a fixed chemical potential, exciton ferromagnetism does not occur.

We shall investigate the behavior of a system with partially noncoincident electron and hole Fermi surfaces for arbitrary reservoir capacity. Here we shall suppose that the separation of the Fermi surfaces in the regions $\Delta\Omega_1$ and $\Delta\Omega_2$ is sufficiently large. A more realistic situation that considered above is one in which the gap is formed only in the region of coincidence of the Fermi surfaces; that is, it becomes anisotropic. The self-consistency equations can be written in the following form (we omit the intermediate calculations):

$$\begin{aligned}\Delta_+ \ln\gamma+\Delta_- \ln\delta&=\Delta_+[\ln|\Delta_+|+\psi(|\Delta_+|)],\\ \Delta_- \ln\gamma+\Delta_+ \ln\delta&=\Delta_-[\ln|\Delta_-|+\psi(|\Delta_-|)],\end{aligned} \tag{25}$$

where

$$\psi(x)=\theta\left(\frac{\mu}{|x|}-1\right)\operatorname{Arch}\frac{\mu}{|x|}. \tag{26}$$

We emphasize that Δ_{s0} and Δ_{t0}, which enter the system (25) through γ and δ, depend on the effective density of states at the Fermi level, since in the derivation of (25) the integration extends only over the region with coincident sections of the Fermi surface:

$$N_{\text{eff}}=N(0)(1-2v), \tag{27}$$

where $N(0)$ is the ordinary density of states at the Fermi level. The system (25) must be supplemented by the equation of electrical neutrality, which for arbitrary capacity β of the reservoir (whose role is played by the noncoincident sections of the Fermi surface) has the form

$$(1+\beta)\mu_0-\beta\mu=\tfrac{1}{2}[(\mu^2-\Delta_+^2)^{1/2}+(\mu^2-\Delta_-^2)^{1/2}], \tag{28}$$

μ_0 is the position of the chemical potential in the metallic phase, μ in the exciton-dielectric phase, and $\beta = N_p/N(0)$, where N_p is the density of states in the reservoir. We shall carry out an investigation of the nonmagnetic solutions of the system (25): $\Delta_+=\Delta_-=\Delta_s$. After simple calculations, one can obtain the following results. When $\beta<1$, a solution exists over the range $0\leq\mu_0/\Delta_{s0}\leq\frac{1}{2}$ and is everywhere single-valued:

$$\Delta=\Delta_{s0},\quad \mu=\frac{1+\beta}{\beta}\mu_0 \tag{29a}$$

when $\mu_0/\Delta_{s0}\leq\beta/(1+\beta)$,

$$\Delta^2=\Delta_{s0}^2\left(\frac{1+\beta}{1-\beta}\right)\left(1-2\frac{\mu_0}{\Delta_{s0}}\right),\quad \mu=\frac{\Delta_{s0}-(1+\beta)\mu_0}{1-\beta} \tag{29b}$$

when $\beta/(1+\beta)<\mu_0/\Delta_{s0}\leq\frac{1}{2}$. When $\beta=1$, the solution has the form of a step:

$$\begin{aligned}&\Delta=\Delta_{s0},\quad \mu=2\mu_0 \quad \text{for} \quad 0\leq\mu_0/\Delta_{s0}<{}^1/_2,\\ &\Delta=0,\quad \mu=\mu_0 \quad \text{for} \quad \mu_0/\Delta_{s0}\geq{}^1/_2.\end{aligned} \tag{30}$$

Finally, when $\beta>1$ there occurs a region of double-valuedness of the solution:

$$\begin{aligned}&\Delta=\Delta_{s0},\quad \mu=\frac{1+\beta}{\beta}\mu_0 \quad \text{for} \quad 0\leq\frac{\mu_0}{\Delta_{s0}}\leq\frac{\beta}{1+\beta},\\ &\Delta^2=\Delta_{s0}^2\left(\frac{1+\beta}{\beta-1}\right)\left(2\frac{\mu_0}{\Delta_{s0}}-1\right) \quad \text{for} \quad \frac{\beta}{1+\beta}\geq\frac{\mu_0}{\Delta_{s0}}\geq\frac{1}{2}.\end{aligned} \tag{31}$$

It is seen that when $\beta>1$, there appears a falling section of the $\Delta(\mu_0)$ relation, but the solution $\Delta=\Delta_{s0}$ is found to be energetically more advantageous, just as for a fixed chemical potential.[2]

We shall find the line of phase transition of the second kind from the exciton-dielectric state, with a single type of pairing, to a state with two types of pairing ($\Delta_t \neq 0$, if the initial state was a singlet). By use of the method proposed in Ref. 2, one can obtain the following results: when $\beta<1$, the line of phase transition of the second kind exists for $\beta/(1+\beta)<\mu_0/\Delta_{s0}<\frac{1}{2}$ and is given by the expression

$$\Delta_{t0}=\Delta_{s0}\exp\left[\left(2-\frac{\Delta_{s0}}{\mu_0}\right)\Big/\left(1-\frac{\beta}{1+\beta}\frac{\Delta_{s0}}{\mu_0}\right)\right]. \tag{32}$$

For $\mu_0/\Delta_{s0}<\beta/(1+\beta)$, when in the initial (singlet) state the solution is $\Delta_s=\Delta_{s0}$, a region of magnetic solutions does not exist, since the transition line behaves in a nonphysical manner; this was discussed in detail in Ref. 1 in the investigation of an undoped exciton dielectric. When $\beta>1$, magnetic solutions with $\Delta_t\neq 0$ are realized on the nonphysical branch when, in the initial (singlet) phase, Δ_s increases with increase of μ_0. But for the solutions $\Delta_s=\Delta_{s0}$, instability with respect to triplet pairing does not occur, and the most advantageous solution is the nonmagnetic $\Delta_+=\Delta_-=\Delta_{s0}$.

The phase diagram of the system is given in Fig. 6.

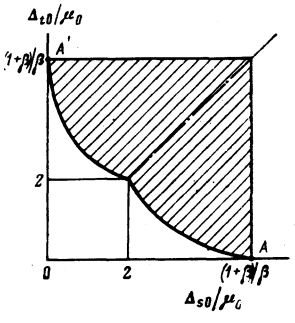

FIG. 6. Phase diagram of an exciton ferromagnet in the presence of a nonmagnetic reservoir of arbitrary capacity.

The shaded region of magnetic solutions for $\beta<1$ is bounded on one side by the line of second-order transitions (32), on the other by the line of first-order transitions $\mu_0/\Delta_{s0}=\beta/(1+\beta)$. The point A is a tricritical point and is defined by the coordinates $\Delta_{t0}/\mu_0=0$, $\Delta_{s0}/\mu_0=(1+\beta)/\beta$. When $\beta>1$, as has already been mentioned, there are no magnetic solutions in the system. As β tends to zero, the point A tends to infinity; and in the limit $\beta=0$, the result (32) reduces to the corresponding result of Ref. 2.

6. CONCLUSION

Above, the kinetic properties of an exciton ferromagnet have been studied. It was shown that the temperature variations of the absorption coefficient of US and of the rate of NSR have a characteristic maximum (Figs. 1–3). It is caused by the fact that in the coherence factors (6) and (10) there is a plus sign in front of the term containing Δ_0^2, in contrast to superconductors in the case of absorption of ultrasound. Therefore on lowering of the temperature below T_s, the coherence factors increase initially because of the appearance of Δ_s, and below T_c once again because of the appearance of the second parameter Δ_t. At low temperatures, the number of carriers decreases, while the parameters Δ_s and Δ_t vary only slightly. Therefore there are maxima on the temperature variations of the coefficients of absorption (such as the absorption of US and of nuclear spin). Numerical calculations have shown that in the case of ultrasound this maximum lies somewhat below the Curie temperature. It is just such a variation that has been observed experimentally in Ni.[8]

But in the case of nuclear-spin relaxation, the maximum, in all cases calculated (Figs. 2 and 3), fell directly at T_c. Such behavior of the rate of NSR has been observed, as was mentioned above, in Ni (Ref. 9) and also in Sc_3In (Ref. 11) and $ZrZn_2$.[12] It should be noted, however, that in $ZrZn_2$ the maximum of the relaxation rate was observed not at the Curie temperature ($T_c=21$ K) but higher, at temperature $T\sim 50$ K, i.e., at the temperature of a conjectured structural transition.[16] In this connection we mention once more that in a comparison of the results of the present calculation with experimental data, only qualitative agreement should be expected.

We point out also one additional interesting fact. It was shown in Ref. 10 that the temperature variations of NSR in the exciton phase with only CDW or SDW states differ from each other. This can be seen from the expression (10) for the coherence factor. When $\Delta_s=0$, which corresponds to the presence of SDW, the sign in front of Δ_t^2 becomes negative, and the coherence factor becomes the same as in the case of US for superconductors. This implies that the temperature variation of NSR in an exciton antiferromagnet should be monotonic.

Borsa and Lecander[17] measured the temperature variation of the spin-lattice relaxation rate in the antiferromagnetic compound CrB_2. A calculation of the band structure of CrB_2 (Ref. 18) showed the presence of plane sections of the Fermi surfaces of electrons and holes; this makes possible, as in pure Cr,[19] the existence of SDW.

The results of the measurements[17] showed, in contrast to the expected monotonic variation, an abrupt increase of the rate of NSR at the Néel temperature T_N. Such a variation can be explained, within the framework of the electron-hole pairing model, by supposing that CDW originates at a certain temperature above T_N, while below T_N the existence of CDW and SDW is possible. The absence of ferromagnetic properties can be explained by the fact that the CDW and SDW are in opposite phase to each other.[20] Furthermore, as has been shown in the present paper, even the presence of a reservoir does not prevent the possibility of coexistence of CDW and SDW, if its capacity is less than unity.

We point out also that certain properties of the ferromagnetic semiconductor $GaMo_5S_6$ (presence of a structural transition preceding the magnetic, the temperature behavior of the magnetic susceptibility, an anomaly in the temperature variation of the resistance at the structural transition point[21]) are apparently described well by the exciton-ferromagnetism model. It is therefore to be expected that the features of the kinetic properties considered above should be observed also in $GaMo_5S_6$.

We note one further fact. In the above calculations of kinetic properties, we took into account only the contribution of single-particle excitations and neglected all collective effects, including the contribution of spin excitations (magnons). Processes due to excitation of magnons may play an important role in ferromagnets with localized moments (ferrodielectrics) in which the concentration of free carriers is small. But in the case of band ferrromagnetism, to which exciton magnetism is related, the relative contribution of magnons to the kinetic phenomena considered is small in comparison with electrons.

We shall make an estimate of the relative contribution of magnons, for example to the absorption of US. Since the magnon spectrum of a ferromagnetic metal (and consequently also of an exciton ferromagnet) in principle does not differ from the magnon spectrum of a ferrodielectric,[22] we may use for the absorption coefficient α_m of US by virtue of excitation of magnons the corresponding result for ferrodielectrics[23]:

$$\alpha_m \sim \frac{T_0}{r_0^4 \tau^{1/2} MS^2}\left(\frac{T}{T_c}\right)^2 \omega, \quad T_0=\frac{S}{a}, \quad r_0=\frac{R_0}{a}, \quad \tau=(T_c-T)/T_c. \tag{33}$$

Here a is the lattice constant, R_0 is the interaction radius of the spins (usually $R_0/a\sim 3$ to 4), M is the mass of the ions, S is the velocity of sound, and ω is its frequency. On dividing α_m by the absorption coefficient of US by virtue of conduction electrons, $\alpha_n\sim(mv_F/MS)\omega$ (m is the effective mass of the electrons, v_F is the Fermi velocity), we find that α_m/α_n becomes of order of magnitude unity when $(T_c-T)/T_c\sim 10^{-4}$. But at such temperatures formula (33) is no longer valid, and α_m will be still further decreased because of strong damping of the magnons. We found above, however, that peculiarities in the absorption of US are already appearing at $(T_c-T)/T_c\sim 0.1$. A similar situation occurs also

in the case of relaxation of nuclear spin.

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[1)] Thus the experimental data[8,9] for Ni in which the coupling constant is not small, agree with the results obtained below.
[2)] We note that an experimental measurement of the spin-lattice relaxation time is always made in the presence of an external magnetic field.
[3)] We are considering the case of strong absorption, in which $\delta \ll v_F/T_s$; δ is the depth of penetration of the field.

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Size effects in the magnetoresistance of antimony whisker crystals

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We investigated the magnetoresistance of antimony whisker crystals in the form of thin platelets with thicknesses $d = 0.1$–2.6 μm, at temperatures 4.2–80 K and in magnetic fields 0–70 kOe at various orientations of the magnetic field relative to the measuring current and the surface of the platelets. Similar measurements were made for a bulk ($d = 1$ mm) sample having the same crystallographic orientation as the platelet. For a transverse magnetic field H⊥I perpendicular to the surface of the platelet, the character of the dependence of the resistance in the magnetic field and on the temperature in thin platelets and in the bulk sample is the same. A substantial difference in the behavior of the resistance of the bulk sample and the thin platelets was observed only when the field was parallel to the platelet surface (this leads, in particular, to a strong resistance anisotropy, not observed in the bulk sample, of thin platelets in a magnetic field). The function $\rho_{\parallel}(H)$ has two singularities in fields corresponding to $r \approx d$ and $2r \approx d$ (r is the Larmor radius of the electrons). In fields with $r > d$, a logarithmic increase of the resistance with increasing field is observed. In contrast to the bulk sample, an increase of the magnetoresistance with increasing temperature is observed for thin platelets when the magnetic field is oriented parallel to the surface. The experimental results confirmed most clearly and reliably the principal ideas of the theory of static skin effect.

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1. INTRODUCTION

A theoretical analysis of galvanomagnetic size effects, both for the case of diffuse reflection of electrons from the surface of the metal and for the case of specular and almost specular reflection, under the condition $\gamma = r/l \ll 1$, was carried out by Azbel' and Peschanskiĭ[1-4] ($r = cp_F/eH$ is the Larmor radius of the electron trajec-

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